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Optimization of the total production time by splitting complex manual assembly processes

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Abstract. In this article the minimization of the learning content in the total processing time is studied. Research is based on manual automotive wiring harness assembly, with unstable demand, fluctuating order quantities and enormous product variety. Such instability in manual production environment results that assembly is always at the start-up or learning phase, thus, operational times are greater than standard and operational efficiency is significantly reduced. Since a lot of research is done on learning time calculation, there is still lacking studies that address learning time reduction in such production situation. The methodology proposed in this article addresses reduction of learning time by splitting and simplifying complex assemblies of automotive wiring harnesses. Experimental results from the company indicate that this approach enables to optimize learning time and increase operational efficiency.

Keywords: manual assembly, manufacturing optimization, operating time, wiring harness, learning curve.

1 Introduction

Last decades in automotive and other industries show decreasing production order quantities and increasing product variety [21, 35] and this appears to be a significant trend [41]. This extremely affects companies in the supply chain of automotive manufacturer. Therefore, companies providing parts for the automobiles encounter variety of production problems and need extreme flexibility. As a result, supplier companies produce enormous variety of different products and the customer demand is fluctuating and changing rapidly for each product. What is more, product life cycle became very short and products constantly are changing, i.e., new different products can be introduced to the production, thus replacing or supplementing current production set. Fluctuations increase setup time in the automated assembly, but in the manual assembly fluctuations not only increase setup time, but also increase the learning time for the operators to learn the manual task or operation.

Even the manual assembly is being widely replaced by robotic and automated equipment; manufacturing fields with manual assembly still exist. Such a manufacturing field is the automotive wiring harness industry [42]. Wiring harness manufacturers producing a big variety of more complex products with lower production volumes naturally perform manual assembly. Even in the demand-based manufacturing systems, where order quantities are stable and large [13] the learning phase is soon completed and have little impact on the production performance. However, when the order quantities are small, intermittent, it is not possible to complete the learning, so the production is always at the start-up phase, even if the manufacturing is regular. Therefore, the manual assembly time is not stable, changes and is much higher than the expected steady-state operating time. And still, the supplier must be flexible and efficient to fulfill the customer orders with precise delivery, perfect quality and low product cost. By default, the way to achieve this is to keep extra personnel, additional manufacturing area and extra equipment and work overtime [17] just to cover ramp-ups and demand fluctuations. It is already reported, that learning time is being increased unnecessarily in this way [24].

Learning effect is known for decades [43, 44] and initially was dedicated for production improvement prediction [7], but current trends in manufacturing leads to application of learning curves (LC) for operation time prediction at the beginning of the production cycle [4]. Many authors report benefit of LC application. Learning curve application based on limited production data was used for better allocation of labor resources [39]. Gabel and Riedmiller [14] reported the LC application to be an effective tool for the production planning and scheduling of work assignments. Rigorous research of the shoe manufacturing company [2, 3] gave results that learning curve application leads to improved production schedules.

Some other authors [12, 33] addressed the production optimization by using learning models and made some comparisons. Reported results proved that impact of the learning effects increases as the order quantity decreases and number of operations increases. Also, comparisons with the traditional line balancing showed, that LC based planning methods are realistic and provide more accurate results. Some recent researches apply LC for worker reliability modelling [15], process time estimation for wiring harness assembly [25, 27], uncertainties in scheduling [29], allocation of training hours [30].

Effects of the learning on the assembly line balancing are also addressed especially in mixed model assembly lines. Authors Chakravarty and Shtub proposed [8] methodology to determine number of work stations required, task and cycle time assignments for assembly cost minimization. Model was further improved by considering stochastic or deterministic task time variability [9]. Such an optimization approach was finalized into mixed-model assembly line model with learning effect [11] which enabled to minimize the cost of the line start-up by selecting appropriate number of work cells task assignments etc.

Regarding the single model assembly line, the methodology to evaluate learning effect was also proposed [10]. This work addressed the design of an assembly line with long cycle time, few products and learning effect and proposed iterative approach to minimize total assembly costs.

From this point of view, there are two major gaps in literature regarding learning curve application is assembly. First, many authors report benefit of the LC application, however the most of them consider learning time as the natural part of overall operating time just to be calculated, planned and balanced rather than the time that needs to be eliminated or reduced. The solution of splitting and simplifying the assembly by organizing parallel or line assembly is based on production volume only [1] and even if learning is considered [10, 11], this learning is still being perceived as factor to be planned and balanced rather than to be eliminated. Therefore, currently existing methods for unique manufacturing situations such as bowl phenomenon, short-cycle production etc. might be not suitable for overall assembly time reduction and process improvement, since they do not address complexity elimination and learning time reduction. Systematic literature review of LC application [16] also emphasized the need for LC research on industry-specific learning.

As it is stated in the beginning, specific manufacturing situations with extreme flexibility, short production cycles, relatively low production volumes and high product variety still demand for cost reduction, however common measures such robotic assembly or mixed model assembly lines cannot be employed due to high production complexity (i.e., wiring harness assembly) and small order quantities (order quantities are too small to cover investment costs). Such production provides only limited production data for the modelling and this data will be still insufficient to conduct full statistical analysis. However, LC can be applied even on such data with acceptable results [28]. Therefore, there is an obvious need for a marginal method to address these issues and to propose quite cheap methodology to reduce total assembly time and costs. In this regard, the goal of this research is to create the learning time reduction methodology and show that manual assembly efficiency can be improved by splitting complex manual assembly of the automotive wiring harness even for relatively small production orders, i.e., organizing short-cycle production line instead of job-shop production. Article is organized in the following order: the mathematically proved LC application methodology is presented with applied splitting effect; then calculations and experimental data from the wiring harness manufacturer are presented and discussed.

