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Exploring new exact solutions in the conformable time-fractional discrete coupled NLSE using a novel approach*

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Abstract. This investigation focuses on the conformable time-fractional discrete coupled nonlinear Schrödinger system (CTFCDNLSEs). This system incorporates a fractional order represented as a conformable derivative. Through the application of the fractional transformation method (FTM), a set of novel analytical discrete solutions is derived. These solutions are characterized by an array of mathematical functions, including trigonometric, hyperbolic, and rational functions. Among these solutions, discrete fractional bright solitons, dark solitons, combined solitons, and periodic solutions stand out. To demonstrate the influence of the fractional-order parameter on the dynamics of fractional discrete solitons, graphical representations are provided. These findings are significant for exploring complex nonlinear discrete physical phenomena.

Keywords: fractional transformation approach, discrete soliton solution, CTFCDNLSE.

1 Introduction

Within the realm of nonlinear sciences, the exploration of discrete solitons has consistently garnered significant interest among researchers. These discrete solitons have been observed across a diverse spectrum of systems of physical and biological nature, encompassing quantum chains, molecular particles, Einstein–Bose condensates, electrical lattices, and more recently, within light-based structures [2, 4, 11, 12, 16]. Over the past few decades, researchers have made a noteworthy observation regarding the nature of differential–difference equations (DDEs). Unlike fully discretized difference equations, DDEs are characterized as partially (or wholly) discretized semidiscrete equations [8, 10]. They involve discrete spatial variables while maintaining continuous time. The pursuit of exact solutions for nonlinear DDEs (NDDEs) has garnered considerable interest from researchers, prompting the development of numerous analytical methods for obtaining these solutions. Some of the techniques that have been employed include the modified

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Kudryashov method [5, 13, 20], the amplitude ansatz technique [18], the expa function approach [9], the modified simple equation method [3], the variational principle method [17], the Hirota bilinear and extended sinh-Gordon equation expansion schemes [15, 25], etc. [19].

There are various types of fractional derivatives, including the Caputo fractional derivative, Atangana–Baleanu Caputo fractional derivative, the local fractional derivative, the M-truncated fractional derivative, β-fractional derivative, conformable derivative, and so on. However, the mathematical properties of the conformable derivative have contributed to its extensive application in recent research studies [1].

Recent research has extended nonlinear differential–difference equations (NDDEs) to their fractional-order derivative. Fractional NDDEs (FNDDEs) have gained considerable prominence in various scientific disciplines including physics, chemistry, biology, and engineering. The derivation of analytical solutions for FNDDEs holds paramount significance in the representation and modeling of numerous physical phenomena. These phenomena encompass particle vibrations within lattices, electrical current flow in networks, and the propagation of pulses in biological systems [7, 14, 21, 22].

A distinctive aspect of this research is the development of a novel approach for deriving optical solutions for FNDDEs by utilizing the fractional transformation method. This technique enables us to solve a range of nonlinear equations and will facilitate the search for various novel fractional solutions to nonlinear problems.

This method also provides a direct, straightforward and computationally feasible approach for deriving exact traveling wave solutions to nonlinear equations. These solutions can be instrumental in advancing the comprehension of FNDDEs. Moreover, this transformation can be extended to other types of nonlinear wave equations. The fractional transformations approach found that the solutions need to be of rational form and include both trigonometric and hyperbolic forms. The primary objective of this work is to develop an efficient auxiliary method for a class of nonlinear model equations.

The CTFCDNLSEs were introduced as a mathematical framework for describing the phenomenon of self-trapping discrete solitons within the field of nonlinear optics [23].

Given that fractional-order models may accurately reflect the dependent process of function development, we suggest the CTFCDNLSEs as

$$\begin{aligned} \mathrm{i}D_t^\alpha \psi_n + (\psi_{n-1} + \psi_{n+1})\bigl[1 + \beta\bigl(|\psi_n|^2 + |\phi_n|^2\bigr)\bigr] - 2\psi_n = 0, \\ \mathrm{i}D_t^\alpha \phi_n + (\phi_{n-1} + \phi_{n+1})\bigl[1 + \beta\bigl(|\psi_n|^2 + |\phi_n|^2\bigr)\bigr] - 2\phi_n = 0. \end{aligned} \tag{1}$$

Here the variables ψ_n and ϕ_n represent complex entities, each associated with a discrete site indexed by the integer value n, and $\beta = \pm 1$. When $\alpha = 1$, Eq. (1) reduces to the discrete coupled nonlinear Schrödinger equations (NLSEs) as previously described in Dai et al.'s work. In the special case where $\psi_n = 0$ (or $\phi_n = 0$), Eq. (1) transforms into the fractional Ablowitz–Ladik (AL) equations. Moreover, when $\alpha = 1$ and $\psi_n = 0$ (or $\phi_n = 0$), Eq. (1) simplifies to the AL equations. This governs regulates a multitude of physical processes, including the dynamics of pulses in the realm of nonlinear optics [24].

In Eq. (1), the conformable time-fractional derivative of the order α is defined as the following.

