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NONLINEAR TRANSPORT AND FLUCTUATION CHARACTERISTICS OF DOPED SEMICONDUCTORS

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Abstract

Fluctuation phenomena in doped n-type GaAs, at moderate applied electric fields being influenced by interelectron scattering, are interpreted in terms of effective electron temperature. Electron Fick's diffusion coefficients in longitudinal and transfer direction are estimated.

INTRODUCTION

Fluctuation phenomena in non-equilibrium steady state of a dissipative system are known to be quite sensitive to details of kinetic processes in the system, often being much more sensitive than mean quantities [1]. In particular, data on electric noise in semiconductors subjected to high electric field contain valuable information on electron scattering processes and band structure not easily obtainable, e.g., from nonlinear charge transport. *Noise spectroscopy* [2] of solid state plasma, especially of effects nonlinear in applied electric field strength, is based on this distinctness of fluctuation phenomena.

A remarkable example of sensitiveness of electronic noise to nonlinearity in carrier density is offered by noise properties of doped semiconductors. Recently it was shown [3,4] that noise properties of a doped semiconductor at moderate applied electric fields, in contrary to its current-voltage characteristic, can be remarkably influenced by interelectron scattering. In particular, the available experimental results [5,6] on microwave noise measured for doped n-type GaAs (impurity density exceeding 10^{17} cm^{-3}) could not be interpreted, even qualitatively, within the framework of models neglecting interelectron collisions. The comparison with the results of Monte Carlo simulation [3,4] showed that interelectron scattering causes an essential (over a hundred times) increase in the field strength required for the excess electric noise to manifest itself. This example in which the influence of interelectron scattering on noise is well controlled deserves detail investigation in terms of the kinetic theory of fluctuations [7,8] (see also [1,2,9]) which predicts, among other effects, appearance of correlation between occupancies of electronic states in nonequilibrium (even provided the individual collisions between electrons remain uncorrelated). Indeed, the analytical treatment of the "kinetic" (or "additional") correlation created by interparticle collisions in nonequilibrium was shown to be possible in cases when frequent enough interelectron collisions control the shape of electron distribution in energy [10] (see also [2,9]), or both in energy and momentum [11] (see also [2]). However, up to now the obtained analytic expressions for noise temperature and additional correlation contribution have never been used for interpretation of experimental results. The reason was that it was not known in what a range of applied fields and to what extent this or that measured noise spectrum is influenced by interelectron collisions. Unambiguous identification achieved in Refs. 3,4 calls for efforts to analyze the experimental results in terms of the above-mentioned analytical approach.

An investigation of the influence of interelectron scattering on noise spectrum by combining analytic and Monte Carlo methods in the case where experimental results are available is called for also by practical reasons. When interelectron collisions are rare, Price's fluctuation-diffusion relation connects the spectral densities of current fluctuations, in a uniform stationary electron gas, with the diffusion coefficients which enter the expressions for the current caused by a small spatial gradient of the electron density. Price's noise-diffusion relation proved to be very useful while providing information on the hot-electron diffusion properties of lightly doped semiconductors from the noise measurements performed in spatially homogeneous states. One of the predictions of the kinetic theory of fluctuations is the violation of Price's relation in a non-equilibrium electron gas in the case when interelectron collisions cannot be neglected. On the other hand, direct measurements of diffusion coefficients in doped semiconductors, being possible in principle, are difficult in practice and up to now were not performed. Effective methods of Monte Carlo simulation of diffusion process in the case of concentration-dependent distribution function are absent [12]. As a result, quantitative data on hot-electron diffusion coefficients in doped semiconductors are lacking.

In this paper, we use analytic and Monte Carlo methods, as well as the available experimental results on noise, to obtain field dependencies of the electron diffusion coefficients for the case when interelectron collisions influence the electronic noise, and Price's relation cannot be used without precaution.