2 Research methodology

2.1 Notations

n	production volume
p	number of process division
P	maximum number of process divisions
x	number of assembled unit
$y(x)$	learning curve with breaking point
β	assembly time of the first unit
α	the slope coefficient
T_{ict}	steady-state assembly time
x_c	cycle number where steady-state assembly time is reached
c_α	a learning slope of the undivided product

c_T	a steady-state assembly time of the undivided product
c_{xc}	a cycle number where steady-state assembly time is reached for the undivided product
c_{Te}	an additional time to aggregate two separate parts
$T(r)$	objective function of general optimization problem
r	vector of decision variables
G	fully connected feasible domain for variables p and n
$T(p,n)$	total time needed to produce n products when process is divided into p parts (objective function for two-dimensional optimization problem)
$p_{\min}(n)$	minimum function depending on production volume n
$T^{(n)}(p,n)$	average time needed to produce n products when process is divided into p parts (objective function for the second two-dimensional optimization problem, normed time function)

2.2 Process splitting

Let n is the number of fully completed products and $p = 1, 2, \dots, P$ is the number of process divisions. The complex process splitting is depicted in Fig. 1.

If the process is not divided, all assembly operations are being performed in one working station. If the process is divided into several work stations (i.e., assembly line, see Fig. 1), instead of one complex assembly, several simpler processes comes out. The simplicity of each process affects total assembly time regarding the number of parts p and total production quantity n. In the following subsection these effects are presented.

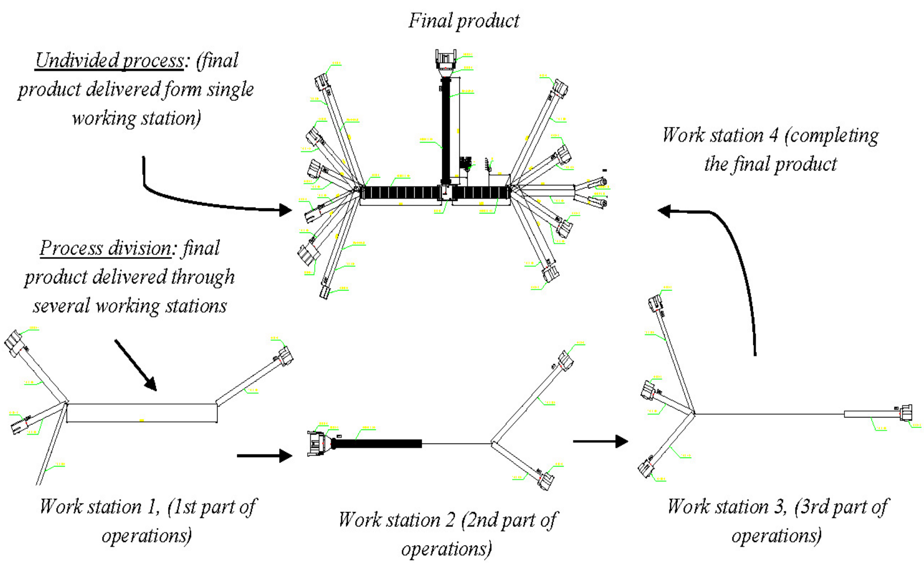

Figure 1. Complex process splitting.

2.3 Learning curve and parameters

The performance of manual skills is improving over time. This improvement is defined by the learning curve, which shows relationship between performance time and number of trials. Wright's LC is the recommended model for a basic and simple learning modelling [18]. However, since Wright's LC is unbounded,Plateau learning curve with break [25] as the basic model for the LC application in this research will be applied:

$$y(x) = \begin{cases} \beta x^{-\alpha}, & x \leqslant x_c, \\ T_{ict}, & x > x_c, \end{cases}$$

where x is number of assembled unit, y is assembly time of x'th unit, β is assembly time of the first unit, α is the slope coefficient, T_{ict} is steady-state assembly time, x_c is cycle number where steady-state assembly time is reached. These parameters fully define the learning curve. In general case, when product assembly is performed at only one separate work center LC parameters are constant.

2.4 Slope coefficient

There are many reports claiming that assemblies with less different operations results le lower complexity. The experimental data from works [37, 38] showed that increasing number of different components resulted slower operator thinking and decision-making time. Component grouping [31] also reduced learning time of the assembly process. The assembled product structure also has big impact on assembly performance [36]. Other researches [22, 34] also claim that complexity of the task affects performance and efficiency of the process. Monfared and Jenab [32] presented expression to calculate task complexity of operation and as a result suggested the LC model which encompasses complexity aspects. To sum up, more operations require more thinking, more learning and vice versa, less operations, less thinking and learning. Therefore, it is obviously that splitting of the complex wiring harness assembly would result lover slope coefficient value for each divided work center. In this research expression to define the slope coefficient as a function of number of divisions p proposed:

$$\alpha(p) = \frac{c_\alpha}{p}, \quad 0 < c_\alpha < 1, \tag{1}$$

where c_α is a learning slope of the undivided product.

2.5 Steady-state assembly time and stabilization cycle number

Some researchers [5, 25, 40] noticed and proved that after certain number of repetitive cycles *xc*, steady-state assembly time is reached. It means that after this number assembly improves no more. Obviously, if one splits the assembly process, it also splits the steady-state operating time. As reported by research [23] it is possible to divide the steady-state assembly time equally for the wiring harness. What is more, steady-state assembly time can be calculated prior to assembly by summing time norms of certain operations [26]. Thus, steady state assembly time is expressed as follows:

$$T_{ict}(p) = \frac{c_T}{p}, \quad c_T > 0, \tag{2}$$

where c_T is a steady-state assembly time of the undivided product.

