NONUNIFORM ESTIMATES IN THE APPROXIMATION BY THE IRWIN LAW

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Abstract. An approximation of a cumulative distribution function by the Irwin cumulative distribution function is considered. The approximating distribution function can be cumulative distribution function of sums (products) of independent (dependent) random variables. Remainder term of the approximation is estimated by the cumulant method. The cumulant method is used by introducing special cumulants which satisfy the V. Statulevičius type condition. The main result is a nonuniform bound for the difference between the cumulative distribution functions in terms of special cumulants.

Key words: cumulative distribution function, characteristic function, cumulants, Irwin law, generalized Rademacher random variables, nonuniform estimates.

1. Introduction

The approximation of cumulative distribution functions of sums (products) of independent (dependent) random variables by normal and Poisson distributions is analysed in many papers. In this paper, we consider the approximation by the Irwin law, because the Irwin law is interesting in probability theory and mathematical statistics.

In 1937, J. O. Irwin [2] considered a random variable $Y = X_1 - X_2$, where $X_1 \sim \mathcal{P}(\lambda)$, $X_2 \sim \mathcal{P}(\lambda)$ are independent and $\mathcal{P}(\lambda)$ is Poisson distribution. In 1946, J. G. Skellam [10] and in 1952, A. de Castro and A. Prekopa [8] generalized this random variable Y, where $X_1 \sim \mathcal{P}(\lambda_1)$ and $X_2 \sim \mathcal{P}(\lambda_2)$ are independent; therefore, this distribution of random variable Y was named the Skellam distribution. Later, in 1962, J. Strakee and J. J. D. van der Gon [11] presented tables of the cumulative distribution function of the Skellam distribution to four digits for some combinations of values of the two parameters. In 2006, D. Karlis and I. Ntzoufras [4] started to examine the random variable Y, when $X_1 \sim \mathcal{P}(\lambda_1)$ and $X_2 \sim \mathcal{P}(\lambda_2)$ are correlated. The Skellam distribution estimates of the parameters obtained by the moment method are

$$\hat{\lambda}_1 = \frac{1}{2}\left(S^2 + \overline{Y}\right), \quad \hat{\lambda}_2 = \frac{1}{2}\left(S^2 - \overline{Y}\right),$$

when $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed by the Skellam distribution, $\overline{Y} = \frac{1}{n}\sum_{i=1}^n Y_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^n \left(Y_i - \overline{Y}\right)^2$. Of course, the estimates of the parameter will exist if $S^2 - \left|\overline{Y}\right| > 0$. And only in 2000, D. Karlis and I. Ntzoufras [3] discussed in detail the properties of the Skellam distribution and obtained the maximum likelihood estimates. The Skellam distribution estimates of the parameters obtained by the maximum likelihood method are

$$\frac{\hat{\lambda}_2 + \bar{y}}{\sqrt{\left(\hat{\lambda}_2 + \bar{y}\right)\hat{\lambda}_2}} \sum_{i=1}^n \frac{I_{y_i+1}\left(2\sqrt{\hat{\lambda}_1\hat{\lambda}_2}\right)}{I_{y_i}\left(2\sqrt{\hat{\lambda}_1\hat{\lambda}_2}\right)} = n, \qquad \text{where} \quad I_z(x) = \left(\frac{x}{2}\right)^z \sum_{k=\max\{0,-z\}}^{\infty} \frac{\left(x^2/4\right)^k}{k!(z+k)!}$$

is a modified Bessel function of the first kind, and

$$\hat{\lambda}_1 = \hat{\lambda}_2 + \bar{y}, \qquad \text{where} \quad \bar{y} = \frac{1}{n}\sum_{i=1}^n y_i.$$

Later, in 2006, D. Karlis and I. Ntzoufras [4] derived Bayesian estimates and used the Bayesian approach for testing the equality of the two parameters of the Skellam distribution.