Definition 1. Let the function $\mathcal{H} : [0, \infty) \to \mathbb{R}$ have the conformable derivative as follows [6]:

$$D_t^\alpha(t) = \lim_{\varphi \to 0} \frac{\mathcal{H}(t + \varphi t^{1-\alpha}) - \mathcal{H}(t)}{\varphi}, \quad \alpha \in (0, 1],\ t > 0.$$

Furthermore, if the aforementioned limit exists, then the function $\mathcal{H}$ becomes α-differentiable.

Theorem 1. *Assume that the functions $\mathcal{H}$, $\mathcal{W}$ are α-differentiable in t and $\alpha \in (0, 1]$. Then we have the following expressions*:

(i) $b_1 D_t^\alpha(\mathcal{H}) + b_2 D_t^\alpha(\mathcal{W}) = D_t^\alpha(b_1\mathcal{H} + b_2\mathcal{W})$ *for all* $b_1, b_2 \in \mathbb{R}$;
(ii) $D_t^\alpha(t^\rho) = \rho t^{\rho-\alpha}$ *for all* $\rho \in \mathbb{R}$;
(iii) $D_t^\alpha(\mathcal{H}\mathcal{W}) = \mathcal{H}D_t^\alpha(\mathcal{W}) + \mathcal{W}D_t^\alpha(\mathcal{H})$;
(iv) $D_t^\alpha(\mathcal{H}/\mathcal{W}) = (\mathcal{H}D_t^\alpha(\mathcal{W}) - \mathcal{W}D_t^\alpha(\mathcal{H}))/\mathcal{W}^2$;
(v) $D_t^\alpha(\mathcal{H}(t)) = t^{1-\alpha}\,\mathrm{d}\mathcal{H}/\mathrm{d}t$.

Theorem 2. *Considering the assumptions made in the preceding theorem,*

$$D_t^\alpha(\mathcal{H}o\mathcal{W})(t) = t^{1-\alpha}\mathcal{W}'(t)\mathcal{H}'\big(\mathcal{W}(t)\big).$$

2 Discrete soliton solutions of the CTFCDNLSE via novel analytical approach

We apply following transformations in Eq. (1) to obtain discrete traveling wave solutions:

$$\begin{aligned} \psi_n &= \mathcal{U}_n\xi_n \exp(\mathrm{i}\mathcal{G}_n), & \xi_n &= dn + c\frac{t^\alpha}{\alpha} + \xi_0, \\ \phi_n &= \mathcal{V}_n\xi_n \exp(\mathrm{i}\mathcal{G}_n), & \mathcal{G}_n &= pn + q\frac{t^\alpha}{\alpha} + \delta, \end{aligned} \tag{2}$$

and

$$\psi_{n\pm1} = \mathcal{U}_{n\pm1}\xi_n \exp\big(\mathrm{i}(\mathcal{G}_n \pm p)\big), \qquad \phi_{n\pm1} = \mathcal{V}_{n\pm1}\xi_n \exp\big(\mathrm{i}(\mathcal{G}_n \pm p)\big). \tag{3}$$

In the context of seeking discrete optical solutions, we consider the real functions $\mathcal{U}_n$ and $\mathcal{V}_n$, as well as the parameters d and c, which are associated with the pulse width and group velocity, respectively. Additionally, the parameters p and q characterize the wave number and frequency of the carrier. The constants ξ_0 and δ represent the initial phases.

Taking into account the above relations and substituting Eqs. (2)–(3) into Eq. (1), we can proceed to separate the real and imaginary components, leading to the following outcome:

$$\begin{aligned} &-q\mathcal{U}_n + \cos p(\mathcal{U}_{n-1} + \mathcal{U}_{n+1})\big(1 + m\beta\big(\mathcal{U}_n^2 + \mathcal{V}_n^2\big)\big) - 2\mathcal{U}_n = 0, \\ &cD_{\xi_n}^\alpha \mathcal{U}_n - \sin p(\mathcal{U}_{n-1} - \mathcal{U}_{n+1})\big(1 + m\beta\big(\mathcal{U}_n^2 + \mathcal{V}_n^2\big)\big) = 0, \\ &-q\mathcal{V}_n + \cos p(\mathcal{V}_{n-1} + \mathcal{V}_{n+1})\big(1 + m\beta\big(\mathcal{U}_n^2 + \mathcal{V}_n^2\big)\big) - 2\mathcal{V}_n = 0, \\ &cD_{\xi_n}^\alpha \mathcal{V}_n - \sin p(\mathcal{V}_{n-1} - \mathcal{V}_{n+1})\big(1 + m\beta\big(\mathcal{U}_n^2 + \mathcal{V}_n^2\big)\big) = 0, \end{aligned} \tag{4}$$

where $m = \exp(\mathrm{i}\mathcal{G}_n)\exp(-\mathrm{i}\mathcal{G}_n)$.

3 The FTM

We propose the following ansatz for Eq. (4):

$$\mathcal{U}_n = \frac{A_0 + A_1\mathcal{L}^2\xi_n}{A_2 + \mathcal{L}^2(\xi_n)}, \qquad \mathcal{V}_n = \frac{F_0 + F_1\mathcal{L}^2\xi_n}{F_2 + \mathcal{L}^2(\xi_n)}, \tag{5}$$

where $\xi_n = dn + c(t^\alpha/\alpha) + \xi_0$. Here the values of the constants A_0, A_1, A_2, F_0, F_1, and F_2 will be determined subsequently.