It is assumed in this research that stabilization number remains the same in spite of divisions:

$$x_c(p) = c_{xc} > 0, \tag{3}$$

where c_{xc} is a cycle number where steady-state assembly time is reached for the undivided product.

This assumption is based on the idea that main factor reduced by simplification is slope of the learning curve α, since simplification reduces time needed to improve knowledge-based skills of the operator. Another group of improvements are directly related to motor skills which improve in a very slight amount after each repetitive cycle. This improvement will not or will be only very slight affected by labor division.

2.6 Assembly time of the first unit

Stabilization cycle number, slope coefficient (which is already known) and steady state assembly time (which is calculated) enable to calculate assembly of the first unit prior to start of assembly:

$$\beta(p) = T_{ict}(p) x_c(p)^{\alpha(p)}.$$

2.7 Aggregation time

Since the process is divided, additional time to aggregate separate parts of assembly is needed i.e., the more divisions, the more aggregation time is needed and thus it is also a function based on number of divisions:

$$T_e(p) = c_{Te}(p-1), \quad c_{Te} > 0, \tag{4}$$

where c_{Te} is an additional time to aggregate two separate parts.

3 Total assembly time optimization

The time needed to produce n products when process is divided into p parts

$$T(p,n) = p\int_0^n y(x,p)\,\mathrm{d}x + T_e(p)p,$$

where

$$y(x,p) = \begin{cases} \beta(p)x^{-\alpha(p)}, & x \leqslant x_c(p), \\ T_{ict}(p), & x > x_c(p), \end{cases} \tag{5}$$

by integrating total time is:

$$T(p,n) = T_1(p,n) + T_2(p), \tag{6}$$

where

$$T_1(p,n) = c_T \frac{pnt(p,n)}{p - c_\alpha}, \qquad T_2(p) = c_{Te} p(p-1),$$
$$t(p,n) = \left(\frac{c_{xc}}{n}\right)^{c_\alpha/p}.$$

When $x \leqslant x_c(p)$ (see Section 2.5) moreover $T_1(p,n) > 0$ and $T_2(p,n) \geqslant 0$, hence $T(p,n) > 0$, when

$$(p,n) \in D = \{p > 1,\ n > 1\}$$

and $0 < c_\alpha < 1$, $c_{Te}, c_T, c_{xc} > 0$.

Now we are about to state the assembly time optimization problem of several variables. This optimization problem can be posed as constrained optimization problem in the same form as a huge number of common design problems in engineering:

$$\text{minimize } T(r) \tag{7}$$

subject to

$$(p,n) \in G \cap \{p < f(n)\}, \tag{8}$$
$$c_\alpha \in (0,1), \tag{9}$$
$$c_{xc}, c_{Te}, c_T \in \mathbb{R}^1_+, \tag{10}$$

where $T(r)$ is objective function, $r = (p, n, c_\alpha, c_{Te}, c_{xc}, c_T)$ is a vector of decision variables, and (8), (9), (10) are the constraints which define the convex and fully connected feasible domain $G \subset \mathbb{R}^6_+$, i.e., we have to find the number $T_{\min}$ and vector r_0 that

$$T_{\min} = \min_{r \in G} T(r) \qquad r_0 = \arg\min_{r \in G} T(r).$$

State of art of global optimization employs variety of deterministic and stochastic methods. In cases of global optimization problem incorporating probabilistic (stochastic) elements or objective function is given as a "black box" computer code, stochastic approaches can often deal with problems better than the deterministic algorithms [45]. As we do not have probabilistic variables and objective function has clear analytical expression, deterministic optimization methods will be used for this problem. However, this problem can be still very hard to solve, as the number of decision variables is large. There are several reasons [19] for this difficulty:

- the problem "terrain" may be riddled with local optima;
- it might be very hard to find a feasible point (i.e., r_0 which satisfy all equalities and inequalities (8), (9), (10)), in fact, the feasible set which needn't even be fully connected, could be empty;

- stopping criteria used in general optimization algorithms are often arbitrary;
- optimization algorithms might have very poor convergence rates;
- numerical problems could cause the minimization algorithm to stop all together or wander.

It has been known for a long time [6, 20], that if the $T(r)$ is convex, then the first three problems disappear: any local optimum is, in fact, a global optimum; feasibility of convex optimization problems can be determined unambiguously, at least in principle; and very precise stopping criteria are available using duality. However, convergence rate and numerical sensitivity issues remain a potential problem.

Due to reasons stated before we will analyze problems (7)–(10) analytically when parameters $c_\alpha, c_{Te}, c_{xc}, c_T$ are received from process monitoring i.e., they are constants and satisfy constrains (9) and (10).

Let us consider the one-dimensional optimization problem:

$$\min_p T(p,n), \quad (p,n) \in G \subseteq D, \tag{11}$$

i.e., we will find $T_{\min}(n) = \min_p T(p,n)$ and $p_{\min}(n) = \arg\min_p T(p,n)$, $p_{\min}(n) \in \mathbb{R}^1_+$.