The Skellam distribution has many applications in different fields. D. Karlis and I. Ntzoufras [5] applied the Skellam distribution for modeling the difference of the number of goals in football games. They also modeled the difference in the decayed, missing and filled teeth (DMFT) index before and after treatment in article [4]. This index is one of the most common methods in oral epidemiology for assessing dental caries prevalence as well as dental treatment needs among populations and has been used for about 75 years. Y. Hwang, J. Kim and I. Kweon [1] introduced the Skellam distribution as a sensor noise model for CCD (charge-coupled device) or CMOS (complementary metal–oxide–semiconductor) cameras. This is derived from the Poisson distribution of photons that determine the sensor response. They showed that the Skellam distribution can be used to measure the intensity difference of pixels in the spatial domain, as well as in the temporal domain. In addition, they showed that Skellam parameters are linearly related to the intensity of the pixels. On applications of the Skellam law in economics, see [6].

We use the cumulant method for the approximation by the Irwin law. The cumulant method is widely described in the L. Saulis' and V. Statulevičius' monography [9].

2. Irwin Law

We define the Irwin law as an instance of the Skellam distribution:

Definition 2.1 *A random variable Y is said to have Irwin distribution if $Y = X_1 - X_2$, where $X_1 \sim \mathcal{P}(\lambda)$, $X_2 \sim \mathcal{P}(\lambda)$ and $\mathcal{P}(\lambda)$ is Poisson distribution.*

Evidently, for $k \in \mathbb{Z}$,

$$\mathbf{P}\{Y = k\} = \mathrm{e}^{-2\lambda} \sum_{s=\max\{0,-k\}}^{\infty} \frac{\lambda^{2s+k}}{s!\,(s+k)!}.$$

The characteristic function of the Irwin law is

$$g(t) = \mathbf{E}\mathrm{e}^{\mathrm{i}tY} = \sum_{k=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}tk}\mathbf{P}\{Y = k\} = \mathrm{e}^{-2\lambda} \sum_{k=-\infty}^{\infty} \sum_{s=\max\{0,-k\}}^{\infty} \mathrm{e}^{\mathrm{i}tk} \frac{\lambda^{2s+k}}{s!\,(s+k)!}.$$

Denote $m = s + |k|$, then we obtain

$$g(t) = \mathrm{e}^{-2\lambda} \sum_{m=0}^{\infty} \sum_{s=0}^{\infty} \mathrm{e}^{\mathrm{i}t(m-s)} \frac{\lambda^{s+m}}{s!m!} = \mathrm{e}^{-2\lambda} \cdot \mathrm{e}^{\lambda \mathrm{e}^{\mathrm{i}t}} \cdot \mathrm{e}^{\lambda \mathrm{e}^{-\mathrm{i}t}} = \mathrm{e}^{\lambda\left(\mathrm{e}^{\mathrm{i}t} + \mathrm{e}^{-\mathrm{i}t} - 2\right)} = \mathrm{e}^{2\lambda(\cos t - 1)}.$$

Applying the expansion

$$\ln g(t) = \lambda\left(\mathrm{e}^{\mathrm{i}t} + \mathrm{e}^{-\mathrm{i}t} - 2\right) = \lambda\left(\sum_{m=1}^{\infty} \frac{(\mathrm{i}t)^m}{m!} + \sum_{r=1}^{\infty} \frac{(-1)^r(\mathrm{i}t)^r}{r!}\right) = 2\lambda \sum_{m=1}^{\infty} \frac{(\mathrm{i}t)^{2m}}{(2m)!},$$

we obtain the Irwin law cumulants

$$\Gamma_{2n-1} = 0, \quad \Gamma_{2n} = 2\lambda \qquad n \in \mathbb{N}. \tag{1}$$

By the expansion

$$g(t) = \exp\left\{\lambda\left(\mathrm{e}^{\mathrm{i}t} + \mathrm{e}^{-\mathrm{i}t} - 2\right)\right\} = \sum_{n=0}^{\infty} \frac{\lambda^n \left(\mathrm{e}^{\mathrm{i}t} + \mathrm{e}^{-\mathrm{i}t} - 2\right)^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(2\lambda)^n}{n!} \left(\sum_{m=1}^{\infty} \frac{(\mathrm{i}t)^{2m}}{(2m)!}\right)^n = 1 + \sum_{r=1}^{\infty} \left(\sum_{m=1}^{r} \frac{(2\lambda)^m}{m!} \sum_{\substack{l_1+\cdots+l_m=2r \\ l_j \in 2\mathbb{N},\ j=\overline{1,m}}} \frac{(2r)!}{l_1! \cdots l_m!}\right) \frac{(\mathrm{i}t)^{2r}}{(2r)!},$$

we obtain the Irwin law moments

$$\mathbf{E}Y^{2r-1}=0,\quad \mathbf{E}Y^{2r}=\sum_{m=1}^{r}\frac{(2\lambda)^m}{m!}\sum_{\substack{l_1+\dots+l_m=2r\\ l_j\in 2\mathbb{N},\ j=\overline{1,m}}}\frac{(2r)!}{l_1!\cdots l_m!}\qquad r\in\mathbb{N}. \tag{2}$$

