WHAT EARLY HUSSERL CAN TELL US ABOUT MATHEMATICAL INTUITION*

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Abstract. *The main objective of this article is to demonstrate that the utter rejection of Husserl's early philosophy of mathematics has been unjustified. More specifically, I argue that he anticipates both ones of the contemporary definitions and applications of mathematical intuition. In order to establish this I will first provide an analysis of the role of intuition in early Husserl's philosophy of mathematics and then show that it bears significant resemblance to the definition and application of intuition in the work of Charles Parsons, who is commonly credited as the flag-bearer of contemporary proponents of mathematical intuition.*

Keywords: *Edmund Husserl, mathematical intuition, Charles Parsons, concept of number.*

Introduction

The common reading of Husserl's early work tends to be faultfinding: it is an immature attempt to marry psychology and the definition of the concept of number, in which he is reproducing the ideas of his teachers, Carl Stumpf and Franz Brentano, rather than offering original insights. My aim in this paper is to show that such a quick dismissal of Husserl's early work is largely unwarranted and that his work is of relevance to the contemporary philosophy of mathematics. I will argue that Husserl, in his definition and application of mathematical intuition, anticipated the contemporary use of the concept of perceptual intuition as the means by which we grasp elementary mathematical concepts. To this end I will focus on relating Husserl's views to those of Charles Parsons, for it is in his work that the concept of perceptual intuition has recently been most thoroughly developed. It will be shown that Husserl applies the concept to the same cases – concrete perceptions – and for the same reasons – to explain how we have immediate knowledge of a certain body of mathematical truths. After making some general remarks on the definition of intuition used in this paper I will examine the role that intuition plays in Husserl's definition of the concept of number and how it relates to Parsons' views.

* This article is based on a talk given in the conference "Suprasti pasaulį, kalbą ir save" in Vilnius University on May 16, 2014.

1. Mathematical intuition: preliminary considerations

The concept of mathematical intuition first became prominent after Kant, who considered intuition (*Anschauung)* to be one of the central features of human cognition. As Parsons notes, in the late nineteenth and early twentieth centuries "Kant offered a kind of a paradigm of a philosophical conception of intuition applied to mathematics" (Parsons 1980: 193). A philosopher who falls decisively under this paradigm is Edmund Husserl. While Husserl's fully-developed notion of intuition first appears in *Logical Investigations,* we can find its origins in *Philosophy of Arithmetic,* in which it is crucial to his definition of the concept of number.

Although the Kantian use of intuition no longer dominates the contemporary philosophy of mathematics, some philosophers are nevertheless sympathetic to it, most prominently Parsons. While Husserl (both early and later in his career) and Parsons deviate considerably from the purely Kantian concept, the following three aspects of it are common to both:

- (i) intuition is an epistemic capacity that is a necessary component of our cognition, and it is primarily involved in connecting concrete instances of objects with the abstract (or quasi-abstract) concepts to which they correspond;
- (ii) intuitive knowledge that yields from intuition is immediate, as opposed to mediate, inferential knowledge;
- (iii) intuition is primarily understood as intuition *of* objects, rather than intuition *that* a proposition is true.

The last characterization should be clarified. It is common to differentiate between intuition *of* and intuition *that*, where the former is part of a cognitive process that is involved in grasping a distinct object and the latter one is a process involved in coming to know – or seeing the plausibility of – some proposition. Contemporary epistemology pays much more attention to propositional intuition, though this need not be all there is to it. Intuition *of* is often emphasized by those who evoke the analogy between intuition and perceptual knowledge in the process of grasping mathematical concepts. On such a view, a necessary condition for an intuition to arise is the agent's direct awareness of the object. This is true not only of Husserl but also of philosophers such as Parsons, Penelope Maddy and, to a lesser extent, Mark Steiner, for all of their conceptions of intuition involve explicit connection to the object.[1] What is specific to all of these views is that despite the differences in their proposed ontologies of mathematical entities, intuition is the crucial element in the agent's coming to have knowledge of a certain mathematical object.

