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MODELLING OF NON LINEAR BAR STOCHASTICAL SYSTEMS

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Abstact. Simple and practical analysis of probabilistic safety indexes of nonlinear bar systems is under consideracion.The nodal epizodic loading of systems and the left-side truncated probability distribution curves of their member resistances are taken into account. The analysis of long term member safety is based on dynamic models of stochastical sequence of its performance function. The evaluation of member safety indexes by the methods of cumulative coefficient of correlation and limit episodic action effects is presented.

1. Introduction

We have set ourselves to encourage the civil and mechanical engineers to introduce the probabilistic estimation of structural quality of principal load bearing nonlinear bar systems subjected to storm, snow, traffic, crane, wave slam or seismic random loading and overloading. Sudden failures of such systems, exposed not only to random reiterated transient and variable in time episodic or multicycle long term actions but to aggressive environmental conditions, are very dangerous.

A wide range of applied design issues can be neither formulated nor. solved within the deterministic nonlinear systems. The introduction of the theory of probabilistic reliability into the structural design leads to safety and durability increase of construction and civil engineering works.

Due to many uncertainties the probabilistic reliability analysis of nonlinear bar systems is rather complicated what leads to the increase psychological barrier for engineers. Therefore, an introduction of simplified, practical and reliable analysis methods into the design practice is indispensable. A simultancous occurrence of two or more random episodic actions and decrease of system member resistances must be taken into account.

The theoretical indexes of prior and posterior probabilistic reliability of systems, presented in this article, are obtained excluding any estimation of structural failure due to human errors. The mathematical models can be applied in other fields of natural sciences.

2. Statistical estimates of member action effects and resistances

As a rule, loading of bar systems is nodal. If the nodal force vector is $\{F\} = (F_1, F_2, \ldots, F_m)^T$, the member action effect vector is

$$\{E\} = (E_1, E_2, \ldots, E_j, \ldots, E_n) = [\alpha]\{F\}. \qquad (1)$$

Here $[\alpha] = [K]\,[A]^T([A]\,[K]\,[A]^T)^{-1}$ is the effect influence matrix, where [K] is the member stiffness matrix and [A] is the static equilibrium matrix. Thus, the action effect of structural member "j" can be written in the form:

$$E_j = \{\alpha_j\}^T\{E\}, \qquad (2)$$

where $\{\alpha_j\}^T$ is transposed row of the influence matrix $[\alpha]$.

The mean, variance and cross covariation of the action effect probability distribution are:

$$E_{jm} = \{\alpha_j\}^T\{F_m\} = \sum_{i=1}^{m} (\alpha_{ji} F_i)_m, \qquad (3)$$

$$s^2 E_j = \{\alpha_j^2\}^T\{s^2 F\} = \sum^{m} (\alpha_{ji}^2 s^2 F_i), \qquad (4)$$

$$\mathrm{cov}(E_k, E_l) = \sum_{i=1}^{m} \alpha_{ki}\,\alpha_{li}\, s^2 F_i + \sum\sum_{i \neq j} \alpha_{ki}\,\alpha_{lj}\, sF_i\, sF_j, \qquad (5)$$

where $s^2 F_i$ is the variance of a random external force F_i.

In many cases the members may attain the limit state only in the case of a simultaneous occurrence of several random episodic indepen-dent loads. The time of their action Δt_c is comparatively short, the reiterated period $t_{\lambda c}$ is sufficiently large and the probability of a combi-nation of their maximum values is quite little.

By A.Rzhanitzin (1), reiteration number of "e" loads combination for the useful lifetime "t_r" is

$$r_e = t_r \prod_{c=1}^{e} \frac{\Delta t_c}{t_{\lambda c}} \sum_{c=1}^{e} \frac{1}{\Delta t_c}. \qquad (6)$$

The stochastical estimates of the resistance "R" of tension and comp-ression or buckling members

$$R_m,\ s^2R \text{ and } \mathrm{cov}(R_k, R_e)$$

of nonlinear systems depend on their structural solution. The resistance "R" is fixed random vector, i, e. it is random only at design (fixed) cut of stochastic process.

3. Instantaneous member safety index

The member performance function can be expressed by the formula:

$$Z_t = R_t - E_t, \qquad (7)$$

where $\mathbf{R}_t$ and $\mathbf{E}_t$ are the random vectors of its resistance and action effect at any cut "t" of a stochastic process. The member instantaneous safety index

$$P_t\{1\} = P_t\{\mathbf{Z}_t > 0\} = \int_0^{\infty} g_{Rt}(\mathbf{R})\, G_{Et}(\mathbf{R})\, d\mathbf{R}, \qquad (8)$$

where $g_{Rt}(\mathbf{R})$ is the density function of its resistance $\mathbf{R}_t$ and $G_{Et}(\mathbf{R})$ is the distribution function of its effect $\mathbf{E}_t$.

