COMMUNICATING MATHEMATICAL IDEAS IN A DIGITAL WRITING ENVIRONMENT: THE IMPACTS ON MATHEMATICAL REASONING FOR STUDENTS WITH AND WITHOUT LEARNING DISABILITIES

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Abstract

Mathematical reasoning is often underdeveloped in students with learning disabilities (LD). Problem solving and reasoning represent one of the most important aspects of a mathematics curriculum. The purpose of the present study *was to examine how communication through writing in mathematics via a digital environment impacts the mathematical reasoning of students with and without LD*. Specifically, the research was guided by research questions: (1) *What differences exist between the mathematical reasoning of students with and without LD prior to and after using a digital writing environment? (2) How does use of the digital writing environment differ between students with and without LD? (3) To what extent does student reasoning change overtime when emphasis is placed on communication via writing in mathematics?* Oral reading and math fact fluency as well as participants' working memory were used as academic variables. The Math Reasoning Inventory (MRI; Burns, 2012) is a formative assessment designed to evaluate mathematical reasoning through a face-to-face interview. The primary focus is on core numerical reasoning strategies and understanding. Subjects participated in an intervention focused on communication in mathematics through the use of digital writing tools in a computer-based mathematics program. Results reveal that communication through writing or peer-based discussions around mathematics can impact students' reasoning skills. Reasoning is a fundamental skill in mathematics and remains an area in which students with LD continuously struggle; therefore, interventions focused on advancing student reasoning will be increasingly pivotal to mathematics education. Students in this study showed improvement in different areas of reasoning over the course of the intervention and responded differently to the types of writing environments that were offered. Regardless of which digital environment was preferred by students with and without LD, results demonstrated that incorporating writing into mathematics to communicate mathematical information benefited all students in this study and has the potential to impact mathematics education.

Key words: *mathematical reasoning, students with or without LD, digital environment*

Internationally there is an increasing emphasis in the development of a numerate population that can use mathematics effectively in everyday life, at home, work and in the community (Diezmann, Lowrie, & Kozak, 2007). These initiatives focus on directly targeting instruction as well as centering on educating those who have difficulty with essential mathematics (Hanushek, Peterson, & Woessmann, 2010; Jitendra, 2005). Recent reports demonstrate that the mathematics performance of elementary and secondary students is rapidly progressing in some countries; however, in most countries student progress has been slow, with little or no gains (Hanushek et al., 2010; National Assessment of Educational [NAEP], 2013). Many countries may not be adequately preparing students with the levels of mathematic knowledge necessary to enter a competitive 21stcentury workplace (Hanushek, et al., 2010).For example, eighth grade students in the United States showed no significant improvement in mathematical proficiency on a national achievement test from 2011 to 2013 (National Assessment of Educational Progress [NAEP], 2013). Approximately two-thirds of students demonstrated only partial mastery of prerequisite mathematical knowledge and skills (NAEP, 2013). Furthermore, the progress of eighth grade students with disabilities is particularly troubling as there was a slight decrease in the average scores from 2011-2013 (NAEP, 2013). Although there have been initiatives focused on improving or enhancing mathematical education, particular attention has been placed on problem solving and reasoning within mathematics (Martin & Kasmer, 2010; National Council for Teachers of Mathematics [NCTM], 2000; CCSS; National Governors Associationetal., 2010).

Problem solving and reasoning represent one of the most important aspects of a mathematics curriculum. Knowing how to solve mathematical problems enhances an individual's ability to function in the context of everyday situations and work settings (Bottge & Hasselbring, 1993), and assessments conducted at all levels (state, national, and international) over past 30 years indicate students are notably deficient in their ability to solve mathematical problems (Kilpatrick et al., 2001). Although many students have difficulty problem solving, research indicates that children at-risk for mathematics difficulty (MD) or those identified with a learning disability (LD) evidence significant challenges in even solving one-step problems (Berch & Mazzocco, 2007; Jitendra et al., 2005).

To be successful in mathematics, students must have adequate short-term memory, good organization skills, and use strategies to facilitate learning (McLoughlin & Lewis, 2007). Unfortunately, students with learning disabilities often have problems with short-term memory, language reasoning, and metacognition (Hallahan, Kauffman, & Pullen, 2009). As on might expect, these difficulties can have a negative impact on overall mathematical performance. For example, weak abstract reasoning skills directly impact mathematical understanding and problem-solving, and students often require support to navigate the curriculum and demonstrate proficiency (Steele & Steele, 2003). Furthermore, working memory and language are associated to fact retrieval, calculation, word problem solving, and strategy use; therefore, difficulty in memory or language deficiency contributes to the lack of development in strong mathematics skills in these areas (Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012; Fuchs, Fuchs, & Compton, 2011; Fuchs, et al., 2008; Gersten, Jordan, & Flojo, 2005; Hitch & McAuley, 1991; Passolunghi & Siegel, 2004; Raghubar, Barnes, & Hetch, 2010; Wilson & Swanson, 2001). Thus, these students often have lower than expected scores on mathematics achievement tests (Geary, Nugent, Hoard, & Byrd-Craven, 2007; Judge & Watson, 2011), plateau in their math achievement at grade 5 or 6 (Cawley, Baker-Kroczynski, & Urban, 1992), and reports show a decline in mathematics scores overtime (Allsopp, McHatton, & Farmer, 2010; NAEP, 2010)

