Nonlinear Analysis: Modelling and Control, Vol. 29, No. 2, 349–364
https://doi.org/10.15388/namc.2024.29.34648

Vilnius University Press

Practical fixed-time stabilization for discrete-time impulsive switched port-controlled Hamiltonian systems*

Xiangyu Chen, Weiwei Sun[1], Xinci Gao, Dehai Yu

Institute of Automation, Qufu Normal University,
Qufu 273165, China
wwsun@hotmail.com

Received: April 28, 2023 / **Revised:** November 7, 2023 / **Published online:** February 19, 2024

Abstract. This paper is concerned with practical fixed-time (FT) stabilization problem of discrete-time impulsive switched port-controlled Hamiltonian systems (DISPCH). First, starting with discrete-time port-controlled Hamiltonian systems, a novel controller is presented to achieve practical FT stability of the obtained closed-loop system. Moreover, in order to well handle the abrupt changes at switch moments in practical switched systems, another novel controller is presented in terms of positive-order Lyapunov functions approach and range dwell time method to make discrete-time impulsive switched port-controlled Hamiltonian system practical FT stable. Ultimately, the validity of proposed methods is illustrated by simulations.

Keywords: discrete-time Hamiltonian systems, practical fixed-time stabilization, impulsive switched systems.

1 Introduction

Switched dynamic systems, which represent a particular type of hybrid dynamical systems, have been widely concerned by researchers recently; see [17, 21]. In practice, many actual physical systems can be represented in the form of switching systems, such as network systems [5], multiagent systems [29], electrical power systems [4], etc. However, sometimes, impulsive phenomena are inevitable in switched systems when they are switching among their subsystems. Since sudden changes at particular moments generally lead to the instability of such systems, the stability analysis problem becomes very essential for impulsive switched systems. In [7], the impulsive and switching hybrid systems were introduced, and several asymptotical stability criteria were established for these systems. Based on impulsive dynamical linear systems, finite-time stabilization control problem is tackled in [1]. In [10], time-delayed impulsive control was introduced to deal with the stability problem of discrete-time systems.

*This research was supported in part by the National Natural Science Foundation of China under grant 62073189 and grant 62173207, and in part by the Taishan Scholar Project of Shandong Province under grant tsqn202211129.

[1]Corresponding author.

Hamiltonian systems, which represent a special kind of nonlinear systems, have been investigated by researchers in a wide range of fields, such as flexible-joints robots [20], physical science [27], electrical networks [31], and the like. The primary reason is their appropriate forms, which have clear physical characteristics. Consequently, the theoretical analysis of Hamiltonian systems is of great practical significance and has attracted wide attention, for instance, [12, 13, 24, 28, 32]. A dynamic output controller of challenging control objectives for port-Hamiltonian systems was proposed in [28]. The adaptive control problem was researched for stochastic Hamiltonian systems in [24]. In [13], for uncertain Hamiltonian systems, the finite-time stabilization problem was solved by using sliding mode control approach. For switched affine nonlinear systems, an event-triggered stabilisation problem was studied via a Hamiltonian approach in [32]. In [12], for PCH systems, H_∞ control and fixed-time stabilization problems were investigated.

Recently, with the widespread utilization of computers to control these systems, the control design and stability analysis for discrete-time Hamiltonian systems have already obtained wide attention, and a large amount of remarkable achievements have been achieved, for instance, [6, 9, 25, 26]. In [26], a discrete formulation of port-Hamiltonian systems was derived. For discrete-time port-Hamiltonian systems, a new dynamic model with discrete-time Dirac structures was given in [9] by using collocation methods. With the development of stochastic systems, Cordoni et al. [6] introduced discrete stochastic port-Hamiltonian systems based on symplectic variational integrators. For discrete time-delay Hamiltonian systems with uncertain item, fusion estimation problem was investigated in [25].

Among various realistic applications, achieving rapidly convergence within finite time and global asymptotic stability play an significant role. Due to their fast convergence and good robustness, there have been many literatures in regard to finite-time stabilization in [2, 22, 23, 30], and global asymptotic stability for all kinds of difference models in [8, 14–16]. However, with respect to finite-time stability, settling time heavily relies on the original condition, and for global asymptotic stability, the speed of stabilization sometimes may be too long. As a result, when the fast convergence is required and accurate initial value is not available in advance, these current finite-time and global asymptotic stabilities fail to obtain an anticipated performance, which mostly limits the realistic applications. Therefore, an unique finite-time stabilization problem, called fixed-time stabilization problem, has attracted a large amount of scholars' attention. Many significant results about FT stability have been achieved, such as [11, 18, 19]. The definition of FT stability was proposed by Polyakov et al. [18], who also studied the FT stabilization of linear control systems by nonlinear feedback design. For finite-time and FT stabilization problems, an implicit Lyapunov function approach was considered in [19]. As for port-Hamiltonian systems, FT stabilization problem was studied in [11].

From the above discussions the remarkable thing is that FT stabilization problem has given rise to some academic interest, but there are few researches about this aspect for discrete-time impulsive switched port-controlled Hamiltonian systems. In this paper, the primary contributions are summed up as follows:

(i) A new class of discrete-time model is set up for PCH systems considering impulsive switch;

(ii) Practical FT stabilization problem is considered in the DISPCH systems, in contrast with finite-time stabilization, settling time functions here eliminate the reliance of original conditions;
(iii) This paper discusses the impulsive effects during switched Hamiltonian systems switching among its subsystems, which can well reflect the engineering practice.

The paper is structured as follows. Section 2 presents the model of DISPCH system and preliminaries about practical FT stabilization problem. In Section 3, two sufficient conditions are given for practical FT stabilization of the DPCH system and the DISPCH system, respectively. Simulation results are shown to demonstrate the effectiveness of presented methods in Section 4. Section 5 gives the conclusion.

Notations. $\mathbb{N}$, $\mathbb{N}_+$, and $\mathbb{R}$ stand for sets of nonnegative integers, positive integers, and real numbers, respectively, and $\mathbb{R}_+ = [0, +\infty)$. Let $w(t) \in \mathcal{K}_\infty$ mean that the function $w(t)$ belongs to the class $\mathcal{K}_\infty$, and let $\{N_l\}$ be a sequence of number. $\|\cdot\|$ stands for the usual Euclidean norm. I_n denotes the $(n \times n)$-dimensional identity matrix. The n- and $(m \times n)$-dimensional real Euclidean space are denoted respectively by $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$. Let A^{T} stand for the transpose of matrix A. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ stand for the maximum and minimum eigenvalues, respectively.

