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Finite-time stabilization of discontinuous fuzzy inertial Cohen–Grossberg neural networks with mixed time-varying delays*

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Abstract. This article aims to study a class of discontinuous fuzzy inertial Cohen–Grossberg neural networks (DFICGNNs) with discrete and distributed time-delays. First of all, in order to deal with the discontinuities by the differential inclusion theory, based on a generalized variable transformation including two tunable variables, the mixed time-varying delayed DFICGNN is transformed into a first-order differential system. Then, by constructing a modified Lyapunov–Krasovskii functional concerning with the mixed time-varying delays and designing a delayed feedback control law, delay-dependent criteria formulated by algebraic inequalities are derived for guaranteeing the finite-time stabilization (FTS) for the addressed system. Moreover, the settling time is estimated. Some related stability results on inertial neural networks is extended. Finally, two numerical examples are carried out to verify the effectiveness of the established results.

Keywords: inertial Cohen–Grossberg neural networks, fuzzy logics, discrete and distributed time-varying delays, Lyapunov–Krasovskii functional, finite-time stabilization.

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1 Introduction

1.1 Previous works

In 1986, Babcock et al. [3] proposed the inertial neural networks (INNs) for the first time. Neural networks with inertial items have been successfully applied to chaos and bifurcation control [23]. Since the states of such inertial systems are of second-order derivatives, the corresponding dynamic behaviors are more complicated to deal with compared to systems with first-order derivative of states [15]. So, stability analysis of the INNs is necessary. In recent years, stability of INNs model and its generalizations have been widely considered; see [7, 13, 19–21].

The concept of finite-time stabilization (FTS) proposed by Haimo [6] means that the solutions of the system reach the equilibrium point in finite time. The time function indicating when the trajectories reach the equilibrium point, variously known as the settling-time, has a great importance in practice. FTS is of major interest to many applications such as secure communications [17] or finite-time attitude tracking for spacecrafts [4]. Time delays, especially, the time-varying delays may turn expected dynamics of the proposed neural network into some undesired complex dynamical behaviors. So, the FTS analysis for the time-delayed system will be difficult. This thanks to the pioneer work of Moulay et al. [16]. After that, on the basis work of Moulay, during the past several years, many efforts have been devoted to the delayed neural networks. See, to name a few, [1, 27, 28].

Despite of many FTS results on the delayed neural networks, there is few work on FTS of INNs with discontinuous activations though the discontinuous phenomenon often occur in neural networks; see [22]. Moreover, it is well known that time delays are often inevitable and time-varying delays may turn expected dynamics of the proposed neural network into some undesired complex dynamical behaviors. In reality, discrete (time-varying) delay and distributed delay always occur simultaneously. In general, the results of the stability analysis and synchronization analysis for delayed neural networks contain delay-dependent and delay-independent criteria. However, the former can derive less conservativeness and take more advantages in the practical applications. For more details, see [9]. But there are few delay-dependent criteria derived for the delayed INNs and few delay-dependent criteria ensuring the FTS of time-varying delayed INNs have been derived.

Hence, it is meaningful to further propose a new framework and study the FTS of the discontinuous INNs with mixed time-varying delays and derive some new delay-dependent criteria to ensure the FTS of discontinuous INNs with mixed time-varying delays. This is the first key purpose.

On the other hand, some inconveniences can inevitably be encountered in the mathematical modeling of practical problems, for example, the uncertainty, the approximation and the vagueness. Fuzzy logic systems can approximate any nonlinear functions. During the past several years, based on the pioneer work of Yang and Yang [25] in 1996, stability analysis of fuzzy neural networks with delays were extensively considered by researchers; see [10–12, 18] and the references therein. However, the best of authors knowledge, there is only few research that investigated the fuzzy INNs (see [8, 24]), not only that the fuzzy inertial Cohen–Grossberg neural networks. Still, the results established in [8,24] are based on the delay-dependent criteria.

Thus, how to take the fuzzy logics into account and further derive some new delay-dependent criteria to guarantee the FTS of DFICGNNs with mixed time delays is the second key purpose.

Based on the pioneer works and addressed two key purposes mentioned above, in this paper, we aim to investigate the FTS of DFICGNNs with mixed time delays via discontinuous state-feedback controllers. Our works mainly aim to put forward to some new delay-dependent criteria for the proposed DFICGNNs and put forward a new approach to further study the dynamic behaviors of fuzzy INNs.

1.2 Major contributions

In contrast to the previous works on the INNs, the major contributions of this paper are reflected in the subsequent key aspects:

- Taking the inertial items, fuzzy logics, CG terms, discontinuous activations and discrete and distributed time-varying delays into consideration, the consider mixed time-vary delayed DFICGNN is a more general case compared to the continuous INNs without fuzzy logics [13, 14] and the fuzzy INNs without mixed time-varying delays [8, 24].
- A mixed-time-varying delayed feedback control law is designed, which can help achieve FTS effectively. Compared with the previous designed delayed feedback control, which can only cope with the discrete delays and discrete time-varying delays, it takes more advantages.
- Some new delay-dependent criteria, which possess less conservativeness, are derived, which can further illustrate that the delays can affect the FTS of the neural system.

2 System description and preliminaries

2.1 System description

Consider the following DFICGNNs with mixed time delays:

$$\begin{aligned}
\ddot{x}_i(t) = {}& -a_i(t)\dot{x}_i(t) - b_i\big(t, x_i(t)\big)\Bigg[k_i\big(t, x_i(t)\big) - \sum_{j=1}^{n} d_{ij}\nu_j - \sum_{j=1}^{n} c_{ij}(t) f_j\big(x_j(t)\big) \\
& - \sum_{j=1}^{n} h_{ij}(t) f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) - \bigwedge_{j=1}^{n} T_{ij}\nu_j - \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\
& - \bigwedge_{j=1}^{n} \rho_{ij}(t) \int\limits_{t-\delta_{ij}(t)}^{t} f_j\big(x_j(s)\big)\,\mathrm{d}s - \bigvee_{j=1}^{n} \beta_{ij}(t) f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\
& - \bigvee_{j=1}^{n} \omega_{ij}(t) \int\limits_{t-\delta_{ij}(t)}^{t} f_j\big(x_j(s)\big)\,\mathrm{d}s - \bigvee_{j=1}^{n} S_{ij}\nu_j - I_i(t)\Bigg],
\end{aligned} \tag{1}$$

where $i, j \in \mathbb{I} \triangleq \{1, 2, \ldots, n\}$, $n \geqslant 2$ is the number of neurons in the network, $x_i(t)$ denotes the state of the ith unit at time t, the second derivative is called an inertial term of system (1). $a_i(t) > 0$ are damping coefficient; $b_i(\cdot)$ denotes an amplification function; $k_i(\cdot)$ is the behaved function; c_{ij} is the elements of feedback templates; h_{ij} is the elements of feed-forward templates; α_{ij}, ρ_{ij} and ω_{ij}, β_{ij} are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, respectively; T_{ij} and S_{ij} are fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively; ν_j and I_i denote input and bias of the jth and ith neuron, respectively; f_j are the activation functions, which are assumed to be discontinuous; $\tau_{ij}(t)$ and $\delta_{ij}(t)$ correspond to the discrete time-varying delay and the distributed time-varying delay at time t satisfying $0 \leqslant \tau_{ij}(t) \leqslant \tau$ and $0 \leqslant \delta_{ij}(t) \leqslant \delta$, where $\tau = \max_{1 \leqslant i,j \leqslant n} \sup_{t \in \mathbb{R}} |\tau_{ij}(t)|$ and $\delta = \max_{1 \leqslant i,j \leqslant n} \sup_{t \in \mathbb{R}} |\delta_{ij}(t)|$, τ and δ are nonnegative constants. Let $\xi = \max\{\tau, \delta\}$.

The initial conditions of system (1) are

$$x_i(s) = \phi_i^x(s), \quad \dot{x}_i(s) = \psi_i^x(s), \quad s \in [-\xi, 0].$$

Throughout the paper, we always use $i, j \in \mathbb{I}$, unless otherwise stated.

Remark 1. The proposed neural system includes the inertial items, fuzzy logics, CG terms, discontinuous activations and discrete and distributed time-varying delays. Thus, the presented results are obtained in a more general framework and are more practical than the aforementioned previous results cited in the references such as [8, 13, 14, 24].

Design the following generalized variable transformation:

$$y_i(t) = \mu_i \dot{x}_i(t) + \omega_i x_i(t), \tag{2}$$

where μ_i and ω_i are positive constants. Then system (1) can be rewritten as

$$\begin{cases} \dot{x}_i(t) = -\frac{\omega_i}{\mu_i} x_i(t) + \frac{1}{\mu_i} y_i(t), \\ \dot{y}_i(t) = -\widetilde{a}_i(t) y_i(t) + \omega_i \widetilde{a}_i(t) x_i(t) - \mu_i b_i(t, x_i(t)) \big[k_i\big(t, x_i(t)\big) \\ \qquad - \sum_{j=1}^n d_{ij}\nu_j - \sum_{j=1}^n c_{ij}(t) f_j\big(x_j(t)\big) \\ \qquad - \sum_{j=1}^n h_{ij}(t) f_j\big(x_j\big(t - \tau_{ij}(t)\big)\big) \\ \qquad - \bigwedge_{j=1}^n T_{ij}\nu_j - \bigwedge_{j=1}^n \alpha_{ij}(t) f_j\big(x_j\big(t - \tau_{ij}(t)\big)\big) \\ \qquad - \bigwedge_{j=1}^n \rho_{ij}(t) \int_{t-\delta_{ij}(t)}^t f_j\big(x_j(s)\big)\,\mathrm{d}s - \bigvee_{j=1}^n S_{ij}\nu_j \\ \qquad - \bigvee_{j=1}^n \beta_{ij}(t) f_j\big(x_j\big(t - \tau_{ij}(t)\big)\big) \\ \qquad - \bigvee_{j=1}^n \omega_{ij}(t) \int_{t-\delta_{ij}(t)}^t f_j\big(x_j(s)\big)\,\mathrm{d}s - I_i(t)\big], \end{cases} \tag{3}$$

where $\widetilde{a}_i(t) = a_i(t) - \omega_i/\mu_i$.