The function $\mathcal{L}$ in ansatz form (5) is a function of ξ_n and has the following cases.

Case I: $\mathcal{L}(\xi_n) = \xi_n$. Equation (5) turns into

$$\mathcal{U}_n = \frac{A_0 + A_1\xi_n^2}{A_2 + \xi_n^2}, \qquad \mathcal{V}_n = \frac{F_0 + F_1\xi_n^2}{F_2 + \xi_n^2}, \tag{6}$$

and

$$\xi_{n\pm1} = \xi_n \pm d. \tag{7}$$

Taking the derivative of Eq. (6) once with respect to ξ_n leads to

$$D^\alpha_{\xi_n}\mathcal{U}_n = \frac{2(-A_0 + A_1A_2)\xi_n}{(A_2 + \xi_n^2)^2}, \qquad D^\alpha_{\xi_n}\mathcal{V}_n = \frac{2(-F_0 + F_1F_2)\xi_n}{(F_2 + \xi_n^2)^2}. \tag{8}$$

Inserting Eqs. (6)–(8) into Eq. (4) and putting each coefficient of ξ_n^l, $l = 0, 1, 2, \dots,$ equal to zero, we derive a few algebraic equations, and by solving them using MAPLE software, we can achieve the following results.

Set 1:

$$c = -4\left(mF_1^2 + \frac{1}{2}\right)\sin p\, d, \qquad \beta = 1,$$
$$q = 4\cos p\, mF_1^2 + 2\cos p - 2,$$
$$A_1 = F_1, \qquad A_2 = \frac{A_0}{F_1}, \qquad F_0 = F_1F_2,$$

where A_0, F_1, F_2, p, d, ξ_0, and δ are arbitrary constants.

Thus, discrete soliton solutions of Eq. (1) are

$$\psi_n = \frac{A_0 + F_1\xi_n^2}{\frac{A_0}{F_1} + \xi_n^2}\exp(\mathrm{i}\mathcal{G}_n), \qquad \phi_n = \frac{F_1F_2 + F_1\xi_n^2}{F_2 + \xi_n^2}\exp(\mathrm{i}\mathcal{G}_n), \tag{9}$$

where

$$\xi_n = dn + \left(-4\left(mF_1^2 + \frac{1}{2}\right)\sin p\, d\right)\frac{t^\alpha}{\alpha} + \xi_0$$

and

$$\mathcal{G}_n = pn + \left(4\cos p\, mF_1^2 + 2\cos p - 2\right)\frac{t^\alpha}{\alpha} + \delta.$$

Set 2:

$$q = -2, \qquad \beta = -1,$$

$$A_1 = \pm\frac{\sqrt{-m(mF_1^2+1)}}{m}, \qquad A_2 = \pm\frac{A_0 m}{\sqrt{-m(mF_1^2+1)}}, \qquad F_0 = F_1F_2,$$

where A_0, F_1, F_2, p, d, c, ξ_0, and δ are arbitrary constants.

Thus, discrete soliton solutions of Eq. (1) are

$$\begin{aligned} \psi_n &= \frac{A_0 \pm \frac{\sqrt{-m(mF_1^2+1)}}{m}\xi_n^2}{\pm\frac{A_0 m}{\sqrt{-m(mF_1^2+1)}} + \xi_n^2}\exp(\mathrm{i}\mathcal{G}_n), \\ \phi_n &= \frac{F_1F_2 + F_1\xi_n^2}{F_2 + \xi_n^2}\exp(\mathrm{i}\mathcal{G}_n), \end{aligned} \tag{10}$$

where

$$\xi_n = dn + c\frac{t^\alpha}{\alpha} + \xi_0 \quad \text{and} \quad \mathcal{G}_n = pn - 2\frac{t^\alpha}{\alpha} + \delta.$$

Case II: $\mathcal{L}(\xi_n) = \sin\xi_n$. Equation (5) turns into

$$\mathcal{U}_n = \frac{A_0 + A_1\sin^2\xi_n}{A_2 + \sin^2\xi_n}, \qquad \mathcal{V}_n = \frac{F_0 + F_1\sin^2\xi_n}{F_2 + \sin^2\xi_n}, \tag{11}$$

and

$$\sin\xi_{n\pm1} := \sin(\xi_n \pm d) = \sin\xi_n\cos d \pm \cos\xi_n\sin d. \tag{12}$$

Then

$$\begin{aligned} \mathcal{U}_{n\pm1} &= \frac{A_0 + A_1(\sin\xi_n\cos d \pm \cos\xi_n\sin d)^2}{A_2 + (\sin\xi_n\cos d \pm \cos\xi_n\sin d)^2}, \\ \mathcal{V}_{n\pm1} &= \frac{F_0 + F_1(\sin\xi_n\cos d \pm \cos\xi_n\sin d)^2}{F_2 + (\sin\xi_n\cos d \pm \cos\xi_n\sin d)^2}. \end{aligned} \tag{13}$$

Taking the derivative of Eq. (11) once with respect to ξ_n leads to

$$\begin{aligned} D^\alpha_{\xi_n}\mathcal{U}_n &= \frac{2(-A_0 + A_1A_2)\cos\xi_n\sin\xi_n}{(A_2 + \sin^2\xi_n)^2}, \\ D^\alpha_{\xi_n}\mathcal{V}_n &= \frac{2(-F_0 + F_1F_2)\cos\xi_n\sin\xi_n}{(F_2 + \sin^2\xi_n)^2}. \end{aligned} \tag{14}$$

Inserting Eqs. (11)–(14) into Eq. (4) and putting each coefficient of $\sin^l\xi_n$, $l = 0, 1, 2, \ldots,$ equal to zero, we derive a few algebraic equations, and solving them using MAPLE software, we can achieve the following results.