2 Problem formulation and preliminaries

2.1 DISPCH systems

Consider the following DISPCH system:

$$\begin{aligned} x_{k+1} - x_k &= T_{\sigma_k}\big(J_{\sigma_k}(x_k) - R_{\sigma_k}(\|x_k\|)\big)\bar{\nabla} H_{\sigma_k}(x_k, x_{k+1}) \\ &\quad + T_{\sigma_k} G_{\sigma_k} u_{\sigma_k,k}, \quad k \neq N_1, \\ x_{k+1} &= g_{\sigma_k}(x_k), \quad k = N_1, \end{aligned} \tag{1}$$

where $x_k = x(k) \in \mathbb{R}^n$ stands for the system state, $u_{\sigma_k,k} = u_{\sigma_k}(k) \in \mathbb{R}^m$ represents system control input. $\sigma_k = \sigma(k) : \mathbb{N} \mapsto P = \{1, 2, 3, \dots, M\}$ with $M \in \mathbb{N}_+$ stands for the switching signal, and P as the index set. Based on σ_k, switching sequence is obtained as $\{x_0\colon (i_0, N_0), (i_1, N_1), \dots, (i_p, N_p), \dots, i_p \in P, N_p \in \mathbb{N}, p \in \mathbb{N}\}$, the switching time sequence is denoted by $\{N_l\}$, $l = 0, 1, 2, \dots$. For any $i \in P$, $u_{i,k}$ is the control input of the ith subsystem. T_i is the sampling period of the ith subsystem. $J_i(x_k) = -J_i^{\mathrm{T}}(x_k) \in \mathbb{R}^{n\times n}$ and $R_i(\|x_k\|) = R_i^{\mathrm{T}}(\|x_k\|) \geqslant 0 \in \mathbb{R}^{n\times n}$ represent the natural interconnection and damping matrices of the ith subsystem, respectively. $g_i(x_k) : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a vector-valued function, $G_i \in \mathbb{R}^{n\times m}$ represents the input gain matrix of the ith subsystem and is assumed to have full column rank. $H_i(x_k) : \mathbb{R}^n \mapsto \mathbb{R}_+$ represents the discrete-time Hamiltonian function for the ith subsystem with $x_k = 0$ being its minimum point and $H_i(x_k) > 0$ for all $x_k \neq 0$. $\bar{\nabla} H_i(x_k, x_{k+1}) : \mathbb{R}^n \mapsto \mathbb{R}^n$ represents the discrete gradient of $H_i(x_k)$, which satisfies

$$\begin{aligned} \bar{\nabla}^{\mathrm{T}} H_i(x_k, x_{k+1})(x_{k+1} - x_k) &= H_i(x_{k+1}) - H_i(x_k), \\ \lim_{x_{k+1}\to x_k} \bar{\nabla} H_i(x_k, x_{k+1}) &= \nabla H_i(x_k). \end{aligned} \tag{2}$$

Remark 1. Compared with continuous PCH systems, in the DISPCH system (1) in this paper, the time derivative is substituted by its forward Euler approximation, and the gradient term is replaced by any discrete gradient as long as it satisfies Eq. (2).

In subsequent analysis, the Hamiltonian function $H_i(x_k)$ of the ith subsystem of (1) is described as

$$H_i(x_k) = \varrho_i \cdot \|x_k\|^2, \quad 0 < \varrho_i \leqslant 1. \tag{3}$$

According to (2) and (3), we can obtain the discrete gradient corresponding to (3) as

$$\bar{\nabla} H_i(x_k, x_{k+1}) = \varrho_i \cdot (x_k + x_{k+1}).$$

The impulsive function is chosen as

$$g_i(x_k) = d_i x_k, \tag{4}$$

where d_i is a given constant.

Remark 2. According to switching sequence $\{x_0\colon (i_0, N_0), (i_1, N_1), \dots, (i_p, N_p), \dots, i_p \in Q,\ N_p \in \mathbb{N},\ p \in \mathbb{N}\}$, the i_pth subsystem is active if $k \in [N_p + 1,\ N_{p+1}]$.

Assumption 1. The sequence $\{N_l\}$ satisfies $N_l \in \mathbb{N}$, $l = 0, 1, 2, \dots$, and $N_0 < N_1 < N_2 < \cdots < N_p < \cdots$ with $N_{p+1} - N_p > 1$.

Assumption 2. The damping matrix $R_i(\|x_k\|)$ eigenvalues' suprema is finite, that is, there exists a constant ς such that $\lambda_{\max} R_i(\|x_k\|) \leqslant \varsigma$ with $x_k \to \infty$.

The preliminaries about practical FT stabilization are briefly reviewed in the following section.

2.2 Practical FT stabilization

Definition 1. (See [18].) For all initial condition $x_0 \in \mathbb{R}^n$, if there exists a constant $\varepsilon > 0$ and settling time function $\tilde{T}(\varepsilon, x_0) < +\infty$ such that $\|x_k\| \leqslant \varepsilon$ is satisfied for any $k \geqslant \tilde{T}(\varepsilon, x_0)$, then system (1) can be called practical finite-time stable.

Definition 2. (See [18].) If system (1) is practical finite-time stable and there exists an upper bound of settling time function, which means that there exists a constant $\tilde{T}_{\max} > 0$ such that $\tilde{T}(\varepsilon, x_0) \leqslant \tilde{T}_{\max}$ for any $x_0 \in \mathbb{R}^n$, then the system (1) is called practical fixed-time stable.

Remark 3. In general, researchers consider the practical FT stabilization problem for discrete-time systems rather than FT stabilization. Because practical FT stabilization leads to $\|x_k\| \leqslant \varepsilon$ for all $k \geqslant \tilde{T}_{\max}$, $\|x_k\|$ converges to a region instead of 0. This is more meaningful for discrete-time systems. Therefore, we here consider practical FT stabilization problem of system (1).

Based on the above analysis, the main objective here is that devises state feedback controllers, which make the DISPCH system practical FT stable under some switching conditions.

Then we will give a lemma to be used in the following proof.

Lemma 1. (*See* [3].) *For system* (1) *and any* $i \in P$, *let there exist function* $V_i : \mathbb{R}^n \mapsto \mathbb{R}_+$, $\alpha \in \mathcal{K}_\infty$, *and three constants* c, q, b *satisfying* $c > 0$, $q > 0$, $b \geqslant 1$, *respectively, and let the following statements hold*:

(i) $\alpha(\|x_k\|) \leqslant V_i(x_k)$,
(ii) $V_i^c(x_{k+1}) - V_i^c(x_k) \leqslant -q(V_i(x_k) \cdot V_i(x_{k+1}))^c$, $k \neq N_l$,
(iii) $V_i(x_{N_l+1}) \leqslant bV_j(x_{N_l})$ *for all* $i, j \in P$, $l = 0, 1, \dots$.