NOISE AND DIFFUSION IN CASE OF FREQUENT INTERELECTRON COLLISIONS

It follows from the kinetic theory of fluctuations in non-equilibrium system (see [9]) that, at sufficiently low (but still microwave) frequencies and for nearly elastic electron scattering by the lattice, both the longitudinal and transverse noise temperatures are expressible in terms of conductivities and electron and lattice temperatures in two special limiting cases [10,11]. Important for us is the case in which so-called effective electron temperature exists, interelectron collisions governing the electron distribution in energy but not in momentum. This is the case when inequality $\tau_p << \tau_{ee} << \tau_\varepsilon$ holds, τ_p and τ_ε being, respectively, the electron momentum and energy relaxation times due to electron collisions with the lattice (with impurities, phonons, etc.), and τ_{ee}^{-1} being the frequency of interelectron collisions. The second case is so-called drifted Maxwellian distribution of electrons which would be realized provided interelectron collisions control the electron distribution both in energy and in momentum.

In both cases, at frequencies $\omega\tau_\varepsilon << 1$, the longitudinal (along the steady current caused by the applied electric field **E**) noise temperature is given by the same expression [10,11]:

$$T_{\parallel} = \frac{T}{T-T_0}\left[\frac{T}{4}\left(\frac{\sigma_{\parallel}}{\sigma}+\frac{\sigma}{\sigma_{\parallel}}+2\right)-T_0\right], \tag{1}$$

where $T(E^2)$ is the electron temperature, T_0 the lattice temperature; the steady current is $j = \sigma(T)\, E$, the differential conductivity $\sigma_{\parallel} = dj/dE$. The transverse noise temperature in both cases equals the electron temperature:

$$T_{\perp} = T\,. \tag{2}$$

Expressions (1), (2) enable to verify, to some extent, validity of the approximations leading to them. If quantities σ, $\sigma_{\parallel}$, $T_{\perp}$, and $T_{\parallel}$ are measured and/or computed through simulation procedures, the validity of the relation (1) for the given situation can be checked up. Noticeable deviations would definitely mean that neither electron temperature approximation works nor the drifted Maxwellian distribution is realized.

The difference between the two cases reveals itself through expressions for the electron diffusion coefficient entering Fick's law. In the electron temperature case [13,14] (see also [2,9]):

$$D_{\parallel} = D_{\perp}\left[1+\left(\frac{\sigma_{\parallel}}{\sigma}-1\right)\left(1+\frac{1}{2}\frac{d\ln\sigma}{d\ln T}\right)\right], \qquad D_{\perp} = \frac{T\sigma}{e^2 n_0}\,, \tag{3}$$

while in the drifted Maxwellian case the relationship between longitudinal and transverse diffusion coefficients is very simple (see [11]):

$$D_{\parallel} = \frac{\sigma_{\parallel}}{\sigma}D_{\perp}, \qquad D_{\perp} = \frac{T\sigma}{e^2 n_0}\,. \tag{4}$$

These expressions enable us to calculate the field dependencies of electron diffusion coefficients from those of conductivity provided electron temperature is known, e.g., from transverse noise temperature measurement or Monte Carlo calculation.

The spectral density of longitudinal velocity fluctuations contains the contribution of additional correlation, $\Delta_{\parallel}$ ([8], see also [2,9]):

$$S_{v\parallel}(\omega) = 4\left(D_{\parallel}-\Delta_{\parallel}/2\right), \qquad S_{v\perp}(\omega) = 4D_{\perp}\,. \tag{5}$$

Thus the additional correlation is responsible for violation of Price's relation. In the electron temperature case [10] (see also [2,9]):

$$\frac{\Delta_{\perp}}{2D_{\parallel}} = -\left(\frac{\sigma_{\parallel}}{\sigma}-1\right)\frac{\frac{T}{2(T-T_0)}\left(\frac{\sigma_{\perp}}{\sigma}-1\right)-\frac{d\ln\sigma}{d\ln T}}{2\frac{\sigma_{\perp}}{\sigma}+\left(\frac{\sigma_{\perp}}{\sigma}-1\right)\frac{d\ln\sigma}{d\ln T}}, \qquad (6)$$

while for the drifted Maxwellian distribution [11] (see also [2]):

$$\frac{\Delta_{\perp}}{2D_{\perp}} = -\frac{T}{4(T-T_0)}\left(\frac{\sigma_{\perp}}{\sigma}+\frac{\sigma}{\sigma_{\perp}}-2\right). \qquad (7)$$

Expressions (6) and (7) determine to what extent Price's relation is violated in both the electron temperature and drifted Maxwellian distribution cases.