The initial conditions of system (3) are

$$x_i(s) = \phi_i^x(s), \quad y_i(s) = \mu_i \psi_i^x(s) + \omega_i \phi_i^x(s) \triangleq \varphi_i^y(s), \quad s \in [-\xi, 0].$$

Remark 2. Two tunable variables μ_i, ω_i are introduced instead of one variable to express the transformation. Currently, lots of previous results for INNs are obtained based on variable transformation with $\mu_i = 1$ or $\mu_i = \omega_i = 1$. In order to further reduce the conservativeness, as verified in [8], two free-weight coefficients μ_i, ω_i can be introduced to the transformation.

Next, the controlled DFICGNNs are obtained as

$$\begin{cases} \dot{x}_i(t) = -\frac{\omega_i}{\mu_i} x_i(t) + \frac{1}{\mu_i} y_i(t), \\ \dot{y}_i(t) = -\widetilde{a}_i(t) y_i(t) + \omega_i \widetilde{a}_i(t) x_i(t) - \mu_i b_i\big(t, x_i(t)\big)\big[k_i\big(t, x_i(t)\big) \\ \qquad - \sum_{j=1}^n d_{ij}\nu_j - \sum_{j=1}^n c_{ij}(t) f_j\big(x_j(t)\big) \\ \qquad - \sum_{j=1}^n h_{ij}(t) f_j\big(x_j\big(t - \tau_{ij}(t)\big)\big) \\ \qquad - \bigwedge_{j=1}^n T_{ij}\nu_j - \bigwedge_{j=1}^n \alpha_{ij}(t) f_j\big(x_j\big(t - \tau_{ij}(t)\big)\big) \\ \qquad - \bigwedge_{j=1}^n \rho_{ij}(t) \int_{t-\delta_{ij}(t)}^t f_j\big(x_j(s)\big)\,\mathrm{d}s - \bigvee_{j=1}^n S_{ij}\nu_j \\ \qquad - \bigvee_{j=1}^n \beta_{ij}(t) f_j\big(x_j\big(t - \tau_{ij}(t)\big)\big) \\ \qquad - \bigvee_{j=1}^n \omega_{ij}(t) \int_{t-\delta_{ij}(t)}^t f_j\big(x_j(s)\big)\,\mathrm{d}s - I_i(t)\big] + u_i(t), \end{cases} \tag{4}$$

where $u_i(t)$ are the feedback control laws to be designed later.

Throughout this paper, we assume that the activation functions satisfy the following conditions:

(A1) For each $i \in \mathbb{I}$, $f_i : \mathbb{R} \to \mathbb{R}$ is piecewise continuous.

(A2) For each $i \in \mathbb{I}$, there exist nonnegative constants $\mathcal{A}_i$, $\mathcal{B}_i$ such that $\sup_{\gamma_i \in K[f_i](x_i)} |\gamma_i| \leqslant \mathcal{A}_i |x_i| + \mathcal{B}_i$ for all $x_i \in \mathbb{R}$.

Here $K[f_j](u) \triangleq \bigcap_{\delta>0} \bigcap_{\mu(\mathbb{N})=0} \overline{\mathrm{co}}[f_j(t, B(u,\delta) \setminus \mathbb{N})]$, $\overline{\mathrm{co}}[E]$ denotes the closure of the convex hull of set E, $\mu(\mathbb{N})$ denotes the Lebesgue measure of set $\mathbb{N}$, and $B(u,\delta)$ is the open ball with the center at $u \in \mathbb{R}$ and the radius $\delta \in \mathbb{R}$.

2.2 Basic definitions and lemmas

Notations. Let $\mathbb{R}$ be the space of real number, $\mathbb{R}_+$ be the set of all nonnegative real numbers and $\mathbb{R}^n$ denote the n-dimensional Euclidean space. Consider the column vectors $x = (x_1, x_2, \dots, x_n)^\top \in \mathbb{R}^n$ and $\|x\| = \sqrt{x^\top x}$, where the superscript $\top$ represents the transpose operator. A continuous function $\nu : \mathbb{R} \to \mathbb{R}$ belongs to the class K if it is strictly increasing and $\nu(0) = 0$. $C([a,b], \mathbb{R}^n)$ denotes the space of all continuous functions $\varphi : [a,b] \to \mathbb{R}^n$ with uniform norm $\|\varphi\| = \sup_{a \leqslant t \leqslant b} |\varphi(t)|$. $\mathrm{sgn}(\cdot)$ denote the sign function.

Define $f^+ = \sup_{t\in\mathbb{R}} |f(t)|$, $f^- = \inf_{t\in\mathbb{R}} |f(t)|$, where $f(t)$ is a bounded and continuous function. Let Λ be an open subset of $C([-\xi, 0], \mathbb{R}^n)$ containing 0.

Definition 1. (See [16].) The origin of system (4) is finite-time stable, where $u_i(t) = 0$, if

(i) The origin of system (4) is stable;

(ii) The origin of system (4) is finite-time convergent, i.e., for any initial state $\varphi(s) \in \Lambda$, there exists $0 \leqslant T(\varphi) < +\infty$ such that every solution of system (1) satisfies $x(t,\varphi) = 0$ for all $t \geqslant T(\varphi)$.

The functional $T_0(\varphi) = \inf\{T(\varphi) \geqslant 0\colon\ x(t,\varphi) = 0\ \forall t \geqslant T(\varphi)\}$ is called the settling-time of system.

Lemma 1. (*See* [16].) *Let there exist a continuous function* $V : [0,+\infty) \times \Lambda \to \mathbb{R}_+$ *and two functions* ν, r *of class* K *for the controlled system* (4) *such that*

(i) $V(t,0) = 0$, $\nu(\|x\|) \leqslant V(t,x)$, $t \in [0,+\infty)$;
(ii) $D^+V(t,x) \leqslant -r(V(t,x))$ *with* $\int_0^\varepsilon \mathrm{d}z/r(z) < +\infty$ *for all* $\varepsilon > 0$, $x \in \Lambda$.

Then system (4) *is finite-time stable with a settling time satisfying the inequality* $T_0(\varphi) \leqslant \int_0^{V(0,\varphi)} \mathrm{d}z/r(z)$. *In particular, if* $r(V) = \lambda V^\rho$, *where* $\lambda > 0$, $\rho \in (0,1)$, *then the settling time satisfies the inequality*

$$T_0(\varphi) \leqslant \int\limits_0^{V(0,\varphi)} \frac{\mathrm{d}z}{r(z)} = \frac{V^{1-\rho}(0,\varphi)}{\lambda(1-\rho)}.$$

Lemma 2. (*See* [5, *Chain Rule*].) *Assume that* $V(x)$ *is* C*-regular* (*regular, positive definite and radially unbounded*) *and* $y(t)$ *is absolutely continuous on any compact subinterval of* $[0,+\infty)$. *Then* $V(x(t))$ *and* $x(t)$ *are differentiable for almost everywhere* $t \in [0,+\infty)$, *and* $\mathrm{d}V(x(t))/\mathrm{d}t = \xi^\top \dot{x}(t)$ *for all* $\xi \in \partial V\big(x(t)\big)$, *where* $\partial V(x) = \overline{\mathrm{co}}[\lim_{k\to\infty} \nabla V(x_k) : x_k \to x,\ x_k \notin \mathbb{N},\ x_k \notin \Omega]$. *Here* $\Omega \subset \mathbb{R}^n \times \mathbb{R}$ *is the set of points,* V *is not differentiable, and* $\mathbb{N} \subset \mathbb{R}^n \times \mathbb{R}$ *is an arbitrary set with measure zero.*

Lemma 3. (*See* [26].) *Suppose* x *and* y *are two states of system* (1)*, then the following inequalities hold*:

$$\left|\bigwedge_{j=1}^{n} \alpha_{ij} g_j(x_j) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(y_j)\right| \leqslant \sum_{j=1}^{n} |\alpha_{ij}|\big|g_j(x_j) - g_j(y_j)\big|,$$
$$\left|\bigvee_{j=1}^{n} \beta_{ij} g_j(x_j) - \bigvee_{j=1}^{n} \beta_{ij} g_j(y_j)\right| \leqslant \sum_{j=1}^{n} |\beta_{ij}|\big|g_j(x_j) - g_j(y_j)\big|.$$

For the sake of convenience, we provide the following basic assumptions:

(A3) For any $i \in \mathbb{I}$, there exists positive constant $\overline{b}_i$ such that $b_i(t, x_i(t)) \leqslant \overline{b}_i$.
(A4) For each $x \in \mathbb{R}$, $k_i(\cdot, x)$ is continuous, $k_i(t,0) = 0$ and there exists a continuous function $\Delta_i(t) > 0$ such that $|k_i(t,x)| \leqslant \Delta_i(t)|x|$, $x \in \mathbb{R}$.
(A5) The discrete time-varying delays $\tau_{ij}(t)$ are continuously differentiable function and satisfy $\dot{\tau}_{ij}(t) \neq 1$ for $i,j \in \mathbb{I}$.
(A6) The distributed time-varying delays $\delta_{ij}(t)$ are continuously differentiable function for $i,j \in \mathbb{I}$.

3 Finite-time stabilization analysis

In this section, we will consider the finite-time stabilization for the proposed DFICGNNs.

Design the following delayed feedback control law:

$$u_i(t) = \operatorname{sgn}\bigl(y_i(t)\bigr)\left(-\lambda_i - \sigma_i\bigl|x_i(t)\bigr| - \gamma_i\bigl|x_i\bigl(t-\tau_{ji}(t)\bigr)\bigr| - \eta_i\int\limits_{t-\delta_{ji}(t)}^{t}\bigl|x_i(s)\bigr|\,\mathrm{d}s\right), \quad (5)$$

where $i \in \mathbb{I}$, λ_i, σ_i, γ_i and η_i are gain coefficients to be determined.