Then system (1) *is called practical FT stable with all switching sequences* σ_k *satisfying*

$$N_{l+1} - N_l - 1 = T_1 \in [\theta_{\min}, \theta_{\max}], \quad l = 0, 1, \dots,$$

where $\theta_{\min} = 1/(qZ_L\alpha^c(\varepsilon))$, $\theta_{\max} = b/(q\alpha^c(\varepsilon))$, $Z_L = \sum_{j=1}^{L}(1/b)^{jc}$, $T_l \in \mathbb{R}$ *is the dwell time,* $\varepsilon > 0$ *is a given constant, and* L *is a given positive integer. Meanwhile, the settling time* $\tilde{T}(\varepsilon, x_0) \leqslant \tilde{T}_{\max} \leqslant L(\theta_{\max} + 1) + 1$.

3 Main results

In this section, the practical FT stabilization problem for system (1) is studied. Before dealing with practical FT stabilization for DISPCH systems, we consider discrete-time PCH (DPCH) systems without impulsive switch first.

The DPCH system is given as follows:

$$x_{k+1} - x_k = T\bigl(J(x_k) - R\bigl(\|x_k\|\bigr)\bigr)\bar{\nabla}H(x_k, x_{k+1}) + TGu_k, \tag{5}$$

where $x_k = x(k) \in \mathbb{R}^n$ represents the system state, $u_k = u(k) \in \mathbb{R}^m$ stands for system control input. T is the sampling period. $J(x_k) = -J^{\mathrm{T}}(x_k) \in \mathbb{R}^{n\times n}$ stands for the interconnection matrix, and $R(\|x_k\|) = R^{\mathrm{T}}(\|x_k\|) \geqslant 0 \in \mathbb{R}^{n\times n}$ represents damping matrix. $R(\|x_k\|)$ satisfies Assumption 2. Suppose the input gain matrix $G \in \mathbb{R}^{n\times m}$ has full column rank. $H(x_k) : \mathbb{R}^n \mapsto \mathbb{R}_+$ stands for the discrete-time Hamiltonian function, which also meets the conditions as shown in system (1). $\bar{\nabla}H(x_k, x_{k+1}) : \mathbb{R}^n \mapsto \mathbb{R}^n$ stands for the discrete gradient of $H(x_k)$, which satisfies Eq. (2) without regarding to switching signal.

3.1 Practical FT stabilization for DPCH systems

Corresponding to system (1), we put forward the desired Hamiltonian function as follows:

$$H(x_k) = \varrho\|x_k\|^2, \quad 0 < \varrho \leqslant 1. \tag{6}$$

According to (2) and (6), we can obtain the discrete gradient

$$\bar{\nabla}H(x_k, x_{k+1}) = \varrho(x_k + x_{k+1}). \tag{7}$$

Theorem 1. *For the DPCH system* (5)*, $\varepsilon > 0$ is a given constant, and suppose Hamiltonian function of system* (5) *is as shown in* (6)*. If there exists a symmetric matrix $K \in \mathbb{R}^{n\times n}$ that satisfies the following statements*

$$\lambda_{\min}\bigl(R(\|x_k\|) - K\bigr) \geqslant 0, \qquad (8)$$
$$\lambda_{\max}\bigl(R(\|x_k\|) - K\bigr) \leqslant \frac{2}{(2+\|x_k\|)\varrho T},$$

then closed-loop system corresponding to system (5) *is practical FT stable under the following state feedback control law:*

$$u_k = (G^T G)^{-1} G^{\mathrm{T}}\left(\varrho K - \frac{1}{T} I_n\right) x_k. \qquad (9)$$

Meanwhile, the settling time function $\tilde{T}(\varepsilon, x_0)$ satisfies

$$\tilde{T}(\varepsilon, x_0) \leqslant \tilde{T}_{\max} = \frac{2}{\varepsilon}.$$

Proof. According to Eq. (3), Lyapunov function is selected as

$$V(x_k) = H(x_k) = \varrho\|x_k\|^2.$$

By substituting the controller (9) into system (5) the following equation can be obtained:

$$\begin{aligned} x_{k+1} - x_k &= T\bigl(J(x_k) - R(\|x_k\|)\bigr)\bar{\nabla} H(x_k, x_{k+1}) + TGu_k \\ &= T\bigl(J(x_k) - R(\|x_k\|)\bigr)\bar{\nabla} H(x_k, x_{k+1}) + (\varrho TK - I_n)x_k. \end{aligned} \qquad (10)$$

When $x_k = 0$, it is effortless to derive that closed-loop system of (5) is practical FT stable. Combining $x_k = 0$ with Eq. (10), we can obtain

$$x_{k+1} = \varrho T\bigl[J(0) - R(0)\bigr]x_{k+1}. \qquad (11)$$

According to the arbitrariness of constant ϱ and T, we will get all the later states are equal to zero, which means the system is practical FT stable.

When $x_k \neq 0$, without loss of generality, we assume that $x_{k+1} \neq 0$. Multiplying both sides of Eq. (10) by x_{k+1}^T and substituting the discrete gradient into it, we have

$$\begin{aligned} x_{k+1}^{\mathrm{T}}(x_{k+1} - x_k) &= Tx_{k+1}^{\mathrm{T}}(J(x_k) - R(\|x_k\|))\varrho(x_{k+1} + x_k) \\ &\quad + Tx_{k+1}^{\mathrm{T}}\left(\varrho K - \frac{1}{T} I_n\right)x_k. \end{aligned}$$

Noticing skew-symmetry property of the matrix $J(x_k)$, we can get $x_{k+1}^{\mathrm{T}} J(x_k) x_{k+1} = 0$. The equation implies

$$\begin{aligned} & x_{k+1}^{\mathrm{T}} x_{k+1} + \varrho T x_{k+1}^{\mathrm{T}} R(\|x_k\|) x_{k+1} \\ &\quad = x_{k+1}^{\mathrm{T}} x_k + \varrho T x_{k+1}^{\mathrm{T}}\bigl(J(x_k) - R(\|x_k\|)\bigr)x_k \\ &\qquad + \varrho T x_{k+1}^{\mathrm{T}} K x_k - x_{k+1}^{\mathrm{T}} x_k. \end{aligned}$$

Combining the semipositive definitiveness of the matrix $R(x_k)$, we can get $x_{k+1}^{\mathrm{T}} R(x_k) x_{k+1} \geqslant 0$. Then one can obtain

$$x_{k+1}^{\mathrm{T}} x_{k+1} \leqslant x_{k+1}^{\mathrm{T}} x_k + \varrho T x_{k+1}^{\mathrm{T}} \big(J(x_k) - R\big(\|x_k\|\big)\big) x_k \\ + \varrho T x_{k+1}^{\mathrm{T}} K x_k - x_{k+1}^{\mathrm{T}} x_k.$$