CHECK OF ELECTRON TEMPERATURE APPROACH

It is quite interesting to find the ratio $\Delta/2D$ *for a realistic situation.* It can be done for doped GaAs at 80 K for which both experimental results on noise are available [5,6] and detailed Monte Carlo simulation in the framework of the adequate model was performed recently [3,4], the results well coinciding with the experimental data. Electron density $n_0 = 3\ 10^{17} cm^{-3}$ is high enough to expect the electron energy distribution to be controlled by interelectron collisions provided the applied electric field is not too high. Direct simulation confirms this conjecture (see Fig.1). The field dependence of electron temperature available from the mean electron energy, as well as of the transverse noise temperature available from simulation of current fluctuations, is presented in Fig.2.

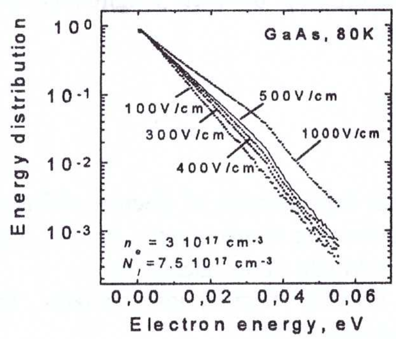

Fig.1. Electron energy distribution up to ~ 500 V/cm is almost exponential (simulation data). N_I - ionized impurity density.

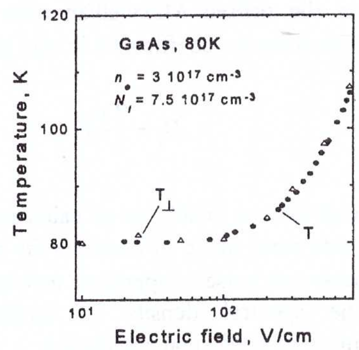

Fig.2. Field dependence of electron temperature T and transverse noise temperature $T_{\perp}$ obtained from Monte Carlo simulation.

The conductivity dependence on electric field is also obtainable: to have sufficiently accurate data in the moderate field range we performed detail measurements of the current-voltage characteristic of the samples in question and compared with the simulation data. The results are presented in Fig.3a.

Data of Figs.2 and 3a enable to calculate the right-hand side of Eq.(1) (Fig.4). On the other hand, the longitudinal noise temperature is known from the experiment and from Monte Carlo simulation of fluctuations (Fig.3b, cf. Ref. 15). To have more accurate data in the moderate field range, we repeated measurements of the longitudinal noise temperature on similar samples. Three curves are presented in Fig.4: the experimental one, also that obtained from the simulation and that calculated as the right-hand side of Eq. (1). All three curves coincide well enough in the field region in question thus confirming the self-consistence of the electron temperature approach in that region.