2. Mathematical intuition and *Philosophy of Arithmetic*

I now turn to the analysis of the application of mathematical intuition in *Philosophy of Arithmetic* (henceforth *PA*)*,* Husserl's first philosophical work. It is common to refer to *PA* as the writing of the immature Husserl, who tries to salvage the ill-fated psycholo-

[1] This is not to say that the theories of Husserl, Parsons, or anyone else mentioned here, do not allow for intuition of propositions, however the general form of intuition that is applied is primarily the intuition of objects.

gistic approach to the concept of number. What contributed largely to the negative reception of this work was Frege's critical review of *PA,* which has by now come close to having the status of a manifesto of anti-psychologism in mathematics. It seems to be a common way of thinking amongst analytic philosophers that since Husserl himself turns his back on psychologism in his later work, *Philosophy of Arithmetic* is not worth paying much attention to. Those who nevertheless focus on the correspondence of Husserl and Frege tend to conclude that it was precisely Frege's criticism that influenced Husserl to reject psychologism and turn to Platonism.

However sound Frege's criticisms may be, it is unfortunate that Husserl's work has come to be understood primarily in relation to Frege's discontent with psychologism. If we understand *PA* in this way, we risk misrepresenting Husserl's position by relying too heavily on Frege's criticism. Furthermore, it could be argued that Frege mistakenly takes the general aim of Husserl's philosophy of mathematics to provide a psychologistic account of the nature of numbers, in the vein of Mill. It rather could be, as Tieszen puts it, "an investigation into the a priori conditions for the possibility of the consciousness and knowledge of number" (Tieszen 1994: 108).

Mill's conviction was that numbers were nothing but our subjective representations of aggregates of units, and that such representations, and in turn our concepts, are crafted by the psychological processes involved in, say, counting. Nothing of this sort can be found in *PA.* If one is patient with Husserl's at times obscure prose, it is clear that he is attempting to give an epistemological account of our concepts of numbers rather than define their ontology. There is no talk of the nature of numbers or number representations and definitely no mention that they are in any sense of the term subjective. In this respect, as David Bell notes, "adoption of the methodological constrains which in part determine the nature of descriptive psychology is entirely motivated" (Bell 1990: 61).

Indeed one of the prominent impressions that one may have while reading *PA* is its contra-Fregean nature. Husserl, contesting the logicist approach to the definition of number, states: "no concept can be thought without foundation in a concrete intuition." (Husserl 2003: 83). That is to say, in order to demonstrate the soundness of arithmetical knowledge, one must show how its concepts can be traced to concrete cases, or phenomenal experiences in which the number concepts appear. Intuition (*Anschauung*) will thus play a pivotal role in Husserl's early project as an epistemic capacity that allows us to perceive abstract mathematical concepts in concrete instances.

The focus on epistemology, rather than ontology or semantics, in Husserl's early writings was highly influenced by his work with Carl Stumpf, under whom he studied in Halle from 1887 to 1901. It was Stumpf's idea that the question of the knowledge of the concept has priority over the question of the content or essence of the concept, for it is only through the careful attendance to the origin of our knowledge of the concept that we are able to bring about the content of it.[2] Guided by such methodological pro-

[2] In *On the Psychological Origin of Space Representations,* a book studied closely by Husserl, Stumpf writes: "The question 'Whence arises a representation?'

visions, Husserl thus claims that the major task of *PA* is to deal with the "origin of the concept of number" (Husserl 2003: 311) and is convinced that once the origin of our knowledge of the concept of number is illuminated, the essence of this concept will be obvious and will require no further considerations[3].

Husserl begins his analysis by distinguishing between authentic and inauthentic concepts of numbers. In short, we form an *authentic* concept by directly perceiving countable objects and by way of intuition get to the concept of number. In contrast, an *inauthentic* concept of number is generated in the mind without direct perceptual evidence – for example through symbolic representation. Since authentic concepts require direct perceptual experience, we can approach only the elementary truths of arithmetic in this way. To surpass this limitation, Husserl builds more complex concepts of number from the elementary concepts and in so doing he is able to accommodate more sophisticated arithmetic.

