THE ANALYSIS OF THE SECOND PILLAR PENSION FUNDS AND THE ROLE OF EXPECTATIONS

Algirdas Bartkus*
Vilnius University

Abstract. *This paper continues the analysis of the second pillar pension funds and is based on the results that were published in June 2013 in the journal "Organizations and Markets in Emerging Economies", under the title "On Future Pensions from the Second Pillar Pension Funds". The results of the previously published study that one needs to keep in mind for the full perception of the material are also presented here. The main result of this paper is interpretation and exhaustive quantitative analysis of cointegrating relationship among social insurance contributions transferred into the second pillar pension funds and assets value of these funds. More specifically, this paper explains what is represented by the equilibrium error and submits a mathematical model under rational expectations with detailed comments.*

Key words: *pension funds, PAYG pensions, error correction model, cointegration.*

1. Introduction

The goal of the study is twofold: to investigate the possibility of the second pillar pensioners to offset the losses from PAYG pensions with surplus funded pension and to define the meaning of cointegrating relationship between the assets and the transfers. The second pillar pension funds that operate in Lithuania should not be confused with the general notion of pension funds that prevails in most of the world. They do not act as a provider of the main income for the elderly, neither are they a source of supplementary income. They provide the replacement for the part of the PAYG pension. If a person decides to participate in these funds, the State Social Insurance Fund Board of Lithuania transfers the portion of PAYG contributions into individual account of the participant. The person who participates in these funds bears not only the risks that are associated with them, but also resigns from the corresponding part of PAYG pension, which is directly proportional to the part of contributions that were

* Corresponding author: Department of Quantitative Methods and Modelling, Faculty of Economics, Vilnius University, Saulėtekio ave. 9 II, room 813, LT-10222, Vilnius, Lithuania, phone: +37052366277, e-mail: algirdas.bartkus@ef.vu.lt

transferred into pension funds and to the length of his or her participation. Lithuanian pension funds are classified into four groups according to their risk-level. There are funds that invest mainly in stocks (70-100% of assets), funds with an average share of stocks (30-70% of assets), funds with a small share of stocks (up to 30% of assets) and there are conservative funds that do not invest in stocks. In this study I have assigned arbitrary names for them: risky, balanced, moderate and conservative, respectively.

Literature review is not presented here as this paper aims to clarify the interpretation of the error correction model that was already estimated and presented in the previous paper on the same subject. The previous paper included a summary of other authors' research. References here are exactly the references from the previous paper.

2. Descriptive analysis of data

Net value of pension fund assets per participant and the transfers from the social insurance agency to a particular pension fund per participant is the main data. Sometimes the key insights may be gathered from a simple descriptive data analysis. The plots below display cumulative transfers and value of pension funds assets for four types of pension funds and one plot for the whole pension funds system. Grey shades indicate the periods when the value of assets was higher than that of cumulative transfers.

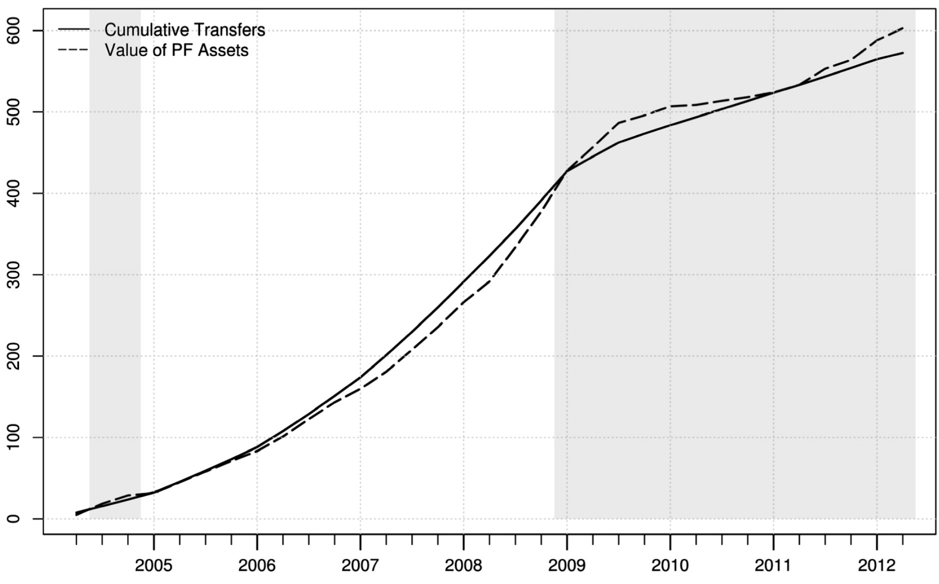

FIG. 1. **Cumulative transfers into conservative funds and value of assets in conservative funds (M Litas)**

Source: data provided by the State Social Insurance Fund Board and the Central Bank of the Republic of Lithuania