Further, we derive

$$\|x_{k+1}\| \leqslant \big\| \varrho T \big(J(x_k) - R\big(\|x_k\|\big)\big) x_k + \varrho T K x_k \big\|. \tag{12}$$

Based on the trajectory of system (5), one can calculate

$$\begin{aligned}
&V^{-1/2}(x_{k+1}) - V^{-1/2}(x_k) \\
&\quad = \big(\sqrt{\varrho} \cdot \|x_{k+1}\|\big)^{-1} - \big(\sqrt{\varrho} \cdot \|x_k\|\big)^{-1} \\
&\quad \geqslant \big(\sqrt{\varrho} \cdot \|\varrho T \big(J(x_k) - R\big(\|x_k\|\big)\big) x_k + \varrho T K x_k\|\big)^{-1} - \big(\sqrt{\varrho} \cdot \|x_k\|\big)^{-1} \\
&\quad \geqslant \big(\sqrt{\varrho} \cdot \big\| \big(x_k x_k^{\mathrm{T}}\big)^{-1} x_k x_k^{\mathrm{T}} \big(\varrho T \big(J(x_k) - R\big(\|x_k\|\big)\big) x_k + \varrho T K x_k\big) \big\|\big)^{-1} \\
&\qquad - \big(\sqrt{\varrho} \cdot \|x_k\|\big)^{-1}.
\end{aligned}$$

According to the compatibility of matrix norms, one has

$$\begin{aligned}
&V^{-1/2}(x_{k+1}) - V^{-1/2}(x_k) \\
&\quad \geqslant \big(\sqrt{\varrho} \cdot \big\| \varrho T x_k^{\mathrm{T}} \big(J(x_k) - R\big(\|x_k\|\big)\big) x_k + \varrho T x_k^{\mathrm{T}} K x_k \big\|\big)^{-1} \\
&\qquad \times \big\| \big(x_k x_k^{\mathrm{T}}\big)^{-1} x_k x_k^{\mathrm{T}} x_k \big(x_k^{\mathrm{T}} x_k\big)^{-1} \big\|^{-1} - \big(\sqrt{\varrho} \cdot \|x_k\|\big)^{-1} \\
&\quad \geqslant \|x_k\| \big(\sqrt{\varrho} \cdot \big\| \varrho T x_k^{\mathrm{T}} R\big(\|x_k\|\big) x_k - \varrho T x_k^{\mathrm{T}} K x_k \big\|\big)^{-1} - \big(\sqrt{\varrho} \cdot \|x_k\|\big)^{-1} \\
&\quad \geqslant \|x_k\| \big(\sqrt{\varrho} \cdot \|2 x_k^{\mathrm{T}} \big(2 + \|x_k\|\big)^{-1} x_k\|\big)^{-1} - \big(\sqrt{\varrho} \cdot \|x_k\|\big)^{-1} \\
&\quad \geqslant \big(2 + \|x_k\|\big) \big(2\sqrt{\varrho} \cdot \|x_k\|\big)^{-1} - \big(\sqrt{\varrho} \cdot \|x_k\|\big)^{-1} \\
&\quad \geqslant (2\sqrt{\varrho})^{-1}.
\end{aligned}$$

Furthermore, it is obvious that

$$\frac{1}{\|x_k\|} - \frac{1}{\|x_0\|} \geqslant \sum_{i=0}^{k-1} \frac{1}{\|x_{i+1}\|} - \frac{1}{\|x_i\|} \geqslant \frac{k}{2},$$

and hence,

$$\|x_k\| \leqslant \frac{2\|x_0\|}{2 + k\|x_0\|} \leqslant \frac{2}{k}. \tag{13}$$

For given $\varepsilon > 0$, it follows from (13) that $\|x_k\| \leqslant \varepsilon$ for $k \geqslant 2/\varepsilon$, which implies that closed-loop system of (5) is practical FT stable with $\tilde{T}(\varepsilon, x_0)$ satisfying

$$\tilde{T}(\varepsilon, x_0) \leqslant \tilde{T}_{\max} = \frac{2}{\varepsilon}.$$

□

Remark 4. The conditions in Theorem 1 are reasonable. We can choose the sampling period T small enough and design appropriate constant matrix K to make inequality (8) satisfied.

Remark 5. $\tilde{T}_{\max}$ is related to the attractive region $\{x_k\colon \|x_k\| \leqslant \varepsilon\}$. The value of $\tilde{T}_{\max}$ increases with the decrease of the value of ε.

Remark 6. In order to demonstrate the practical FT stabilization problem of DPCH system (5), the positive-order Lyapunov function is used in Theorem 1, which is distinct from usual Lyapunov function.

Based on the obtained results, we further study practical FT stabilization of DISPCH systems, which can effectively describe the switching characteristics.

3.2 Practical FT stabilization for DISPCH systems

The practical FT stabilization control of DISPCH system (1) is discussed as follows.

Theorem 2. *For the DISPCH system* (1)*, $\varepsilon > 0$ is a given constant, L is a given positive integer. Suppose that for any $i \in P$, the Hamiltonian function $H_i(x_k)$ is given as* (3)*. Let there exist a symmetric matrix $K_i \in \mathbb{R}^{n\times n}$ that satisfies the following statements*:

$$\lambda_{\min}\bigl(R_i(\|x_k\|) - K_i\bigr) \geqslant 0,$$
$$\lambda_{\max}\bigl(R_i(\|x_k\|) - K_i\bigr) \leqslant \frac{2}{(2+\|x_k\|)\varrho_i T_i},$$

and let the switching time sequence satisfy

$$N_{l+1} - N_l - 1 = T_l \in [\theta_{\min}, \theta_{\max}], \quad l = 0, 1, \dots, \tag{14}$$

where $\theta_{\min} = 2/(\sqrt{\varrho_{\min}} Z_L \varepsilon)$, $\theta_{\max} = 2b/(\sqrt{\varrho_{\min}}\varepsilon)$, $Z_L = \sum_{j=1}^{L}(1/b)^{j/2}$, $\varrho_{\min} = \min_{1\leqslant i\leqslant M}\{\varrho_i\}$, $b = \max_{1\leqslant i\leqslant M}\{b_i, 1\}$, $b_i = \varrho_i d_i^2/\varrho_j$, and $T_l \in \mathbb{R}$ is the dwell time. Then closed-loop system of system (1) *is practical FT stable under state feedback control law*

$$u_{i,k} = \bigl(G_i^T G_i\bigr)^{-1} G_i^T\left(\varrho_i \cdot K_i - \frac{1}{T_i} I_n\right) x_k. \tag{15}$$