Set 1:

$$c = -2\sin p\cos d\sin d, \qquad \beta = 1, \qquad q = 2\cos p - 2,$$
$$A_0 = A_1A_2, \qquad F_1 = \pm \mathrm{i}A_1, \qquad F_2 = \mp\frac{\mathrm{i}F_0}{A_1},$$

where A_1, A_2, F_0, p, d, ξ_0, and δ are arbitrary constants.

Thus, discrete soliton solutions of Eq. (1) are

$$\psi_n = \frac{A_1A_2 + A_1\sin^2\xi_n}{A_2 + \sin^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \qquad \phi_n = \frac{F_0 \pm \mathrm{i}A_1\sin^2\xi_n}{\mp\frac{\mathrm{i}F_0}{A_1} + \sin^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n),$$

where

$$\xi_n = dn - 2\sin p\cos d\sin d\frac{t^\alpha}{\alpha} + \xi_0 \quad \text{and} \quad \mathcal{G}_n = pn + (2\cos p - 2)\frac{t^\alpha}{\alpha} + \delta.$$

Set 2:

$$\beta = -1, \qquad q = 2\cos p - 2,$$
$$A_0 = A_1A_2, \qquad F_1 = \pm\mathrm{i}A_1, \quad F_2 = \mp\frac{\mathrm{i}F_0}{A_1},$$

where A_1, A_2, F_0, p, d, c, ξ_0, and δ are arbitrary constants.

Thus, discrete soliton solutions of Eq. (1) are

$$\psi_n = \frac{A_1A_2 + A_1\sin^2\xi_n}{A_2 + \sin^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \qquad \phi_n = \frac{F_0 \pm \mathrm{i}A_1\sin^2\xi_n}{\mp\frac{\mathrm{i}F_0}{A_1} + \sin^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \tag{15}$$

where

$$\xi_n = dn + c\frac{t^\alpha}{\alpha} + \xi_0 \quad \text{and} \quad \mathcal{G}_n = pn + (2\cos p - 2)\frac{t^\alpha}{\alpha} + \delta.$$

Case III: $\mathcal{L}(\xi_n) = \sinh\xi_n$. Equation (5) turns into

$$\mathcal{U}_n = \frac{A_0 + A_1\sinh^2\xi_n}{A_2 + \sinh^2\xi_n}, \qquad \mathcal{V}_n = \frac{F_0 + F_1\sinh^2\xi_n}{F_2 + \sinh^2\xi_n}, \tag{16}$$

and

$$\sinh\xi_{n\pm1} := \sinh(\xi_n \pm d) = \sinh\xi_n\cosh d \pm \cosh\xi_n\sinh d. \tag{17}$$

Then

$$\begin{aligned} \mathcal{U}_{n\pm1} &= \frac{A_0 + A_1(\sinh\xi_n\cosh d \pm \cosh\xi_n\sinh d)^2}{A_2 + (\sinh\xi_n\cosh d \pm \cosh\xi_n\sinh d)^2}, \\ \mathcal{V}_{n\pm1} &= \frac{F_0 + F_1(\sinh\xi_n\cosh d \pm \cosh\xi_n\sinh d)^2}{F_2 + (\sinh\xi_n\cosh d \pm \cosh\xi_n\sinh d)^2}. \end{aligned} \tag{18}$$

Taking the derivative of Eq. (16) once with respect to ξ_n leads to

$$\begin{aligned} D^{\alpha}_{\xi_n}\mathcal{U}_n &= \frac{2(-A_0 + A_1A_2)\cosh\xi_n\sinh\xi_n}{(A_2+\cosh^2\xi_n)^2}, \\ D^{\alpha}_{\xi_n}\mathcal{V}_n &= \frac{2(-F_0 + F_1F_2)\cosh\xi_n\sinh\xi_n}{(F_2+\cosh^2\xi_n)^2}. \end{aligned} \tag{19}$$

Inserting Eqs. (16)–(19) into Eq. (4) and putting each coefficient of $\sinh^l\xi_n$, $l = 0, 1, 2, \dots,$ equal to zero, we derive a few algebraic equations, and solving them using MAPLE software, we can achieve the following results.

Set 1:

$$c = -2\sin p\cosh d\sinh d, \qquad \beta = 1, \qquad q = 2\cos p - 2,$$
$$A_0 = A_1A_2, \qquad F_1 = \pm \mathrm{i}A_1, \qquad F_2 = \mp\frac{\mathrm{i}F_0}{A_1},$$

where A_1, A_2, F_0, p, d, ξ_0, and δ are arbitrary constants.