In practical analysis one must take into account the left-side truncated probability distribution curves of member resistance and the same to increase an analysis accuracy. In this case, the formula (8) can be written in the form:

$$P_t\{1\} = \frac{1}{1 - G_{R/R_1}(R)} \int_{R_1}^{\infty} g_R(c)\, G_E(c)\, dc, \qquad (9)$$

where $\mathbf{R}_1$ is the member effect, caused by erection forces.

When the probability distribution law of the member performance function by (7) obeys the normal or lognormal one, the instantaneous safety index of high reliable nonlinear systems must be made more accurate according to the formula:

$$P_t\{1\} = 1 - 460 \exp(-4.3\,\beta), \qquad (10)$$

where β is the Gaussian reliability factor [2].

4. Long term member safety index

4.1 Domination of permanent actions over episodic ones

The use of continuos non-stationary stochastic processes considerably complicated the structural safety analysis of nonlinear systems. Therefore, it is reasonable to consider a stochastic sequence of structural member performance (Fig.1).

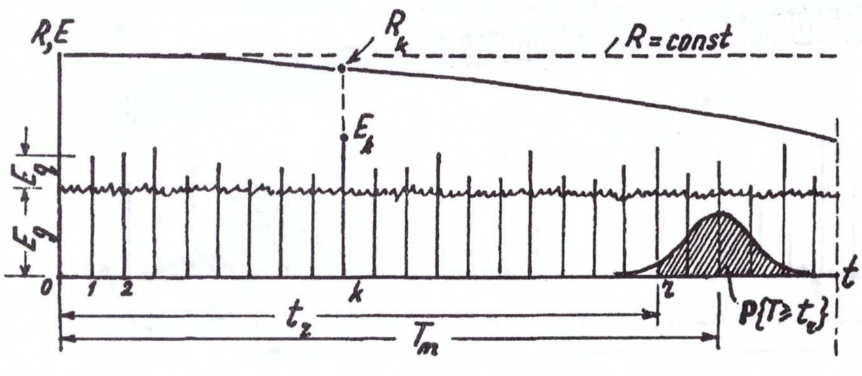

Fig.1. Approximate dynamic model for the long term safety analysis by the method of the cumulative coefficient of autocorrelation.

If permanent actions dominate over random transient episodic ones, the maxima probability distribution law is closely connected with the normal one.

Since an analysis sequence consists of "r" stochastically dependent cuts, connected in series, the probabilistic long term safety can be calculated by the method of cumulative coefficient of correlation. This index is

$$P_i\{ T \geq t_2 \} = P\{(Z_1 > 0) \cap \ldots \cap (Z_r > 0)\} \approx$$
$$\approx \rho P_z\{ 1 \} + (1 - \rho)\exp[- \sum_{k=1}^{r} (1 - P_k\{ 1 \})], \qquad (11)$$

where "ρ" is the cumulative coefficient of correlation of the stochastical sequence cuts; $P_k\{ 1 \}$ is the instantaneous safety index by (8) or (9); r is the number of reiteration actions [3].

The simultaneous action effect of two stochastically independent random episodic components is of the highest level. In this case, the long term safety index is

$$P\{ T \geq t_r \} \approx P_1\{ T \geq t_r \} * P_2\{ T \geq t_r \} * P_{12}\{ T \geq t_r \}, \qquad (12)$$

where the probability $P_i\{ T \geq t_r \}$ is calculated by (11).

4.2. Domination of episodic actions over permanent ones

If episodic actions dominate over permanent ones, the member long term safety index can be evaluated by the limit action effect method. According to Fig.2 dynamic model, the limit episodic action effect is

$$E_{lim} = R_m - E_{gm} - \beta_1 (s^2 R + s^2 E_g)^{1/2}, \qquad (13)$$

where $\beta_1 \approx 1.64$ is the Gaussian reliability factor.