Theorem 1. *Suppose that assumptions* (A1)–(A6) *are satisfied and the following assumption hold*:

(A7) $\limsup\limits_{t\to+\infty}\left\{\dfrac{1}{\mu_i} - \widetilde{a}_i(t)\right\} \leqslant 0, \quad \widetilde{a}_i(t) = a_i(t) - \dfrac{\omega_i}{\mu_i},$

$$\begin{aligned}\varLambda_i = \liminf_{t\to+\infty}\Biggl\{&-\lambda_i + \mu_i\bar{b}_i\bigl|I_i(t)\bigr| + \mu_i\bar{b}_i\sum_{j=1}^{n}\bigl[\mathcal{B}_j\bigl(\bigl|c_{ij}(t)\bigr| + \bigl|h_{ij}(t)\bigr| \\ &+ \bigl|\alpha_{ij}(t)\bigr| + \bigl|\beta_{ij}(t)\bigr| + \bigl|\omega_{ij}(t)\bigr|\bigl|\delta_{ij}(t)\bigr| + \bigl|\rho_{ij}(t)\bigr|\bigl|\bigl|\delta_{ij}(t)\bigr|\bigr) \\ &+ |\nu_j|\bigl(|d_{ij}| + |T_{ij}| + |S_{ij}|\bigr)\bigr]\Biggr\} \geqslant 0,\end{aligned}$$

$$\begin{aligned}\limsup_{t\to+\infty}\Biggl\{&-\sigma_i - \frac{\omega_i}{\mu_i} + \omega_i\bigl|\widetilde{a}_i(t)\bigr| + \mu_i\bar{b}_i\varDelta_i(t) \\ &+ \sum_{j=1}^{n}\left[\mu_j\bar{b}_j\mathcal{A}_i\bigl|c_{ji}(t)\bigr| + \frac{|h_{ji}(\varphi_{ji}^{-1}(t))|}{1-\dot{\tau}_{ji}(\varphi_{ji}^{-1}(t))} + \bigl|\delta_{ji}(t)\bigr|\right]\Biggr\} \leqslant 0,\end{aligned}$$

$$\limsup_{t\to+\infty}\left\{-\gamma_i + \sum_{j=1}^{n}\bigl[-\bigl|h_{ij}(t)\bigr| + \mu_i\bar{b}_i\mathcal{A}_j\bigl(\bigl|h_{ij}(t)\bigr| + \bigl|\alpha_{ij}(t)\bigr| + \bigl|\beta_{ij}(t)\bigr|\bigr)\bigr]\right\} \leqslant 0,$$

$$\limsup_{t\to+\infty}\left\{-\eta_i + \sum_{j=1}^{n}\bigl[\mu_j\bar{b}_i\mathcal{A}_i\bigl(\bigl|\rho_{ji}(t)\bigr| + \bigl|\omega_{ji}(t)\bigr|\bigr) + \dot{\delta}_{ji}(t)\bigr]\right\} \leqslant 0.$$

Then the closed-loop system (4) *is FTS, and the settling-time satisfies* $T_0 = V(0)/\varDelta$, $\varDelta = \min_{i\in\mathbb{I}}\{\varDelta_i\}$, *where*

$$\begin{aligned}V(0) = {}&\sum_{i=1}^{n}\bigl(\bigl|x_i(0)\bigr| + \bigl|y_i(0)\bigr|\bigr) + \sum_{i=1}^{n}\sum_{j=1}^{n}\int\limits_{-\tau_{ij}(0)}^{0}\frac{|h_{ij}(\varphi_{ij}^{-1}(s))|}{1-\dot{\tau}_{ij}(\varphi_{ij}^{-1}(s))}\bigl|x_j(s)\bigr|\,\mathrm{d}s \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}\int\limits_{-\delta_{ij}(0)}^{0}\int\limits_{s}^{0}\bigl|x_j(u)\bigr|\,\mathrm{d}u\,\mathrm{d}s.\end{aligned}$$

Proof. Due to the presence of discontinuities in system (4), by using set-valued map and differential inclusion theory, we have the following differential inclusion system corresponding to systems (4):

$$\begin{cases} \dot{x}_i(t) \in -\frac{\omega_i}{\mu_i}x_i(t) + \frac{1}{\mu_i}y_i(t), \\ \dot{y}_i(t) \in -\widetilde{a}_i(t)y_i(t) + \omega_i\widetilde{a}_i(t)x_i(t) - \mu_i b_i\bigl(t, x_i(t)\bigr)\bigl[k_i\bigl(t, x_i(t)\bigr) \\ \qquad - \sum_{j=1}^n d_{ij}\nu_j - \sum_{j=1}^n c_{ij}(t)K[f_j]\bigl(x_j(t)\bigr) \\ \qquad - \sum_{j=1}^n h_{ij}(t)K[f_j]\bigl(x_j\bigl(t-\tau_{ij}(t)\bigr)\bigr) \\ \qquad - \bigwedge_{j=1}^n T_{ij}\nu_j - \bigwedge_{j=1}^n \alpha_{ij}(t)K[f_j]\bigl(x_j\bigl(t-\tau_{ij}(t)\bigr)\bigr) \\ \qquad - \bigwedge_{j=1}^n \rho_{ij}(t)\int_{t-\delta_{ij}(t)}^t K[f_j]\bigl(x_j(s)\bigr)\,\mathrm{d}s \\ \qquad - \bigvee_{j=1}^n S_{ij}\nu_j - \bigvee_{j=1}^n \beta_{ij}(t)K[f_j]\bigl(x_j\bigl(t-\tau_{ij}(t)\bigr)\bigr) \\ \qquad - \bigvee_{j=1}^n \omega_{ij}(t)\int_{t-\delta_{ij}(t)}^t K[f_j]\bigl(x_j(s)\bigr)\,\mathrm{d}s - I_i(t)\bigr] + u_i(t). \end{cases} \tag{6}$$

By the measurable selection lemma stated in [2], if $(x(t), y(t))^\top$ is the Filippov solutions of system (6), then there exists a measurable function $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_n)^\top : [-\xi, +\infty) \to \mathbb{R}^n$, where $\gamma_j(t) \in K[f_j](x_j(t))$ such that

$$\begin{cases} \dot{x}_i(t) = -\frac{\omega_i}{\mu_i}x_i(t) + \frac{1}{\mu_i}y_i(t), \\ \dot{y}_i(t) = -\widetilde{a}_i(t)y_i(t) + \omega_i\widetilde{a}_i(t)x_i(t) - \mu_i b_i(t, x_i(t))\bigl[k_i\bigl(t, x_i(t)\bigr) \\ \qquad - \sum_{j=1}^n d_{ij}\nu_j - \sum_{j=1}^n c_{ij}(t)\gamma_j(t) \\ \qquad - \sum_{j=1}^n h_{ij}(t)\gamma_j\bigl(t-\tau_{ij}(t)\bigr) \\ \qquad - \bigwedge_{j=1}^n T_{ij}\nu_j - \bigwedge_{j=1}^n \alpha_{ij}(t)\gamma_j\bigl(t-\tau_{ij}(t)\bigr) \\ \qquad - \bigwedge_{j=1}^n \rho_{ij}(t)\int_{t-\delta_{ij}(t)}^t \gamma_j(s)\,\mathrm{d}s - \bigvee_{j=1}^n S_{ij}\nu_j \\ \qquad - \bigvee_{j=1}^n \beta_{ij}(t)\gamma_j\bigl(t-\tau_{ij}(t)\bigr) \\ \qquad - \bigvee_{j=1}^n \omega_{ij}(t)\int_{t-\delta_{ij}(t)}^t \gamma_j(s)\,\mathrm{d}s - I_i(t)\bigr] + \overline{u}_i(t) \end{cases} \tag{7}$$

hold for almost all $t \in [-\xi, +\infty)$, where $\overline{u}_i(t) \in K[u_i(t)]$, $K[u_i(t)] = K[\mathrm{sgn}(y_i(t))] \times (-\lambda_i - \sigma_i x_i(t) - \gamma_i|x_i(t-\tau_{ji}(t))| - \eta_i \int_{t-\delta_{ji}(t)}^t |x_i(s)|\,\mathrm{d}s)$ and

$$K\bigl[\mathrm{sgn}\bigl(y_i(t)\bigr)\bigr] = \begin{cases} \{1\}, & x_i(t) > 0, \\ [-1, 1], & x_i(t) = 0, \\ \{-1\}, & x_i(t) < 0. \end{cases}$$

Consider the following Lyapunov–Krasovskii candidate functional:

$$V(t) = \sum_{i=1}^n \bigl(\bigl|x_i(t)\bigr| + \bigl|y_i(t)\bigr|\bigr) + \sum_{i=1}^n\sum_{j=1}^n \int_{t-\tau_{ij}(t)}^t \frac{|h_{ij}(\varphi_{ij}^{-1}(s))|}{1-\dot{\tau}_{ij}(\varphi_{ij}^{-1}(s))}\bigl|x_j(s)\bigr|\,\mathrm{d}s$$
$$+ \sum_{i=1}^n\sum_{j=1}^n \int_{-\delta_{ij}(t)}^0 \int_{t+s}^t \bigl|x_j(u)\bigr|\,\mathrm{d}u\,\mathrm{d}s,$$

where φ_{ij}^{-1} is the inverse function of $\varphi_{ij}(t) = t - \tau_{ij}(t)$.