Reasoning plays a crucial role in mathematics and has been emphasized in many standards or practices in recent years. Mathematical reasoning has been defined as the ability to understand and make sense of mathematical concepts in a logical way in order to form a conclusion or judgment (Merriam-Webster, 2014; CCSSM, 2010). NCTM (2009) suggests that, "being able to reason is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and at all grade levels, students should recognize and expect that mathematics makes sense" (p 2–3). Unfortunately, studies have shown that students with LD have weaker reasoning skills in comparison to typically developing peers (Bressette, 2011; Layton & Lock, 2003). Given that many national and international standards emphasize mathematical problem solving and reasoning across all grade levels, students with LD must establish a proficient ability to reason in mathematics to comprehend the foundational mathematical skills.

Communication and mathematical reasoning are closely intertwined as reasoning requires an individual to formulate and represent a given mathematics problem, explain, and justify the solution or argument about the problem (Aleven, Koedinger, & Popescu, 2003; Aleven, McLaren, Roll, & Koedinger, 2006; Kilpatrick, Swafford & Findell, 2001). However, this becomes problematic when students do not or cannot communicate their mathematical reasoning in a coherent manner. Writing in mathematics is one mode in which students can communicate their reasoning, expand understanding beyond calculations, build on partial knowledge, and improve problem-solving (Aleven et al., 2003; Cooper, 2012; Trafton & Trickett, 2001). Research has demonstrated that writing can develop thought processes, assist in identifying errors, encourage problem solving and reinforce the ability to define, classify, or summarize, which are useful for engaging in mathematics (Connolly, 1989; King, 1982) Moreover, writing incorporates the use of drawing represented through pictures, which may benefit students wholack the necessary language to express their mathematical ideas (Baxter, Woodward, & Olson, 2005). Although writing in mathematics can be supported using various methods, the rapid development of technological innovations has become a widely used tool in education (Cooper, 2012; Zemelman, Daniels, & Hyde, 2012).

Technology offers many benefits to assist instructional practices and meet the educational needs of all children (Kramarski & Mizrachi, 2006;Cemal Nat, Walker, Bacon, Dastbaz, & Flynn, 2011; Noeth & Volkov, 2004; U.S. Department of Education, 2010a). Studies examining the use of technology in mathematics have reported an increase in student engagement, the promotion of higher-order thinking skills, drill and practice opportunities, improved motivation, and positive impacts on achievement (Ke, 2008; Li & Ma, 2010; Lim, 2008; Cemal Nat et al., 2011; Ota & DuPaul, 2002). Specific studies have examined the use of technology in mathematics for students with LD and reported gains in achievement as well as increased motivation and engagement in learning (Allsopp et al., 2010; Nordness, Haverkost, & Volberding, 2011; Okolo, 1992). Although many types of technology tools have been studied, few have looked specifically at the use of technology to support writing. Zemelman et al. (2012) identified the use of technology such as blogs, chats, or forums as authentic writing environments that can facilitate communication about mathematics; however, little empirical evidence exists on the impacts of writing in mathematics through the use of technology and more specifically how this communication impacts mathematical reasoning.

The purpose of the present study *was to examine how communication through writing in mathematics via a digital environment impacts the mathematical reasoning of students with and without LD*. Specifically, the research was guided by two primary **research questions**: (1) *What differences exist between the mathematical reasoning of students with and without LD prior to and after using a digital writing environment? (2) How does use of the digital writing environment differ between students with and without LD? (3) To what extent does student reasoning change overtime when emphasis is placed on communication via writing in mathematics?*

Methods

Participants

Participants included 31 elementary students in grades 3, 4, and 5 and the majority of participants were male (61.3%). Well over three-fourths of the sample was Caucasian (83.9%) and the rest of the sample consisted of African American (9.7%), Asian (3.2%), and multiracial

(3.2%) participants. Slightly more than one third of the sample (41.9%)was verified with a primary or secondary eligibility category of learning disability (LD). Of those participants identified as primary or secondary LD, 31% were verified in reading, 23% in math, 23% in writing, and 15% were general LD. Only one participant qualified for free/reduced lunch, and all of the students had English as a native language.

Setting

School Overview. The study was conducted at two private schools located in North Texas. One school serves students in Pre-kindergarten through 5th grade and the other school is comprised of students in grades 2-12 with learning disabilities or differences. Participants engaged in the intervention using either a computer lab or computers in their classroom for 45 minutes, two times per week, in addition to regular mathematics instruction.