Meanwhile, settling time function $\tilde{T}(\varepsilon, x_0)$ satisfies

$$\tilde{T}(\varepsilon, x_0) \leqslant \tilde{T}_{\max} = L(\theta_{\max} + 1) + 1.$$

Proof. The Lyapunov function is selected as

$$V_i(x_k) = H_i(x_k) = \varrho_i \cdot \|x_k\|^2. \tag{16}$$

When $k \neq N_l$, substituting the controller (15) into the ith subsystem of system (1), the following equation can be obtained:

$$x_{k+1} - x_k = T_i\bigl(J_i(x_k) - R_i\bigl(\|x_k\|\bigr)\bigr)\bar{\nabla} H_i(x_k, x_{k+1}) \\ + T_i\left(\varrho_i K_i - \frac{1}{T_i} I_n\right)x_k. \tag{17}$$

Under the condition $k \neq N_l$, when $x_k = 0$, it is effortless to derive that closed-loop system of (1) is practical FT stable. Combining $x_k = 0$ with Eq. (17), we can obtain

$$x_{k+1} = \varrho_i T\bigl(J_i(0) - R_i(0)\bigr)x_{k+1}.$$

According to the arbitrariness of constant a_i and T_i, we will get that all the later states are equal to zero, so the system is practical FT stable.

Under the condition $k \neq N_l$, when $x_k \neq 0$, without loss of generality, we assume $x_{k+1} \neq 0$. Multiplying both sides of Eq. (10) by x_{k+1}^{T} and substituting the discrete gradient into it, we have

$$x_{k+1}^{\mathrm{T}}(x_{k+1} - x_k) = \varrho_i T_i x_{k+1}^{\mathrm{T}}(J_i(x_k) - R_i\bigl(\|x_k\|\bigr))(x_k + x_{k+1}) \\ + T_i x_{k+1}^{\mathrm{T}}(\varrho_i K_i - \frac{1}{T} I_n)x_k.$$

Noticing the skew-symmetry of the matrix $J_i(x_k)$, we can get $x_{k+1}^{\mathrm{T}} J_i(x_k) x_{k+1} = 0$. This implies

$$x_{k+1}^{\mathrm{T}} x_{k+1} + T_i x_{k+1}^{\mathrm{T}} R_i\bigl(\|x_k\|\bigr)x_{k+1} \\ = x_{k+1}^{\mathrm{T}} x_k + \varrho_i T_i x_{k+1}^{\mathrm{T}}(J_i(x_k) - R_i\bigl(\|x_k\|\bigr))x_k \\ + \varrho_i T_i x_{k+1}^{\mathrm{T}} K_i x_k - x_{k+1}^{\mathrm{T}} x_k.$$

According to the semipositive definitiveness of the matrix $R(x_k)$, we can get that $x_{k+1}^{\mathrm{T}} R_i(x_k) x_{k+1} \geqslant 0$. By calculating the difference $V_i^{1/2}(x_{k+1}) - V_i^{1/2}(x_k)$ the following inequality can be obtained:

$$\begin{aligned} &V_i^{1/2}(x_{k+1}) - V_i^{1/2}(x_k) \\ &\quad\leqslant \sqrt{\varrho_i}\bigl\|\varrho_i T_i\bigl(J_i(x_k) - R_i\bigl(\|x_k\|\bigr)\bigr)x_k + \varrho_i T_i K_i x_k\bigr\| - \sqrt{\varrho_i}\|x_k\| \\ &\quad\leqslant \sqrt{\varrho_i}\bigl\|\varrho_i T_i x_k^{\mathrm{T}}\bigl(J_i(x_k) - R_i\bigl(\|x_k\|\bigr)\bigr)x_k + \varrho_i T_i x_k^{\mathrm{T}} K_i x_k\bigr\| \cdot \|x_k\|^{-1} - \sqrt{\varrho_i}\|x_k\| \\ &\quad\leqslant 2\sqrt{\varrho_i}\|x_k\|\bigl(2 + \|x_k\|\bigr)^{-1} - \sqrt{\varrho_i}\|x_k\| \\ &\quad\leqslant -\frac{\sqrt{\varrho_i}}{2}\|x_k\|\sqrt{\varrho_i}\|x_{k+1}\| \leqslant -\frac{1}{2}\bigl(V_i(x_k)V_i(x_{k+1})\bigr)^{1/2}, \end{aligned}$$

which implies that the ith subsystem satisfies

$$V_i^{1/2}(x_{k+1}) - V_i^{1/2}(x_k) \leqslant -\frac{1}{2}\bigl(V_i(x_k) \cdot V_i(x_{k+1})\bigr)^{1/2}.$$

When $k = N_l$, according to impulsive function (4), the following equation can be obtained:

$$x_{N_l+1} = g_i(x_{N_l}) = d_i x_{N_l}.$$

Combining with (16), we can obtain

$$V_i^{1/2}(x_{N_l+1}) = \sqrt{\varrho_i}\|d_i x_{N_l}\|, \qquad V_j^{1/2}(x_{N_l}) = \sqrt{\varrho_j}\|x_{N_l}\|.$$

Next, we get

$$V_i^{1/2}(x_{N_l+1}) \leqslant \sqrt{\varrho_i}\|d_i x_{N_l}\| \leqslant \frac{\sqrt{\varrho_i}|d_i|}{\sqrt{\varrho_j}} V_j^{1/2}(x_{N_l}) \leqslant \sqrt{b} V_j^{1/2}(x_{N_l}),$$

where $b = \max_{1\leqslant i\leqslant M}\{b_i, 1\}$ with $b_i = \varrho_i d_i^2/\varrho_j$. Thus, for any $i, j \in Q$, the following inequality is satisfied:

$$V_i(x_{N_l+1}) \leqslant bV_j(x_{N_l}).$$

According to Lemma 1, it is effortless to derive that system (1) is practical FT stable. In this case, we choose $c = 1/2$, $q = 1/2$, and $\tilde{T}(\varepsilon, x_0)$ satisfies

$$\tilde{T}(\varepsilon, x_0) \leqslant \tilde{T}_{\max} = L(\theta_{\max} + 1) + 1 = L\left(\frac{2b}{\varepsilon\sqrt{\varrho_{\min}}} + 1\right) + 1,$$

where ε is a given constant, $\varrho_{\min} = \min_{1\leqslant i\leqslant M}\{\varrho_i\}$, and L is a given positive integer. □

Remark 7. L is a given positive integer, and the value of L is related to the number of switches between different subsystems. Specifically, the value of L is less than the number of switches. In fact, during the proof of Lemma 1, such condition is needed to obtain $\alpha(\|x_k\|) \leqslant V_i(x_k) \leqslant \alpha(\varepsilon)$ with ε being a given positive constant, and then one can derive that $\|x_k\| \leqslant \varepsilon$ after a fixed-time interval.