In addition to what has been mentioned about Husserl's use of intuition in the previous section, one further clarification must be made. Since Husserl's goal is to define the concept of number, or, more precisely, how do we come to know numbers, he is interested in cases where we immediately perceive or experience an instance of it. In many cases the mind has to actively "purify" the perceptual representations of the irrelevant content in order to grasp a basic concept. As an example of this, let us consider an agent's perception of two cups on the table. Husserl claims that in order to arrive at the numerical concept 'two' the agent has to be able to get rid of all the irrelevant features, such as, say, where the cups are, what colour they are, that one of them has a chipped edge, and so on, until she arrives at the simple intuition of two objects. The mind could arrive at such an intuition by, for example, ignoring or disregarding parts of a given complex whole; or combining or unifying singular parts of a given complex whole.

What is specific to Husserl's early views on the sort of intuition involved in the generation of elementary arithmetic concepts is that these primitive mental activities appear parallel to intuition. Since Husserl grounds his analysis of the concept of number in everyday perceptual experiences, he assumes that aside from being able to intuit, the mind must have a certain apparatus to simplify the intuition. And while the mental activities involved in simplifying the intuition can be conceptually distinguished from intuition, they nevertheless do not appear separately and thus are an integral part of the process through which we come to know number concepts.

Though perceptual awareness is crucial in defining a concept of number, it is not enough for the formation of the concept of, say, 'the number three', that I am able to perceive three books stacked by my bedside, for I can perceive the objects without actu-

is of course […] to be clearly distinguished from the other question, 'What is its knowledge content, once we have it?'. However these two questions are methodologically related, insofar as the question of the origin of a representation leads us to the separate parts of which it is composed, and therefore yields a more precise grasp of its content" (Stumpf 1873: 3-4)

[3] What is meant here by 'origin of the concept' should be understood as a question about the origin of the knowledge of the concept, rather than the origin of the concept itself. Husserl main aim is to analyze our knowledge of number-concepts, rather than how concepts are created.

ally grouping them and thus recognizing that a certain number can be attributed to them. Thus the first step in Husserl's analysis is noting that the agent has a perceptual representation[4] of a determinate collection of objects including all of its specific features, such as its context and the particular properties of the perceived objects. In order to arrive at the numerical concept, however, I must be able to grasp the collection of objects as a unified whole having a distinct numerical value, and this is precisely where intuition plays a crucial role. By perceiving a concrete instance of objects, I intuit that a certain abstract numerical property can be ascribed to them, and it is in this way that we arrive at knowledge of distinct numbers.

As noted above, Husserl's early conception of intuition goes hand in hand with a number of primitive mental activities, and the one that plays a central role in the generation of number concepts is *collective connection* (*kollektive Verbindung*). Collective connection is a way in which our mind reduces the initial perceptual representations to more basic intuitions. What is crucial here is that while I grasp a certain collection of distinct objects I am able to see them as belonging to one sort[5]. As Willard puts it, "in such a case, a new and distinctive type of whole is <…> present to me with my field of consciousness: a totality or a multiplicity – a concrete unity of x number of objects" (Willard 2003: xviii). Once I have grasped the collection of objects as a unified whole I am able to intuit the numerical property that ought to be ascribed to them. The role of collective combination along with intuition is thus to present the consciousness with an objective representation of a numerical property that can be applied to a multiplicity of perceived units.

One arrives at a fully abstract object (i.e., a quantity stripped of even its distinct numerical property) by performing yet another mental procedure, that of *abstraction*, by which one bracket all the remaining concrete parts of the collection of concrete objects perceived until he arrives at a concept of an indeterminate multiplicity. What follows from this is that a number in itself is primarily a certain multiplicity of units, a mere featureless 'something' (Husserl 2003: 123). A distinct numerical quantity, on the other hand, is given to us by intuition and collective combination and is viewed by Husserl as property of that multiplicity. It is important to note here that Husserl is not describing how number concepts are created. His analysis requires that one already be equipped with a certain concept of number, intuition merely connects the particulars with such concepts.