Online Learning Environment. The Math Learning Companion (MLC) program is a computer-based instructional program designed as a supplemental curriculum for students with learning differences in grades 3-8. MLC has 73 lessons grouped into one of seven modules: Math Foundations 1, 2, and 3; Number Sense; Algebra; Geometry; and Data Analysis. Each lesson entails six components: (1) Real World (instructional set), (2) Vocabulary (introduction of new mathematical terms), (3) Instruction (explicitly delivered), (4) Try It (guided practice), (5) Game (independent practice), and (6) Quiz (10-items randomly selected that align with lesson content). The curriculum framework for MLC is based on *HELPMath*©, which has demonstrated statistically significant effects on an ELL population (Tran, 2005), and in 2012, this study met the What Works Clearinghouse evidence standards without reservations. The classroom teachers were given an overview of the program, lesson components, and asked to assign their class a curriculum sequence of eight lessons based on what the students were learning in the classroom.

Measures

Several measures were included to assess differences between groups across multiple variables, including demographics, academics, writing environment behavior, and mathematical reasoning. Each measure is described below in detail.

Demographic variables. A demographic checklist containing10 items was completed by the homeroom teacher of each participant. These items included information on the participant's gender, grade, free/reduced lunch status, primary language spoken, and special education status (e.g., primary and secondary eligibility categories). Because our settings consisted of private schools that may or may not serve students with learning differences, the federal definition for learning disability was provided to the teachers to ensure that students primary or secondary eligibility of LD aligned. All data reported on the checklist were teacher report via a file review of each student.

Academic variables. Oral reading and math fact fluency as well as participants' working memory were used as academic variables. The DIBELS-DORF (Good & Kaminski, 2002) was used as the measure of oral reading fluency. DIBELS-DORF has demonstrated adequate reliability with test-retest reliability scores ranging from .92-.97. Three grade-level reading passages were administered to each participant. Each passage was timed at one minute and the median score of correct words per minute represents their oral reading. Mathematical content knowledge was assessed using a curriculum aligned test that consisted of 30 items and was administered on the computer in MLC prior to and upon completion of the intervention. Math fact fluency was assessed using brief timed curriculum based measurement (CBM) math fact probes (Fox, Howell, Morehead, & Zucker, 1993; addition; subtraction; and multiplication facts). Probes were given to participants as a paper/pencil task and they were instructed when to start and stop. Correct digits per minute for the addition, subtraction, multiplication, and division timings in mathematics were calculated and recorded for analysis.

Math Reasoning Inventory. The Math Reasoning Inventory (MRI; Burns, 2012) is a formative assessment designed to evaluate mathematical reasoning through a face-to-face interview. The primary focus is on core numerical reasoning strategies and understanding. Because students respond to questions by explaining their thought processes, the interviewer can record both the students' accuracy and the strategies they used to solve problems. Participants were administered the 10 items from the Whole Numbers (Cronbach's Alpha = .81; Bernbaum-Wilmont, 2012) subtest of the MRI during this study. Participants were provided a visual representation of the problem via a notecard, followed by a scripted question, and then asked to answer the problem without the use of pencil and paper. Once participant responses were noted, they were asked, "How did you figure this out?" Researchers then recorded verbatim what participants were saying in an open documentation section of the MRI. Interventionists viewed all instructional videos related to administration, as well as practiced delivering and scoring of the MRI prior to administering it in the study.

Use of digital writing environment. Participant behavior was represented by the frequency of digital writing tool use within MLC as well as minutes engaged in the online mathematics program. Data were collected from the online mathematics program through a daily download of students' "click" behavior. Each time a participant clicked on the notepad or the wall, that behavior was recorded and downloaded. In conjunction with the click data, the number of participant notes and wall posts were also tabulated to give both a frequency of times the participant opened each tool and a frequency of actual notes/posts taken.

Procedures

Subjects participated in an intervention focused on communication in mathematics through the use of digital writing tools (i.e., a notepad and peer-mediated wall) in a computer-based mathematics program. Participants worked in the MLC program twice weekly for 45 minutes, completing a total of 8 lessons. These lessons were assigned by the grade-level teacher and participants completed them in the same order. Over the eight lessons participants were trained on using the digital writing tools embedded in MLC to communicate information. Participants completed four levels of intervention, consisting of two lessons each. Participants were provided a scripted training on use of the digital notepad including a word-processing and drawing feature, use of a peer-mediated wall (similar to blogging), and a note-taking strategy. They were provided the opportunity to practice using each tool and were granted the opportunity to ask questions. Different requirements were given to participants that included: taking notes in the digital notepad, posting comments to peers, responding to the questions or comments of peers, and using a note-taking strategy. New demands were placed on participants every two lessons of the intervention. Figure 1 provides additional details of the intervention levels.