In particular, the Irwin law moments are

$$\begin{cases}\alpha_1=0,\\ \alpha_2=2\lambda,\\ \alpha_3=0,\\ \alpha_4=12\lambda^2+2\lambda,\\ \alpha_5=0,\\ \alpha_6=120\lambda^3+60\lambda^2+2\lambda,\\ \dots\dots\dots\dots\dots\dots\dots\dots\end{cases} \tag{3}$$

3. Special cumulants in approximation by the Irwin law

To approximate the symmetric cumulative distribution function $F(x)$ of a random variable X by the cumulative distribution function of the Irwin law, we use special cumulants

$$\widetilde{\Gamma}_k=\frac{\mathrm{d}^k}{\mathrm{d}z^k}\ln\mathbf{E}\left(1+z+\sqrt{z^2+2z}\right)^X\bigg|_{z=0},\qquad k\in\mathbb{N}, \tag{4}$$

where $z=\cos t-1$ and $f_X(t)=\mathbf{E}e^{\mathrm{i}tX}=\mathbf{E}\left(1+z+\sqrt{z^2+2z}\right)^X$.

It is easy to find connection between $\widetilde{\Gamma}_k$ and $\Gamma_m=\frac{\mathrm{d}^m}{\mathrm{d}(\mathrm{i}t)^m}\ln f_X(t)\Big|_{t=0}$, $m=1,2,3,\dots,k$, $k=1,2,3,\dots$:

$$\widetilde{\Gamma}_k=k!\sum_{r=1}^{k}\frac{(-1)^{k+r}\cdot\Gamma_{2r}\cdot 2^r}{(2r)!}\times$$
$$\times\sum_{\substack{l_1+\dots+l_{2r}=k-r\\ l_j\in\mathbb{N}\cup\{0\},\ j=\overline{1,2r}}}\frac{C_{l_1}\cdots C_{l_{2r}}\cdot\left|(2l_1-1)\cdots(2l_{2r}-1)\right|}{(2l_1+1)\cdots(2l_{2r}+1)2^{\sum_{i=1}^{2r}(3l_i-1)\cdot\mathbf{1}_{\{l_i\neq 0\}}}}, \tag{5}$$

where $C_0=1$, $C_N=\frac{(2N-2)!}{(N-1)!N!}$ are the Catalan numbers, when $N\in\mathbb{N}$ and $\mathbf{1}_A$ the indicator function of a set A.

In particular,

$$\begin{cases}\widetilde{\Gamma}_1=\Gamma_2,\\ \widetilde{\Gamma}_2=\frac{1}{3}\Gamma_4-\frac{1}{3}\Gamma_2,\\ \widetilde{\Gamma}_3=\frac{1}{15}\Gamma_6-\frac{1}{3}\Gamma_4+\frac{4}{15}\Gamma_2,\\ \dots\dots\dots\dots\dots\dots\dots\dots\end{cases} \tag{6}$$

It is easy to make sure that for the Irwin law

$$\widetilde{\Gamma}_k=\begin{cases}2\lambda, & \text{if } k=1,\\ 0, & \text{if } k\geqslant 2.\end{cases} \tag{7}$$

4. Nonuniform estimates of the remainder term

In this section, we will find nonuniform estimates of the remainder term in the approximation by the Irwin law. For that purpose, we will use a lemma:

Lemma 4.1 [7] *If X and Y are integer-valued random variables with cumulative distribution functions $F(x)$, $G(x)$ and characteristic functions $f(t)$, $g(t)$, then*

$$(1+|x|)\big|F(x)-G(x)\big| \leqslant \frac{1}{8\pi}\left(4\int_{-\pi}^{\pi}\left|\frac{f(t)-g(t)}{\sin\frac{t}{2}}\right|\mathrm{d}t\right.$$

$$\left.+\int_{-\pi}^{\pi}\left|\frac{f(t)-g(t)}{\sin^2\frac{t}{2}}\right|\mathrm{d}t+2\int_{-\pi}^{\pi}\left|\frac{(f(t)-g(t))'}{\sin\frac{t}{2}}\right|\mathrm{d}t\right). \tag{8}$$