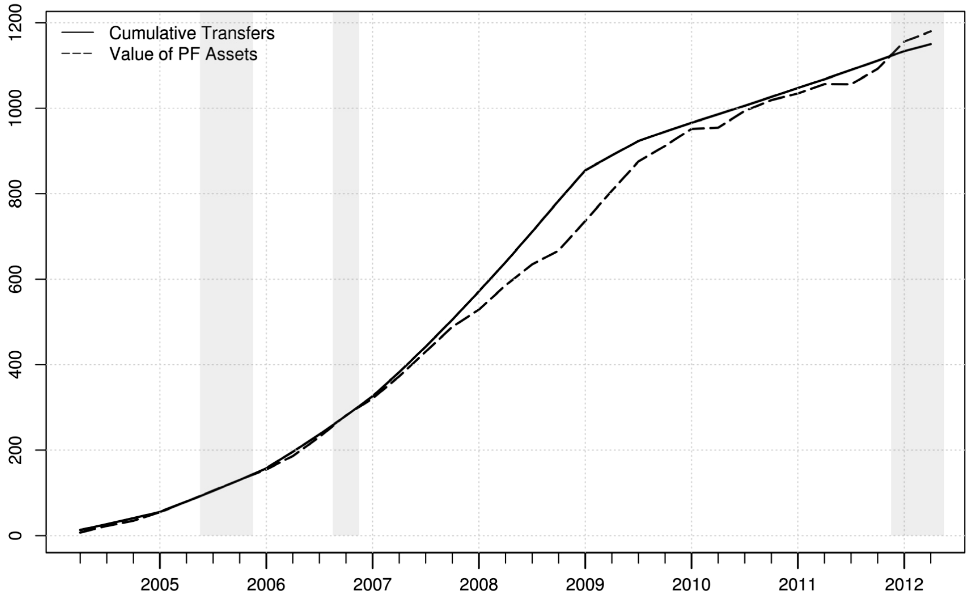

FIG. 2. **Cumulative transfers into moderate funds and value of assets in moderate funds (M Litas)**

Source: data provided by the State Social Insurance Fund Board and the Central Bank of the Republic of Lithuania

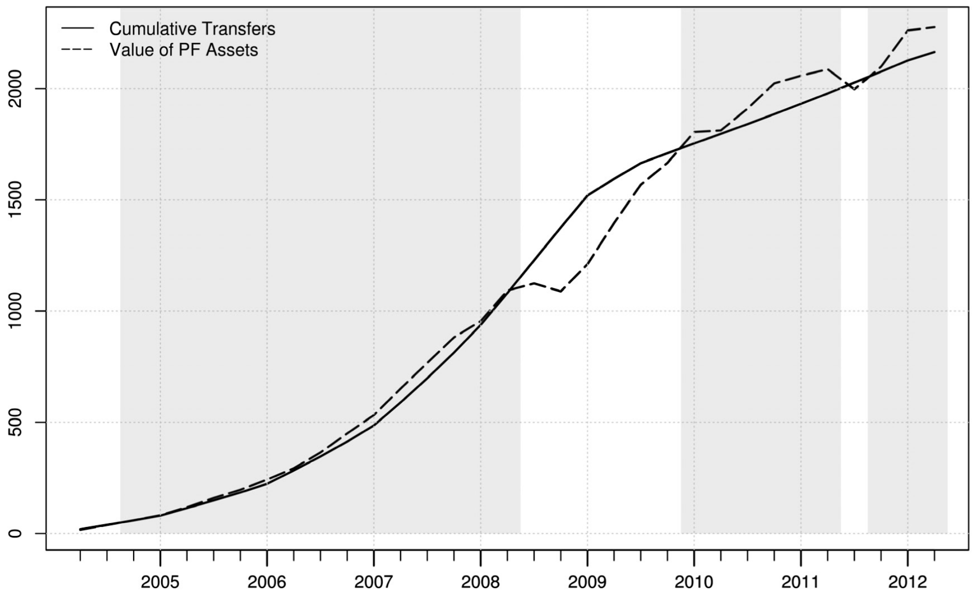

FIG. 3. **Cumulative transfers into balanced funds and value of assets in balanced funds (M Litas)**

Source: data provided by the State Social Insurance Fund Board and the Central Bank of the Republic of Lithuania

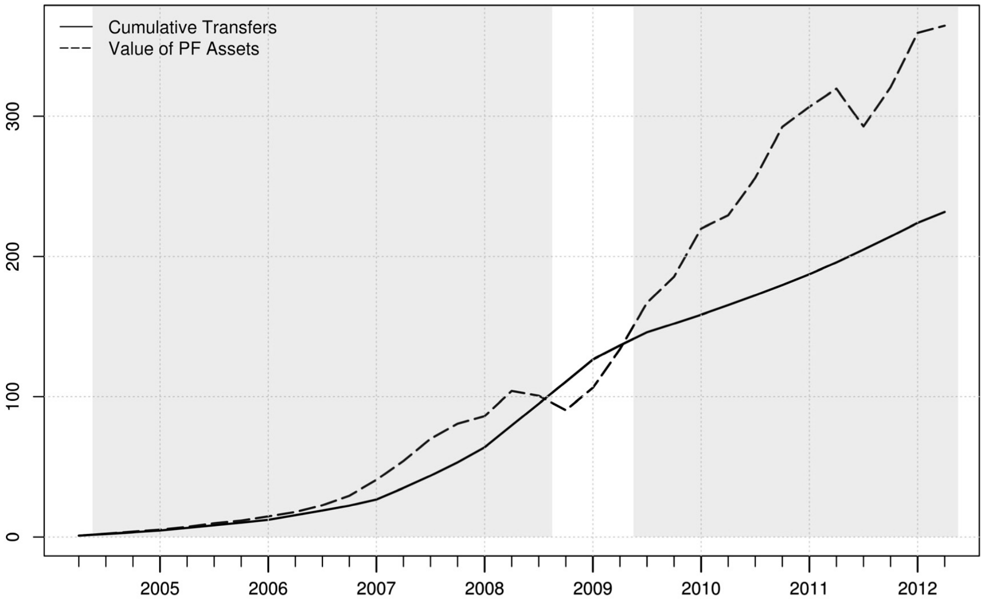

FIG. 4. **Cumulative transfers into risky funds and value of assets in risky funds (M Litas)**