Data Analysis

Data were analyzed using several distinct steps. First, data were entered, cleaned, and descriptive statistics were generated to provide an overview of the sample. Second, specific non-parametric statistical analyses were conducted to address each research question. A Mann-Whitney U test was used to examine group differences on the MLC-based pre- and posttest, and the frequency of notepad and wall clicks and notes/posts. A Wilcoxon signed ranks test was conducted to determine the gains over the course of the program on the MLC pre- and posttest. Finally, effect sizes for both the Mann Whitney U and the Wilcoxon signed rank test were computed by dividing the standardized test statistic by the square root of the sample size, providing an indicator of the probability that an observation from one group will be higher

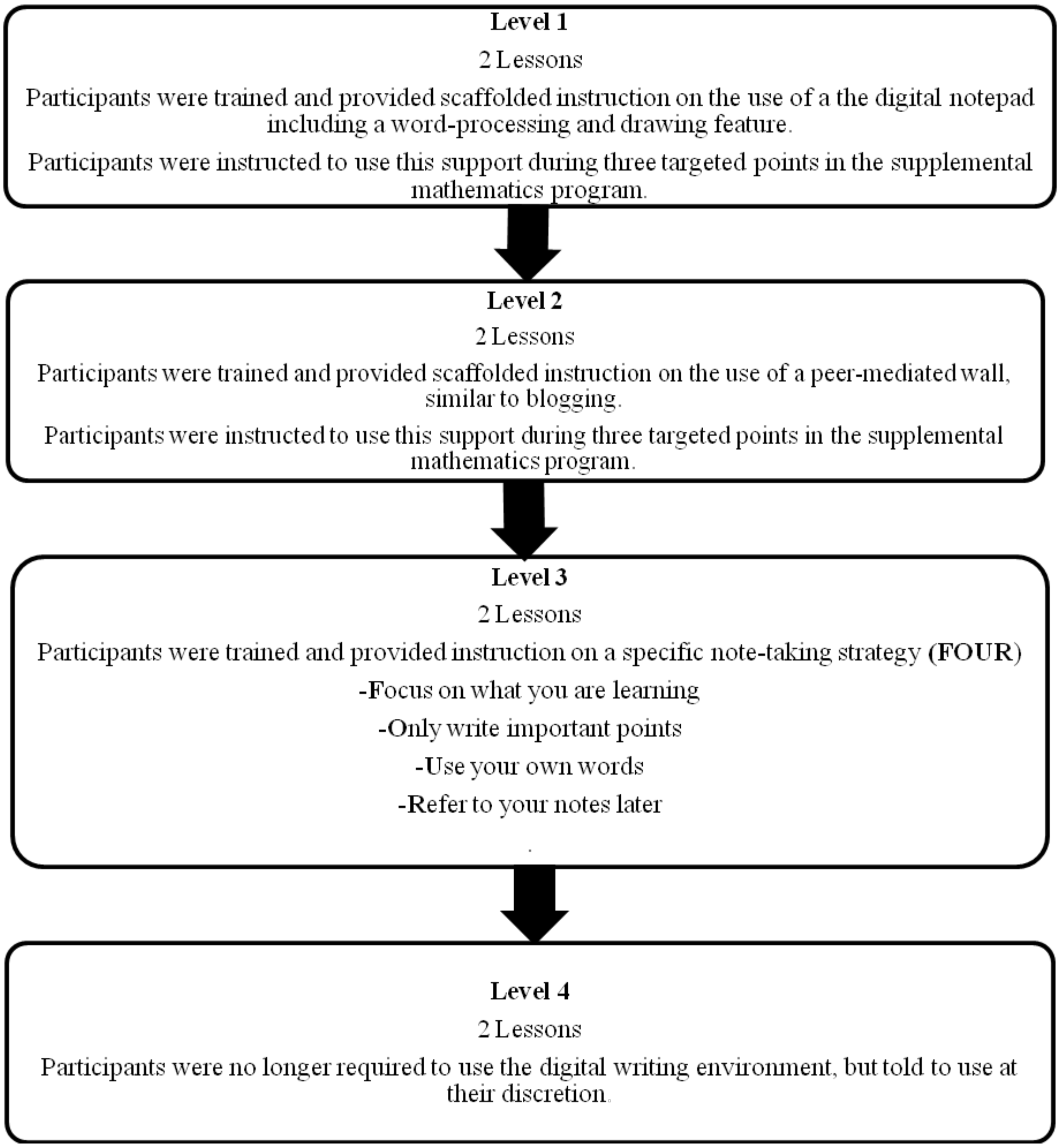

Figure 1. Intervention Levels for Writing in Mathematics

than an observation from another (Conroy, 2014). Criteria for interpreting the magnitude of the effect sizes were based on Hopkins (1997) recommendations (*ES* =.10-.30 = small; .30-.50= medium; .50-.70 = large; .70-.90 = very large; .90-1.00 = nearly perfect).

Prior to analysis of the MRI, a systematic process was used to code participant data. A coding dictionary was generated for each of the three components of the MRI (student answer, student explanation, and student reasoning). *Student answer* pertained to the correctness of the answer and was comprised of four categories (correct, incorrect, self-corrected, or did not answer). *Student explanation* assessed student's methods for solving a problem (used the standard algorithm, used another method specific to the problem, gave other reasonable explanation, or guessed). Finally, *Student reasoning* was represented by seven categories (no attempt at reasoning, guess attempt but incorrect answer, guess attempt and correct answer, partial attempt but had a reasoning breakdown, complete reasoning with a calculation error, complete reasoning and correct answer, or entirely wrong process). See Table 1 for examples of participant responses for each MRI category.