If $c_\alpha < 1$, $c_{xc} > n$, $c_{Te} > 0$ are constant, then for every n and for the sufficiently large P there exits unique minimum of the function $T(p,n)$ by p, i.e.,

$$p_{\min}(n) = \arg\min_{1 \leqslant p \leqslant P} T(p,n) \tag{12}$$

and

$$\min_{1 \leqslant p \leqslant P} T(p,n) = T\big(p_{\min}(n), n\big).$$

From equation (6):

$$\frac{\partial T}{\partial p} = \frac{\partial T_1}{\partial p} + \frac{\partial T_2}{\partial p},$$

where

$$\frac{\partial T_1}{\partial p} = -c_T \frac{nt(p,n)}{(p-c_\alpha)}\left[\ln\big(t(p,n)\big) + \frac{c_\alpha}{(p-c_\alpha)}\right], \qquad \frac{\partial T_2}{\partial p} = c_{Te}(2p-1).$$

For (11) we conclude that $\partial T_1/\partial p|_{p=1} < 0$ and function $\partial T_1/\partial p < 0$ is strictly monotonously increasing for every n by p. In addition, $\partial T_1/\partial p$ have zero horizontal asymptote, because

$$\lim_{p\to\infty} \frac{\partial T_1}{\partial p} = 0.$$

For (12) we conclude that $T_2'(1,n) > 0$ and $T_2'(p,n) > 0$ is strictly monotonously increasing for every n by p, therefore the solution of optimization problem (11) is simplified to solution of equation

$$-\frac{\partial T_1}{\partial p} = \frac{\partial T_2}{\partial p},$$

which has a unique solution $p_{\min}(n)$ if P is sufficiently large and then the minimal time is:

$$T_{\min}(n) = T\bigl(p_{\min}(n), n\bigr). \tag{13}$$

Note that $T(p,n)$ is strictly monotone increasing by n because

$$\frac{\partial T}{\partial n} = c_T t(p,n) > 0. \tag{14}$$

Hence function (13) is strictly monotone increasing too.

Let us consider two-dimensional optimization problem:

$$\min_{p,n} T(p,n), \quad (p,n) \in G \subseteq D, \tag{15}$$

i.e., we will find $T_{\min} = \min_{p,n} T(p,n)$ and $r_1 = \arg\min_{p,n} T(p,n)$, $r_1 = (p_1, n_1) \in \mathbb{R}_+^2$. From (14) we have

$$|\operatorname{grad} T| = \sqrt{\left(\frac{\partial T}{\partial p}\right)^2 + \left(\frac{\partial T}{\partial n}\right)^2} > 0 \quad \forall (p,n) \in G \tag{16}$$

because $\partial T/\partial n \neq 0$. Detailed calculation shown that $\partial^2 T/\partial p^2$ and Hessian (determinant of Hesse matrix) of function $T(p,n)$ have different signs

$$\frac{\partial^2 T}{\partial p^2} > 0, \qquad \mathrm{H}[T] = \left|\begin{pmatrix} \frac{\partial^2 T}{\partial p^2} & \frac{\partial^2 T}{\partial p \partial n} \\ \frac{\partial^2 T}{\partial p \partial n} & \frac{\partial^2 T}{\partial n^2} \end{pmatrix}\right| < 0 \quad \forall (p,n) \in G.$$

Hence function $T(p,n)$ is concave and have no global minima at interior of convex domain G. From the concavity of function $T(p,n)$ and (16) follows that point at which $T(p,n)$ reached minimal value $T_{\min} = \inf_{\bar{G}} T = T(p_g, n_g)$ is located at contour ∂G of domain G ($(p_g, n_g) \in \partial G$) [20], i.e., $r_1 = (p_g, n_g)$. This situation is demonstrated in Fig. 2.

The time considered in previous equations encompasses total assembly time, but the average time for each assembled part remains unknown and cannot be compared. Therefore, let us consider the normed time function which addresses the average time required for each assembled unit with different p and n:

$$T^{(n)}(p,n) = \frac{T(p,n)}{n} = c_T \frac{p t(p,n)}{p - c_\alpha} + c_{Te} \frac{p(p-1)}{n}, \tag{17}$$

then

$$\frac{\partial T^{(n)}}{\partial p} = \frac{1}{n} \frac{\partial T}{\partial p}, \tag{18}$$

$$\frac{\partial T^{(n)}}{\partial n} = -c_T c_\alpha \left[\frac{c_{Te} p(p-1)}{n^2 c_T c_\alpha} + \frac{t(p,n)}{n(p - c_\alpha)}\right] < 0. \tag{19}$$

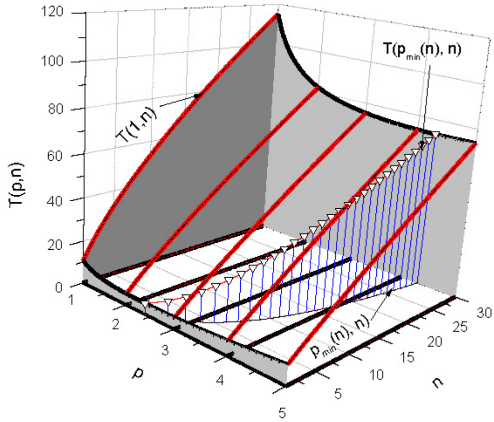

Figure 2. Trajectory plot of the functions $T(p,n)$, $T_{\min}(n) = T(p_{\min}(n),n)$ and $p = p_{\min}(n)$, when $c_\alpha = 0.3$, $c_{Te} = 0.3$, $c_{xc} = 80$, $c_T = 2$.

It can be proved (proof is analogical) that the function (17) has a unique minimum for every n

$$T^{(n)}_{\min}(n) = \min_{1\leqslant p\leqslant P} T^{(n)}(p,n) = T^{(n)}\bigl(p^{(n)}_{\min}(n),n\bigr),$$
$$p^{(n)}_{\min}(n) = \arg\min_{1\leqslant p\leqslant P} T^{(n)}(p,n), \tag{20}$$

and from (18) follows that $p^{(n)}_{\min}(n) \equiv p_{\min}(n)$. Function $T^{(n)}(p,n)$ (for every p) is strictly monotone decreasing (19), but then function (20) is strictly monotone decreasing too.