It is easy to verify that $V(t)$ is C-regular. Calculating the time derivative of $V(t)$ along the trajectory of system (7), it follows from Lemma 2 and assumption (A5) that

$$
\begin{aligned}
\dot V(t) = {} & \sum_{i=1}^{n} \operatorname{sgn}\big(x_i(t)\big)\left[-\frac{\omega_i}{\mu_i}x_i(t) + \frac{1}{\mu_i}y_i(t)\right] \\
& + \sum_{i=1}^{n} \operatorname{sgn}\big(y_i(t)\big)\Bigg\{-\widetilde{a}_i(t)y_i(t) + \omega_i\widetilde{a}_i(t)x_i(t) - \mu_i b_i\big(t, x_i(t)\big) \\
& \times \Bigg[k_i\big(t, x_i(t)\big) - \sum_{j=1}^{n} d_{ij}\nu_j - \sum_{j=1}^{n} c_{ij}(t)\gamma_j(t) - \sum_{j=1}^{n} h_{ij}(t)\gamma_j\big(t-\tau_{ij}(t)\big) \\
& - \bigwedge_{j=1}^{n} T_{ij}\nu_j - \bigwedge_{j=1}^{n} \alpha_{ij}(t)\gamma_j\big(t-\tau_{ij}(t)\big) - \bigwedge_{j=1}^{n} \rho_{ij}(t) \int\limits_{t-\delta_{ij}(t)}^{t} \gamma_j(s)\,\mathrm{d}s - \bigvee_{j=1}^{n} S_{ij}\nu_j \\
& - \bigvee_{j=1}^{n} \beta_{ij}(t)\gamma_j\big(t-\tau_{ij}(t)\big) - \bigvee_{j=1}^{n} \omega_{ij}(t) \int\limits_{t-\delta_{ij}(t)}^{t} \gamma_j(s)\,\mathrm{d}s - I_i(t)\Bigg] + \overline{u}_i(t)\Bigg\} \\
& + \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{|h_{ij}(\varphi_{ij}^{-1}(t))|}{1-\dot\tau_{ij}(\varphi_{ij}^{-1}(t))}\big|x_j(t)\big| - \sum_{i=1}^{n}\sum_{j=1}^{n} \big|h_{ij}(t)\big|\big|x_j\big(t-\tau_{ij}(t)\big)\big| \\
& + \sum_{i=1}^{n}\sum_{j=1}^{m} \delta'_{ij}(t) \int\limits_{t-\delta_{ij}(t)}^{t} \big|x_j(u)\big|\,\mathrm{d}u + \sum_{i=1}^{n}\sum_{j=1}^{m} \int\limits_{-\delta_{ij}(t)}^{0} \big|x_j(s)\big|\,\mathrm{d}s \\
& - \sum_{i=1}^{n}\sum_{j=1}^{n} \int\limits_{-\delta_{ij}(t)}^{0} \big|x_j(t+s)\big|\,\mathrm{d}s,
\end{aligned}
$$

which, together with (7), assumptions (A2)–(A4), Lemma 3, gives

$$
\begin{aligned}
\dot V(t) \leqslant {} & \sum_{i=1}^{n}\left[\frac{1}{\mu_i} - \widetilde{a}_i(t)\right]\big|y_i(t)\big| + \sum_{i=1}^{n}\Bigg\{-\sigma_i - \frac{\omega_i}{\mu_i} + \omega_i\big|\widetilde{a}_i(t)\big| + \mu_i\overline{b}_i\varDelta_i(t) \\
& + \sum_{j=1}^{n}\left[\mu_i\overline{b}_j\mathcal{A}_i\big|c_{ji}(t)\big| + \frac{|h_{ji}(\varphi_{ji}^{-1}(t))|}{1-\dot\tau_{ji}(\varphi_{ji}^{-1}(t))} + \big|\delta_{ji}(t)\big|\right]\Bigg\}\big|x_i(t)\big| \\
& + \sum_{i=1}^{n}\Bigg\{-\gamma_i + \sum_{j=1}^{n}\big[-\big|h_{ij}(t)\big| + \mu_i\overline{b}_i\mathcal{A}_j\big(\big|h_{ij}(t)\big| + \big|\alpha_{ij}(t)\big| + \big|\beta_{ij}(t)\big|\big)\big]\Bigg\} \\
& \times \big|x_i\big(t-\tau_{ji}(t)\big)\big| \\
& + \sum_{i=1}^{n}\Bigg\{-\lambda_i + \mu_i\overline{b}_i\big|I_i(t)\big| + \mu_i\overline{b}_i\sum_{j=1}^{n}\big[\mathcal{B}_j\big(\big|c_{ij}(t)\big| + \big|h_{ij}(t)\big| + \big|\alpha_{ij}(t)\big| + \big|\beta_{ij}(t)\big|
\end{aligned}
$$

$$+ \left|\omega_{ij}(t)\right|\left|\delta_{ij}(t)\right| + \left|\rho_{ij}(t)\right|\left|\delta_{ij}(t)\right|\bigr) + |\nu_j|\bigl(|d_{ij}| + |T_{ij}| + |S_{ij}|\bigr)\bigr]\Biggr\}$$

$$+ \sum_{i=1}^{n}\Biggl\{-\eta_i + \sum_{j=1}^{n}\bigl[\mu_j\bar{b}_j\mathcal{A}_i\bigl(\left|\rho_{ji}(t)\right| + \left|\omega_{ji}(t)\right|\bigr) + \dot{\delta}_{ji}(t)\bigr]\Biggr\}\int\limits_{t-\delta_{ji}(t)}^{t}\left|x_i(s)\right|\mathrm{d}s.$$

According to assumption (A6), we can have $\dot{V}(t) \leqslant -\Delta$, where $\Delta = \min_{i\in\mathbb{I}}\{\Delta_i\}$ and

$$\begin{aligned}\Delta_i = {} & \lambda_i - \mu_i\bar{b}_i\left|I_i(t)\right| \\ & - \mu_i\bar{b}_i\sum_{j=1}^{n}\bigl[\mathcal{B}_j\bigl(\left|c_{ij}(t)\right| + \left|h_{ij}(t)\right| + \left|\alpha_{ij}(t)\right| + \left|\beta_{ij}(t)\right| \\ & + \left|\omega_{ij}(t)\right|\left|\delta_{ij}(t)\right| + \left|\rho_{ij}(t)\right|\left|\delta_{ij}(t)\right|\bigr) + |\nu_j|\bigl(|d_{ij}| + |T_{ij}| + |S_{ij}|\bigr)\bigr].\end{aligned}$$

Then there exists a constant $T_0 = V(0)/\Delta$ such that $V(t) = 0$ for all $t \geqslant T_0$. Therefore, according to Definition 1, system (1) is finite-time stabilizable under the designed control law (5). □

Remark 3. During the past several years, some delayed control laws have been designed to help achieve FTS for the delayed neural networks and INNs. But, compared with the previous delayed control laws, which can only help achieve FTS for neural networks with discrete delays (see [11]), the designed mixed time-varying delayed control law cannot only cope with the discrete delays, but also the distributed delays. From this point of view, the proposed control strategy is more generalized.

Remark 4. From Theorem 1 one can see that the FTS can achieve is based on the derived delay-dependent criteria. This fact can further show that the delays can affect the FTS of INNs. Thus, the established results in this paper can include and extend the previous works on the INNs based on the delay-independent criteria such as [20, 21].

Let $\delta_{ij}(t) \equiv 0$, $i, j \in \mathbb{I}$, and consider the following DFICGNNs with discrete time delays:

$$\begin{aligned}\ddot{x}_i(t) = {} & -a_i(t)\dot{x}_i(t) - b_i\bigl(t, x_i(t)\bigr)\Biggl[k_i\bigl(t, x_i(t)\bigr) - \sum_{j=1}^{n}d_{ij}\nu_j \\ & - \sum_{j=1}^{n}c_{ij}(t)f_j\bigl(x_j(t)\bigr) - \sum_{j=1}^{n}h_{ij}(t)f_j\bigl(x_j\bigl(t-\tau_{ij}(t)\bigr)\bigr) \\ & - \bigwedge_{j=1}^{n}T_{ij}\nu_j - \bigwedge_{j=1}^{n}\alpha_{ij}(t)f_j\bigl(x_j\bigl(t-\tau_{ij}(t)\bigr)\bigr) \\ & - \bigvee_{j=1}^{n}\beta_{ij}(t)f_j\bigl(x_j\bigl(t-\tau_{ij}(t)\bigr)\bigr) - \bigvee_{j=1}^{n}S_{ij}\nu_j - I_i(t)\Biggr].\end{aligned} \tag{8}$$

By using the variable transformation (2), set-valued map, differential inclusion theory and measurable selection lemma, we can have the following loop-closed system:

$$\begin{cases} \dot{x}_i(t) = -\frac{\omega_i}{\mu_i} x_i(t) + \frac{1}{\mu_i} y_i(t), \\ \dot{y}_i(t) = -\widetilde{a}_i(t) y_i(t) + \omega_i \widetilde{a}_i(t) x_i(t) - \mu_i b_i(t, x_i(t)) \big[k_i(t, x_i(t)) \\ \qquad - \sum_{j=1}^n d_{ij}\nu_j - \sum_{j=1}^n c_{ij}(t)\gamma_j(t) - \sum_{j=1}^n h_{ij}(t)\gamma_j(t - \tau_{ij}(t)) \\ \qquad - \bigwedge_{j=1}^n T_{ij}\nu_j - \bigwedge_{j=1}^n \alpha_{ij}(t)\gamma_j(t - \tau_{ij}(t)) \\ \qquad - \bigvee_{j=1}^n S_{ij}\nu_j - \bigvee_{j=1}^n \beta_{ij}(t)\gamma_j(t - \tau_{ij}(t)) - I_i(t) \big] + \widetilde{u}_i(t), \end{cases} \tag{9}$$

where $\widetilde{u}_i(t)$ are the feedback control laws to be designed later.