Table 1. Examples of MRI responses in each category

Type of Reasoning	Question	Student Answer	Student Explanation	Student Reasoning
No attempt at reasoning	7000/70	Did not answer	N/A	I don't know how to answer this
Guess attempt, incorrect	7000/70	Incorrect (7)	Guessed, did not explain, or gave faulty explanation	Well, um I think if you line up the 7's and bring it down the 0's cross out and then you have 7 left.
Guess attempt, correct	7000/70	Correct	Gave other reasonable explanation	I just divided them in my head
Partial attempt, reasoning breakdown	7000/70	Incorrect (0)	Guessed, did not explain, or gave faulty explanation	Well, I begin by setting up the problem and then crossing out the 0's to divide and then subtract the 7's so you get 0
Complete reasoning, calculation error	7000/70	Incorrect (10)	Used standard algorithm	I know that you add 0's to anything multiplied by 100 so it is 10.
Complete reasoning, correct	7000/70	Correct	Used other method specific to problem	Because I know 70 x 100 is 7000
Entirely wrong process	7000/70	Incorrect (81)	Guessed, did not explain, or gave faulty explanation	When you think about it, it can't be in the 90s because 18 is more than 10, but it can't be in the 100s because you are taking away the 0's, so it has to be in the 80s

Next, training and reliability was established between coders. Reliability was completed by two researchers on 11% of the sample, averaging 95.8% reliability. All disagreements were discussed until unanimity was reached. Finally, researchers independently coded the remaining MRI interviews. If the participant did not answer the question under the first category, the remaining categories were not coded. Therefore, a maximum of 180 responses could have been coded under the first category for participants without LD and 130 responses for participants with LD. Once the data were coded into these categories, chi-square tests were performed on the MRI pre- and posttest to determine differences in reasoning related to *student answer* (i.e., correctness of problems), *student explanation (*i.e., type of explanation) and *student reasoning* between participants with LD and without LD from pre to posttest. Cramer's (Phi) effect sizes were computed to determine the magnitude of difference between groups. Criteria for interpreting the magnitude of the effect sizes were based on Rea & Parker (1992) recommendations ($V < .10$ = negligible associate; V =.10 — 0.20 = weak association; V= 0.2 — 0.4 =moderate association; V .40 — .60 = relatively strong association; V = .60 -.80 strong association*;* V = 0.8 — 1.0 = very strong association).

Results

Academic Variables

Descriptive statistics were analyzed and Mann-Whitney U statistics were conducted to determine the differences between participants with and without LD on academic variables (see Table 2). As expected, participants without LD scored significantly higher than those with LD on oral reading fluency and math fact fluency — multiplication, and marginally higher on math fact fluency — addition. Participants without LD also scored significantly higher than those with LD on the MLC pretest and posttest. Wilcoxon signed ranks tests were conducted to determine the change from MLC pretest to posttest for both groups of participants. The gains made from pre to posttest for participants both without and with LD approached statistical significance over the course of the intervention ($z = 1.699$, $p < .10$, ES = 0.40 and $z = -1.648$, $p < .10$, ES = 0.34, respectively).

Reasoning Prior to Intervention

Prior to the intervention, chi-square analyses revealed significant differences between participants with and without LD in the MRI categories *student answer, student explanation,* and *student reasoning.* For *student answer,* results showed significant differences with medium effects for those problems that were answered($\chi^2 = 18.210$, df = 1, $p < .001$, ES = .26) as participants without LD answered problems correctly more often than individuals with LD. In the *student explanation* category, significant differences with medium effects were also present ($\chi^2 = 8.670$, df = 2, $p < .05$, ES = .23) as participants without LD used a standard algorithm, gave a reasonable explanation, or used a method specific to the problem more often that participants with LD. For *student reasoning,* significant differences with large effects were found ($\chi^2 = 29.517$, df = 6, $p < .001$, ES = .33) with the biggest difference in participants without LD communicating complete reasoning and providing the correct answer more often than individuals with LD (see Table 2).

Table 2. Descriptive statistics and independent sample t-tests for academic variables

	Without LD (n = 18)		With LD (n = 13)			
Variable	Mean	SD	Mean	SD	Mann-Whitney U	ES
Math Fact Fluency						
Addition	27.03	12.48	19.31	10.5	-1.926^{+}	-0.35
Subtraction	14.78	7.99	10.73	5.5	-1.403	--
Multiplication	22.14	12.06	12	8.24	-2.223*	-0.40
Division (N = 8, 7)	9	6.2	5.57	4.14	-1.043	--
Oral Reading Fluency	138.778	49.66	84.39	45.55	-2.682**	-0.48
MLC Pretest	20.33	5.35	14.46	3.86	-3.077**	-0.55
MLC Posttest	21.83	5.53	16.69	5.45	-2.328*	-0.42