Assume that a random variable X with the symmetric cumulative distribution function $F(x)$ and a characteristic function $f(t)$ have all finite moments $\mathbf{E}X^k$, $k=1,2,3,\dots$. Then $\ln f(t)$ can be represented in the form

$$\ln f(t)=\sum_{k=1}^{\infty}\widetilde{\Gamma}_k\frac{z^k}{k!},$$

where $z=\cos t-1$.

We denote the Irwin law cumulative distribution function with parameter λ by $G(x)$.

Theorem 4.1 *Let cumulants $\widetilde{\Gamma}_k$ of integer-valued random variable X with the symmetric cumulative distribution function $F(x)$ and the variance $\mathbf{D}X=2\lambda$, satisfy the V. Statulevičius type condition:*

$$\left|\widetilde{\Gamma}_k\right| \leqslant \frac{(k-1)!}{\Delta^{k-1}}2\lambda,\ \ \Delta>0,\ \ k=2,3,4,\dots, \tag{9}$$

then

$$(1+|x|)\big|F(x)-G(x)\big| \leqslant \frac{22\lambda}{\pi\Delta(1-\delta)}, \tag{10}$$

where

$$\Delta \geqslant 2\max\left\{\frac{1}{\delta};\frac{1}{1-\delta}\right\},\ \ 0<\delta<1. \tag{11}$$

Proof of Theorem 4.1. From the inequality $|\mathrm{e}^w-1|\leqslant|w|\,\mathrm{e}^{|w|}$, $w\in\mathbb{C}$, we obtain

$$\big|f(t)-g(t)\big|=\left|\mathrm{e}^{\ln f(t)}-\mathrm{e}^{\ln g(t)}\right|=\big|g(t)\big|\left|\mathrm{e}^{\ln f(t)-\ln g(t)}-1\right|$$

$$\leqslant\big|g(t)\big|\big|\ln f(t)-\ln g(t)\big|\,\mathrm{e}^{\left|\ln f(t)-\ln g(t)\right|}. \tag{12}$$

Notice that $\widetilde{\Gamma}_1=2\lambda$ and $\ln g(t)=2\lambda z$, where $z=\cos t-1$; therefore, from (9), we obtain

$$\big|\ln f(t)-2\lambda z\big|=\left|\sum_{k=1}^{\infty}\widetilde{\Gamma}_k\frac{z^k}{k!}-2\lambda z\right|\leqslant\sum_{k=2}^{\infty}\left|\widetilde{\Gamma}_k\right|\frac{|z|^k}{k!}\leqslant\lambda\frac{|z|^2}{\Delta}\sum_{k=2}^{\infty}\left|\frac{z}{\Delta}\right|^{k-2}.$$

Since

$$|z|=|\cos t-1|=2\sin^2\frac{t}{2}\leqslant 2,$$

and $\Delta\geqslant\frac{2}{\delta}$, $0<\delta<1$, then $\left|\frac{z}{\Delta}\right|\leqslant\delta$. Therefore,

$$\big|\ln f(t)-\ln g(t)\big|\leqslant\lambda\frac{|z|^2}{\Delta}\sum_{k=2}^{\infty}\left|\frac{z}{\Delta}\right|^{k-2}\leqslant\frac{\lambda|z|^2}{\Delta(1-\delta)}\leqslant\frac{4\lambda}{\Delta(1-\delta)}\sin^4\frac{t}{2}. \tag{13}$$

Since

$$\big|g(t)\big| = \mathrm{e}^{2\lambda(\cos t-1)} = \mathrm{e}^{-4\lambda\sin^2\frac{t}{2}}, \tag{14}$$

from (12), (13) we obtain that

$$\big|f(t)-g(t)\big| \leqslant \frac{4\lambda}{\Delta(1-\delta)}\sin^4\frac{t}{2}\exp\left\{-4\lambda\sin^2\frac{t}{2}\left(1-\frac{1}{\Delta(1-\delta)}\right)\right\}. \tag{15}$$