4 Two illustrative examples

The effectiveness of presented methods for practical FT stabilization control of DPCH systems and DISPCH systems is proven by two simulation examples, respectively.

Example 1. Consider the following DPCH system:

$$x_{k+1} - x_k = T\big(J(x_k) - R(\|x_k\|)\big)\bar{\nabla}H(x_k, x_{k+1}) + TGu_k, \tag{18}$$

where $x_k = [x_k^1, x_k^2]^{\mathrm{T}} \in \mathbb{R}^2$ is system's state, $u_k = [u_k^1, u_k^2]^{\mathrm{T}} \in \mathbb{R}^2$ represents system's control input, T is the sampling period, and

$$H(x_k) = \frac{1}{2}\|x_k\|^2, \quad \bar{\nabla}H(x_k, x_{k+1}) = \frac{1}{2}(x_k + x_{k+1}),$$
$$J(x_k) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R(\|x_k\|) = \begin{bmatrix} 3 & 0 \\ 0 & \frac{2}{2+\|x_k\|} + 2 \end{bmatrix}.$$

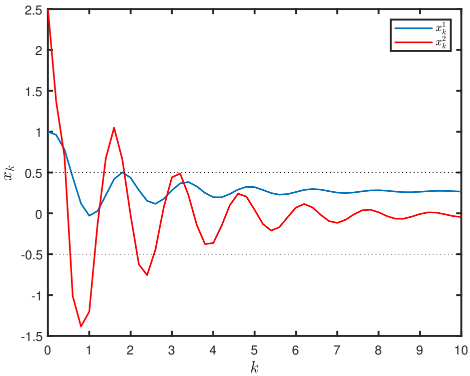

Figure 1. State responses x_k of (18).

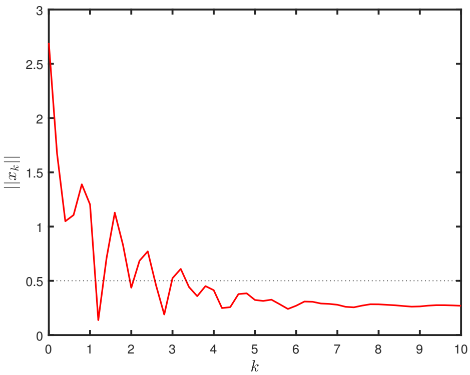

Figure 2. The norm of x_k of (18).

According to Theorem 1, we choose $T = 1/4$, $\varepsilon = 0.5$, and

$$K = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}.$$

From (9) the practical FT controller is

$$\begin{aligned} u_k &= (G^T G)^{-1} G^{\mathrm{T}} \left(a \cdot K - \frac{1}{\mathrm{T}} I_n \right) \bar{\nabla} H(x_k, x_{k+1}) \\ &= \begin{bmatrix} -\frac{5}{2} & 0 \\ 0 & -3 \end{bmatrix} x_k = \begin{bmatrix} -\frac{5}{2} x_k^1 \\ -3x_k^2 \end{bmatrix}. \end{aligned}$$

Meanwhile, $\tilde{T}(\varepsilon, x_0)$ satisfies

$$\tilde{T}(\varepsilon, x_0) \leqslant \tilde{T}_{\max} = \frac{2}{\varepsilon} = 4.$$

We select the initial condition as $x_0 = [1,\ 0.7]^{\mathrm{T}}$. Figures 1 and 2 respectively display the state responses and state norm for DPCH system (18). It can be obtained from Fig. 2 that state norm of DPCH system (18) satisfies $\|x_k\| \leqslant 0.5$ approximately 3.5 s later, which means that the state of system (18) converges to the attractive region $\{x_k\colon \|x_k\| \leqslant \varepsilon\}$ and remain inside. Thus, simulation results display that this presented method has good practical FT stability performance.

Example 2. Consider the following DISPCH system:

$$\begin{aligned} x_{k+1} - x_k &= T_p \big(J_p(x_k) - R_p \big(\|x_k\| \big) \big) \bar{\nabla} H_p(x_k, x_{k+1}) \\ &\quad + T G_p u_{p,k}, \quad k \neq N_1, \\ x_{k+1} &= g_p(x_k), \quad k = N_1,\ p = 1, 2, \end{aligned} \tag{19}$$

where $x_k = [x_k^1, x_k^2]^{\mathrm{T}} \in \mathbb{R}^2$ is the state, $u_{p,k} = [u_{p,k}^1, u_{p,k}^2]^{\mathrm{T}} \in \mathbb{R}^2$ is the control input, $\{N_l\}$ with $l = 0, 1, 2, \dots$ is the switching time sequence, and

$$H_1(x_k) = H_2(x_K) = \frac{1}{2} \|x_k\|^2,$$

$$\bar{\nabla} H_1(x_k, x_{k+1}) = \bar{\nabla} H_2(x_k, x_{k+1}) = \frac{1}{2}(x_k + x_{k+1}),$$

$$g_1(x_k) = \frac{2}{3}x_k, \qquad J_1(x_k) = \begin{bmatrix} 0 & \frac{1}{4} \\ -\frac{1}{4} & 0 \end{bmatrix},$$

$$g_2(x_k) = \frac{1}{2}x_k, \qquad J_2(x_k) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

$$R_1\big(\|x_k\|\big) = \begin{bmatrix} \frac{4}{3+\|x_k\|} + 2 & 1 \\ 1 & 4 \end{bmatrix}, \qquad R_2\big(\|x_k\|\big) = \begin{bmatrix} 3 & 0 \\ 0 & \frac{2}{2+\|x_k\|} + 2 \end{bmatrix},$$

$$G_1 = G_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

According to Theorem 2, we choose $T_1 = T_2 = 1/4$, $\varepsilon = 0.5$, and

$$K_1 = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}.$$

From (15) the practical FT controller of the first subsystem is

$$\begin{aligned} u_{1,k} &= \big(G_1^{\mathrm{T}} G_1\big)^{-1} G_1^{\mathrm{T}} \left(\varrho_1 \cdot K_1 - \frac{1}{T_1} I_n\right) x_k \\ &= \left(\frac{1}{2} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} - 4 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{bmatrix} x_k^1 \\ x_k^2 \end{bmatrix} = \begin{bmatrix} -3x_k^1 + \frac{1}{2}x_k^2 \\ -2x_k^2 + \frac{1}{2}x_k^1 \end{bmatrix}, \end{aligned}$$

the practical FT controller of the second subsystem is

$$\begin{aligned} u_{2,k} &= \big(G_2^{\mathrm{T}} G_2\big)^{-1} G_2^{\mathrm{T}} \left(\varrho_2 \cdot K_2 - \frac{1}{T_2} I_n\right) x_k \\ &= \left(\frac{1}{2} \cdot \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} - 4 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) \begin{bmatrix} x_k^1 \\ x_k^2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2}x_k^1 \\ -3x_k^2 \end{bmatrix}. \end{aligned}$$