Thus, discrete soliton solutions of Eq. (1) are

$$\psi_n = \frac{A_1A_2 + A_1\sinh^2\xi_n}{A_2+\sinh^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \qquad \phi_n = \frac{F_0 \pm \mathrm{i}A_1\sinh^2\xi_n}{\mp\frac{\mathrm{i}F_0}{A_1}+\sinh^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n),$$

where

$$\xi_n = dn - 2\sin p\cosh d\sinh d\frac{t^{\alpha}}{\alpha} + \xi_0 \quad \text{and} \quad \mathcal{G}_n = pn + (2\cos p - 2)\frac{t^{\alpha}}{\alpha} + \delta.$$

Set 2:

$$\beta = -1, \qquad q = 2\cos p - 2,$$
$$A_0 = A_1A_2, \qquad F_1 = \pm \mathrm{i}A_1, \qquad F_2 = \mp\frac{\mathrm{i}F_0}{A_1},$$

where A_1, A_2, F_0, p, d, c, ξ_0, and δ are arbitrary constants.

Thus, discrete soliton solutions of Eq. (1) are

$$\psi_n = \frac{A_1A_2 + A_1\sinh^2\xi_n}{A_2+\sinh^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \qquad \phi_n = \frac{F_0 \pm \mathrm{i}A_1\sinh^2\xi_n}{\mp\frac{\mathrm{i}F_0}{A_1}+\sinh^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \tag{20}$$

where

$$\xi_n = dn + c\frac{t^{\alpha}}{\alpha} + \xi_0 \quad \text{and} \quad \mathcal{G}_n = pn + (2\cos p - 2)\frac{t^{\alpha}}{\alpha} + \delta.$$

Case IV: $\mathcal{L}(\xi_n) = \cosh\xi_n$. Equation (5) turns into

$$\mathcal{U}_n = \frac{A_0 + A_1\cosh^2\xi_n}{A_2+\cosh^2\xi_n}, \qquad \mathcal{V}_n = \frac{F_0 + F_1\cosh^2\xi_n}{F_2+\cosh^2\xi_n}, \tag{21}$$

and

$$\cosh\xi_{n\pm1} := \cosh(\xi_n \pm d) = \cosh\xi_n\cosh d \pm \sinh\xi_n\sinh d. \tag{22}$$

Then

$$\mathcal{U}_{n\pm1} = \frac{A_0 + A_1(\cosh\xi_n \cosh d \pm \sinh\xi_n \sinh d)^2}{A_2 + (\cosh\xi_n \cosh(d) \pm \sinh\xi_n \sinh d)^2},$$
$$\mathcal{V}_{n\pm1} = \frac{F_0 + F_1(\cosh\xi_n \cosh d \pm \sinh\xi_n \sinh d)^2}{F_2 + (\cosh\xi_n \cosh d \pm \sinh\xi_n \sinh d)^2}. \tag{23}$$

Taking the derivative of Eq. (21) once with respect to ξ_n leads to

$$D^{\alpha}_{\xi_n}\mathcal{U}_n = \frac{2(-A_0 + A_1A_2)\cosh\xi_n \sinh\xi_n}{(A_2 + \cosh^2\xi_n)^2},$$
$$D^{\alpha}_{\xi_n}\mathcal{V}_n = \frac{2(-F_0 + F_1F_2)\cosh\xi_n \sinh\xi_n}{(F_2 + \cosh^2\xi_n)^2}. \tag{24}$$

Inserting Eqs. (21)–(24) into Eq. (4) and putting each coefficient of $\cosh^l\xi_n$, $l = 0, 1, 2, \dots$, equal to zero, we derive a few algebraic equations, and solving them using MAPLE software, we can achieve the following results.

Set 1:

$$c = -2\sin p\cosh d\sinh d, \qquad \beta = 1, \qquad q = 2\cos p - 2,$$
$$A_0 = A_1A_2, \qquad F_1 = \pm \mathrm{i}A_1, \qquad F_2 = \mp\frac{\mathrm{i}F_0}{A_1},$$

where A_1, A_2, F_0, p, d, ξ_0, and δ are arbitrary constants.

Thus, discrete soliton solutions of Eq. (1) are

$$\psi_n = \frac{A_1A_2 + A_1(\cosh^2\xi_n)}{A_2 + \cosh^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \qquad \phi_n = \frac{F_0 \pm \mathrm{i}A_1\cosh^2\xi_n}{\mp\frac{\mathrm{i}F_0}{A_1} + \cosh^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \tag{25}$$

where

$$\xi_n = dn - 2\sin p\cosh d\sinh d\frac{t^\alpha}{\alpha} + \xi_0 \quad \text{and} \quad \mathcal{G}_n = pn + (2\cos p - 2)\frac{t^\alpha}{\alpha} + \delta.$$

Set 2:

$$\beta = -1, \quad q = 2\cos p - 2,$$
$$A_0 = A_1A_2, \qquad F_1 = \pm\mathrm{i}A_1, \quad F_2 = \mp\frac{\mathrm{i}F_0}{A_1},$$

where A_1, A_2, F_0, p, d, c, ξ_0, and δ are arbitrary constants.

