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KINK-EXCITATION OF *N*-SYSTEM UNDER SPATIO -TEMPORAL NOISE

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Abstract

Random walk of the nonlinear localized excitations in a dissipative *N*-system, i.e., the influence of the irregular perturbations on the kink-shaped excitations in a system characterized by nonlinearities of "*N*-type", is analyzed. The "evolution" of the randomly walking excitation is described by the one-dimensional PDE (partial differential equation) of the parabolic type. The analysis of the considered excitations is performed for the case of the disturbing torque which is randomly distributed in space and time, and makes up the white Gaussian noise. An iterative scheme of perturbation technique is presented to derive the randomly perturbed solutions of the considered evolution equation in a general case of *N*-system. The average characteristics of the "steady state" of the randomly walking kink-excitations are examined in detail. The explicit expressions that describe the considered random walk are presented for the particular case of the kink-shaped excitations of the free electron gas in semiconductors.

1. INTRODUCTION

Nonlinear localized excitations, i.e., the highly localized solitary states of nonlinear system, are widely known in various fields of physics as well as in many chemical and biological systems [1-12]. The dynamical and stochastic properties of such excitations, i.e., the influence of regular as well as random perturbations on nonlinear excitations, have been intensively analyzed theoretically in the last few decades, mainly in the case of conservative systems [1-5].

The basic evolution equations that describe the solitary excitations in essentially dissipative systems are the nonlinear PDEs of the parabolic type. The evolution equations of such type are frequently employed in plasma physics, hydrodynamics, liquid crystals physics, superconductivity, solid-state physics, chemistry, biophysics, etc. [6-12]. In the most general case they make up a system of

PDEs and the solutions of the considered equations depend on many spatial variables. An analytical derivation of the needed solutions is rather complicated in such cases and the numerical simulations should be employed in examination of the dynamical and stochastic properties of the excitations. Evidently, the most simple models that are used in theoretical analysis of the nonlinear excitations are one-dimensional ones, described by a single spatial variable. The one-dimensional evolution equations are those that are usually employed in a semi-qualitative analysis of the solitary excitations [7, 8]. Moreover, an explicit analytical description of such excitations is also possible in such case, even in the presence of perturbations. A relatively simple model of this kind is presented by the following evolution equation:

$$\frac{\partial u}{\partial t} + \hat{\Lambda} u = b\, f(z,t)\,, \qquad (1.1)$$

where $\hat{\Lambda}$ is the nonlinear operator,

$$\hat{\Lambda} = -D\,\frac{\partial^2}{\partial z^2} + P(.)\,\frac{\partial}{\partial z} + R(.)\,, \qquad (1.2)$$

the functions *P(u)* and *R(u)* describe the nonlinearities of the system under consideration and the parameter $D = l_S^2 / t_S$ denotes the "diffusion coefficient", where l_S and t_S indicate the characteristic length and the characteristic time of the system, respectively. The function *f(z, t)* describes the disturbing torque and the formal parameter $b << 1$ indicates the smallness of the torque *f*. Substituting $b = 0$ into Eq. (1.1) one derives the unperturbed solutions $u_0(z,t)$ that describe the free (undisturbed) excitations. Obviously, the "profile" of the function $u_0(z)$, which characterizes the type of the excitation, depends on the shape of the nonlinear characteristics *P(u)* and *R(u).* In the present report we will be interested in *N*-systems, i.e., the systems that are described by *R-u* characteristic of *N*-type. Relatively simple systems of *N*-type are well known in superconductivity [13], solid state physics [10, 14], liquid crystal physics [15] etc. In addition, the simplified models of the considered *N*-type are useful in analytical considerations of the pattern formation in biological systems, reaction-diffusion media etc. (see Refs. [6, 8, 9,12]). The free localized excitations of *N*-system are fronts and pulses, moving or static ones, which are described by the kink-like and soliton-like profiles respectively. It is well known (see e. g. [12, 16]) that the "pulse-excitations" of *N*-system of the considered type (1.1) are unstable in respect to the small perturbations, and the additional conditions are required to guarantee the stability of such excitations.

The "kink-state", on the other hand, is stable in a relatively wide range of the controlling parameters of the system involved. Thus, in the present report we will be interested in the kink-shaped excitations of the *N*-system governed by Eq. (1.1). For brevity, the kink-excitation will be referred to as "*K*-excitation". The free *K*-excitation, i.e., the steady localized state of a homogeneous infinite *N*-system, is characterized by the parameters u_m and u_M (let there be $u_m < u_M$) which indicate the extreme values of the kink-shaped "field" $u_0(z)$. They are obtained from the equation $R(u) = 0$ and indicate the saddle-points of the free ($b = 0$) evolution equation in the phase plane (u_0, du_0 / dz), namely, ($u_m, 0$) and ($u_M, 0$).