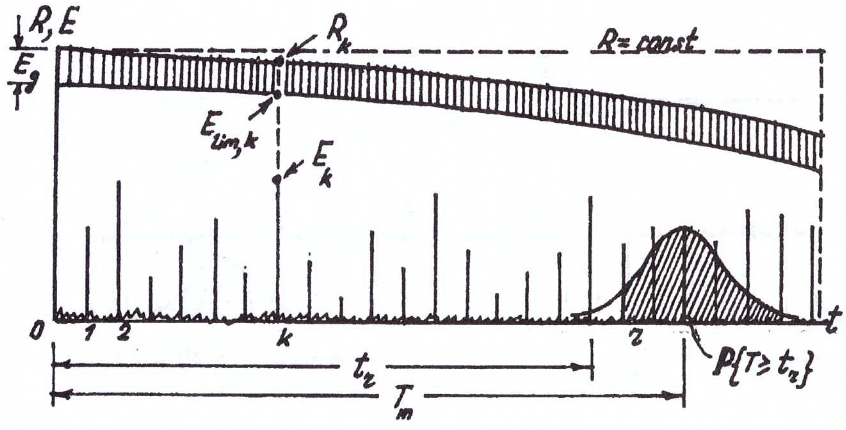

Fig.2. Approximate dynamic model for the long term safety analysis by the method of the limit episodic action effect

According to the method of limit episodic action effect ,the member long term safety index is

$$P\{T \geq t_r\} = P\{ \bigcap_{k=1}^{r} (E_k < E_{lim,k}) \}. \qquad (14)$$

Where E_k and $E_{lim,k}$ are the episodic action effect and its limit value at the cut "k" of the stochastic sequence.

When the episodic action obeys the Gumbel or Weibull probability distribution law the equation (14) is written in the forms as follows:

$$P\{T \geq t_r\} \approx \exp\left[-\sum_{k-1}^{r} \exp\left(\frac{a - E_{lim,k}}{b}\right)\right], \qquad (15)$$

$$P\{T \geq t_r\} \approx \exp\{-\sum_{k-1}^{r} [-\exp(CE^{\alpha}_{lim,k})]\}. \qquad (16)$$

where a,b,c and α are the probability distribution parameters of episodic action effects.

If nonlinear member is subjected to two episodic actions the long term safety index can be calculated by the formula :

$$P\{T \geq t_r\} \approx \exp[-\sum_{k=1}^{n_{12}} Q_{12k}\{1\}] \qquad (17)$$

Here the probability of member limit state under a single application of two loads combination can be evaluated by the formula :

$$Q_{12k}\{1\} = \int_{0}^{E_{lim,k}} g_{E_1}(E)[1 - G_{E_2}(E)]de \qquad (18)$$

where g_{E_1} and G_{E_2} (E) are the density and distribution function of the action effects E_1 and E_2, respectively. If the first of their is distributed according to Gumbel and the second-to Waybull law, these functions can be expres-sed by the formulae :

$$g_{E_1}(E) = \frac{1}{b} \exp\left(\frac{a-E}{b} - \exp\frac{a-E}{b}\right), \qquad (19)$$

$$G_{E_2}(E) = 1 - \exp[- c(N_{lim,k} - N)^{\alpha}] . \qquad (20)$$

5. System safety index

System limit state is reached either by plastic failure or by buckling of its bars or by brittle fracture of welded joints. Therefore ,the global safety of high reliability systems is characterised by the probability

$$P = 1 - Q\left\{\bigcup_{c=1}^{m}\left[\bigcap_{j=1}^{n}\left(Z_{tij} \le 0\right)\right]\right\}, \qquad (21)$$

where Z_{tij} is the member performance function of the failure path " i " at the level " j " calculated by the branch-and-bound method [4].

The reliability analysis of statically determinate systems is less complicated. The analysis model of such systems consists of "**s**" conventional elements, connected in series, because they attain a limit state immediatly after a failure of one element. Thus ,the safety index of statically determinate system can be calculated by the formula :

$$P = P\{Z_{t1} > 0\} * P\{(Z_{t2} > 0) \mid (Z_{t1} > 0)\}\ldots$$
$$\ldots P\{(Z_{ts} > 0) \bigcap_{i=1}^{s-1} (Z_{ti} > 0)\} \qquad (22)$$

It needs to remember, there is close stochastical connection among some structural bars. Therefore, the number of conventional elements is less as the amount of actual members of nonlinear systems.

6. Conclusions

1. The structural safety analysis of nonlinear bar stochastical systems must be based on the probabilistic dynamic of their member performance functions. It is expedient to evaluate the truncated probability distribution of member resistance in the realisation of models.

2. The long term structural safety index of system members exposed to random transient reiterated and variable in time episodic loads can be calculated by the methods of cumulative coefficient of correlation or limit episodic action effects methods ,if dead actions dominate over episodic ones or vice versa ,respectively.

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