Let as state and analyze the second two-dimensional optimization problem:

$$\min_{p,n} T^{(n)}(p,n), \quad (p,n) \in G \subseteq D, \tag{21}$$

i.e., we will find $T^{(n)}_{\min} = \min_{p,n} T^{(n)}(p,n)$ and $r_2 = \arg\min_{p,n} T^{(n)}(p,n)$, $r_2 = (p_2,n_2) \in \mathbb{R}^2_+$. From (19) we have

$$\left|\operatorname{grad} T^{(n)}\right| > 0 \quad \forall (p,n) \in G \tag{22}$$

because $\partial T^{(n)}/\partial n \neq 0$. Detailed calculation shown that

$$\frac{\partial T^{(n)2}}{\partial p^2} > 0 \quad \forall (p,n) \in G$$

and that Hessian of function $T^{(n)}(p,n)$ is positive

$$\mathrm{H}\bigl[T^{(n)}\bigr] > 0 \quad \forall (p,n) \in G,$$

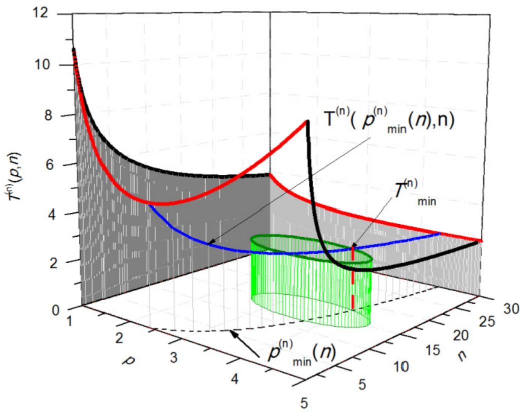

Figure 3. Trajectory plot of the functions $T^{(n)}(p,n)$, $T^{(n)}_{\min}(n) = T^{(n)}(p^{(n)}_{\min}(n),n)$ and $p = p^{(n)}_{\min}(n)$, when $c_\alpha = 0.3$, $c_{Te} = 0.3$, $c_{xc} = 80$, $c_T = 2$.

hence function $T^{(n)}(p,n)$ is convex and have no global minima at interior of convex domain G. From the convexity of function $T^{(n)}(p,n)$ and (22) follows that point at which $T^{(n)}(p,n)$ reached minimal value $T^{(n)}_{\min} = \inf_{\bar{G}} T^{(n)} = T^{(n)}(p_g,n_g)$ is located at contour ∂G of domain G ($(p_g,n_g) \in \partial G$) [20], i.e., $r_2 = (p_g,n_g)$. This situation is demonstrated in Fig. 3.

4 Application in manufacturing

Proposed model was applied at the certain wiring harness manufacturer. This Scandinavian company has production facilities in the Central Europe, North America and Eastern Asia, working approximately 2000 employees. Study was performed in company's facility in Lithuania. This factory produces enormous variety of different harness (more than four thousand) for auto industry and the customer demand is fluctuating and changing rapidly for each product. Demand of the wiring harnesses sharply differs from each other: from one piece per year, to several hundred per month. Final assembly operation times (OT) differ from 1 minute to 200 hours and depend on size of the wiring harness. What is more, these wiring harness constantly changing. Each month from two, to three hundred new wiring harnesses are being introduced to production, thus replacing or supplementing current products. Also, the manufacturing is demand-based, i.e., no possibilities to produce to stock and to synchronize production in that way.

Due to high complexity of products, variety and instability common process improvement approaches mixed-model assembly lines etc. can be hardly applied. As a result, several types of manufacturing layouts are employed, from assembly line to singular prototype production. The whole manufacturing system is mostly based on the short-cycle production. After project is done, the assembly tooling is dismantled and scraped.

In order to conduct experiment and to test the proposed model, first, parameter values were estimated. Parameter values of c_T and c_{Te} were determined according to standard

Table 1. Common operations in the wiring harness assembly (electrical centers).

Operation	Standard time, min
Placing of braches/wires	0.06
Number of hoses to be pulled	0.1
Assembling of label	0.12
Plugging terminal into housing	0.04
Assembling of shrink sleeve	0.2
Assembling of cable ties	0.06
Pick-place of component	0.04
Assembling of connector for electrical test	0.1
Number of packing's	1.05

time of the operations (which represent time values excluding learning time). Typical operations used in wiring harness assembly presented in Table 1.

Therefore, each wiring harness is continuously divisible, i.e., if in a full wiring harness there are 800 terminals to be plugged into housing, this number can be easily distributed into two, three or more working stations.

For the experiment a certain complex wiring harness product (electrical center) was selected consisting of 570 circuits and 83 connectors (housings). Steady state assembly time $c_T = 2$ h and aggregation time$c_{Te} = 0.3$ h

Much more complicated issue was to estimate parameters c_α and c_{xc}. As it is stated in the beginning, the company is producing more than 4000 different wiring harness products in its manufacturing system. There are several families of different wiring harnesses: 1) power cable systems; 2) common wiring harnesses; 3) engine bay harnesses; 4) electrical centers. For this experiment, electrical center wiring harnesses are selected. Another important remark regarding wiring harness is that two different wiring harness from the same family might show the same (or similar) quantities of operations and steady-state assembly time, but regarding the assembly operator would have to learn them as totally different components (due to different electrical layout). However, the slope coefficient of these two products would be the same, since this is the same product family, same operations and the same-steady state assembly time. This is very important point, since it enables to measure one product and apply estimated parameters to another. As the major function of wiring harness is electrical (even the assembly is mechanical process), each product must pass electrical test. The company is using electrical test system which is connected into a computer network and each completed and tested product is stored as a separate record in the main database. This enables to use large historical production data for process parameter estimation. In addition to this, many direct production data measurements were performed during experiment as well. The main objective of production data monitoring was to collect enough production data for learning curve fitting and parameter estimation and empirically test functions (1), (3) and (5).