Source: data provided by the State Social Insurance Fund Board and the Central Bank of the Republic of Lithuania

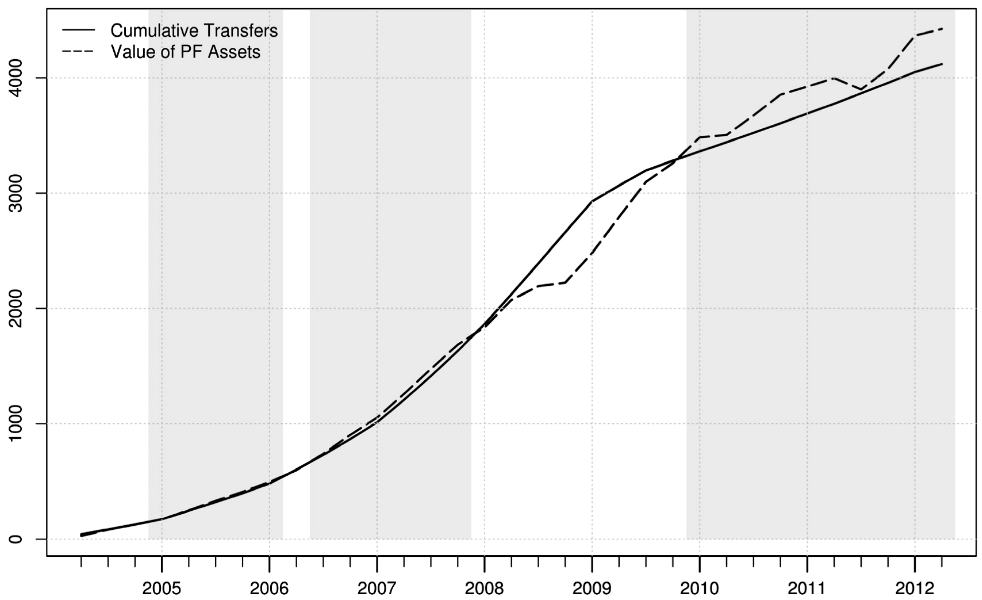

FIG. 5. **Cumulative transfers into all funds and value of assets in all funds (M Litas)**

Source: data provided by the State Social Insurance Fund Board and the Central Bank of the Republic of Lithuania

Simple descriptive analysis reveals that the value of assets did not differ very much from the cumulative transfers with the exception for risky funds – these funds experienced a very high growth starting with the 3rd quarter of 2009. The second very important insight comes from the observation that the external shock associated with financial crisis hit harder those funds which invested large portions of capital into stocks and other risky assets. Visual inspection of data allows us to form one proposition, which will be refined in the forthcoming pages, and to raise one question. After the initial inspection of data you can suspect that if cumulative transfers are I(2) variable, then the value of assets should also be I(2) variable. Is this true? But what is much more important, can we carry out the trends observed in the series in the forthcoming periods?

3. Model for the pension funds assets

The evolution of assets in the previous study was represented by the following set of equations:

$$
\begin{aligned}
A_2 &= \delta \exp(\varepsilon_1)\tau r_2 \\
A_3 &= \delta^2 \exp\left(\sum_{i=1}^{2}\varepsilon_i\right)\tau r_3 \left[1+\frac{r_2}{\delta \exp(\varepsilon_2)}\right] \\
A_4 &= \delta^3 \exp\left(\sum_{i=1}^{3}\varepsilon_i\right)\tau r_4 \left[1+\frac{r_3}{\delta \exp(\varepsilon_3)}+\frac{\prod_{i=2}^{3} r_i}{\delta^2 \exp\left(\sum_{i=2}^{3}\varepsilon_i\right)}\right] \\
&\vdots \\
A_t &= \delta^{t-1} \exp\left(\sum_{i=1}^{t-1}\varepsilon_i\right)\tau r_t \left[1+\frac{r_{t-1}}{\delta \exp(\varepsilon_{t-1})}+\ldots+\frac{\prod_{i=2}^{t-1} r_i}{\delta^{t-2} \exp\left(\sum_{i=2}^{t-1}\varepsilon_i\right)}\right]
\end{aligned}
\tag{1}
$$

A_t is the level of pension funds assets, δ is the long run growth rate of wages, ε_t denote permanent innovations in the stochastic trend for the wages. r_s stand for the growth rate of pension funds assets defined as $(1 + R_t) = r_t$, while τ is contributions rate for pension funds. The product of the first two factors is in fact the level of wages. Taking logs of (1), denoting the log of assets as a_t, the log of returns as ρ_t, the log of contributions rate for pension funds transfers as τ^* and the log of cumulative growth factor (the expression in square brackets in (1)) as κ_t, we get:

$$
\begin{aligned}
a_2 &= w_1 + \tau^* + \rho_2 \\
a_3 &= w_2 + \tau^* + \rho_3 + \kappa_3 \\
&\vdots \\
a_t &= w_{t-1} + \tau^* + \rho_t + \kappa_t
\end{aligned}
\tag{2}
$$