Noticing $d_1 = 2/3$, $d_2 = 1/2$, $\varrho_1 = \varrho_2 = 1/2$, $\varrho_{\min} = 1/2$, by choosing $L = 4$, $\varepsilon = 1$, we have $b_1 = 4/9$, $b_2 = 1/4$, $b = \max_{1 \leqslant i \leqslant 2}\{b_i, 1\} = 1$, and $Z_L = \sum_{j=1}^{4} 1^{j/2} = 4$. According to (14), we further get

$$\theta_{\min} = \frac{2}{\sqrt{\varrho_{\min}} Z_L \varepsilon} = \frac{2 \cdot \sqrt{2}}{\sum_{j=1}^{4} 1^{j/2} \cdot 0.5} = \sqrt{2},$$

$$\theta_{\max} = \frac{2b}{\sqrt{\varrho_{\min}} \varepsilon} = \frac{2 \cdot 1 \cdot \sqrt{2}}{0.5} = 4\sqrt{2}.$$

The setting time function is

$$\tilde{T}(\varepsilon, x_0) \leqslant \tilde{T}_{\max} = L(\theta_{\max} + 1) + 1 = 4 \cdot (4\sqrt{2} + 1) + 1 = 16\sqrt{2} + 5.$$

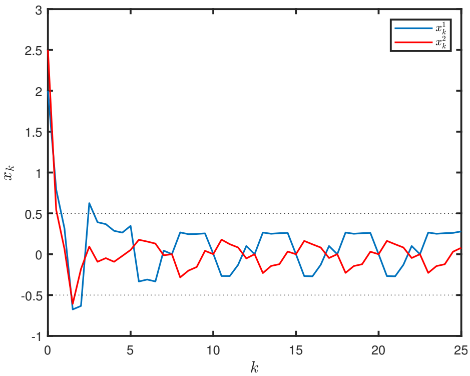

Figure 3. State responses x_k of (19).

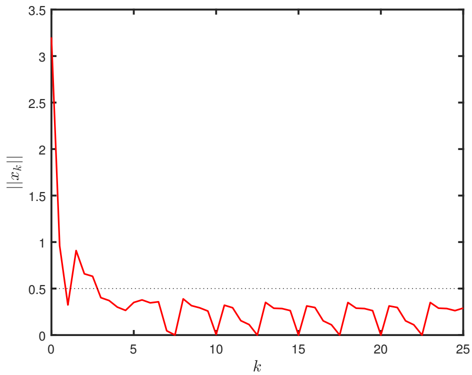

Figure 4. The norm of x_k of (19).

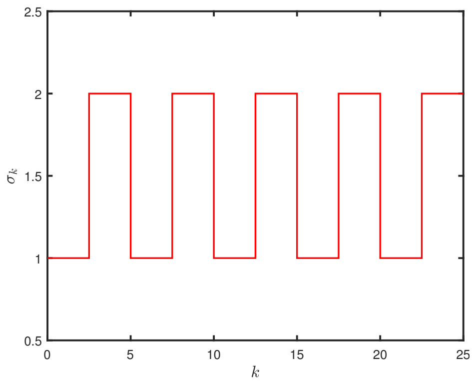

Figure 5. Switching signal σ_k.

Let $N_{l+1} - N_l - 1 = T_l \in [\theta_{\min}, \theta_{\max}]$, $l = 0, 1, \dots$. Figure 5 shows the switching signal σ_k of system (19). We select the initial condition as $x_0 = [2\,2.5]^{\mathrm{T}}$. Figures 3 and 4 respectively show state responses and state norm of DISPCH system (18). We can obtain from Fig. 4 that state norm of the DISPCH system (19) satisfies $\|x_k\| \leqslant 0.5$ approximately 3 s later, which means that the state of system (19) converges to the attractive region $\{x_k\colon \|x_k\| \leqslant \varepsilon\}$ and remains inside. Thus, the simulation results confirm that the presented method is competent for stabilizing DISPCH systems.

5 Conclusion

The practical FT stabilization problem of DPCH systems and DISPCH systems have been discussed, respectively. Starting with a class of DPCH systems, a novel controller is presented to make the DPCH systems practical FT stable. Using positive-order Lyapunov functions method and range dwell time technique, another novel controller is presented to make the DISPCH systems practical FT stable. Specific simulation results illustrate the validity of the presented methods. When using the presented method, the settling time functions have an upper bound, which is unrelated to initial states. Sometimes, the upper bound of setting-time function may be over estimated. Hence, further work needs to be done to reduce the conservativeness in estimating the upper bound.

References

1. F. Amato, R. Ambrosino, C. Cosentino, D. Tommasi, Finite-time stabilization of impulsive dynamical linear systems, *Nonlinear Anal., Hybrid Syst*, **5**(1):89–101, 2011, https://doi.org/10.1016/j.nahs.2010.10.001.
2. S.P. Bhat, D.S. Bernstein, Continuous finite-time stabilization of the translational and rotational double integrators, *IEEE Trans. Autom. Control*, **43**(5):678–682, 1998, https://doi.org/10.1109/9.668834.
3. G. Chen, Y. Yang, F. Deng, On practical fixed-time stability of discrete-time impulsive switched nonlinear systems, *Int. J. Robust Nonlinear Control*, **30**(17):7822–7834, 2020, https://doi.org/10.1002/rnc.5216.
4. J. Chen, C. Dou, L. Xiao, Z. Wang, Fusion state estimation for power systems under DoS attacks: A switched system approach, *IEEE Trans. Syst. Man Cybern.*, **49**(8):1679–1687, 2019, https://doi.org/10.1109/TSMC.2019.2895912.
5. Y. Chen, Z. Wang, J. Hu, Q.-L. Han, Synchronization control for discrete-time-delayed dynamical networks with switching topology under actuator saturations, *IEEE Trans. Neural Networks Learn. Syst.*, **32**(5):2040–2053, 2021, https://doi.org/10.1109/TNNLS.2020.2996094.
6. F.G. Cordoni, L.D. Persio, R. Muradore, Discrete stochastic port-Hamiltonian systems, *Automatica*, **137**:110122, 2022, https://doi.org/10.1016/j.automatica.2021.110122.
7. Z. Guan, D.J. Hill, X. Shen, On hybrid impulsive and switching systems and application to nonlinear control, *IEEE Trans. Autom. Control*, **50**(7):1058–1062, 2005, https://doi.org/10.1109/TAC.2005.851462.
8. Ö. Karabacak, A. Kıvılcım, R. Wisniewski, Almost global stability of nonlinear switched systems with time-dependent switching, *IEEE Trans. Autom. Control*, **65**(7):2969–2978, 2020, https://doi.org/10.1109/TAC.2019.2927934.
9. P. Kotyczka, L. Lefèvre, Discrete-time port-Hamiltonian systems: A definition based on symplectic integration, *Syst. Control Lett.*, **133**:104530, 2019, https://doi.org/10.1016/j.sysconle.2019.104530.
10. L. Li, C. Li, W. Zhang, Time-delayed impulsive control for discrete-time nonlinear systems with actuator saturation, *Nonlinear Anal. Model. Control*, **24**(5):804–818, 2019, https://doi.org/10.15388/NA.2019.5.7.
11. X. Liu, X. Liao, Fixed-time stabilization control for port-Hamiltonian systems, *Nonlinear Dyn.*, **96**(2):1497–1509, 2019, https://doi.org/10.1007/s11071-019-04867-0.
12. X. Liu, X. Liao, Fixed-time H_∞ control for port-controlled Hamiltonian systems, *IEEE Trans. Autom. Control*, **64**(7):2753–2765, 2020, https://doi.org/10.1109/TAC.2018.2874768.