Function (5) with measured production data is presented in Fig. 4. These experiments confirm functions (5) consistency with the raw assembly data (couple examples are presented in Fig. 4 to illustrate this). Also, production data monitoring confirms the existence of stabilization point. Another finding confirms the assumption made in (3). Even for

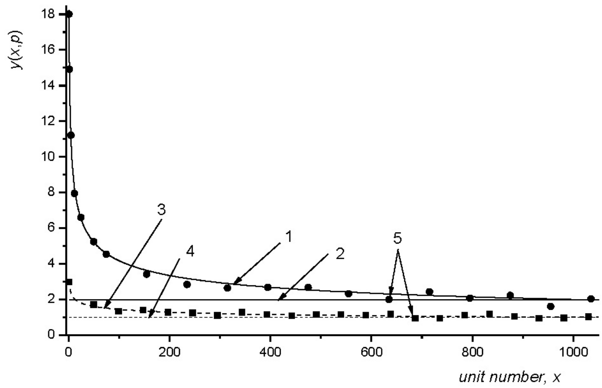

Figure 4. Dependences $y(x,p)$ on unit number x and monitored production data, $c_\alpha = 0.32$, $c_T = 2$, $c_{xc} = 1000$. 1 – $y(x,1)$, 2 – $T_{ict}(1)$, 3 – $y(x,2)$, 4 – $T_{ict}(2)$, 5 – monitored production data.

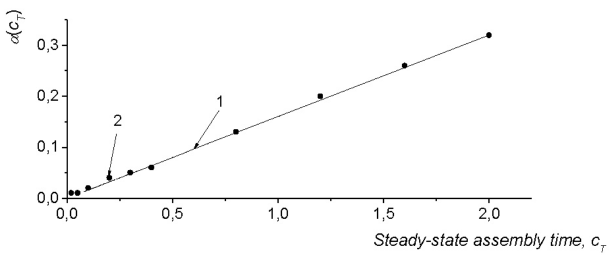

Figure 5. 1 – linear α dependency on stabilization time c_T, 2 – estimated values from production monitoring.

a very simple task assembly time slightly improves in the same way as for complex task i.e., until the same stabilization point. This means that for this manufacturing experiment any artifacts of the model should be avoided.

The next issue was to test assumption made in equation (1), i.e., that slope coefficient is linearly dependant on steady-state assembly time. In order to do this, several slope coefficients were estimated for different steady-state assembly times c_T, but for the same product family. The results are presented in Fig. 5. Measured results confirm linear relationship between steady-state assembly time and slope coefficient for electrical center wiring harness family.

After the values of the parameters are estimated and calculated, several assembly experiments performed to obtain total production times, with the different number of divisions. There were five production runs performed to complete 50 pieces of wiring harnesses. What is more, for this certain product, the presented production time one-dimension minimization procedure (11) was performed and optimal division number calculated for selected batch size of $n = 50$. Calculation results and monitored production data is depicted in Fig. 6.

Also, measured production runs were used to track additional predictions of the model for different batch sizes. With the other production volumes, the results are the same,

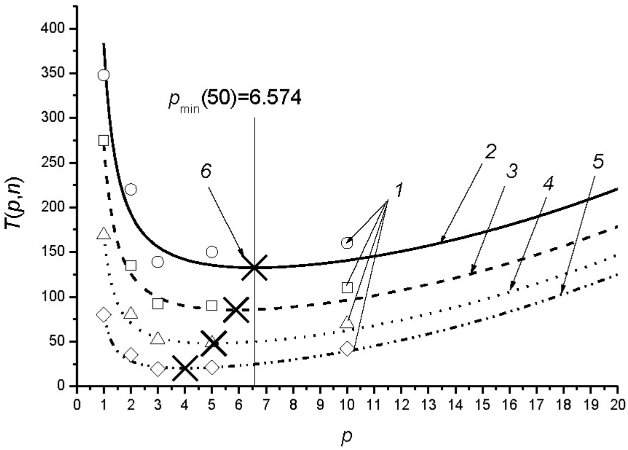

Figure 6. Calculated total assembly time and production data. 1 – production data, 2 – $T(p,50)$; 3 – $T(p,30)$; 4 – $T(p,15)$; 5 – $T(p,5)$; 6 – calculated minimum points. Parameters: $c_\alpha = 0.32$, $c_T = 2$, $c_{xc} = 1000$, $c_{Te} = 0.3$, $P = 20$.

therefore, several values of n (5, 15 and 30) were selected for graphical representation. Production data and calculated minimums are presented in Fig. 6.

For this particular product the optimal division number exists for any different n, however only one of experiments ($n = 15$) hit the minimum point. Even some of the data points lies on the model lines, in general, proposed model is only roughly consistent with the measured production data. This complicates the main conclusion that monitored production data empirically validates the proposed model. Therefore, additional insights are needed to justify these discrepancies. While studying results from the Fig. 6, higher values of p and n are resulting worse actual data sets. Also, while studying separate parts of process division (recall Figs. 4 and 5) results showed good consistency between real world data and model assumptions. This proposes an explanation that with higher labor divisions and production quantity the impact of random error increases. With higher labor divisions causes some discrepancies between work stations – more divisions, more possibilities for variable c_{Te} to vary. Also, higher production volumes n result less learning time, therefore any abnormal operations (dropped housing during assembly, jammed terminal, tangled wires and etc.) are visible. Also, in a real setting assumptions (1)–(3) and (4) might vary across individual divisions, some operators learn faster than others, some operations might be more complicated than another even they are continuously divided. The outcome of these factors is learning slope discrepancies. Therefore, all these reasons lead to result that in general proposed model is only roughly consistent with the real-world production data. On the other hand, with the all simplifications and assumptions made, huge amount of manual work and random errors, the model provided fairly good results.