Thus, discrete soliton solutions of Eq. (1) are

$$\psi_n = \frac{A_1A_2 + A_1(\cosh^2\xi_n)}{A_2 + \cosh^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \qquad \phi_n = \frac{F_0 \pm \mathrm{i}A_1\cosh^2\xi_n}{\mp\frac{\mathrm{i}F_0}{A_1} + \cosh^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \tag{26}$$

where

$$\xi_n = dn + c\frac{t^\alpha}{\alpha} + \xi_0 \quad \text{and} \quad \mathcal{G}_n = pn + (2\cos p - 2)\frac{t^\alpha}{\alpha} + \delta.$$

Case V: $\mathcal{L}\xi_n = \cos\xi_n$. Equation (5) turns into

$$\mathcal{U}_n = \frac{A_0 + A_1\cos^2\xi_n}{A_2 + \cos^2\xi_n}, \qquad \mathcal{V}_n = \frac{F_0 + F_1\cos^2\xi_n}{F_2 + \cos^2\xi_n}, \tag{27}$$

and

$$\cos\xi_{n\pm1} := \cos(\xi_n \pm d) = \cos\xi_n\cos d \mp \sin\xi_n\sin d. \tag{28}$$

Then

$$\begin{aligned}\mathcal{U}_{n\pm1} &= \frac{A_0 + A_1(\cos\xi_n\cos d \mp \sin\xi_n\sin d)^2}{A_2 + (\cos\xi_n\cos d \mp \sin\xi_n\sin d)^2},\\ \mathcal{V}_{n\pm1} &= \frac{F_0 + F_1(\cos\xi_n\cos d \mp \sin\xi_n\sin d)^2}{F_2 + (\cos\xi_n\cos d \mp \sin\xi_n\sin d)^2}.\end{aligned} \tag{29}$$

Taking the derivative of Eq. (11) once with respect to ξ_n leads to

$$\begin{aligned}D^{\alpha}_{\xi_n}\mathcal{U}_n &= \frac{2(-A_0 + A_1A_2)\cos\xi_n\sin\xi_n}{(A_2 + \cos^2\xi_n)^2},\\ D^{\alpha}_{\xi_n}\mathcal{V}_n &= \frac{2(-F_0 + F_1F_2)\cos\xi_n\sin\xi_n}{(F_2 + \cos^2\xi_n)^2}.\end{aligned} \tag{30}$$

Inserting Eqs. (27)–(30) into Eq. (4) and putting each coefficient of $\cos^l\xi_n$, $l = 0, 1, 2, \dots,$ equal to zero, we derive a few algebraic equations, and solving them using MAPLE software, we can achieve the following results.

Set 1:

$$c = -2\sin p\cos d\sin d, \qquad \beta = 1, \qquad q = 2\cos p - 2,$$
$$A_0 = A_1A_2, \qquad F_1 = \pm\mathrm{i}A_1, \qquad F_2 = \mp\frac{\mathrm{i}F_0}{A_1},$$

where A_1, A_2, F_0, p, d, ξ_0, and δ are arbitrary constants.

Thus, discrete soliton solutions of Eq. (1) are

$$\psi_n = \frac{A_1A_2 + A_1\cos^2\xi_n}{A_2 + \cos^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \qquad \phi_n = \frac{F_0 \pm \mathrm{i}A_1\cos^2\xi_n}{\mp\frac{\mathrm{i}F_0}{A_1} + \cos^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \tag{31}$$

where

$$\xi_n = dn - 2\sin p\cos d\sin d\frac{t^\alpha}{\alpha} + \xi_0 \quad\text{and}\quad \mathcal{G}_n = pn + (2\cos p - 2)\frac{t^\alpha}{\alpha} + \delta.$$

Set 2:

$$\beta = -1, \qquad q = 2\cos p - 2,$$
$$A_0 = A_1A_2, \qquad F_1 = \pm\mathrm{i}A_1, \qquad F_2 = \mp\frac{\mathrm{i}F_0}{A_1},$$

where A_1, A_2, F_0, p, d, c, ξ_0, and δ are arbitrary constants.

Thus discrete soliton solutions of Eq. (1) are

$$\psi_n = \frac{A_1A_2 + A_1\cos^2\xi_n}{A_2 + \cos^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n), \qquad \phi_n = \frac{F_0 \pm \mathrm{i}A_1\cos^2\xi_n}{\mp\frac{\mathrm{i}F_0}{A_1} + \cos^2\xi_n}\exp(\mathrm{i}\mathcal{G}_n),$$

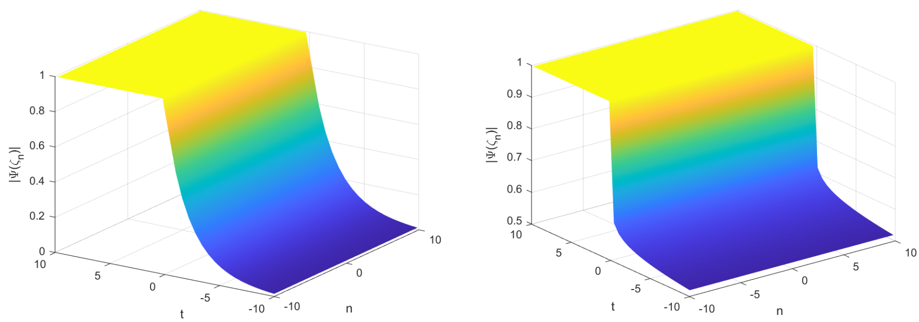
where

$$\xi_n = dn + c\frac{t^\alpha}{\alpha} + \xi_0 \quad \text{and} \quad \mathcal{G}_n = pn + (2\cos p - 2)\frac{t^\alpha}{\alpha} + \delta.$$