A perceptual account of knowledge of number is surely incapable of dealing with all numerical concepts, since there is a limit to the number of distinct items we can explicitly notice or focus our attention on. In order to tackle this issue, Husserl introduces inauthentic concepts of number, which he defines as presentations via signs.[6]

[4] This need not be limited to actual perceptual experience i.e. Husserl also considers cases of counting where the objects involved are, say, imaginary or merely in our mind and not materially presented to us. Direct perception here is replaced with imagination. See Husserl 2003: 17.

[5] "Disregarding the properties that are different, we retain those that are common to all, as those which may belong to the concept in question" (Husserl 2003: 19).

[6] "If a content is not directly given to us, as what it in fact is, but only given via signs that uniquely characterize it, then our presentation of that content is not an authentic, but rather, a symbolic one" (Husserl 2003: 193).

As Bell notes, Husserl's theory of symbolic representation has three essential features: (1) the signs themselves are perceptible items; whether written or spoken they can be the content of concrete representations; (2) they comprise a recursive progression, so that any possible combination of numerals has a unique place in this series, and so that the series of numerals is generated recursively, in that there is an effective procedure, which enables us to generate any later term in the series on the basis of earlier terms; (3) one or more of the earliest signs in the series must be correlated with the authentic concept of number (Bell 1990: 56).

We may summarize Husserl's definition of number as follows. Both authentic concepts of numbers and determinate numerical quantities are arrived at by perceptual intuition: I have direct contact with them in that I can immediately and assuredly say both that this aggregate of objects is a multiplicity and that the property 'having n units' applies to this multiplicity. On the other hand my ability to grasp larger numbers is based on my ability to recognize their place in the arithmetical system of signs and, as Bell notes, my understanding of large numbers reaches no further than my ability to correctly identify numerals and their arithmetical signs (Bell 1990: 57).

What Husserl's analysis of the concept of number points to is that we cannot define number separately from the cognitive processes in which we come to grasp it. As noted above, such a definition becomes more of a specification of a priori conditions in which we come to grasp it, rather than a formal description of the concept, as, for example, the one that Frege proposes. In this sense, the non-inferential and immediate knowledge of number concepts that we arrive to by way of intuition is of crucial importance.

It is noteworthy that Husserl's use of intuition is not limited to mathematics alone. Indeed, he stresses that it is applicable in any case of *immediate cognition* and it surely should not be confined to the context of our knowledge of numbers. This observation will be especially important to his later work. In *Logical Investigations* and onwards intuition gets developed into an independent faculty and, one of its forms, namely, categorical intuition, is explicitly intended to account for our knowledge of all ideal objects, whether mathematical or otherwise.

Because Husserl (perhaps due to his insistence upon the priority of epistemology) does not touch upon ontological issues concerning numbers, that is, it is unclear whether he takes them to be ideal objects, we cannot fully equate his conception of intuition in *PA* with that in *Logical Investigations.* However, it seems plausible to state that we can see the headwaters of this orientation already in the *PA*. It is defined in a similar fashion, namely as an immediate cognition of sorts, and it is as essential to our knowledge of numbers in *PA* as it is in *Logical Investigations.*

3. Contemporary applications of mathematical intuition: the access challenge

Let us now turn to more recent use of intuition in philosophy of mathematics. While in contemporary epistemology intuition often figures in issues dealing with immediate and non-discursive knowledge whatever ontology the proponent has accepted, mathematical intuition is primarily seen in the context of Platonism. More precisely, it is used as means in answering the epistemic issues arising from Benacerraf's dilemma. The dilemma first sketched in Paul Benacerraf's "Mathematical Truth", roughly summarized, states that no interpretation of mathematical truth encompasses both a coherent semantics and epistemology. Put in other words, if the interpretation of mathematical truth satisfies the requirements of a homogenous semantic theory, "in which the semantics for the statements of mathematics parallel the semantics for the rest of the language" (Hale, Wright 2006: 1), it will clash with a reasonable epistemology.