Corollary 1. *Suppose that assumptions* (A1)–(A5) *are satisfied, and the following assumption hold*:

(A8) $$\limsup_{t\to+\infty} \left\{ \frac{1}{\mu_i} - \widetilde{a}_i(t) \right\} \leqslant 0,$$

$$\widetilde{\Lambda}_i = \liminf_{t\to+\infty} \Bigg\{ -\widetilde{\lambda}_i + \mu_i \bar{b}_i |I_i(t)| + \mu_i \bar{b}_i \sum_{j=1}^n \big[|\nu_j| (|d_{ij}| + |S_{ij}| + |T_{ij}|) + \mathcal{B}_j (|c_{ij}(t)| + |h_{ij}(t)| + |\alpha_{ij}(t)| + |\beta_{ij}(t)|) \big] \Bigg\} \geqslant 0,$$

$$\limsup_{t\to+\infty} \Bigg\{ -\widetilde{\sigma}_i - \frac{\omega_i}{\mu_i} + \omega_i |\widetilde{a}_i(t)| + \mu_i \bar{b}_i \Delta_i(t) + \sum_{j=1}^n \mu_j \bar{b}_j |c_{ji}(t)| \mathcal{A}_i + \sum_{j=1}^n \frac{|h_{ji}(\varphi_{ji}^{-1}(t))|}{1 - \dot{\tau}_{ji}(\varphi_{ji}^{-1}(t))} \Bigg\} \leqslant 0,$$

$$\limsup_{t\to+\infty} \Bigg\{ -\widetilde{\gamma}_i + \sum_{j=1}^n \big[-|h_{ji}(t)| + \mu_j \bar{b}_j \mathcal{A}_i (|h_{ji}(t)| + |\alpha_{ji}(t)| + |\beta_{ji}(t)|) \big] \Bigg\} \leqslant 0.$$

Then the closed-loop system (9) *is FTS via the following designed control laws:*

$$\widetilde{u}_i(t) = \operatorname{sgn}(y_i(t)) \big(-\widetilde{\lambda}_i - \widetilde{\sigma}_i |x_i(t)| - \widetilde{\gamma}_i |x_i(t - \tau_{ji}(t))| \big), \tag{10}$$

where $i \in \mathbb{I}$, $\widetilde{\lambda}_i$, $\widetilde{\sigma}_i$ and $\widetilde{\gamma}_i$ are gain coefficients to be determined. Moreover, the settling-time is estimated as follows: $\widetilde{T}_0 = \widetilde{V}(0)/\widetilde{\Delta}$, $\widetilde{\Delta} = \min_{i\in\mathbb{I}}\{\widetilde{\Delta}_i\}$, *where*

$$\widetilde{V}(0) = \sum_{i=1}^n (|x_i(0)| + |y_i(0)|) + \sum_{i=1}^n \sum_{j=1}^n \int_{-\tau_{ij}(0)}^{t} \frac{|h_{ij}(\varphi_{ij}^{-1}(s))|}{1 - \dot{\tau}_{ij}(\varphi_{ij}^{-1}(s))} |x_j(s)| \,\mathrm{d}s.$$

Proof. Consider the following Lyapunov–Krasovskii candidate functional:

$$\widetilde{V}(t) = \sum_{i=1}^n (|x_i(t)| + |y_i(t)|) + \sum_{i=1}^n \sum_{j=1}^n \int_{t-\tau_{ij}(t)}^{t} \frac{|h_{ij}(\varphi_{ij}^{-1}(s))|}{1 - \dot{\tau}_{ij}(\varphi_{ij}^{-1}(s))} |x_j(s)| \,\mathrm{d}s, \tag{11}$$

where φ_{ij}^{-1} is the inverse function of $\varphi_{ij}(t) = t - \tau_{ij}(t)$. Calculating the time derivative of $\widetilde{V}(t)$ along the trajectory of system (9), we can have

$$\begin{aligned}\dot{\widetilde{V}}(t) &= \sum_{i=1}^{n} \operatorname{sgn}\bigl(x_i(t)\bigr)\left[-\frac{\omega_i}{\mu_i}x_i(t) + \frac{1}{\mu_i}y_i(t)\right] + \sum_{i=1}^{n} \operatorname{sgn}\bigl(y_i(t)\bigr)\Biggl\{-\widetilde{a}_i(t)y_i(t) \\ &\quad + \omega_i\widetilde{a}_i(t)x_i(t) - \mu_i b_i\bigl(t, x_i(t)\bigr)\Biggl[k_i\bigl(t, x_i(t)\bigr) - \sum_{j=1}^{n} d_{ij}\nu_j - \sum_{j=1}^{n} c_{ij}(t)\gamma_j(t) \\ &\quad - \sum_{j=1}^{n} h_{ij}(t)\gamma_j\bigl(t - \tau_{ij}(t)\bigr) - \bigwedge_{j=1}^{n} T_{ij}\nu_j - \bigwedge_{j=1}^{n} \alpha_{ij}(t)\gamma_j\bigl(t - \tau_{ij}(t)\bigr) \\ &\quad - \bigvee_{j=1}^{n} S_{ij}\nu_j - \bigvee_{j=1}^{n} \beta_{ij}(t)\gamma_j\bigl(t - \tau_{ij}(t)\bigr) - I_i(t)\Biggr] + \widetilde{u}_i(t)\Biggr\} \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{|h_{ij}(\varphi_{ij}^{-1}(t))|}{1 - \dot{\tau}_{ij}(\varphi_{ij}^{-1}(t))}\bigl|x_j(t)\bigr| - \sum_{i=1}^{n}\sum_{j=1}^{n} \bigl|h_{ij}(t)\bigr|\bigl|x_j\bigl(t - \tau_{ij}(t)\bigr)\bigr|.\end{aligned}$$

By using a similar method with that in Theorem 1, we get

$$\begin{aligned}\dot{\widetilde{V}}(t) &\leqslant \sum_{i=1}^{n}\left[\frac{1}{\mu_i} - \widetilde{a}_i(t)\right]\bigl|y_i(t)\bigr| + \sum_{i=1}^{n}\Biggl\{-\widetilde{\sigma}_i - \frac{\omega_i}{\mu_i} + \omega_i\bigl|\widetilde{a}_i(t)\bigr| + \mu_i\bar{b}_i\varDelta_i(t) \\ &\quad + \sum_{j=1}^{n} \mu_j\bar{b}_j\bigl|c_{ji}(t)\bigr|\mathcal{A}_i + \sum_{j=1}^{n} \frac{|h_{ji}(\varphi_{ji}^{-1}(t))|}{1 - \dot{\tau}_{ji}(\varphi_{ji}^{-1}(t))}\Biggr\}\bigl|x_i(t)\bigr| \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}\bigl\{-\widetilde{\gamma}_i + \mu_j\bar{b}_j\mathcal{A}_i\bigl(\bigl|h_{ji}(t)\bigr| + \bigl|\alpha_{ji}(t)\bigr| + \bigl|\beta_{ji}(t)\bigr|\bigr) - \bigl|h_{ji}(t)\bigr|\bigr\} \\ &\quad \times \bigl|x_i\bigl(t - \tau_{ji}(t)\bigr)\bigr| \\ &\quad + \sum_{i=1}^{n}\Biggl\{-\widetilde{\lambda}_i + \mu_i\bar{b}_i\bigl|I_i(t)\bigr| + \mu_i\bar{b}_i\sum_{j=1}^{n}\bigl[|\nu_j|\bigl(|d_{ij}| + |S_{ij}| + |T_{ij}|\bigr) \\ &\quad + \mathcal{B}_j\bigl(\bigl|c_{ij}(t)\bigr| + \bigl|h_{ij}(t)\bigr| + \bigl|\alpha_{ij}(t)\bigr| + \bigl|\beta_{ij}(t)\bigr|\bigr)\bigr]\Biggr\}.\end{aligned}$$

From Assumption (A8) it follows that $\dot{\widetilde{V}}(t) \leqslant -\widetilde{\varDelta}$, where $\widetilde{\varDelta} = \min_{i\in\mathbb{I}}\{\widetilde{\varDelta}_i\}$ and

$$\begin{aligned}\widetilde{\varLambda}_i = \liminf_{t\to+\infty}\Biggl\{-\widetilde{\lambda}_i + \mu_i\bar{b}_i\bigl|I_i(t)\bigr| + \mu_i\bar{b}_i\sum_{j=1}^{n}\bigl[|\nu_j|\bigl(|d_{ij}| + |S_{ij}| + |T_{ij}|\bigr) \\ + \mathcal{B}_j\bigl(\bigl|c_{ij}(t)\bigr| + \bigl|h_{ij}(t)\bigr| + \bigl|\alpha_{ij}(t)\bigr| + \bigl|\beta_{ij}(t)\bigr|\bigr)\bigr]\Biggr\}.\end{aligned}$$

Then there exists a constant $\widetilde{T}_0 = \widetilde{V}(0)/\widetilde{\Delta}$ such that $\widetilde{V}(t) = 0$ for all $t \geqslant \widetilde{T}_0$. Therefore, according to Definition 1, system (8) is finite-time stabilizable under the designed control law (10).

The proof is completed. □

For $i, j \in \mathbb{I}$, let $\tau_{ij}(t) \equiv 0$ and $\delta_{ij}(t) \equiv 0$, and further consider the following DFICGNNs:

$$\begin{aligned}\ddot{x}_i(t) = -a_i(t)\dot{x}_i(t) - b_i\big(t, x_i(t)\big)\Bigg[&k_i\big(t, x_i(t)\big) - \sum_{j=1}^{n} d_{ij}\nu_j \\ &- \sum_{j=1}^{n} c_{ij}(t) f_j\big(x_j(t)\big) - \bigwedge_{j=1}^{n} T_{ij}\nu_j - \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j\big(x_j(t)\big) \\ &- \bigvee_{j=1}^{n} S_{ij}\nu_j - \bigvee_{j=1}^{n} \beta_{ij}(t) f_j\big(x_j(t)\big) - I_i(t)\Bigg]. \end{aligned} \tag{12}$$

By using the variable transformation (2), set-valued map, differential inclusion theory and measurable selection lemma, we can have the following loop-closed system:

$$\begin{cases} \dot{x}_i(t) = -\frac{\omega_i}{\mu_i} x_i(t) + \frac{1}{\mu_i} y_i(t), \\ \dot{y}_i(t) = -\widetilde{a}_i(t) y_i(t) + \omega_i \widetilde{a}_i(t) x_i(t) \\ \qquad - \mu_i b_i(t, x_i(t))\big[k_i\big(t, x_i(t)\big) - \sum_{j=1}^{n} d_{ij}\nu_j \\ \qquad - \sum_{j=1}^{n} c_{ij}(t)\gamma_j(t) - \bigwedge_{j=1}^{n} T_{ij}\nu_j - \bigwedge_{j=1}^{n} \alpha_{ij}(t)\gamma_j(t) \\ \qquad - \bigvee_{j=1}^{n} S_{ij}\nu_j - \bigvee_{j=1}^{n} \beta_{ij}(t)\gamma_j(t) - I_i(t)\big] + \widehat{u}_i(t), \end{cases} \tag{13}$$

where $\widehat{u}_i(t)$ are the feedback control laws to be designed later.