$^{+}p < .10$, $^{*}p < .05$, $^{**}p < .01$

Reasoning Following Intervention

After the use of a digital writing intervention, chi-square analyses identified significant in the MRI categories *student answer, student explanation,* and *student reasoning.* For *student answer,* results showed significant differences with medium effects (χ^2 = 12.709, df = 1, p < .01, ES = .21) as participants without LD continued to answer more problems correctly than participants with LD. In the *student explanation* category, significant differences with medium effects were noted (χ^2 = 9.949, df = 2, p < .01; ES = .26) with the biggest differences present in the categories of using a method specific to the problem and providing other reasonable explanations. For *student reasoning,* significant differences with large effects were found (χ^2 = 38.680, df = 6, p < .001, ES = .37) with the biggest difference continuing to be that participants without LD communicated complete reasoning and provided the correct answer more often than participants with LD (see Table 3).

Table 3. Student Explanations of Reasoning Pre and Post test

		Pretest		Posttest	
		Students with LD	Students without LD	Students with LD	Students without LD
Category	n (%)	(n = 115)[a]	(n = 162)[a]	(n = 114)[a]	(n = 172)[a]
No attempt at reasoning		2 (1.7%)	4 (2.5%)	14 (12.2%)	2 (1.2%)
Guess attempt at communicating reasoning, but incorrect answer		24 (20.9%)	22 (13.6%)	12 (10.5%)	23 (13.4%)
Guess attempt at communicating reasoning, correct answer		3 (2.6%)	2 (1.2%)	2 (1.6%)	4 (2.3%)
Partial attempt at communicating reasoning, breakdown in mathematical process		28 (24.3%)	23 (14.2%)	21 (18.4%)	25 (14.5%)
Complete reasoning communicated, but had calculation error		20 (17.4%)	32 (19.6%)	18 (15.8%)	27 (15.7%)
Complete reasoning communicated, correct answer provided**		20 (17.4%)	71 (43.8%)	30 (26.3%)	86 (50.0%)
Entirely wrong process, therefore reasoning communication was off base		18 (15.7%)	8 (4.9%)	17 (14.9%)	5 (2.9%)

Note. p<.001 a Data are reflective of recorded student responses, some students may not have answered the question and these were coded as missing, thus not included in analyses

Change in Reasoning Skills

Individuals with LD demonstrated significant changes over the course of the intervention in all three MRI categories. In the category *student answer*, participants were more likely to either answer correctly or not answer the question at all (χ^2 = 29.794, df = 3, p < .001; ES= .28, p < .001). For *student explanation*, participants were less likely to guess on the posttest or use the standard algorithm (χ^2 = 32.058, df = 3, p < .001, ES = .32, p < .001). Under the category *student reasoning*, participants were more likely to show no attempt at reasoning or use complete reasoning and get the answer correct and had fewer instances where they guessed and got the answer incorrect ($\chi^2 = 77.109$, df = 6, p < .001, ES = .35, p < .001). Results across all three categories indicate a shift in the reasoning from guessing and answering incorrectly to either answering questions correctly or refusing to answer if they do not understand the problem at hand.

For participants without LD, chi-square analyses for all three MRI categories showed a significant difference from pretest to posttest as well. For the category *student answer* ($\chi^2 = 63.297$, df = 3, p < .001; ES= .34, p < .001), participants were more likely to answer correctly or not answer at all on the posttest than on the pretest. Participants were also less likely to use the standard algorithm on the posttest than on the pretest in the category *student explanation* ($\chi^2 = 47.698$, df = 3, p < .001, ES = .31, p < .001). Under *student reasoning* ($\chi^2 = 100.575$, df = 6, p < .001, ES = .33, p < .001), participants were significantly more likely to provide complete reasoning and a correct answer on the posttest than on the pretest.

Use of Digital Writing Environment

Descriptive statistics were examined for the use of MLC, including the mean and standard deviation of the number of minutes spent in each intervention level of the program (i.e. Levels 1-4) for participants with and without LD. The mean amount of time for students with LD was higher for each level than the mean amount of time for students without LD. However, Mann-Whitney U statistics were not significant at each level, indicating that students with and without LD spent comparable time in the program.

The weighted frequencies of notes taken on the notepad and the wall for students with and without LD were also examined. Overall, students with LD used the wall more often than students without LD, and students without LD used the notepad more often than students with LD. Use of the notepad was similar between groups at Level 1 (i.e., students with LD took an average of 10.31 notes and students without LD took an average of 11.22 notes); however, by Level 4, students without LD took approximately twice as many notes as students with LD. The wall was only required at Level 2 students with LD (M = 6.77) had more wall entries at this level than those without LD (M = 5.06) and this pattern continued despite wall entries no longer being required (see Table 4).