Nevertheless, it is obvious that assembly of the complex products at the single working station is inefficient due to large learning time and even splitting into two parts reduce total production time significantly, however too much work stations ends up with increased total production time what, finally, proves the existence of optimum.

5 Discussion

In this section detailed discussion regarding model applicability in general is presented. The main optimization problem (generalized) is presented in (7) with constrains (8), (9) and (10). However, this problem is quite difficult to solve and its application in manufacturing might be even more complicated. What is more, in real setting variables c_α, c_{Te}, c_{xc} and c_T appear to be estimated (or calculated) from production process monitoring rather than optimized by solving optimization problem. Therefore, optimization problem (7) can be simplified into three different problems at least:

- one-dimension optimization (to find optimal number of process divisions p);
- two-dimensional optimization problem (to find optimal number of process divisions p and optimal production volume n);
- two-dimensional optimization problem with normed function (to find p and n).

The simplest problem regarding its solution and applicability is the one-dimensional optimization to find optimal number of divisions p. Since assumptions (1)–(3) and (4) are made, the results might be artifactual in some extent. The one of the potential sources of artifact is the simplification made in (3). However, only if x_c is largely affected (hardly realistic, marginal situation) by labor division, such artifact would exist. Otherwise, slight variations of x_c will not significantly affect results.

This optimization problem was tested in real manufacturing situation. While ultimate results proposed only rough consistency with real data sets, on the other hand, it can be concluded that with such a simple model provided fairly good results. It is very important result while considering short cycle production lines in unstable manufacturing environment when there is simply to less time for complex calculations and difficult combinatorial problems.

Two-dimensional optimization problem is the extension of one-dimensional optimization. As it can be seen in Fig. 2, the time function is monotone and no minimum regarding production size n exist. In other words, minimum point exists only for certain production quantity n.

The second two-dimensional optimization problem (with normed time) shows optimization possibility if the certain set of feasible combination of p and n exists (see Fig. 3). There is no global minimum, however the lowest value of the function $T^{(n)}_{\min} = \inf_{\bar{G}} T^{(n)} = T^{(n)}(p_g, n_g)$. The existence of the feasible subset G is also very important result of this research, since it enables to combine proposed methodology with other models and methods, i.e., with mixed-model line balancing and etc. In other words, the feasible set G can be estimated by other methods and then additionally evaluated with the proposed model regarding complexity aspects.

6 Conclusions

In this research the optimization of the total production time by minimization learning time by splitting complex processes presented. The main optimization problem stated also several additional optimization problems solved to validate proposed method. Experimental data from wiring harness manufacturing company confirm the splitting to be an effective measure for the total production time reduction and process improvement even with splitting into two parts. Additional analysis shows the possibility to combine proposed methodology with other line balancing methods by evaluating complexity aspects and reducing total assembly time in such way. This should be investigated in the future research.

References

1. E. Aguirre, L. Ferriere, B. Raucent, Robotic assembly of wire harnesses: Economic and technical justification, *J. Manuf. Syst.*, **16**(3):220–231, 1997.
2. M.J. Anzanello, F.S. Fogliatto, Learning curve modeling of work assignment in mass customized assembly lines, *Int. J. Prod. Res.*, **45**(13):2919–2938, 2007.
3. M.J. Anzanello, F.S. Fogliatto, Scheduling learning dependent jobs in customized assembly lines, *Int. J. Prod. Res.*, **48**(22):6683–6699, 2010.
4. M.J. Anzanello, F.S. Fogliatto, Learning curve models and applications: Literature review and research directions, *Int. J. Ind. Ergonom.*, **41**(5):573–583, 2011.
5. N. Baloff, Extension of the learning curve – some empirical results, *Oper. Res. Quart.*, **22**(4): 329–340, 1971.
6. A. Ben-Tal, A. Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*, MPS/SIAM Ser. Optim., SIAM, Philadelphia, 2001.
7. F.V. Bevis, C. Finniear, D.R. Towill, Prediction of operator performance during learning of repetitive tasks, *Int. J. Prod. Res.*, **8**(4):293–305, 1970.
8. A.K. Chakravarty, A. Shtub, Balancing mixed model lines with in-process inventories, *Manag. Sci.*, **31**(9):1161–1174, 1985.
9. A.K. Chakravarty, A. Shtub, A cost minimization procedure for mixed model production lines with normally distributed tasks times, *Eur. J. Oper. Res.*, **23**(1):25–36, 1986.
10. A.K. Chakravarty, A. Shtub, Technical note: Dynamic manning of long cycle assembly lines with learning effect, *IIE Trans.*, **18**(4):392–397, 1986.
11. A.K. Chakravarty, A. Shtub, The effect of learning on the operation of mixed-model assembly lines, *Prod. Oper. Manag.*, **1**(2):198–211, 1992.
12. Y. Cohen, G. Vitner, S.C. Sarin, Optimal allocation of work in assembly lines for lots with homogenous learning, *Eur. J. Oper. Res.*, **168**(3):922–931, 2006.
13. F. Estrada, J.R. Villalobos, L. Roderick, Evaluation of just-in-time alternatives in the electric wire-harness industry, *Int. J. Prod. Res.*, **35**(7):1993–2008, 1997.
14. T. Gabel, M. Riedmiller, Distributed policy search reinforcement learning for job-shop scheduling tasks, *Int. J. Prod. Res.*, **50**(1):41–61, 2012.
15. Z.S. Givi, M.Y. Jaber, W.P. Neumann, Modelling worker reliability with learning and fatigue, *Appl. Math. Modelling*, **39**(17):5186–5199, 2015.
16. C.H. Glock, E.H. Grosse, M.Y. Jaber, T.L. Smunt, Applications of learning curves in production and operations management: A systematic literature review, *Comput. Ind. Eng.*, **131**: 422–441, 2019.