If we assume that growth rate of assets does not differ from the predictable growth rate of wages $\delta = r$ and all changes in wages are completely predictable, so that all random shocks are set to zero $\varepsilon_t = 0$, we obtain:

$$\begin{aligned} A_2 &= \delta^2 \tau \\ A_3 &= 2\delta^3 \tau \\ A_4 &= 3\delta^4 \tau \\ &\vdots \\ A_t &= (t-1)\delta^t \tau \end{aligned} \tag{3}$$

In order to obtain a more estimable form of a process we may switch to the log form, so that system of equations (3) becomes:

$$\begin{aligned} a_2 &= 2d + \tau^* \\ a_3 &= 3d + \tau^* + \ln(2) \\ a_4 &= 4d + \tau^* + \ln(3) \\ &\vdots \\ a_t &= td + \tau^* + \ln(t-1) \end{aligned} \tag{4}$$

Obviously, under the above mentioned restrictions, there is a logarithmic trend in data. The following two plots display log of assets in conservative and risky funds, side by side with logarithmic trend.

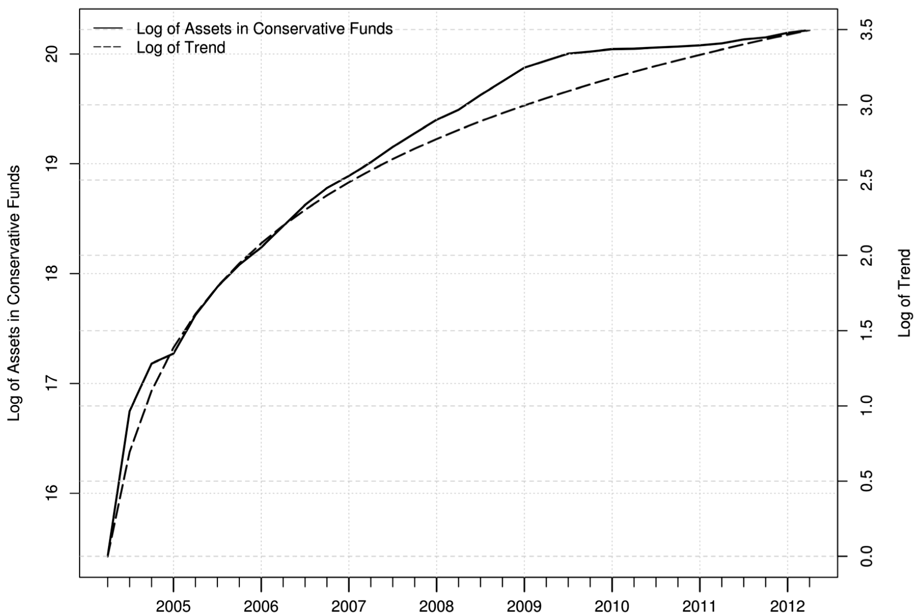

FIG. 6. **Log of assets in conservative funds**

Source: data provided by the State Social Insurance Fund Board of Lithuania

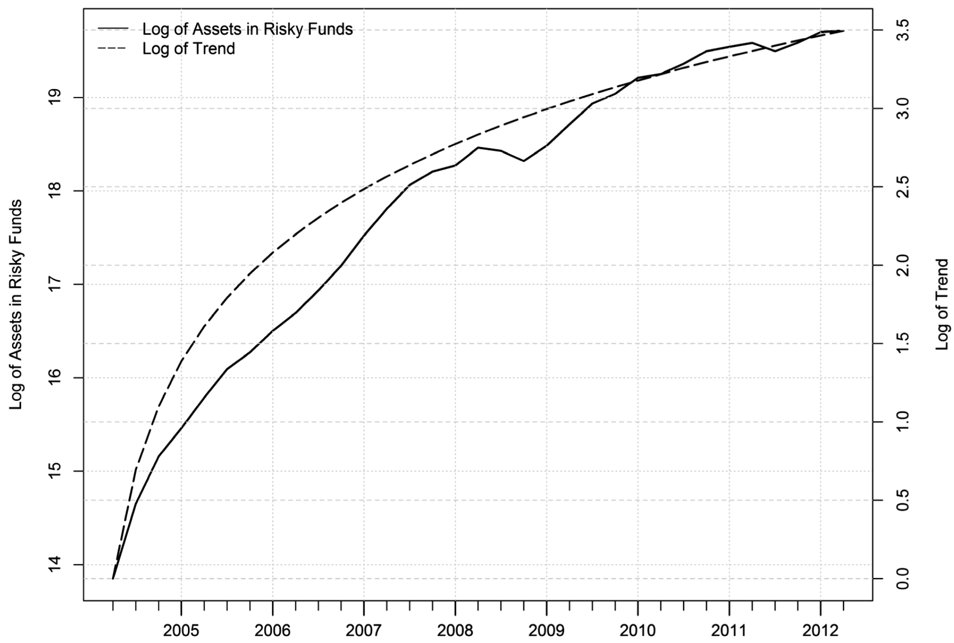

FIG. 7. **Log of assets in risky funds**

Source: data provided by the State Social Insurance Fund Board of Lithuania

It is important to know that the logarithmic trend is present only under the restriction of perfect predictability of wages. At first sight one can get an impression that the value of assets in conservative funds can be well approximated by logarithmic trend. The approximation for risky funds is far worse. The question of fundamental importance is how we should treat the changes in the levels of assets. The deterministic treatment may be erroneous if stochastic component dominates over deterministic and vice versa. Eyeballing the plots suggests that deterministic approximation is worse for funds that invest large portions of assets in risky securities. The logarithmic trends for moderate and balanced funds are not depicted as it is clear that approximations with logarithmic trend would be worse compared with conservative funds and better in comparison to risky funds.