From (11), we obtain $1-\frac{1}{\Delta(1-\delta)} \geqslant \frac{1}{4}$. Therefore,

$$\big|f(t)-g(t)\big| \leqslant \frac{4\lambda}{\Delta(1-\delta)}\sin^4\frac{t}{2}\cdot\mathrm{e}^{-\lambda\sin^2\frac{t}{2}}. \tag{16}$$

Now we estimate $(f(t)-g(t))'$. We have

$$\begin{aligned}(f(t)-g(t))' &= g'(t)\cdot\left(\mathrm{e}^{\ln f(t)-\ln g(t)}-1\right)+g(t)\cdot(\ln f(t)-\ln g(t))'\cdot\mathrm{e}^{\ln f(t)-\ln g(t)}\\ &= -2\lambda\sin t(f(t)-g(t))+g(t)\cdot(\ln f(t)-\ln g(t))'\cdot\mathrm{e}^{\ln f(t)-\ln g(t)},\end{aligned}$$

then

$$\big|(f(t)-g(t))'\big| \leqslant 2\lambda\cdot\big|f(t)-g(t)\big|+\big|g(t)\big|\cdot\big|(\ln f(t)-\ln g(t))'\big|\cdot\mathrm{e}^{\left|\ln f(t)-\ln g(t)\right|}. \tag{17}$$

Since

$$\big|(\ln f(t)-2\lambda z)'\big| \leqslant \big|z'\big|\sum_{k=2}^{\infty}\big|\widetilde{\Gamma}_k\big|\frac{|z|^{k-1}}{(k-1)!} \leqslant \frac{4\lambda}{\Delta(1-\delta)}\sin^2\frac{t}{2}, \tag{18}$$

from the estimates (13), (14), (15), (17) and (18), we obtain that

$$\begin{aligned}\big|(f(t)-g(t))'\big| &\leqslant \frac{4\lambda(2\lambda+1)}{\Delta(1-\delta)}\sin^2\frac{t}{2}\exp\left\{-4\lambda\sin^2\frac{t}{2}\left(1-\frac{1}{\Delta(1-\delta)}\right)\right\}\\ &\leqslant \frac{12\lambda\max\{1,\lambda\}}{\Delta(1-\delta)}\sin^2\frac{t}{2}\mathrm{e}^{-\lambda\sin^2\frac{t}{2}}.\end{aligned} \tag{19}$$

From (16) and (19), we obtain that

$$\int_{-\pi}^{\pi}\left|\frac{f(t)-g(t)}{\sin\frac{t}{2}}\right|\mathrm{d}t \leqslant \frac{8\lambda}{\Delta(1-\delta)}\int_0^{\pi}\sin^3\frac{t}{2}\cdot\mathrm{e}^{-\lambda\sin^2\frac{t}{2}}\,\mathrm{d}t \leqslant \frac{16}{\Delta(1-\delta)}\min\{1,\lambda\}, \tag{20}$$

$$\int_{-\pi}^{\pi}\left|\frac{f(t)-g(t)}{\sin^2\frac{t}{2}}\right|\mathrm{d}t \leqslant \frac{8\lambda}{\Delta(1-\delta)}\int_0^{\pi}\sin^2\frac{t}{2}\cdot\mathrm{e}^{-\lambda\sin^2\frac{t}{2}}\,\mathrm{d}t \leqslant \frac{16}{\Delta(1-\delta)}\min\{1,\lambda\}, \tag{21}$$

$$\begin{aligned}\int_{-\pi}^{\pi}\left|\frac{(f(t)-g(t))'}{\sin\frac{t}{2}}\right|\mathrm{d}t &\leqslant \frac{24\lambda\max\{1,\lambda\}}{\Delta(1-\delta)}\int_0^{\pi}\sin\frac{t}{2}\mathrm{e}^{-\lambda\sin^2\frac{t}{2}}\,\mathrm{d}t \leqslant \frac{48\max\{1,\lambda\}}{\Delta(1-\delta)}\min\{1,\lambda\}\\ &\leqslant \frac{48\lambda}{\Delta(1-\delta)}.\end{aligned} \tag{22}$$

Then from (8), (20), (21) and (22), we obtain

$$(1+|x|)\big|F(x)-G(x)\big| \leqslant \frac{22\lambda}{\pi\Delta(1-\delta)}.$$

Theorem 4.1 is proved.