17. C.H. Glock, M.Y. Jaber, S. Zolfaghari, Production planning for a ramp-up process with learning in production and growth in demand, *Int. J. Prod. Res.*, **50**(20):5707–5718, 2012.
18. E.H. Grosse, C.H. Glock, S. Müller, Production economics and the learning curve: A meta-analysis, *Int. J. Prod. Econ.*, **170**(B):401–412, 2015.
19. H. Hindi, A tutorial on convex optimization, in *Proceedings of the 2004 American Control Conference, Vol. 1*, IEEE, 2004, pp. 3252–3265.
20. J.B. Hiriart-Urruty, C. Lemaréchal, *Fundamentals of Convex Analysis*, Springer, Berlin, Heidelberg, 2001.
21. M. Holweg, The genealogy of LEAN production, *J. Oper. Manag.*, **25**(2):420–437, 2006.
22. E. Kahya, The effects of job performance on effectiveness, *Int. J. Ind. Ergonom.*, **39**(1):96–104, 2009.
23. V. Kleiza, J. Tilindis, Models for optimization of total assembly time and assembly line balancing, in V. Kleiza, Z. Bazaras (Eds.), *Proceedings of 4th International Conference ITELMS' 2009, Panevėžys, Lithuania, May 3–4, 2009*, Technologija, Kaunas, 2009, pp. 100–103.
24. V. Kleiza, J. Tilindis, The effect of learning factors due to low volume order fluctuations in the automotive wiring harness production, *Procedia CIRP*, **19**:129–134, 2014.
25. V. Kleiza, J. Tilindis, Log-linear learning model for predicting a steady-state manual assembly time, *Nonlinear Anal. Model. Control*, **19**(4):592–601, 2014.
26. V. Kleiza, J. Tilindis, Mathematical modeling of production operations and processes for product design and manufacturing, *Appl. Mech. Mater.*, **510**:243–247, 2014.
27. V. Kleiza, J. Tilindis, An almost learning curve model for manual assembly performance improvement, *Nonlinear Anal. Model. Control*, **21**(6):839–850, 2016.
28. V. Kleiza, J. Tilindis, Learning curve parameter estimation beyond traditional statistics, *Appl. Math. Modelling*, **45**:768–783, 2017.
29. Y. Li, X. Yang, Z. Yang, Uncertain learning curve and its application in scheduling, *Comput. Ind. Eng.*, **131**:534–541, 2019.
30. F. Lolli, E. Balugani, R. Gamberini, B. Rimini, Quality cost-based allocation of training hours using learning-forgetting curves, *Comput. Ind. Eng.*, **131**:552–564, 2019.
31. M. Madan, T. Bramorski, R.P. Sundarraj, The effects of grouping parts of ready-to-assemble products on assembly time: An experimental study, *Int. J. Oper. Prod. Man.*, **15**(3):39–49, 1995.
32. S.K. Monfared, K. Jenab, Complexity analysis of an operation in demand-based manufacturing, *Int. J. Prod. Res.*, **49**(17):5303–5315, 2011.
33. R.O. Neidigh, T.P. Harrison, Optimizing lot sizing and order scheduling with non-linear production rates, *Int. J. Prod. Res.*, **48**(8):2279–2295, 2010.
34. D.A. Nembhard, N. Osothsilp, Task complexity effects on between-individual learning/forgetting variability, *Int. J. Ind. Ergonom.*, **29**(5):297–306, 2002.
35. F.T. Piller, Mass customization: Reflections on the state of the concept, *Int. J. Flex. Manuf. Syst.*, **16**(4):313–334, 2004.
36. G.V. Prabhu, M.G. Helander, V.L. Shalin, Effect of product structure on manual assembly performance, *Int. J. Hum. Factor. Man.*, **5**:149–161, 1995.

37. M. Richardson, G. Jones, M. Torrance, Identifying the task variables that influence perceived object assembly complexity, *Ergonomics*, **47**(9):945–964, 2004.

38. M. Richardson, G. Jones, M. Torrance, T. Baguley, Identifying the task variables that predict object assembly difficulty, *Hum. Factors*, **48**(3):511–525, 2006.

39. T.L. Smunt, C.A. Watts, Improving operations planning with learning curves: Overcoming the pitfalls of 'messy' shop floor data, *J. Oper. Manag.*, **21**(1):93–107, 2003.

40. C.J. Teplitz, *The Learning Curve Deskbook: A Reference Guide to Theory, Calculations and Applications*, Quorum Books, New York, 1991.

41. A. Trattner, L. Hvam, C. Forza, Z.N.L. Herbert-Hansen, Product complexity and operational performance: A systematic literature review, *CIRP J. Manuf. Sci. Technol.*, **25**:69–83, 2019.

42. J. Trommnau, J. Kuhnle, J. Siegert, R. Inderka, T. Bauernhansl, Overview of the state of the art in the production process of automotive wire harnesses, current research and future trends, *Procedia CIRP*, **81**:387–392, 2019.

43. T.P. Wright, Factors affecting the cost of airplanes, *J. Aeronaut. Sci.*, **3**(4):122–128, 1936.

44. L.E. Yelle, The learning curve: Historical review and comprehensive survey, *Decision Sci.*, **10**(2):302–328, 1979.

45. A. Žilinskas, A. Zhigljavsky, Stochastic global optimization: A review on the occasion of 25 years of Informatica, *Informatica*, **27**(2):229–256, 2016.