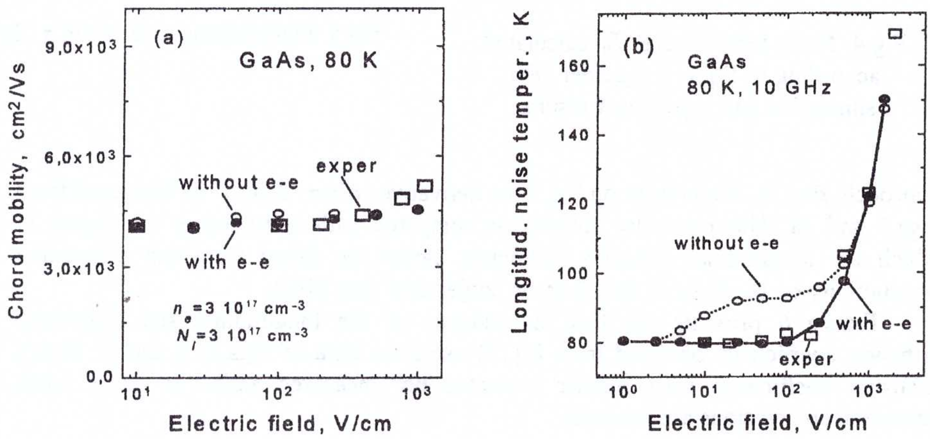

Fig.3. Field dependence of (a) chord mobility and (b) longitudinal noise temperature T [15]: results of measurement, and those of simulation taking into account and neglectin interelectron collisions.

There is no ground for the electron momentum distribution to be controlled by interelectron collisions in the given experimental situation.

ELECTRON DIFFUSION COEFFICIENTS

As the following step we calculate, using the data of Figs.2 and 3, the longitudinal and transverse electron diffusion coefficients within the framework of the electron temperature

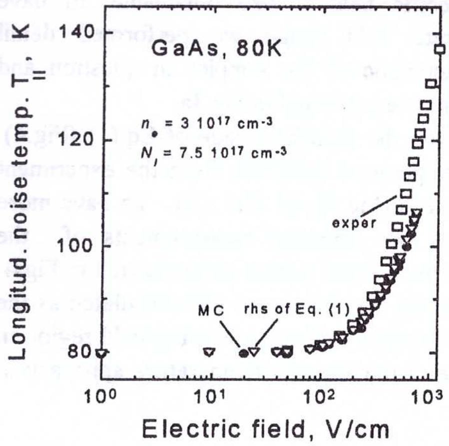

Fig.4. Noise temperature T_{II} calculated according to Eq. (1), together with simulation and experiment results.

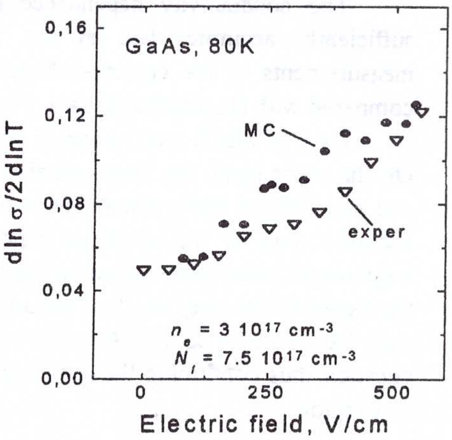

Fig.5. Field dependence of $d\ln\sigma / 2d\ln T$.

approach, Eq.(3). We present on Fig.5 the derivative $d\ln\sigma / 2d\ln T$ as extracted from Figs.2 and 3a. This particular quantity as compared with unity shows the degree in which the longitudinal diffusion coefficient cannot be found from the differential conductivity on the basis of the "Robson conjecture" (see [16]).

Figure 6 presents the field dependence of the longitudinal and transverse diffusion coefficients obtained from Eq.(3) using the data of Figs.2, 3, and 5. Fick's diffusion coefficients were neither evaluated nor measured earlier in cases when interelectron collisions are essential.

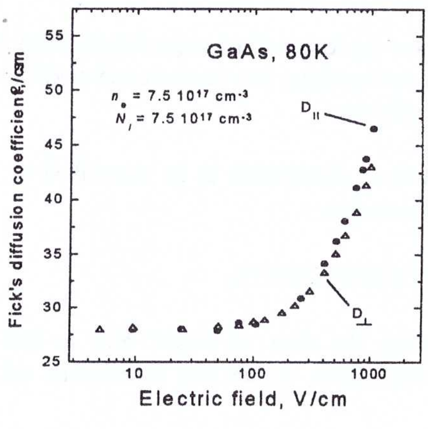

Fig.6. Field dependence of longitudinal and transverse Fick's diffusion coefficients.