We will apply the results of Theorem 4.1 for the approximation of cumulative distribution functions of sums of independent generalized Rademacher random variables by the cumulative distribution function of the Irwin law.

Corollary 4.1 *Let $X_{nk},\ k=1,2,\dots,n,\ n=1,2,\dots,$ denote independent generalized Rademacher random variables:*

$$\mathbf{P}\{X_{nk}=-1\}=\mathbf{P}\{X_{nk}=1\}=\frac{\lambda}{n},\qquad \mathbf{P}\{X_{nk}=0\}=1-\frac{2\lambda}{n},$$

and $F_{S_{nn}}(x)$ are cumulative distribution functions of random variables $S_{nn}=X_{n1}+\dots+X_{nn}$, $G(x)$ is the cumulative distribution function of the Irwin law , then

$$(1+|x|)\left|F_{S_{nn}}(x)-G(x)\right|\leqslant\frac{44\lambda^2}{n\pi(1-\delta)},\tag{23}$$

where

$$n\geqslant 4\lambda\max\left\{\frac{1}{\delta};\frac{1}{1-\delta}\right\},\ \ 0<\delta<1.\tag{24}$$

Proof of Corollary 4.1. Characteristic functions of random variables $S_{nn}=X_{n1}+\dots+X_{nn}$ are

$$f_{S_{nn}}(t)=\left(1+\frac{2\lambda}{n}(\cos t-1)\right)^n.\tag{25}$$

Since

$$\left(1+\frac{2\lambda}{n}(\cos t-1)\right)^n\longrightarrow\exp\{2\lambda(\cos t-1)\},\quad \text{as}\quad n\to\infty,$$

then $\mathbf{P}\{S_{nn}<x\}\Rightarrow G(x)$ as $n\to\infty$.

For $z=\cos t-1$, we obtain

$$\ln f_{S_{nn}}(t)=n\cdot\ln\left(1+\frac{2\lambda}{n}z\right)=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}(k-1)!(2\lambda)^k}{n^{k-1}}\frac{z^k}{k!}.$$

Therefore, cumulants $\widetilde{\Gamma}_k$, $k=1,2,3,\dots$ of random variables S_{nn} are

$$\widetilde{\Gamma}_k=\frac{(-1)^{k-1}(k-1)!(2\lambda)^k}{n^{k-1}}$$

and

$$\left|\widetilde{\Gamma}_k\right|\leqslant\frac{(k-1)!(2\lambda)^k}{n^{k-1}}=\frac{(k-1)!}{\Delta_n^{k-1}}2\lambda,$$

where $\Delta_n=\frac{n}{2\lambda}$, $k=1,2,3,\dots$.

From Theorem 4.1 with $\Delta=\Delta_n=\frac{n}{2\lambda}$, we obtain (23). Corrollary 4.1 is proved.

5. Conclusions

We obtain the nonuniform estimates of the remainder term (10) through the application of the cumulant method to the integer-valued random variable, provided all its moments are finite and its special cumulants satisfy the V. Statulevičius type condition (9) described in this paper. The results from Theorem 4.1 allow approximating cumulative distribution functions of sums (products) of independent (dependent) random variables by the Irwin law. Nonuniform estimates of cumulative distribution functions of sums of independent generalized Rademacher random variables by the Irwin distribution are obtained from Corollary 4.1.

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NETOLYGŪS ĮVERČIAI APROKSIMUOJANT IRWIN DĖSNIU

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Santrauka. Šiame darbe nagrinėjama visus baigtinius momentus turinčių atsitiktinių dydžių skirstinių aproksimacija Irwin skirstiniu. Aproksimacijos liekamasis narys vertinamas kumuliantų metodu. Nagrinėjami specialūs kumuliantai, leidžiantys užrašyti V. Statulevičiaus tipo sąlygas. Gauti sveikąsias reikšmes įgyjančio atsitiktinio dydžio, kurio pasiskirstymo funkcija yra simetrinė ir kurio specialieji kumuliantai tenkina V. Statulevičiaus tipo sąlygą, aproksimacijos Irwin skirstiniu liekamojo nario netolygūs įverčiai.

Reikšminiai žodžiai: pasiskirstymo funkcija, charakteristinė funkcija, kumuliantai, Irwin skirstinys, apibendrintieji Rademacherio atsitiktiniai dydžiai, netolygūs įverčiai.