What Benacerraf takes to be "a coherent semantic theory" is the classical correspondence theory of truth, best formulated in the works of Alfred Tarski. According to the Tarskian interpretation, propositions are true in virtue of their reference to the corresponding objects[7]. However just how should we think of abstract objects that propositions as those of mathematics are to refer to? It is generally taken that if there exist any abstract mathematical objects, they are ideal, outside space-time and thus causally inert. How then are we, physical beings existing entirely in space-time, supposed to merge any contact with these objects? Namely, if what guarantees the truth of mathematical propositions is utterly inaccessible to us, how can we say that we know any of these propositions?[8]

A mathematical Platonist then has to deal with a problem of explaining just how are we supposed to access and thus have knowledge of such mathematical objects. One way to account for this access has thus been by way of the application of mathematical intuition. What this strategy will involve is either the claim that (i) human beings have a special cognitive faculty that allows for the grasping of abstract mathematical objects and in this way puts the cognizer in direct contact with such objects (Cartesian, sensation independent intuition); or, somewhat more modestly, (ii) that intuition enables the recognition of concepts, however it does not put the cognizer in direct contact with the objects themselves (Kantian intuition).

It is generally taken that the first instance of the application of intuition to the access problem is found in the work of Kurt Gödel. Gödel claims that we gain knowledge of mathematical objects in much the same way as we gain knowledge of concrete objects, that is, by experiencing them in a certain way. This "experience of mathematical objects" is precisely what Gödel considers to be mathematical intuition. Just as we perceive and experience concrete objects as actually present and true, we intuit mathematical objects as both actually present and true. Gödel writes:

> But, despite their remoteness from sense-experience, we do have something like a perception of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less

[7] The semantic horn of the dilemma is formulated as follows: "(S) Any theory of mathematical truth [ought to] be in conformity with a general theory of truth <…> which certifies that the property of sentences that the account calls 'truth' is indeed truth" (Benacerraf 1973: 408).

[8] The epistemological horn of the dilemma is formulated as follows: "(E) A satisfactory account of mathematical truth <…> must fit into an over-all account of knowledge in a way that makes it intelligible how we have the mathematical knowledge that we have. An acceptable semantics of mathematics must fit and acceptable epistemology" (Benacerraf 1973: 409).

> confidence in this kind of perception, i.e., in mathematical intuition, than in sense-perception. (Gödel 1964: 271)

How does intuition have access to such objects remains a controversial aspect of Gödelian exegesis, though it is generally taken that Gödel considered our minds to be of the same nature as the abstract mathematical objects, namely immaterial. Thus the claim that mathematical objects are somehow epistemically suspicious simply rests on a faulty and overly naturalistic conception of the mind.

Gödel's view has been criticized by many, to the extent that it has now become more of a piece of philosophical mythology than a legitimate position. Among the many issues one may have with this particular use of intuition, the most prominent ones are the following. Firstly, Gödel's proposal is committed to an radical version of mind-body dualism. Since Gödel is defending a version of mathematical intuition where the contact with the objects in question is direct, he assumes that our minds are such that they have the ability to come into contact with entities of the Platonic realm, and thus themselves must be immaterial and located outside space-time. It seems extremely difficult to defend such a picture of the mind, and a very few Platonist are ready to pay such a great price for the answer of the access problem. The second problem is that even if one agreed with the dualism Gödel's view entails, his proposal does not seem to solve the access problem but rather postpone it. One may be fully justified in asking just how it is the case that our immaterial mind comes into contact with mathematical objects. Merely postulating the faculty of intuition does not seem to be helpful, for the lack of access problem still remains unanswered.

In the light of these and other criticisms, the contemporary use of mathematical intuition has taken a different turn. The new approach may be best characterized by claiming that "we possess a psychological apparatus whose only ultimate sources of information are the naturalistic sources of perception and introspection, but that nevertheless generates intuitive beliefs and thoughts about mathematical objects (or structures or patterns)" (Balaguer 2001: 37).