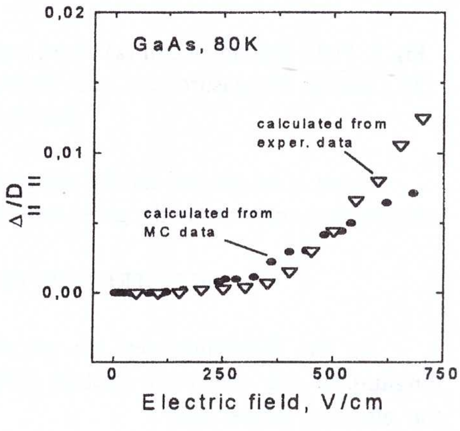

Fig.7. Field dependence of additional correlation contribution.

ESTIMATION OF VIOLATION OF PRICE'S RELATION

Now, one can estimate the degree of violation of Price's relation between diffusion coefficient and current fluctuation spectral density, i.e. the ratio $\Delta_{\parallel}/2D_{\parallel}$ as given by Eq.(6), in the situation in question. Figure 7 presents the field dependence of the ratio $\Delta_{\parallel}/D_{\parallel}$ obtained by using data of Figs.2, 3, and 5. One sees that, though Eq.(6) in general does not contain any small parameter, the ratio $\Delta_{\parallel}/2D_{\parallel}$ in our case is rather small. This result, though nonuniversal, is interesting since it regenerates some possibilities to obtain information about hot-electron diffusion coefficients from noise measurements also in the cases of frequent interelectron collisions.

The smallness of the ratio $\Delta_{\parallel}/2D_{\parallel}$ at moderate fields may have relation to the fact noticed earlier (see [9,17]) that in the electron temperature case, at low bias fields the quantity $\Delta_{\parallel}$ is extremely small. Namely, if bias field is low enough for the deviation from equilibrium with the thermal bath to be small, $\Delta T \equiv T - T_0 << T_0$ *(the warm electrons),* the quantity $\Delta_{\parallel}$ appears only as a correction of the order of $(\Delta T)^2 \propto E^4$, while E^2- terms are typical for corrections to conductivities and diffusion coefficients. Indeed, expanding in E^2, or ΔT, we have

$$\frac{\sigma_{\parallel}}{\sigma} - 1 \approx 2\frac{\Delta T}{T_0}\left(\frac{d\ln\sigma}{d\ln T}\right)_{T=T_0}, \tag{8}$$

so that, in the first approximation in ΔT, the ratio $\Delta_{\parallel}/2D_{\parallel}$ as given by Eq.(6) vanishes (contrary to that given by Eq.(7)). Of course, such a specific behaviour of warm electron fluctuations in the electron temperature approximation does not mean that in this approximation the quantity $\Delta_{\parallel}$ should necessarily remain small also at higher fields.

One can see from Fig.5 that the contribution of the quantity $d\ln\sigma/2d\ln T$, governing the difference in the degree of anisotropy of diffusion coefficient and that of differential conductivity, in the region of fields of interest does not exceed 0.12. So, in our case, with accuracy not less than 12%,

$$\frac{\left(S_{v,\parallel}\right)_{\omega\tau<<1}}{\left(S_{v,\perp}\right)_{\omega\tau<<1}} \approx \frac{D_{\parallel}}{D_{\perp}} \approx \frac{\sigma_{\parallel}}{\sigma_{\perp}}; \tag{9}$$

in particular, with such accuracy the Robson conjecture works.

Of course this does not mean that the same tendency to proportionality of the main kinetic quantities can be expected to survive beyond the limits of validity of inequality $\tau_p << \tau_{ee} << \tau_{\varepsilon}$. The result obtained by Dedulevich *et al.* [17] shows that in the intermediate cases the nonlinearity of the electron distribution with respect to the electron density can lead to efficient violation of Price's relation. We realize that results of this paper, being interesting as such, cannot be safely extrapolated beyond the limits of the electron densities for which the electron temperature approximation holds.

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