4. Solution

In the previous paper cointegration was chosen as the main analytical tool. It is very important to grasp that if the linear combination of nonstationary variables is random walk, variables are not cointegrated. Stationary linear combination of nonstationary variables points out the variables that are cointegrated. There are two types of stationary processes, predictable and unpredictable, and the discrimination between them is crucial. Predicable processes may follow a pattern suggested by a broad class of ARMA models while unpredictable ones resemble a noise. Predictability or unpredictability in this discussion should be seen as a result of properly placed restrictions on the variables of interest. Filtering (described in detail in the previous paper) removed several distortion effects, but the slope of deterministic trend was left as it was. This was done deliberately. One cannot subjectively or without any theoretical advice treat certain patterns as deterministic or stochastic. If a researcher included the log of trend as a regressor in the filtering equations, the filtering would return stationary series. And we would never know whether this was the right treatment of the growth pattern, which was observed in the data. This would imply that the researcher treats a trend as a deterministic without knowing whether it is deterministic or stochastic.

If the order of integration for assets and transfers is the same, purely from the technical point of view, they can be cointegrated. The equilibrium error in this case has a completely different meaning. In order to characterize it, let us divide both sides of the last equation in (1) by FT_{t-1}:

$$\begin{aligned} A_t / FT_{t-1} = r_t[1 + r_{t-1}\delta^{-1}\exp(\varepsilon_{t-1})^{-1} + r_{t-1}r_{t-2}\delta^{-2}\exp(\varepsilon_{t-1}+\varepsilon_{t-2})^{-1} + \ldots \\ + r_{t-1}r_{t-2}\cdots r_2\delta^{-(t-2)}\exp(\varepsilon_{t-1}+\varepsilon_{t-2}+\ldots+\varepsilon_2)^{-1}] \end{aligned} \tag{5}$$

Equation (5) does not necessary mean that equilibrium error will take exactly the form A_t / FT_{t-1}. The equilibrium may also be defined as $A_t / FT_{t-1}^{\beta_{12}}$ or in a log form $a_t - \beta_{12} ft_{t-1}$:

$$\begin{aligned} A_t FT_{t-1}^{-\beta_{12}} = FT_{t-1}^{1-\beta_{12}} r_t[1 + r_{t-1}\delta^{-1}\exp(\varepsilon_{t-1})^{-1} + r_{t-1}r_{t-2}\delta^{-2}\exp(\varepsilon_{t-1}+\varepsilon_{t-2})^{-1} + \ldots \\ + r_{t-1}r_{t-2}\cdots r_2\delta^{-(t-2)}\exp(\varepsilon_{t-1}+\varepsilon_{t-2}+\ldots+\varepsilon_2)^{-1}] \end{aligned} \tag{6}$$

Logarithmic transformation of (6) yields:

$$a_t - \beta_{12} ft_{t-1} = (1-\beta_{12}) ft_{t-1} + \rho_t + \kappa_t \tag{7}$$

Lower-case letters denote logs of variables, ρ_t is a logarithmic rate of return and κ_t is a log of cumulative growth factor (the expression in square brackets in (6)). Coefficient β_{12} has no clear economic meaning with the exception that it shows proportionality between assets and funds transferred. If a coefficient β_{12}, such that the right hand side of (7) is a white noise, exists, the variables should be treated as cointegrated. If β_{12} is positive and larger than 1, a significant portion of assets can be predicted. If β_{12} is less than 1 or negative, the whole increment in the value (right hand side of equation) should be considered as unpredictable and stationary. If this is true, the empirical investigation is necessary for the full conviction about the way things really are. The magnitude, sign and significance of the cointegrating vector parameter will determine whether the initial guess is right or wrong.

Macroeconomic theory often underlines and emphasizes the crucial role of expectations in economy. The investment decisions are not the exception. Rational expectations assume that the prediction error v_t is the difference between the actual value of variable Y_t and the expectation for Y_t, which is formed with the best possible utilization of information available at period $t-1$, denoted by $Y_t^*: v_t = Y_t - Y_t^*$.

For simplicity, let us assume that agents are making decisions how to allocate assets and they are considering one of two alternatives: investment in risk-free securities with defined returns, or risky securities. If the transfers are invested in assets with defined returns, the expectations are not formed, as the actual values are known in advance. This applies to all low-risk investments such as government bonds or deposits in top ranked banks. If, on the contrary, transfers are invested in corporate stocks or any other asset with undefined return, i.e., the return that is not known in advance, the expectation of the return is the main factor that determines the investment decision. If we compare the decisions to invest in risky assets and risk-free assets, we see that expectations play a major role in risky investments and they are of little or no importance, when it comes to risk-free assets.