The particular case of *N*-system of the free electron gas in a semiconductor specimen (Gunn diode) has been already analyzed in [14, 16, 17]. In addition, a perturbation technique has been developed in [17] to describe the random walk of the kink-shaped excitations (Gunn waves) influenced by the spatially homogeneous fluctuations. The perturbation scheme presented in [17] may be simply extended to a more general case of *N*-system discussed above, and the dynamical as well as stochastic properties of *K*-excitations may be examined with the needed accuracy. Let us discuss briefly the perturbation technique required.

2. DESCRIPTION OF THE DISTURBED EXCITATION

Considering a slightly disturbed excitation one starts from the equation

$$\widehat{\Lambda}\, u_0(z) = 0, \tag{2.1}$$

which describes the free excitation and follows directly from (1.1) by taking $b = 0$. The perturbed solution of Eq. (1.1) *u(z, t)*, which describes a slightly disturbed ($b << 1$) excitation, is presented in the following manner:

$$u(x,t) = u_0(x) + \Delta u(x,t), \qquad \Delta u << u_0, \tag{2.2}$$

where $x = z + s(t)$, $s(t)$ indicates the phase shift, and the additional field $\Delta u(x,t)$ describes the distortions of the shape of the excitation, both induced by the torque *f*. In the case of the weak torque f one may expand the quantities s, and Δu in a power series of the small parameter b,

$$A_n = \sum_{r=1}^{n} b^r A^{(r)}, \qquad A = s(t),\ \Delta u(x,t),\ F(x,t)\,. \tag{2.3}$$

The function F in (2.3) indicates the "renormalized" forcing which includes the disturbing torque f as well as all nonlinear terms of Eq. (1.1). The two lowest approximations of the forcing F are expressed by the relations,

$$F^{(1)} = f(x,t)$$

$$F^{(2)} = -\{ \frac{ds^{(1)}}{dt} \frac{\partial \Delta u^{(1)}}{\partial x} + \frac{1}{2} R''[u_0(x)]\,(\Delta u^{(1)})^2 + \frac{1}{2}\frac{\partial}{\partial x}[\,P'[u_0(x)\,](\Delta u^{(1)})^2\ ]\} \tag{2.4}$$

Evidently, Eq. (2.3) describes the *n*th-order approximation of the quantity *A*, and the exact result is obtained by taking the limit $n \to \infty$. Starting from (1.1) used in conjunction with (2.1) and (2.3) one may derive the following relations which describe the needed functions $s^{(n)}(t)$ and $\Delta u_n(\xi, t)$, in a simmilar way as in [17] :

$$s^{(n)}(t) = \left\langle \overline{Y} \middle| u_0' \right\rangle^{-1} \int_0^t d\,t \left\langle \overline{Y}(x) \middle| F^{(n)}(x,t) \right\rangle,$$

$$\Delta u_n(x,t) = \sum_a T_a^{(n)}(t)\, Y_a(x),$$

$$T_a^{(n)}(t) = \int_0^t dt\ \exp[\,l_a(t-t)] \left\langle Y_a(x) \middle| F^{(n)}(x,t) \right\rangle. \tag{2.5}$$

Here $u_0' = du_0 / dx$ and the quantities Y_a and l_a indicate the eigenfunction and the corresponding eigenvalue of the operator $\hat{L}$ which is described by the relation ,

$$\hat{L} = -D\,\frac{d^2}{dz^2} + Q(z)\frac{d}{dz} + U(z)\,,$$

where

$$Q(z) = P[u_0(z)]\,, \qquad U(z) = P'[u_0(z)]\,u_0'(z) + R'[u_0(z)]\,. \tag{2.6}$$

Here prime indicates the derivative: $P' = dP / du_0$, $u'_0 = du_0 / dz$, etc., and the symbol $\langle Y_a \mid j \rangle$ in (2.5) denotes the "scalar product" of the following type,

$$\langle Y_a \mid j \rangle = \int_{-\infty}^{\infty} dx\, Y_a^+(x)\, j\,(x,t)\,, \tag{2.7}$$

where dagger designates the adjoint eigenfunctions which are "orthogonal" to the functions Y_a and make up the complete set. The operator $\hat{L}$ defined by Eqs. (2.6) is intimately related to the translational mode $\overline{Y}(x) = du_0(x) / dx$,

$$\hat{L}\,\overline{Y}(x) = 0\,. \tag{2.8}$$

It has been supposed in the derivation of Eqs. (2.5) that the disturbing torque f was "turned on" at the moment $t = 0$. The prime in the sum (2.5) indicates that the translational mode was omitted.