The most prominent defender of such a conception of mathematical intuition is Parsons, though similar accounts have been given by Jerold Katz and Mark Steiner. In addition to the latter, Stewart Shapiro and Michael Resnik have been proposing something along the lines of a naturalized intuition, even though they prefer the term 'abstraction' over 'intuition'. This naturalized version of intuition is generally considered to be taking inspiration from Kant. However in what follows I will show that in its definition and application it is closer to that of Husserl's. Insofar as Husserl anticipated an application of intuition, in the vein of Parsons, his theory should be of interest to contemporary philosophy, and both the continuities and the discontinuities between his and Parsons' views may be quite telling.[9]

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[9] It must however be mentioned that drawing parallels between Husserl and Parsons views on mathematical intuition has been done before – most notably by Richard Tieszen, who has argued that Husserl's notion of intuition can help Parsons account for the difficulties his theory encounters when faced with potential infinities (Tieszen 1984). However no work has been done, either by Tieszen, Parsons or others to investigate the relation that Parsons' view of intuition could bear to that of early Husserl's put forth in the *PA,* his only work devoted solely to philosophy of mathematics.

Parsons begins his article "Mathematical Intuition" with the claim that if mathematical intuition is to be relevant to philosophy of mathematics, "it should play a role like that of sense-perception in our knowledge in everyday world and physics" (Parsons 1980: 145). This way stating that the central feature of intuition is the analogy between sense perception as a cognitive relation to the physical world, and "something like a perception" giving a similar relation to mathematical objects (Parsons 1980: 145). Such intuition, Parsons is convinced, will be very limited in scope, in the sense that it will only cover the simplest cases of elementary geometry and arithmetic. Here I will focus mainly on his conception of intuition of elementary arithmetical concepts.

The manner in which Parsons presents the notion of intuition is akin to that of Husserl's, namely through providing a genetic analysis of the concept of number. However, while in *PA* the use of intuition was evoked as a necessary condition for arriving to the concept of number, in Parsons' work we may note the reverse – the conditions in which we grasp the concept of number are reconstructed as means to define and illuminate the use of mathematical intuition. However differing the motivations, in both of their works we may find something like an explication of a priori conditions for our knowledge of elementary mathematical concepts.

Contrary to Husserl, Parsons, while focusing mainly on epistemological issues, does not leave ontological considerations aside. At the very least, they are required to see what exactly does intuition provide access to – concrete objects, abstract or some sort of intermediary between the two. In relation to this question, a critical feature of Parsons' project is a three-level ontology, the constituents of which are purely concrete (physical level), quasi-abstract (conceptual level), and purely abstract objects (the objects in the Platonic Heaven). The objects of mathematical intuition are the quasi-concrete, while abstract objects are taken to be causally inert and thus not accessible by any of our epistemic faculties. This is why we cannot have intuitions of natural numbers, however we can have intuitions of quasi-abstract structures that represent the numbers.

A way to understand the distinction better is to see the quasi-concrete objects as a certain structure of concepts and concrete objects as instantiations of parts of that structure. A good illustrative example of this is David Hilbert's conception of finitary mathematics. Hilbert, similarly to Husserl and Parsons, asserts that there is a body of elementary mathematical objects that are known to us intuitively, namely, immediately. Furthermore, Hilbert insists that the existence of such primitive and intuitively known objects is necessary, for they are the underlying conditions of any sort of higher-order reasoning (see Hilbert 2002: 376).

Parsons, following Hilbert, considers such objects to be strings of strokes that he calls types, while the concrete instances of the types are tokens and are understood as objects given to us in concrete perception. What makes up a type is a special geometrical form composed of strokes. According to Parsons, purely abstract entities, such as numbers, are defined by types. Thus if I wanted to define elementary natural numbers in this language, I could say that 1 is defined by a type that has the form of |, 2 is a type that has the form of ||, 3 is a type that has the form of |||, etc. I can recursively generate greater and greater numbers by the process of repetition.

By perceiving a concrete token of the type (say two cats on the fence) the cognizer intuits its type (||). Therefore the role of intuition is, roughly put, to immediately connect some object of perception, a concrete token, with a concept of a type – to make the form of the type immediately clear (see Parsons 1980: 103).