Rational expectations hypothesis for the rate of return is $r_t = r_t^* + v_t$. Priority for rational expectations over adaptive and naïve expectations is given especially for this core assumption, which says that the actual rate of return from the predicted return will differ by the random error. This may be true as Lithuanian assets managers are highly professional and employ all available methods for the allocation of assets. The resulting error may be treated as random and not related to the decisions made by the manager. Adaptive expectations imply a certain fixed adjustment mechanism that can be justified only if managers use the same tools for the allocation of assets and do not change their learning habits. Substituting expression $r_t = r_t^* + v_t$ with equation (5), we obtain a set of equations that allow us to examine the various impacts of expectations on dynamic adjustment of assets:

$$\begin{aligned} A_t / FT_{t-1} = \; & (r_t^* + v_t)[1 + (r_{t-1}^* + v_{t-1})\delta^{-1} \exp(\varepsilon_{t-1})^{-1} \\ & + (r_{t-1}^* + v_{t-1})(r_{t-2}^* + v_{t-2})\delta^{-2} \exp(\varepsilon_{t-1} + \varepsilon_{t-2})^{-1} + \ldots \\ & + (r_{t-1}^* + v_{t-1})(r_{t-2}^* + v_{t-2}) \cdots (r_2^* + v_2)\delta^{-(t-2)} \exp(\varepsilon_{t-1} + \varepsilon_{t-2} + \ldots + \varepsilon_2)^{-1}] \end{aligned} \tag{8}$$

Although equation (8) represents the impact of expectations and prediction errors on the assets, it has one undesirable feature – the factors $\exp(\varepsilon_{t-1})^{-1}$, $\exp(\varepsilon_{t-1} + \varepsilon_{t-2})^{-1}$ and up to $\exp(\varepsilon_{t-1} + \varepsilon_{t-2} + \ldots + \varepsilon_2)^{-1}$ are nonlinear. One way to proceed further is to use proper approximation for this functional form. Plausible approximation using terms no higher than order three is:

$$\begin{aligned}\exp(\varepsilon_{t-1})^{-1} &= \alpha_0^* + \alpha_1^*\varepsilon_{t-1} + \alpha_2^*\varepsilon_{t-1}^2 + \alpha_3^*\varepsilon_{t-1}^3 \\ \exp(\varepsilon_{t-1}+\varepsilon_{t-2})^{-1} &= \beta_0^* + \beta_1^*(\varepsilon_{t-1}+\varepsilon_{t-2}) + \beta_2^*(\varepsilon_{t-1}+\varepsilon_{t-2})^2 + \beta_3^*(\varepsilon_{t-1}+\varepsilon_{t-2})^3 \\ \exp(\varepsilon_{t-1}+\varepsilon_{t-2}+\ldots+\varepsilon_2)^{-1} &= \omega_0^* + \omega_1^*(\sum_{i=2}^{t-1}\varepsilon_i) + \omega_2^*(\sum_{i=2}^{t-1}\varepsilon_i)^2 + \omega_3^*(\sum_{i=2}^{t-1}\varepsilon_i)^3\end{aligned} \qquad (9)$$

Substituting expressions in (8) with the expressions from (9), we obtain the model in the final form:

$$\begin{aligned}A_t / FT_{t-1} = (r_t^* + \nu_t)\{1 + (r_{t-1}^* + \nu_{t-1})\delta^{-1}[\alpha_0^* + \alpha_1^*\varepsilon_{t-1} + \alpha_2^*\varepsilon_{t-1}^2 + \alpha_3^*\varepsilon_{t-1}^3] \\ + (r_{t-1}^* + \nu_{t-1})(r_{t-2}^* + \nu_{t-2})\delta^{-2} \\ [\beta_0^* + \beta_1^*(\varepsilon_{t-1}+\varepsilon_{t-2}) + \beta_2^*(\varepsilon_{t-1}+\varepsilon_{t-2})^2 + \beta_3^*(\varepsilon_{t-1}+\varepsilon_{t-2})^3] + \ldots \\ + (r_{t-1}^* + \nu_{t-1})(r_{t-2}^* + \nu_{t-2})\cdots(r_2^* + \nu_2)\delta^{-(t-2)} \\ [\omega_0^* + \omega_1^*(\sum_{i=2}^{t-1}\varepsilon_i) + \omega_2^*(\sum_{i=2}^{t-1}\varepsilon_i)^2 + \omega_3^*(\sum_{i=2}^{t-1}\varepsilon_i)^3]\}\end{aligned} \qquad (10)$$

The less predictable ratio A_t / FT_{t-1} will be, the higher variance we will observe. The variance of ratio is defined as usual:

$$\mathrm{var}\left(\frac{A_t}{FT_{t-1}}\right) = E\left\{\left[\frac{A_t}{FT_{t-1}} - E\left(\frac{A_t}{FT_{t-1}}\right)\right]^2\right\}$$

The higher the variance, the worse predictions we will get. The less predictable assets will be, the less stability and certainty we will observe in assets and the less clarity we will have on the future pensions from pension funds.