Corollary 2. *Suppose that assumptions* (A1)–(A4) *are satisfied and the following assumption hold*:

(A9)
$$\limsup_{t\to+\infty}\left\{\frac{1}{\mu_i} - \widetilde{a}_i(t)\right\} \leqslant 0,$$

$$\begin{aligned}\widehat{\Lambda}_i = \liminf_{t\to+\infty}\Bigg\{&-\widehat{\lambda}_i + \mu_i\bar{b}_i\big|I_i(t)\big| + \sum_{j=1}^{n} \mu_i\bar{b}_i\big[\mathcal{B}_j\big(\big|c_{ij}(t)\big| \\ &+ \big|\alpha_{ij}(t)\big| + \big|\beta_{ij}(t)\big|\big) + |\nu_j|\big(|d_{ij}| + |T_{ij}| + |S_{ij}|\big)\big]\Bigg\} \geqslant 0,\end{aligned}$$

$$\begin{aligned}\limsup_{t\to+\infty}\Bigg\{&-\widehat{\sigma}_i - \frac{\omega_i}{\mu_i} + \omega_i\big|\widetilde{a}_i(t)\big| + \mu_i\bar{b}_i\Delta_i(t) \\ &+ \mu_j\bar{b}_j\mathcal{A}_i\big(\big|c_{ji}(t)\big| + \big|\alpha_{ji}(t)\big| + \big|\beta_{ji}(t)\big|\big)\Bigg\} \leqslant 0.\end{aligned}$$

Then the closed-loop system (13) *is FTS via the following designed control laws*:

$$\widehat{u}_i(t) = \operatorname{sgn}\bigl(y_i(t)\bigr)\bigl(-\widehat{\lambda}_i - \widehat{\sigma}_i\bigl|x_i(t)\bigr|\bigr), \tag{14}$$

where $i \in \mathbb{I}$, $\widehat{\lambda}_i$ *and* $\widehat{\sigma}_i$ *are gain coefficients to be determined. Moreover, the settling-time is estimated as follows*:

$$\widehat{T}_0 = \frac{\widehat{V}(0)}{\widehat{\Delta}}, \quad \widehat{\Delta} = \min_{i\in\mathbb{I}}\{\widehat{\Delta}_i\}, \quad \widehat{V}(0) = \sum_{i=1}^{n}\bigl(\bigl|x_i(0)\bigr| + \bigl|y_i(0)\bigr|\bigr).$$

Proof. Consider the following Lyapunov–Krasovskii candidate functional:

$$\widehat{V}(t) = \sum_{i=1}^{n}\bigl(\bigl|x_i(t)\bigr| + \bigl|y_i(t)\bigr|\bigr).$$

Calculating the time derivative of $\widehat{V}(t)$ along the trajectory of system (13), we can have

$$\begin{aligned}
\dot{\widehat{V}}(t) = {}& \sum_{i=1}^{n}\operatorname{sgn}\bigl(x_i(t)\bigr)\biggl[-\frac{\omega_i}{\mu_i}x_i(t) + \frac{1}{\mu_i}y_i(t)\biggr] + \sum_{i=1}^{n}\operatorname{sgn}\bigl(y_i(t)\bigr)\biggl\{-\widetilde{a}_i(t)y_i(t) \\
& + \omega_i\widetilde{a}_i(t)x_i(t) - \mu_i b_i\bigl(t, x_i(t)\bigr)\biggl[k_i\bigl(t, x_i(t)\bigr) - \sum_{j=1}^{n} d_{ij}\nu_j - \sum_{j=1}^{n} c_{ij}(t)\gamma_j(t) \\
& - \bigwedge_{j=1}^{n} T_{ij}\nu_j - \bigwedge_{j=1}^{n}\alpha_{ij}(t)\gamma_j(t) - \bigvee_{j=1}^{n} S_{ij}\nu_j - \bigvee_{j=1}^{n}\beta_{ij}(t)\gamma_j(t) - I_i(t)\biggr] \\
& + \operatorname{sgn}\bigl(y_i(t)\bigr)\bigl(-\widehat{\lambda}_i - \widehat{\sigma}_i x_i(t)\bigr)\biggr\}.
\end{aligned}$$

By using a similar method with that in Theorem 1, we have

$$\begin{aligned}
\dot{\widehat{V}}(t) \leqslant {}& \sum_{i=1}^{n}\biggl[\frac{1}{\mu_i} - \widetilde{a}_i(t)\biggr]\bigl|y_i(t)\bigr| + \sum_{i=1}^{n}\biggl\{-\widehat{\sigma}_i - \frac{\omega_i}{\mu_i} + \omega_i\bigl|\widetilde{a}_i(t)\bigr| \\
& + \mu_i\overline{b}_i\Delta_i(t) + \mu_j\overline{b}_j\mathcal{A}_i\bigl(\bigl|c_{ji}(t)\bigr| + \bigl|\alpha_{ji}(t)\bigr| + \bigl|\beta_{ji}(t)\bigr|\bigr)\biggr\}\bigl|x_i(t)\bigr| \\
& + \sum_{i=1}^{n}\biggl\{-\widehat{\lambda}_i + \mu_i\overline{b}_i\bigl|I_i(t)\bigr| + \sum_{j=1}^{n}\mu_i\overline{b}_i\bigl[\mathcal{B}_j\bigl(\bigl|c_{ij}(t)\bigr| + \bigl|\alpha_{ij}(t)\bigr| + \bigl|\beta_{ij}(t)\bigr|\bigr) \\
& + |\nu_j|\bigl(|d_{ij}| + |T_{ij}| + |S_{ij}|\bigr)\bigr]\biggr\}.
\end{aligned}$$

Then there exists a constant $\widehat{T}_0 = \widehat{V}(0)/\widehat{\Delta}$ such that $\widehat{V}(t) = 0$ for all $t \geqslant \widehat{T}_0$. Therefore, according to Definition 1, system (12) is finite-time stabilizable under the designed control law (14).

Up to now, the proof is completed. □

Remark 5. From Theorem 1 and Corollaries 1–2 one can see that the results on FTS of DFICGNNs with mixed time-varying delays, with discrete time delays and without time delays are established. If we make some comparisons between Theorem 1 and Corollaries 1–2, there exist at least three points need to be pointed.

First, from the criteria derived in assumptions (A7), (A8) and (A9) it follows that time-varying delays can affect the FTS of the considered DFICGNNs. That is to say, the delay-dependent criteria derived in this paper can better illustrate the FTS of time-varying delayed INNs. Thus, the delay-independent criteria derived in [8, 13, 19–21, 24] can be extended. Second, compared with Theorem 1 and Corollary 1, we can see that discrete time-varying delays can affect FTS of the considered DFICGNNs, but the distributed time-varying delays can also affect FTS of the considered DFICGNNs. From this point of view, the result established in Theorem 1 can extend some previous related works on INNs and fuzzy neural networks without distributed time-varying delays such as [20, 21] and [8, 11, 24]. Lastly, the activation functions considered in this paper are discontinuous, which are different from the continuous activation functions studied in the previous INNs. Moreover, it is clear to see that assumption (A2) can still hold when the activation functions are continuous.

Remark 6. In contrast to the asymptotic convergence results in [13], global exponential convergence results in [7, 19, 20] and Lagrange exponential stability convergence results in [21], the finite time convergence obtained in this paper can provide faster convergence speed.

4 Numerical examples and simulations

In this section, two numerical examples are given to verify the correctness of the obtained results.

Example 1 [Example used to verify Theorem 1]. Consider the following two-dimensional DFICGNN with discrete and distributed time-delays:

$$\begin{aligned}\ddot{x}_i(t) = &-a_i(t)\dot{x}_i(t) - b_i\big(t, x_i(t)\big)\Bigg[k_i\big(t, x_i(t)\big) - \sum_{j=1}^{n} d_{ij}\nu_j \\ &- \sum_{j=1}^{n} c_{ij}(t) f_j\big(x_j(t)\big) - \sum_{j=1}^{n} h_{ij}(t) f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ &- \bigwedge_{j=1}^{n} T_{ij}\nu_j - \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ &- \bigwedge_{j=1}^{n} \rho_{ij}(t) \int\limits_{t-\delta_{ij}(t)}^{t} f_j\big(x_j(s)\big)\,\mathrm{d}s - \bigvee_{j=1}^{n} \beta_{ij}(t) f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ &- \bigvee_{j=1}^{n} \omega_{ij}(t) \int\limits_{t-\delta_{ij}(t)}^{t} f_j\big(x_j(s)\big)\,\mathrm{d}s - \bigvee_{j=1}^{n} S_{ij}\nu_j - I_i(t)\Bigg], \end{aligned} \tag{15}$$

where $a_1(t) = 5.9 + 1.5\cos t$, $a_2(t) = 5.3 + 2.7\cos t$, $b_1(t,x) = 2 + 1/(2(1+x^2))$, $b_2(t,x) = 2+1/(2(1+x^2))$, $k_1(t,x) = (1.5+0.3\sin t)x$, $k_2(t,x) = (1.5+0.5\cos t)x$,

$$\big(c_{ij}(t)\big)_{2\times 2} = \begin{pmatrix} 0.7+0.2\cos t & 0.9+0.6\sin t \\ -2+0.5\sin t & -1.8+0.3\cos t \end{pmatrix},$$

$$\big(h_{ij}(t)\big)_{2\times 2} = \begin{pmatrix} -0.4+0.1\sin t & 0.7+0.3\cos t \\ -1.6+0.3\cos t & 0.3+0.1\sin t \end{pmatrix},$$

$$\big(\alpha_{ij}(t)\big)_{2\times 2} = \begin{pmatrix} -2.4+0.6\sin t & -1.4+0.7\cos t \\ -1.2+0.8\cos t & -1+0.3\sin t \end{pmatrix},$$

$$\big(\rho_{ij}(t)\big)_{2\times 2} = \begin{pmatrix} -1+0.1\sin t & -0.5+0.2\cos t \\ -0.8+0.2\cos t & -1.7+0.3\sin t \end{pmatrix},$$

$$\big(\beta_{ij}(t)\big)_{2\times 2} = \begin{pmatrix} -0.5+0.3\cos t & -1.9+0.2\sin t \\ -1.7+0.2\cos t & -1.5+0.5\sin t \end{pmatrix},$$

$$\big(\omega_{ij}(t)\big)_{2\times 2} = \begin{pmatrix} -2+0.2\cos t & -1.6+0.2\sin t \\ -0.4+0.3\cos t & -1.5+0.3\sin t \end{pmatrix},$$

$(d_{ij})_{2\times 2} = (T_{ij})_{2\times 2} = (S_{ij})_{2\times 2} = \left(\begin{smallmatrix} 0.1 & 0 \\ 0 & 0.2 \end{smallmatrix}\right)$, $\nu_1 = \nu_2 = 1$, $I_1(t) = 5 + 3.5\sin t$, $I_2(t) = 6.6 - 1.4\cos t$, $\tau_{ij}(t) = 0.5\sin t$, $\dot{\tau}_{ij}(t) \neq 1$, $\delta_{ij}(t) = 0.4\cos t$, $t > 0$, $i,j = 1,2$. Then we have that $\bar{b}_1 = \bar{b}_2 = 5/2$, $\tau = 0.5$, $\dot{\tau}_{ij}(t) = 0.5\cos t \neq 1$, $\delta = 0.4$ and $\varDelta_1(t) = 1.5 + 0.4\sin t$, $\varDelta_2(t) = 1 + 0.9\cos t$. Then assumptions (A3)–(A6) are all satisfied.