Table 4. Weighted frequency of notepad and wall use at each level of the intervention

	Students with LD	Students without LD
Intervention Level	(n = 13)	(n = 18)
Level 1		
Notepad	10.31	11.22
Wall	N/A	N/A
Level 2		
Notepad	2.77	3.44
Wall	6.77	5.06
Level 3		
Notepad	3	5.22
Wall	2.31	0.61
Level 4		
Notepad	0.85	1.61
Wall	0.54	0.11

Discussion

Proficiency in mathematics is problematic when students do not or cannot communicate their mathematical reasoning in a coherent manner. Globally, as new demands are present in mathematics, an emphasis has been placed on students' ability to articulate reasoning. Writing in mathematics can be used to foster communication related to mathematical ideas and provide students an opportunity to demonstrate reasoning. Unfortunately, limited research has been conducted on the impacts of communication through writing in technology-based environments. Furthermore, relatively no evidence is present on how this may impact the mathematical reasoning skills of students, and in particular students with LD. Therefore, the objectives of this study were to identify differences in mathematical reasoning of students with and without LD, understand how mathematical reasoning differs after using a digital environment for writing in mathematics, and explore differences in how students use a digital writing environment.

Results indicate that both before and after being trained on writing in mathematics within a digital environment, individuals with and without LD differed significantly in all three MRI categories (i.e., *student answer, student explanation, student reasoning*). Prior to and after receiving training on communicating mathematical information in the digital writing environment, participants with LD answered fewer questions correctly, were more likely to guess on answers, use an entirely wrong mathematical process, and provide complete reasoning less often than individuals without LD. Although these findings are not surprising given the difficulty that students with LD have with reasoning (Bressette, 2010; Layton & Lock, 2003), it does shed light on how differences in specific reasoning skills manifest themselves across students with and without LD. Specifically, for this sample, participants with LD had a breakdown in mathematical processes, calculation errors, and used an entirely wrong mathematical process to answer a problem.

Despite the significant differences in mathematical reasoning between groups prior to and following the intervention, both groups made significant gains in their quality of reasoning. For example, participants with LD answered correctly more often or chose not to answer, used the standard algorithm, or guessed at the answer less often. They were also less likely to guess and get the problem incorrect, more likely to either reason completely through a problem and obtain a correct answer, or not attempt the problem at all than prior to the intervention. The ability for these students to select when to answer the problem based on knowing whether or not they can solve the problem shows a possible awareness of a lack of understanding a problem that was not present prior to the intervention. For participants without LD, results indicated that they were more likely to get a correct answer and less likely to refuse to answer the problem, they were less likely to use the standard algorithm, and they were more likely to show complete reasoning and get a correct answer on the posttest than on the pretest. This indicates a general improvement in understanding the problems and using successful methods to reason through them.

These findings are similar to other studies that have examined how writing in mathematics can impact the mathematical reasoning skills of students (Baxter et al., 2005; Burns, 2005). However, unlike previous research, this study incorporated the use of a digital writing environment to facilitate writing in mathematics. Participants were provided with explicit instruction on how and when to record their thoughts on a digital notepad and a peer-mediated wall or blogging tool, and were required to incorporate new writing demands every two lessons. As the assimilation of students' reasoning into written assignments and discussions has become an integral part of mathematics teaching (Burns, 2005); these findings suggest that a digital writing environment that includes both a traditional writing environment (e.g. the notepad) and use of social interaction (e.g., peer-mediated wall) are useful tools to foster communication through writing and impact the mathematical reasoning of students with and without LD. These results reflect findings from previous studies which have reported positive academic outcomes student engaged in interventions that explicitly teach communication or self-explanation strategies in the area of mathematics (Aleven et al., 2003; Bielaczyc, Pirolli, & Brown, 1995).

Finally, the differences between use of the notepad and wall for participants with and without LD were clear and become more pronounced over the course of the intervention. Notepad entries were only required at Levels 1 and 3, so use of the notepad on Levels 2 and 4 is substantially lower for both groups. Yet, interestingly, individuals without LD continued to choose this as a preferred method for taking notes versus the peer-mediated wall, which is the opposite of what we anticipated. The wall was only required to be used on Level 2 and use of the wall dropped drastically for students without LD in Levels 3 and 4 after it was no longer required. However, individuals with LD continued to use the wall outside of the requirement and had four times as many entries as students without LD on Level 3 and five times as much as students without LD on Level 4.Although incorporating the use of technology such as blogs, chats, or forums as authentic writing environments to facilitate communication about mathematics has been encouraged (Zemelman et al., 2012), some studies have noted that students with LD have difficulty with social communication (Mitchell, Franklin, Greco, & Bell, 2009). Therefore, it was of particular interest that individuals with LD continued to use the wall for communication about mathematics even though it was not required.