Though Husserl discusses the role of intuition in the formation of the concept of number in much more detail than Parsons, paying careful attention to the processes that come parallel to intuition (as collective combination and alike), in both of their approaches intuition is defined in the same fashion. It is a cognitive process that is essential for the acquisition of elementary mathematical objects. In both of their uses of the term intuition puts the agent into a direct cognitive relationship with an intuited object or, in the case of Husserl, a property of a certain object.

Furthermore, Parsons' use of intuition is much closer to Husserl's than that of Kant's. While Kant uses intuition to designate both immediate cognition and a singular representation, the latter is much more prominent in his epistemology. We may note the reverse in Husserl and Parsons – intuition is understood almost exclusively as immediate cognition, and very rarely used to designate a singular representation.

For Kant intuition is a necessary condition for knowledge of any mathematical objects. Husserl and Parsons, on the other hand, apply it only to a very limited body of mathematical objects. Both Husserl and Parsons claim that it simply is implausible to assume that intuition will be applicable to objects that are too complex to be perceived. Therefore, Husserl evokes the notion of inauthentic concepts of number, namely those that are not directly experienced and thus not given by perceptual intuition. Parsons makes this point very clear with his criticism of Maddy's application of intuition to set theory (Parsons 2007: 167) where he states that set theory is too complex and relies too heavily on non-empirical observations to be an object of mathematical intuition.

All that said, my strive to see similarities between the views of Husserl and Parsons ought not be taken as a claim that Husserl and Parsons' use of intuition is identical. Parsons' evocation of the term is much more nuanced and does not commit itself to such counterintuitive claims as Husserl's insistence that the only true mathematical knowledge we have is that of elementary mathematical concepts and those that can be traced back to such concepts. In this respect, Husserl can be seen as advocating a crude and early version of causal theory of knowledge. The main assumption behind such a view is that in order for some belief to be considered knowledge it has to be properly caused by a truly existing object or state of affairs which the belief addresses. The only proper mathematical knowledge that we can have is that which is caused by the perceivable objects that our beliefs are about, such as a collection of countable items and alike.

Many contemporary epistemologists and philosophers of mathematics are skeptical towards this view (see Collier 1973: 350-352, among others). Among other reasons, it cannot account for a body *a priori* propositions that we plausibly consider to be knowledge. Parsons is thus cautious not to commit himself to such views, stating explicitly that his evocation of intuition does not imply an account for all mathematical knowledge (Parsons 2007: 152).

However, setting this difference aside, I think it is plausible to assume that Husserl nevertheless anticipated the conception and application of mathematical intuition, in the vein of Parsons. It is not in Kant's or Gödel's but in Husserl's early work that we find the first explicit articulation of the relevance of mathematical intuition to the knowledge of elementary mathematical concepts in a sufficiently similar way to the contemporary use of the term.

4. Conclusion and directions for further research

In addressing Husserl's notion of mathematical intuition, I hope to have shown that Husserl anticipated one of the contemporary definitions and uses of the concept. However this need not be the end of the story. One may agree that Husserl indeed anticipates the notion of mathematical intuition, as it is used today, however worry whether there is anything more to this than a curious historical observation. Thus further directions of research will involve seeing whether and how Husserl's use of mathematical intuition could be plausibly applied to contemporary debates in the epistemology of mathematics.

It is often claimed that Husserl's main philosophical objective throughout his career was to unite two competing assumptions about knowledge: (i) knowledge involves ideal objects; (ii) knowledge can only arise from experience (among others see Bell 1990: 23, De Boer 1978: 12, Miller 1982: 89–100). If this is the case we may interpret Husserl as attempting to provide an answer to something resembling the epistemic horn of Benacerraf's dilemma, and his use of mathematical intuition may be related to the contemporary discourse on these grounds.

Though can we plausibly infer that this was Husserl's goal already by the time of writing *PA*? To see this, among other things, we will need to begin with the question of Husserl's ontological commitments in his early period. This is a burdensome task, given that nowhere does Husserl fully state his stance on the issue; one rather has to reconstruct it from controversial hints found all throughout the book. The essential question here is the following: did Husserl consider numbers to be ideal? If yes, what kind of realism does he advocate? Could he be plausibly considered to be a full-blown Platonist already by the time of writing *PA*, or should we see him as someone taking a critical realist stance in the vein of Kant and Brentano?