4 Physical description

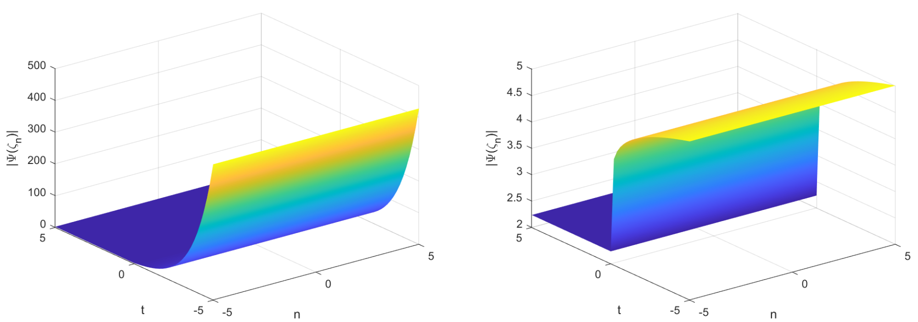
We have effectively obtained a collection of exact soliton solutions by utilizing the FTM in the CTFCDNLSE. Consequently, our computations yield expressions containing undetermined parameters. These solutions illustrate a broad spectrum of waveforms, which can arise by selecting various values for these parameters. These solutions consist of trigonometric and hyperbolic functions.

Figures 1–4 and 5–7 are kink-shape and periodic solitons, respectively. These figures consist of dark and bright solitons. In Fig. 1(a), $\alpha = 0.9$, and in Fig. 1(b), $\alpha = 0.1$. In Fig. 2(a), $\alpha = 0.75$, and in Fig. 2(b), $\alpha = 0.1$. In Fig. 3(a), $\alpha = 0.1$, and in Fig. 3(b),



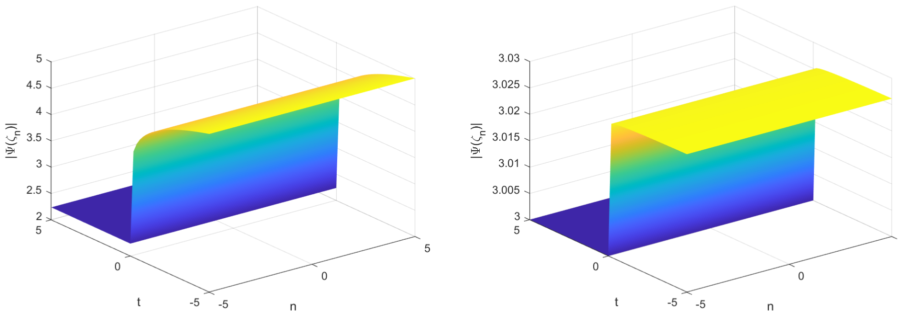
(a) $\alpha = 0.9$ (b) $\alpha = 0.1$

Figure 1. The 3D profiles represents $\mathrm{abs}(\psi_n)$ of Eq. (9) with parameters $A_0 = 3$, $F_1 = 2$, $d = 1$, $p = 0.5$, $\xi_0 = 0.01$, $\delta = 0.05$, and $m = 1$.



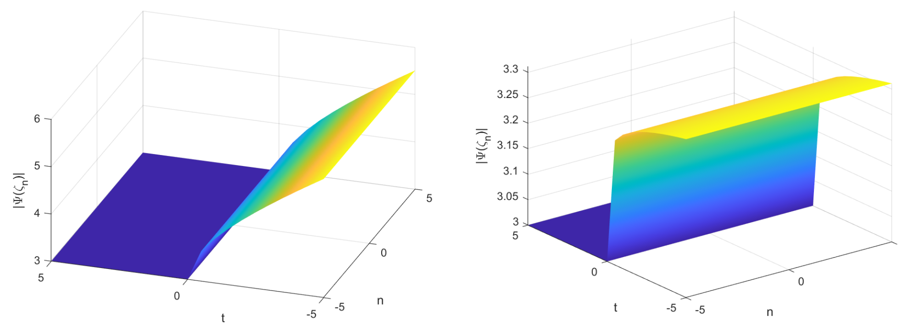
(a) $\alpha = 0.75$ (b) $\alpha = 0.1$

Figure 2. The 3D profiles represents $\mathrm{abs}(\psi_n)$ of Eq. (10) with parameters $A_0 = 3$, $F_1 = 2$, $d = 1$, $p = 0.5$, $\xi_0 = 0.01$, $\delta = 0.05$, and $m = 1$.



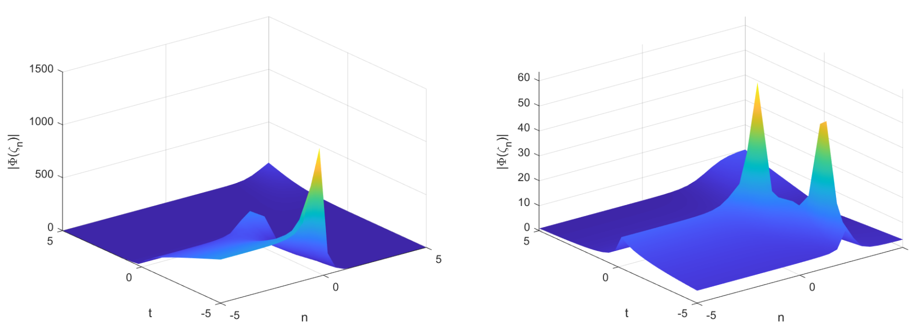
(a) $\alpha = 0.1$ (b) $\alpha = 0.01$

Figure 3. The 3D profiles represents $\mathrm{abs}(\psi_n)$ of Eq. (15) with parameters $A_1 = 3$, $A_2 = -2$, $d = 1$, $p = 0.5$, $\xi_0 = 0.01$, and $\delta = 0.05$.