Define

$$f_1(x) = f_2(x) = \begin{cases} 0.5\tanh x - 0.1, & x \geqslant 0; \\ 0.5\tanh x + 0.1, & x < 0. \end{cases}$$

It is easy to see that the activation function $f_j(x)$ are discontinuous and nonmonotonic. The activation function $f_j(x)$ has a discontinuous point $x = 0$, and $\overline{\mathrm{co}}[f_i(0)] = [f_i^+(0), f_i^-(0)] = [-0.1, 0.1]$, $i = 1,2$. Thus, assumptions (A1) and (A2) all hold if $\mathcal{A}_1 = \mathcal{A}_2 = 0.5$ and $\mathcal{B}_1 = \mathcal{B}_2 = 0.1$.

Furthermore, let $\mu_1 = \mu_2 = 0.4$, $\omega_1 = \omega_2 = 0.5$, $\lambda_1 = 9$, $\lambda_2 = 5.2$, $\sigma_1 = 9.43$, $\sigma_2 = 9.23$, $\gamma_1 = 0.95$, $\gamma_2 = 0.8$, $\eta_1 = 2.5$, $\eta_2 = 2.95$, then it follows from straightforward computation that assumption (A7) holds and $\varLambda_1 \approx 0.79 \geqslant 0$, $\varLambda_2 \approx 1.59 \geqslant 0$.

By using the generalized variable transformation $y_i(t) = 0.4\mathrm{d}x_i(t)/\mathrm{d}t + 0.5x_i(t)$, system (15) can be rewritten as

$$\begin{cases} \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -\frac{0.5}{0.4}x_i(t) + \frac{1}{0.4}y_i(t), \\ \frac{\mathrm{d}y_i(t)}{\mathrm{d}t} = -\widetilde{a}_i(t)y_i(t) + 0.5\widetilde{a}_i(t)x_i(t) - 0.4b_i(t,x_i(t))\big[k_i\big(t,x_i(t)\big) \\ \qquad - \sum_{j=1}^n d_{ij}\nu_j - \sum_{j=1}^n c_{ij}(t)f_j\big(x_j(t)\big) \\ \qquad - \sum_{j=1}^n h_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) - \bigwedge_{j=1}^n T_{ij}\nu_j \\ \qquad - \bigwedge_{j=1}^n \alpha_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) - \bigwedge_{j=1}^n \rho_{ij}(t)\int_{t-\delta_{ij}(t)}^t f_j\big(x_j(s)\big)\,\mathrm{d}s \\ \qquad - \bigvee_{j=1}^n S_{ij}\nu_j - \bigvee_{j=1}^n \beta_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ \qquad - \bigvee_{j=1}^n \omega_{ij}(t)\int_{t-\delta_{ij}(t)}^t f_j\big(x_j(s)\big)\,\mathrm{d}s - I_i(t)\big], \end{cases} \tag{16}$$

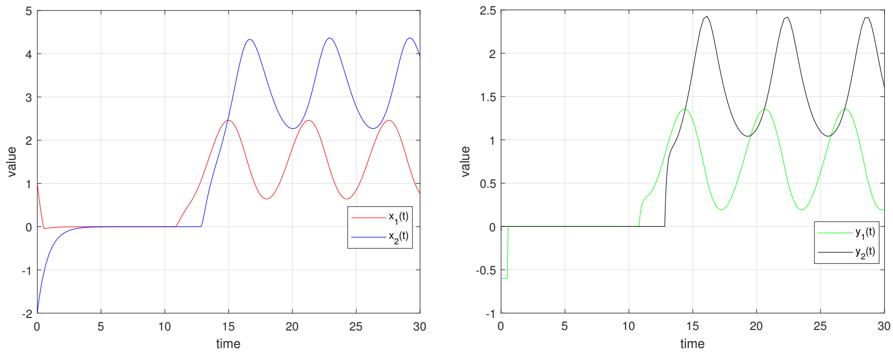

Figure 1. State trajectories of variables $x_1(t)$, $x_2(t)$ and $y_1(t)$, $y_2(t)$ of system (16) with the initial condition $(1,-2)$, $(-2,1.5)$ and without controllers.

where $\widetilde{a}_i(t) = a_i(t) - \omega_i/\mu_i$. Under the initial condition $(\phi_1(s),\phi_2(s)) = (1,-2)$ and $(\psi_1,\psi_2) = (-2,1.5)$, $s \in [-0.5,0]$, the state trajectories of variables $x(t) = (x_1(t),x_2(t))^\top$ and $y(t) = (y_1(t),y_2(t))^\top$ of system (16) are shown in Fig. 1.

Design the following controlled DFICGNNs:

$$\begin{cases} \dot{x}_i(t) = -\frac{0.5}{0.4}x_i(t) + \frac{1}{0.4}y_i(t), \\ \dot{y}_i(t) = -\widetilde{a}_i(t)y_i(t) + 0.5\widetilde{a}_i(t)x_i(t) - 0.4b_i\big(t,x_i(t)\big)\big[k_i\big(t,x_i(t)\big) \\ \qquad - \sum_{j=1}^n d_{ij}\nu_j - \sum_{j=1}^n c_{ij}(t)f_j\big(x_j(t)\big) \\ \qquad - \sum_{j=1}^n h_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) - \bigwedge_{j=1}^n T_{ij}\nu_j \\ \qquad - \bigwedge_{j=1}^n \alpha_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) - \bigwedge_{j=1}^n \rho_{ij}(t)\int_{t-\delta_{ij}(t)}^t f_j\big(x_j(s)\big)\,\mathrm{d}s \\ \qquad - \bigvee_{j=1}^n S_{ij}\nu_j - \bigvee_{j=1}^n \beta_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ \qquad - \bigvee_{j=1}^n \omega_{ij}(t)\int_{t-\delta_{ij}(t)}^t f_j\big(x_j(s)\big)\,\mathrm{d}s - I_i(t)\big] + u_i(t), \end{cases} \tag{17}$$

where delayed feedback control laws are designed as follows:

$$\begin{aligned} u_1(t) &= \operatorname{sgn}\big(y_1(t)\big)\Bigg(-9 - 9.43\big|x_1(t)\big| \\ &\qquad - 0.95\big|x_1(t-0.5\sin t)\big| - 2.5\int\limits_{t-0.4\cos t}^{t}\big|x_1(s)\big|\,\mathrm{d}s\Bigg), \\ u_2(t) &= \operatorname{sgn}\big(y_2(t)\big)\Bigg(-5.2 - 9.23\big|x_2(t)\big| \\ &\qquad - 0.8\big|x_2(t-0.5\sin t)\big| - 2.95\int\limits_{t-0.4\cos t}^{t}\big|x_2(s)\big|\,\mathrm{d}s\Bigg). \end{aligned} \tag{18}$$

Thus, the closed-loop system (16) is FTS via delayed feedback control laws (17) and the settling-time $T_0 \approx 13.038(s)$. This fact is shown by Fig. 2.

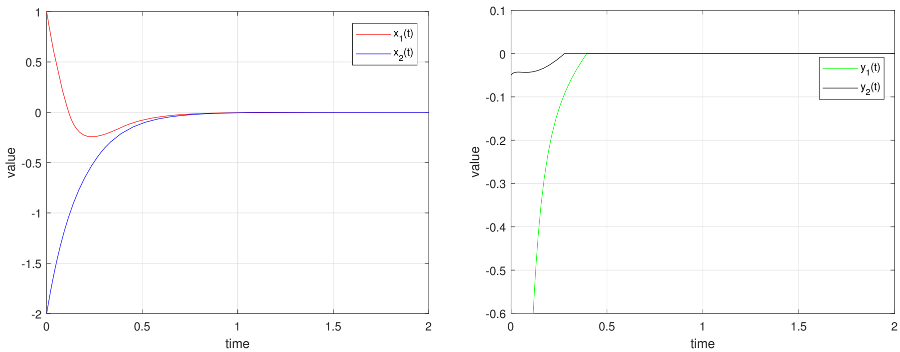

Figure 2. States trajectories $x(t)$ and $y(t)$ of system (17) via delayed feedback control laws (18).