Limitations

Limitations of this study should be acknowledged and addressed in future research. First, the small sample was from two private schools. Because private and public school programs offer various approaches towards mathematics education and online learning, the results and generalizability from this study should be cautiously interpreted as they may not be representative of all elementary students. Replication of this study in other settings and with larger samples is needed to determine how communication in writing through the use of technology impacts mathematical reasoning skills. Furthermore, future researchers should look at including a larger and more diverse sample of youth receiving special education services to explore if there are additional differences among specific categories of youth with disabilities. This may help to better understand how to improve mathematical reasoning in students with disabilities. Next, although the MRI is a validated measure, it does not provide a comprehensive overview of mathematical reasoning due to the subjective nature of some response categories. All responses are communicated verbally; therefore it does not allow for any process to be recorded by the students in writing. Thus, future studies assessing both the process and product of student's mathematical reasoning might incorporate a modified version of the MRI, an additional measure of reasoning which allows the opportunity to articulate reasoning through various modes. Third, because the online curriculum is individualized and self-paced, students reached intervention levels at various times which made it difficult to control for confounding variables (i.e., maturation and teacher instruction). Therefore, it is difficult to say with complete confidence that gains in reasoning were strictly related to the intervention and not because of content teachers chose to focus on in class or length of time in the online program.

Implications

Results reveal that communication through writing or peer-based discussions around mathematics can impact students' reasoning skills. Although additional research is needed, these findings suggest important implications for practitioners and researchers working to improve communication through writing in mathematics as well as the reasoning skills of students, particularly students with LD. First, incorporating training on the use of note-taking, recording thoughts, or processes positively impacts the reasoning skills of students with and without LD. Therefore, teachers should consider this when planning mathematics lessons around reasoning. Second, access to embedded support tools such as a digital notepad or peer-mediated wall are beneficial for students and should be considered as an option for students to communicate mathematical reasoning. Third, students with LD preferred to engage in discussions with peers versus taking notes individually around mathematical concepts; therefore, finding ways to facilitate this dialogue between students is essential. Finally, given that The Common Core State Standards (CCSS, 2010) has placed emphasis on problem solving and reasoning across the eight strands, using measures of reasoning, such as the MRI, to better understand the reasoning skills of students with LD. Moreover, the information from this type of assessment could be used to develop specific goals within the student's Individualized Education Plan (IEP) that pertain to mathematical reasoning.

Enhancing the problem-solving and reasoning skills of students is integral to mathematics instruction as it continues to be a focus of educational systems worldwide. Reasoning is a fundamental skill in mathematics and remains an area in which students with LD continuously struggle; therefore, interventions focused on advancing student reasoning will be increasingly pivotal to mathematics education. Students in this study showed improvement in different areas of reasoning over the course of the intervention and responded differently to the types of writing environments that were offered. Regardless of which digital environment was preferred by students with and without LD, results demonstrated that incorporating writing into mathematics to communicate mathematical information benefited all students in this study and has the potential to impact mathematics education.

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COMMUNICATING MATHEMATICAL IDEAS IN A DIGITAL WRITING ENVIRONMENT: THE IMPACTS ON MATHEMATICAL REASONING FOR STUDENTS WITH AND WITHOUT LEARNING DISABILITIES

Summary

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Mathematical reasoning is often underdeveloped in students with learning disabilities (LD). Technology-based environments have quickly become a strategy to enhance students' reasoning in mathematics. Unfortunately, little research has examined the impact of technology on the reasoning skills of students with LD. This study sought to address this research gap by examining the impacts of a multi-modal writing environment intervention on the mathematical reasoning of students with LD. Recent reports demonstrate that the mathematics performance of elementary and secondary students is rapidly progressing in some countries; however, in most countries student progress has been slow, with little or no gains. To be successful in mathematics, students must have adequate short-term memory, good organization skills, and use strategies to facilitate learning. Unfortunately, students with LD often have problems with short-term memory, language reasoning, and metacognition (Hallahan, Kauffman, & Pullen, 2009). Reasoning plays a crucial role in mathematics and has been emphasized in many standards or practices in recent years. Technology offers many benefits to assist instructional practices and meet the educational needs of all children. The objectives of this study were to identify differences in mathematical reasoning of students with and without LD, understand how mathematical reasoning differs after using a digital environment for writing in mathematics, and explore differences in how students use a digital writing environment. The research was guided by research questions: (1) What differences exist between the mathematical reasoning of students with and without LD prior to and after using a digital writing environment? (2) How does use of the digital writing environment differ between students with and without LD? (3) To what extent does student reasoning change overtime when emphasis is placed on communication via writing in mathematics?

Subjects participated in an intervention focused on communication in mathematics through the use of digital writing tools in a computer-based mathematics program. Writing in mathematics can

be used to foster communication related to mathematical ideas and provide students an opportunity to demonstrate reasoning. Unfortunately, limited research has been conducted on the impacts of communication through writing in technology-based environments. Despite the significant differences in mathematical reasoning between groups prior to and following the intervention, both groups made significant gains in their quality of reasoning. They were also less likely to guess and get the problem incorrect, more likely to either reason completely through a problem and obtain a correct answer, or not attempt the problem at all than prior to the intervention. The ability for these students to select when to answer the problem based on knowing whether or not they can solve the problem shows a possible awareness of a lack of understanding a problem that was not present prior to the intervention. For participants without LD, results indicated that they were more likely to get a correct answer and less likely to refuse to answer the problem, they were less likely to use the standard algorithm, and they were more likely to show complete reasoning and get a correct answer on the posttest than on the pretest. This indicates a general improvement in understanding the problems and using successful methods to reason through them.