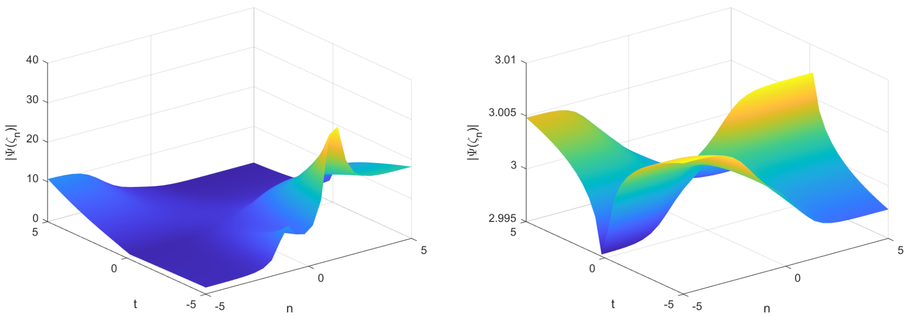
(a) $\alpha = 0.5$ (b) $\alpha = 0.1$

Figure 4. The 3D profiles represents $\mathrm{abs}(\psi_n)$ of Eq. (20) with parameters $A_1 = 3$, $A_2 = -2$, $d = 1$, $p = 0.5$, $\xi_0 = 0.01$, and $\delta = 0.05$.



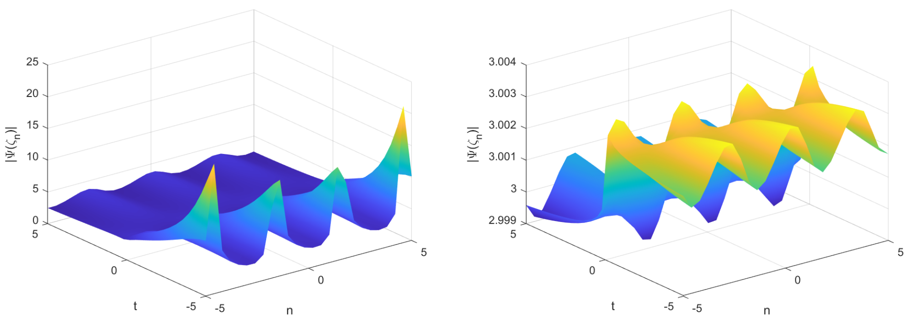
(a) $\alpha = 0.5$ (b) $\alpha = 0.25$

Figure 5. The 3D profiles represents $\mathrm{abs}(\phi_n)$ of Eq. (25) with parameters $A_1 = 3$, $F_0 = -2$, $d = 1$, $p = 0.5$, $\xi_0 = 0.01$, and $\delta = 0.05$.



(a) $\alpha = 0.5$ (b) $\alpha = 0.001$

Figure 6. The 3D profiles represents $\mathrm{abs}(\psi_n)$ of Eq. (26) with parameters $A_1 = 3$, $A_2 = -2$, $F_0 = 2$, $d = 1$, $p = 0.5$, $\xi_0 = 0.01$, and $\delta = 0.05$.



(a) $\alpha = 0.5$ (b) $\alpha = 0.001$

Figure 7. The 3D profiles represents $\mathrm{abs}(\psi_n)$ of Eq. (31) with parameters $A_1 = 3$, $A_2 = -2$, $F_0 = 2$, $d = 1$, $p = 0.5$, $\xi_0 = 0.01$, and $\delta = 0.05$.

$\alpha = 0.01$. In Fig. 4(a), $\alpha = 0.5$, and in Fig. 4(b), $\alpha = 0.1$. In Fig. 5(a), $\alpha = 0.5$, and in Fig. 5(b), $\alpha = 0.25$. In Fig. 6(a), $\alpha = 0.5$, and in Fig. 6(b), $\alpha = 0.001$. In Fig. 7(a), $\alpha = 0.5$, and in Fig. 7(b), $\alpha = 0.001$. Figures 1–7 demonstrate that variations in the fractional parameter α influence the soliton's width and amplitude.

5 Conclusion

The model we investigated was the CTFCDNLSE as a FNDDEs. Applying the fractional transformation technique, we uncovered various soliton families, including trigonometric solitons, hyperbolic solitons, singular solitons, and kink solitons. It is important to highlight that the solutions obtained in this research are innovative and expand the existing knowledge base. This approach is a straightforward method that does not require the use of the homogeneous balance principle. Furthermore, the utilization of 3D profiles provided valuable insights into the behavior and characteristics of these solitons. It should be noted that the order of the derivative, namely the parameter α, influences the determination of these solutions.

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