The differences in variances for risk-free and risky investments may come only from the expectation of A_t / FT_{t-1}. Taking expectations of (10) we obtain a series of terms whose expectations under certain conditions are not equal to zero:

$$\begin{aligned}E(\delta^{-1}r_t^*\nu_{t-1}\alpha_1^*\varepsilon_{t-1}) &= \delta^{-1}E(r_t^*)\alpha_1^*E(\nu_{t-1}\varepsilon_{t-1}) \\ E(\delta^{-2}r_t^*r_{t-1}^*\nu_{t-2}\beta_1^*\varepsilon_{t-2}) &= \delta^{-2}E(r_t^*r_{t-1}^*)\beta_1^*E(\nu_{t-2}\varepsilon_{t-2}) \\ E(\delta^{-2}r_t^*r_{t-2}^*\nu_{t-1}\beta_1^*\varepsilon_{t-1}) &= \delta^{-2}E(r_t^*r_{t-2}^*)\beta_1^*E(\nu_{t-1}\varepsilon_{t-1}) \\ &\vdots \\ E(\delta^{-(t-2)}r_t^*r_{t-2}^*r_{t-3}^*\cdots r_2^*\nu_{t-1}\omega_1^*\varepsilon_{t-1}) &= \delta^{-(t-2)}E(r_t^*r_{t-2}^*r_{t-3}^*\cdots r_2^*)\omega_1^*E(\nu_{t-1}\varepsilon_{t-1}) \\ E(\delta^{-(t-2)}r_t^*r_{t-1}^*r_{t-3}^*\cdots r_2^*\nu_{t-2}\omega_1^*\varepsilon_{t-2}) &= \delta^{-(t-2)}E(r_t^*r_{t-1}^*r_{t-3}^*\cdots r_2^*)\omega_1^*E(\nu_{t-2}\varepsilon_{t-2}) \\ &\vdots \\ E(\delta^{-(t-2)}r_t^*r_{t-1}^*r_{t-2}^*\cdots r_3^*\nu_2\omega_1^*\varepsilon_2) &= \delta^{-(t-2)}E(r_t^*r_{t-1}^*r_{t-2}^*\cdots r_3^*)\omega_1^*E(\nu_2\varepsilon_2)\end{aligned} \qquad (11)$$

The variances of (10) when one invests in risk-free and risky assets differ by the presence of these terms in $E(A_t / FT_{t-1})$. Due to the fact that coefficients α_1^*, β_1^* and up to ω_1^* are negative, variance is lower when covariance between the expectation errors and unpredicted changes in wages is negative and, conversely, when covariance is positive, the variance of A_t / FT_{t-1} is higher.

The returns on assets usually go up when there are indications of positive changes in economy or in separate sectors, e.g., in labour market. Most statistical and econometric tools project these changes into the forthcoming periods. The usage of these methods shapes up the positive correlation between the predicted returns and certain labour market variables, e.g., predicted wages. In the periods when agents expect the growth of wages and the wages grow faster than expected, positive unexpected changes in wages call out, summon positive expectation errors of returns. The unexpected changes in wages can be attributed to many factors, among which external exogenous shocks play a significant role. If exogenous negative shocks hit wages, wages decrease and the returns in turn are less than expected. Summarizing, the expectation errors of returns and unexpected changes in wages are positively correlated. It is necessary to mention one important nuance: to a certain extent, wages display a downward rigidity, the returns – volatility. Very frequent changes and swings in preferences, and dependency on a very large number of factors contributes to the volatility of returns. Beside unexpected changes in nominal wages, there are plenty of other factors that are related to expectation errors. Nevertheless, under rational expectations hypothesis, these errors are related and occur as described above.

If transfers are invested in risk-free assets whose returns are defined, no expectation is formed and all terms in (11) are equal to zero. This is the main difference in the risk-free and risky allocations of transfers.

5. Cointegration analysis revised

The Johansen methodology was used for cointegration analysis. The model assumes VAR(1) process for levels:

$$\Delta y_t = \alpha\beta' \, y_{t-1} + e_t \tag{12}$$

Vector y_t contains estimates of filtered transfers $\hat{z}_t$ and filtered assets $\hat{v}_t$. More information on filtering can be found in the previously published study. VAR(1) specification is sufficient from statistical point of view: the errors are not autocorrelated, more or less normally distributed, but it raises certain awareness from the economic point of view. VAR(1) specification implies ECM with nil lags, which in turn implies that at least growth rates of wages are not autocorrelated, which is not true. Of course, if the equilibrium errors were large and the largest portions of movements in variables were induced by the disequilibrium, this specification could be justified.

TABLE 4. **Estimates of speed of adjustment coefficients with *t* statistics in the parenthesis**

	$\Delta\hat{z}_t$ equation		$\Delta\hat{v}_t$ equation	
	Coefficient	***t* statistic**	**Coefficient**	***t* statistic**
Conservative	0.0299	3.6463	0.0687	(1.2074)
Moderate	-0.0253	(-2.3211)	-0.2334	(-3.1456)
Balanced	-0.0260	(-1.9549)	-0.3146	(-3.4085)
Risky	-0.0624	(-2.6000)	-0.2560	(-1.8234)

Source: calculated by the author

In VAR(1) model, almost all coefficients of α vector are statistically significant under conventional levels, with the exception of a coefficient from $\Delta\hat{v}_t$ equation. In moderate, balanced and risky funds, both variables (net value of assets and transfers) respond to the disequilibrium. The absolute magnitude of the coefficient indicates that the impact of assets is more noticeable than impact of transfers. As it can be seen from (7), disequilibrium may be induced by the unexpected changes in the rate of return ρ or may be rolled over from the past and reflected through the cumulative growth factor κ_t. Of course it is not unexpected that disequilibrium which originates from the surprises in the rate of return is eliminated by the changes in the net value of assets, as the net value of assets is directly the result of ρ. At first sight, what is unusual about the results is that transfers also contribute to the error correction. Several arguments for such behaviour were laid down while discussing the role expectations. The value of assets is formed and shaped by rates of returns and the wages. Transfers are the result of wages. All errors in variables have direct effects on the level of assets and wages. If expectation errors for returns are positively correlated with unexpected changes in wages, the assets and wages should move in the same directions, in order to eliminate disequilibrium. Speed of adjustment coefficients confirm this, as the signs from different equations are identical.