Answering these questions will be an important further step in the analysis of Husserl's early philosophy of mathematics. However, regardless of the result of such investigations, the fact remains: the relative neglect of Husserl's early work is, by and large, unwarranted.

BIBLIOGRAPHY

Balaguer, M. 2001. *Platonism and Anti-Platonism in Mathematics*. New York: Oxford University Press.

Bell, A. 1990. *Husserl,* London: Routledge.

Benacerraf, P. 1973. Mathematical Truth. *The Journal of Philosophy,* 70: 661–679.

Collier, K. W. 1973. Contra the Causal Theory of Knowing. *Philosophical Studies: An International Journal in Philosophy in the Analytic Tradition* 24: 350–352.

De Boer, T. 1978. *The Developments of Husserl's Thought.* Trans. T. Plantinga. The Hague: Martinus Nijhoff.

Frege, G. 1970. Review of Dr. Husserl's Philosophy of Arithmetic. In: J. N. Mohanty, *Edmund Husserl's Logical Investigations*, The Hague: Martinus Nijhoff.

Gödel, K. 1964. What Is Cantor's Continuum Problem? In: P. Benacerraf and H. Putnam, *Philosophy of Mathematics: Selected Readings,* Englewood Hills, N.J.: Prentice-Hall.

Hale, B. and Wright, C. 2002. Benacerraf's Dilemma Revisited. *European Journal of Philosophy* 10: 101–129.

Hilbert, D. 2002. Über das Unendliche. In: J. Heijenroot, *From Frege to Gödel. A Source Book in Mathematical Logic, 1897–1931,* Cambridge: Harvard University Press.

Husserl, E. 2002. *Logical Investigations.* Vol. 1. Trans. J. Findlay. London: Routledge.

Husserl, E. 2003. *Philosophy of Arithmetic*. Dordrecht: Kluwer Academic Publishers.

Parsons, C. 1980. Mathematical Intuition. *Proceedings of the Aristotelian Society,* 80: 145–168.

Parsons, C. 2007. *Intuition in Mathematical Thought and Its Objects*. New York: Cambridge University Press.

Miller, P. J. 1982. *Numbers in Presence and Absence – A Study of Husserl's Philosophy of Mathematics*. The Hague: Martinus Nijhoff.

Stumpf, C. 1873. *Über den Psychologischen Ursprung der Raumvorstellung,* Leipzig: Hirzel.

Tieszen, R. 1994. The Philosophy of Arithmetic: Frege and Husserl. In: L. Happaranta, *Mind, Meaning and Mathematics,* Dordrecht: Kluwer.

Willard, D. 2003. Translator's Introduction. In: E. Husserl, *Philosophy of Arithmetic*, Dordrecht: Kluwer Academic Publishers.

KĄ GALIME IŠ ANKSTYVOJO HUSSERLIO SUŽINOTI APIE MATEMATINĘ INTUICIJĄ?

Rasa Davidavičiūtė

Santrauka. Pagrindinis straipnsnio tikslas yra pademonstruoti, jog ankstyvojo Edmundo Husserlio matematikos filosofijos visiškas atmetimas buvo nepagrįstas. Tiksliau, įrodinėdama, kad jau ankstyvasis Husserlis numato matematinės intuicijos šiuolakinius apibrėžimus ir taikymus. Šiam tikslui pirmiausia pateiksiu intuicijos vaidmens ankstyvojoje Husserlio matematikos filosofijoje analizę, o tada parodysiu, kad ji pasižymi esminiais panašumais su intuicijos apibrėžimu ir taikymu bene ryškiausio iš šiuolaikinių matematinės intuicijos šalininkų Charleso Parsonso darbuose.

Pagrindiniai žodžiai: Edmundas Husserlis, Charlesas Parsonsas, matematinė intuicija, skaičiaus sąvoka.

Įteikta 2014 m. lapkričio 10 d.