Example 2 [Example used to verify Corollary 1]. Let $\delta_{ij}(t) \equiv 0$, $i, j \in \mathbb{I}$, and consider the following DFICGNNs with discrete time delays:

$$\begin{aligned}\ddot{x}_i(t) = -a_i(t)\dot{x}_i(t) - b_i\bigl(t, x_i(t)\bigr)\Biggl[&k_i(t, x_i(t)) - \sum_{j=1}^{n} d_{ij}\nu_j \\ &- \sum_{j=1}^{n} c_{ij}(t) f_j\bigl(x_j(t)\bigr) - \sum_{j=1}^{n} h_{ij}(t) f_j\bigl(x_j\bigl(t - \tau_{ij}(t)\bigr)\bigr) \\ &- \bigwedge_{j=1}^{n} T_{ij}\nu_j - \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j\bigl(x_j\bigl(t - \tau_{ij}(t)\bigr)\bigr) \\ &- \bigvee_{j=1}^{n} \beta_{ij}(t) f_j\bigl(x_j\bigl(t - \tau_{ij}(t)\bigr)\bigr) - \bigvee_{j=1}^{n} S_{ij}\nu_j - I_i(t)\Biggr], \end{aligned} \tag{19}$$

where $a_1(t) = 5.6 + 2.1\cos t$, $a_2(t) = 4.7 + 2.5\sin t$, $b_1(t, x) = 1 + 1/(3(1 + x^2))$, $b_2(t, x) = 1 + 1/(3(1 + x^2))$, $k_1(t, x) = (1.9 + 0.7\sin t)x$, $k_2(t, x) = (2.5 + \cos t)x$,

$$\begin{aligned}\bigl(c_{ij}(t)\bigr)_{2\times2} &= \begin{pmatrix} 1 + 0.2\cos t & 0.8 + 0.6\sin t \\ -1.3 + 0.4\sin t & -0.4 + 0.2\cos t \end{pmatrix}, \\ \bigl(h_{ij}(t)\bigr)_{2\times2} &= \begin{pmatrix} -1 + 0.2\sin t & 0.4 + 0.1\cos t \\ -1.1 + 0.5\cos t & 0.2 + 0.1\sin t \end{pmatrix}, \\ \bigl(\alpha_{ij}(t)\bigr)_{2\times2} &= \begin{pmatrix} -1.3 + 0.5\sin t & -1.5 + 0.6\cos t \\ -1.9 + 0.4\cos t & -0.7 + 0.3\sin t \end{pmatrix}, \\ \bigl(\beta_{ij}(t)\bigr)_{2\times2} &= \begin{pmatrix} -2.4 + \cos t & -1.8 + 0.3\sin t \\ -1.8 + 0.3\cos t & -0.6 + 0.5\sin t \end{pmatrix},\end{aligned}$$

$(d_{ij})_{2\times2} = (T_{ij})_{2\times2} = (S_{ij})_{2\times2} = \left(\begin{smallmatrix} 0.1 & 0 \\ 0 & 0.2 \end{smallmatrix}\right)$, $\nu_1 = \nu_2 = 1$, $I_1(t) = 2 - 1.1\sin t$, $I_2(t) = 3.8 + 1.4\cos t$, $\tau_{ij}(t) = 0.5\sin t$, $\dot{\tau}_{ij}(t) \neq 1$, $t > 0$, $i, j = 1, 2$, and the discontinuous activation functions $f_1(x)$, $f_2(x)$ are the same as those in Example 1. Then

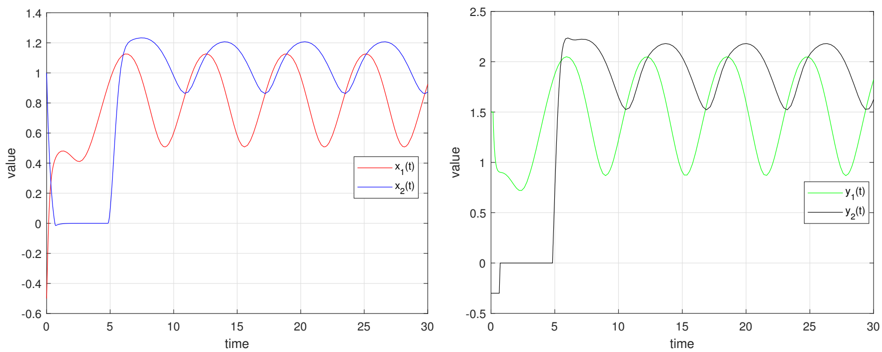

Figure 3. State trajectories of variables $x_1(t)$, $x_2(t)$ and $y_1(t)$, $y_2(t)$ of system (20) with the initial condition $(1,-2)$, $(-2,1.5)$ and without controllers.

we have that $\overline{b}_1 = \overline{b}_2 = 4/3$, $\tau = 0.5$, $\dot{\tau}_{ij}(t) = 0.5\cos t \neq 1$, $\varDelta_1(t) = 1.9 + 0.7\sin t$, $\varDelta_2(t) = 2.5 + \cos t$. Then assumptions (A1)–(A2), (A3)–(A5) are all satisfied.

Furthermore, let $\mu_1 = \mu_2 = 0.6$, $\omega_1 = \omega_2 = 1.8$, $\widetilde{\lambda}_1 = 0.1$, $\widetilde{\lambda}_2 = 0.2$, $\widetilde{\sigma}_1 = 7.58$, $\widetilde{\sigma}_2 = 9.6$, $\widetilde{\gamma}_1 = 0.96$, $\widetilde{\gamma}_2 = 0.86$. Then it follows from straightforward computation that Assumption (A8) holds and $\widetilde{\varLambda}_1 \approx 3.3 \geqslant 0$, $\widetilde{\varLambda}_2 \approx 4.88 \geqslant 0$.

By using the following generalized variable transformation $y_i(t) = 0.6\mathrm{d}x_i(t)\mathrm{d}t + 1.8x_i(t)$, system (19) can be rewritten as

$$\begin{cases} \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -\frac{1.8}{0.6}x_i(t) + \frac{1}{0.6}y_i(t), \\ \frac{\mathrm{d}y_i(t)}{\mathrm{d}t} = -\widetilde{a}_i(t)y_i(t) + 1.8\widetilde{a}_i(t)x_i(t) - 0.6b_i\big(t,x_i(t)\big)\big[k_i\big(t,x_i(t)\big) \\ \qquad - \sum_{j=1}^n d_{ij}\nu_j - \sum_{j=1}^n c_{ij}(t)f_j(x_j(t)) \\ \qquad - \sum_{j=1}^n h_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ \qquad - \bigwedge_{j=1}^n T_{ij}\nu_j - \bigwedge_{j=1}^n \alpha_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ \qquad - \bigvee_{j=1}^n S_{ij}\nu_j - \bigvee_{j=1}^n \beta_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) - I_i(t)\big], \end{cases} \tag{20}$$

where $\widetilde{a}_i(t) = a_i(t) - \omega_i/\mu_i$. Under the initial condition $(\phi_1(s),\phi_2(s)) = (-0.5,1)$ and $(\psi_1,\psi_2) = (1,-0.5)$, $s \in [-0.5,0]$, the state trajectories of variables $x(t) = (x_1(t),x_2(t))^\top$ and $y(t) = (y_1(t),y_2(t))^\top$ of system (20) are shown in Fig. 3.

Design the following controlled DFICGNNs:

$$\begin{cases} \dot{x}_i(t) = -\frac{1.8}{0.6}x_i(t) + \frac{1}{0.6}y_i(t), \\ \dot{y}_i(t) = -\widetilde{a}_i(t)y_i(t) + 1.8\widetilde{a}_i(t)x_i(t) - 0.6b_i\big(t,x_i(t)\big)\big[k_i\big(t,x_i(t)\big) \\ \qquad - \sum_{j=1}^n d_{ij}\nu_j - \sum_{j=1}^n c_{ij}(t)f_j(x_j(t)) \\ \qquad - \sum_{j=1}^n h_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ \qquad - \bigwedge_{j=1}^n T_{ij}\nu_j - \bigwedge_{j=1}^n \alpha_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ \qquad - \bigvee_{j=1}^n S_{ij}\nu_j - \bigvee_{j=1}^n \beta_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) - I_i(t)\big] + \widetilde{u}_i(t), \end{cases} \tag{21}$$

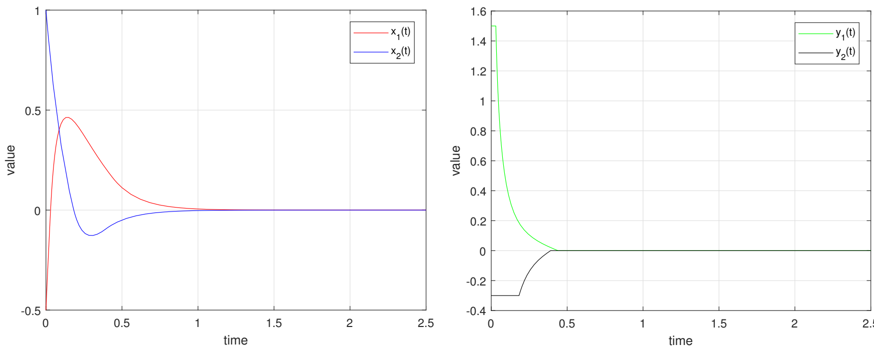

Figure 4. States trajectories $x(t)$ and $y(t)$ of system (21) via delayed feedback control laws (22).

where delayed feedback control laws are designed as follows:

$$\begin{aligned}\widetilde{u}_1(t) &= \operatorname{sgn}\big(y_1(t)\big)\big(-0.1 - 7.58\big|x_1(t)\big| - 0.96\big|x_1(t-0.5\sin t)\big|\big),\\ \widetilde{u}_2(t) &= \operatorname{sgn}\big(y_2(t)\big)\big(-0.2 - 9.6\big|x_2(t)\big| - 0.86\big|x_2(t-0.5\sin t)\big|\big).\end{aligned} \tag{22}$$

Thus, the closed-loop system (21) is FTS via delayed feedback control laws (22) and the settling-time $T_0 \approx 3.456(s)$. This fact is shown by Fig. 4.

Remark 7. Since the INNs considered in [7,13,20,21] or fuzzy INNs considered in [8,24] are the special case of ours, so, the finite time stability of their considered DFICGNNs can be obtained directly by using Theorem 1 and Corollaries 1–2. Therefore, the established FTS results in the paper are more inclusive and generalized.

Remark 8. Theorem 1 and Corollary 1 provide the delay-dependent criteria ensuring the FTS of the considered DFICGNNs with discrete and distributed time-delays. In sharp contrast to the delay-independent criteria derived in the previous works concerning delayed INNs, the delay-dependent criteria derived in the paper take more advantages. Moreover, Examples 1 and 2 can illustrate the influence of time-varying delays.

5 Conclusion

In this paper, we have investigated the finite-time stabilization for a class of discontinuous fuzzy inertial Cohen–Grossberg neural networks with discrete and distributed time-delays. Based on a generalized variable transformation including two tunable variables, differential inclusion theory and by constructing a modified Lyapunov–Krasovskii candidate functional concerning with the mixed delays, delay-dependent criteria formulated by algebraic inequalities are derived to ensure the finite-time stabilization for the addressed system via the designed delayed feedback control law. Moreover, the settling time is estimated. Some related works on inertial neural networks can be extended. Finally, two numerical examples are carried out to verify the effectiveness of the established results. The results established derive less conservativeness and are more inclusive.

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