Despite significance of coefficients, one should be very cautions treating the results as evidence for causal relationships among the variables. Wages are causal to assets, but assets are not causal to wages. The significance of coefficients is the result of cross-correlation among the errors. If the funds invested mainly in Lithuania, one could easily suspect that dynamics of assets and wages would be very similar. If they diversified their investments, the dynamics would be much less similar. But as we see, this is not so – the investments in moderate, balanced and risky funds are diversified, but nevertheless the speed of adjustment coefficients for these funds are significant in both equations and have identical signs. This means that funds that invest significant portion of transfers in risky assets do not offer a protection from the swings in labour market. This may be true, as the majority of developed countries are interconnected and the external shocks from larger economies are easily transmitted into small economies. A different explanation is also possible. Higher returns usually denote positive shocks or the expectations of positive shocks and they may also have positive effects on the main sources of contributions: on wages and employment. That is one of the reasons why the responses of the variables have the same directions.

The adjustment process for conservative funds is completely different. The speed of adjustment coefficient is significant for transfers. This means that disequilibrium is eliminated by the changes of transfers and not by the changes of the net value of assets. This may arise if the changes in the net value of assets do not depend on wage or employment shocks, or respond to these shocks with a substantial lag. In conservative funds large amounts of assets are held in risk-free or low-risk securities. The errors may arise only from the portion of investments with undefined returns. For investments with defined returns no expectations are formed, as the returns are known in advance, though there are no cross-correlations among the errors. Disequilibrium arises from the labour market conditions and is eliminated by the changes of labour market variables, including wages. This also means that investments in risk-free assets offer a reliable protection from swings in labour market.

Cointegrating vectors are normalized with respect to $\hat{v}_t$, so that error correction term is:

$$\beta' y_{t-1} = \begin{bmatrix} 1 & -\beta_{12} \end{bmatrix} \begin{bmatrix} \hat{v}_{t-1} \\ \hat{z}_{t-1} \end{bmatrix} \tag{13}$$

TABLE 5. **Estimates of cointegrating vectors**

	Estimate	***t* value**
Conservative	-24.0165	-4.1367
Moderate	11.1842	2.3274
Balanced	10.1255	2.5708
Risky	6.7663	3.4399

Source: calculated by the author

Parameters of cointegrating vectors show proportionality between the assets and funds transferred. The estimates of β_{12} are significant and negative for moderate, balanced and risky funds. This means that the whole increment in value of assets for these funds should be considered as unpredictable and stationary. This in turn means that pensions assets from these funds are unstable and their value may diminish or increase unexpectedly. The pensioners with identical working history may receive pensions that differ significantly if they leave labour market at different periods. The dependency of pensions on time period when a person leaves the labour market is the undesirable attribute of these funds.

Conservative funds tell a completely different story – the estimate of β_{12} is significant and positive. This means that a significant portion of assets value can be predicted. The value of assets is much more stable in conservative funds, compared with risky funds. Large increment in value of assets for these funds should be considered as predictable. Conservative funds offer protection from unexpected changes in the labour market and they tend to preserve the value of assets. The logarithmic trend that was discussed earlier is most noticeable in these funds. The presence of logarithmic trend in the remaining funds is apparent. Conservative funds offer stability that cannot be offered by the funds that invest large portion of transfers in risky assets.

The origins of differences between pension funds lie in the cross-correlations among the errors that influence the variance of the cointegrating relationship. The investments with defined returns have lower variances compared with those the returns of which are not defined.

6. Warnings and limitations

Rational expectations hypothesis was used as the main tool interpreting the results, but the lack of the data implies that no expectations are formed and econometric models under rational, adaptive or even naïve expectations would be of limited use here. On the other hand, as it is often the case with pensions, the agents do not form expectations not only because of the limited availability of data, but also due to the short-sighted belief that pensions will be paid sometime in the future and this time has not yet come. Of course, one can always argue that a participant in a pension fund has certain subjective beliefs about his or her future pension. And of course this is true. But subjective opinions should never be mistaken as expectations – they are merely the imagination of the participant.

All results are obtained under the assumption that a person earns average wages over a lifetime and has the typical (average) employment history, they cannot be generalised and applied to the individuals who earn significantly more than the average.

The biggest doubts concerned with the validity of results are related to the filtering. Sometimes filtering removes what you did not intended to remove.

7. Conclusions

All increments in the value of assets of moderate, balanced and risky funds must be treated as unpredictable and unstable. Funds that invest significant portion of transfers in risky assets do not offer a protection from the swings in the labour market.

Large increments in value of assets for conservative funds should be considered as predictable. Conservative funds offer protection from unexpected changes in the labour market and they tend to preserve the value of assets.

Conservative funds offer stability, which cannot be offered by the funds that invest large portion of transfers in risky assets. The benefits of diversification are obtained from investments in risk-free assets. The logarithmic trend, which was discussed earlier, is the most noticeable in the conservative funds.

In the long run the participants in conservative funds have the highest chances to offset the losses from PAYG scheme. The possibility to offset the loss with the surplus remains unclear. The main doubt comes from the fact that transfers and assets are cointegrated.

The origins of differences between pension funds lie in the cross-correlations among the errors that influence the variance of the cointegrating relationship.

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