13. X. Lv, Y. Niu, J. Song, Finite-time boundedness of uncertain Hamiltonian systems via sliding mode control approach, *Nonlinear Dyn.*, **104**:497–507, 2021, https://doi.org/10.1007/s11071-021-06292-8.
14. W.-X. Ma, Global behavior of a higher-order nonlinear difference equation with many arbitrary multivariate functions, *East Asian J. Appl. Math.*, **9**(4):643–650, 2019, https://doi.org/10.4208/eajam.140219.070519.
15. W.-X. Ma, Global behavior of a new rational nonlinear higher-order difference equation, *Complexity*, **2019**:2048941, 2019, https://doi.org/10.1155/2019/2048941.
16. W.-X. Ma, Global behavior of an arbitrary-order nonlinear difference equation with a nonnegative function, *Mathematics*, **8**(5):825, 2020, https://doi.org/10.3390/math8050825.
17. M. Philippe, R. Essick, G.E. Dullerud, R.M. Jungers, Stability of discrete-time switching systems with constrained switching sequences, *Automatica*, **72**:242–250, 2016, https://doi.org/10.1016/j.automatica.2016.05.015.
18. A. Polyakov, Nonlinear feedback design for fixed-time stabilization of linear control systems, *IEEE Trans. Autom. Control*, **57**(8):2106–2110, 2012, https://doi.org/10.1109/TAC.2011.2179869.
19. A. Polyakov, D. Efimov, W. Perruquetti, Finite-time and fixed-time stabilization: Implicit Lyapunov function approach, *Automatica*, **51**:332–340, 2015, https://doi.org/10.1016/j.automatica.2014.10.082.
20. R. Reyes-Báez, A.J. van der Schaft, B. Jayawardhana, A family of virtual contraction based controllers for tracking of flexible-joints port-Hamiltonian robots: Theory and experiments, *Int. J. Robust Nonlinear Control*, **30**(8):3269–3295, 2020, https://doi.org/10.1002/rnc.4929.
21. Y. Su, J. Huang, Stability of a class of linear switching systems with applications to two consensus problems, *IEEE Trans. Autom. Control*, **57**(6):1420–1430, 2011, https://doi.org/10.1109/TAC.2011.2176391.
22. L. Sun, G. Feng, Y. Wang, Finite-time stabilization and H_∞ control for a class of nonlinear Hamiltonian descriptor systems with application to affine nonlinear descriptor systems, *Automatica*, **50**(8):2090–2097, 2014, https://doi.org/10.1016/j.automatica.2014.05.031.
23. W. Sun, X. Lv, Practical finite-time fuzzy control for Hamiltonian systems via adaptive event-triggered approach, *Int. J. Fuzzy Syst.*, **22**(1):35–45, 2020, https://doi.org/10.1007/s40815-019-00773-0.
24. W. Sun, L. Peng, Observer-based robust adaptive control for uncertain stochastic Hamiltonian systems with state and input delays, *Nonlinear Anal. Model. Control*, **19**(4):626–645, 2014, https://doi.org/10.15388/NA.2014.4.8.
25. W. Sun, Z. Wang, X. Lv, F.E. Alsaadi, H. Liu, H_∞ fusion estimation for uncertain discrete time-delayed Hamiltonian systems with sensor saturations: An event-triggered approach, *Inf. Fusion*, **86**:93–103, 2022, https://doi.org/10.1016/j.inffus.2022.06.004.
26. V. Talasila, J. Clemente-Gallardo, A.J. van der Schaft, Discrete port-Hamiltonian systems, *Syst. Control Lett.*, **55**(6):478–486, 2006, https://doi.org/10.1016/j.sysconle.2005.10.001.

27. A.J. van der Schaft, B.M. Maschke, Port-Hamiltonian systems on graphs, *SIAM J. Control Optim.*, **51**(2):906–937, 2013, https://doi.org/10.1137/110840091.

28. A. Yaghmaei, M.J. Yazdanpanah, Output control design and separation principle for a class of port-Hamiltonian systems, *Int. J. Robust Nonlinear Control*, **29**(4):867–881, 2019, https://doi.org/10.1002/rnc.4407.

29. H. Yang, B. Jiang, V. Cocquempot, H. Zhang, Stabilization of switched nonlinear systems with all unstable modes: Application to multi-agent systems, *IEEE Trans. Autom. Control*, **56**(9):2230–2235, 2011, https://doi.org/10.1109/TAC.2011.2157413.

30. R. Yang, Y. Wang, Finite-time stability analysis and H_∞ control for a class of nonlinear time-delay Hamiltonian systems, *Automatica*, **49**(2):390–401, 2013, https://doi.org/10.1016/j.automatica.2012.11.034.

31. D. Yu, W. Sun, X. Chen, M. Du, Anti-saturation coordination control of permanent magnet synchronous wind power system, *IEEE Access*, **11**:33428–33441, 2023, https://doi.org/10.1109/ACCESS.2023.3263481.

32. Q. Zhang, W. Sun, C. Qiao, Event-triggered stabilisation of switched nonlinear systems with actuator saturation: A Hamiltonian approach, *Int. J. Syst. Sci.*, **54**(4):849–866, 2023, https://doi.org/10.1080/00207721.2022.2147279.