Evidently, the relations (2.7) are valid if the eigenfunctions Y_a make up a complete set and the closure condition of the eigenfunctions is satisfied. One can see from the explicit expression of the operator $\hat{L}$ given by (2.6) that the required conditions are fulfilled if the following relation is satisfied:

$$\lim_{z \to \pm\infty} \{z^{-1}\, Q(z)\} = 0\,. \tag{2.9}$$

The eigenvalues l_a are real in such a case and the eigenfunctions Y_a are simply expressed through the eigenfunctions X_a of the Hermitian operator $S^{-1}\,\hat{L}\,S$,

$$Y_a(z) = S(z)\; X_a(z)\,, \qquad S(z) = \exp\{(2D)^{-1} \int_{-\infty}^{z} dx\, Q(x)\}\,. \tag{2.10}$$

Doubtless, the requirement (2.9) is fulfilled too if $P(u) \equiv 0$. The operator $\hat{L}$ is Hermitian in this case. Eqs. (2.5), used in conjunction with (2.2), (2.3), (2.4), and (2.6), describe the evolution of the slightly disturbed *K*-excitation in a general case of *N*-system discussed above.

3. RANDOM WALK OF *K*-EXCITATIONS

Clearly, the disturbing torque *f* is irregular in the case of the randomly walking excitation. In the present report we will be interested in the random walk of *K*-excitations influenced by a spatio-temporal noise. We suppose that the disturbing torque *f* fluctuates in space and time, and the following relations are satisfied:

$$< f(z,t) >= 0\,,$$
$$B_{i\,j} =< f(z_i, t_i)\; f(z_j, t_j) > = 2\,s^2\; d(z_i - z_j)\,d(t_i - t_j)\,,$$
$$< f(z_1, t_1)\ldots f(z_n, t_n) > = d_{n,2r}\; \sum B_{i\,j}\ldots B_{ml}\;. \tag{3.1}$$

where r is an integer, Σ indicates the sum of correlators $B_{i\,j}$ covering all pair combinations of indices, the parameter s characterizes the intensity of the noise, and the bracket <> denotes the ensemble average. Thus, the fluctuations are Gaussian and make up the white-noise. It should be noted that in a most general case of the propagating (not static) excitation the spatial variable $z = x - c\,t$ in the equations (1.1) and (3.1) indicates the moving coordinate, where c is the velocity of the free excitation. Evidently, the characteristic properties of the disturbing torque described by Eqs. (3.1) do not depend on the velocity of the moving frame *c*, i.e., the considered fluctuations are also Gaussian and in the rest frame. Hence, our results discussed below are valid in either case of the moving or static excitation.

Now, let us turn to the the averaged charcteristics of the randomly walking *K*-excitations which may be described by use of Eqs. (3.1) and the perturbation technique discussed above.

Ensembles and averages. It follows from Eqs. (2.2) and (2.5) that the randomly disturbed *K*-excitation is described by two irregular functions, $x(t)$ and $\Delta u(x,t)$. Hence, two procedures of the averaging are usually used in description of the mean characteristics of the randomly walking excitations (see, e.g. [14, 18]). They correspond to the averages over two statistical ensembles, both describing the random walk of the excitations. The first one that is called the "small ensemble" incorporates the averaging "relative to the walking excitation" and involves the considered excitations of the randomly distorted shapes only. The averaging over irregular shifts of the phase *s(t)* is ignored in this ensemble. The averages over the small ensemble describe the mean characteristics of the excitations that have "arrived" to a certain point of the system. The averages of the second type are obtained by the averaging over the "complete ensemble", which involves all "random excitations" that are characterized by the different shapes and all available phases of the disturbed excitations. This ensemble may be attributed to the infinite collection of the macroscopically identical *N*-systems characterized by unique realizations of the random torque *f(x,t).* Thus, the complete ensemble includes all "realizations" of the perturbed excitations. In the present report we will concern only with the averages over the complete ensemble. The small ensemble averages of the considered *K*-excitations will be discussed elsewhere.

The following notations will be used in the present analysis: the bracket << >> indicates the averaging (procedure) and the overline “ $=$ ”denotes the average (result).

Averages over complete ensemble. Considering the averages over the complete ensemble one should have in mind that both the profiles and the positions of the randomly disturbed excitations are irregular. By the “position” we mean the nucleus center of *K*-excitation. The "complete averages" will be evaluated in the first-order approximation to b. The first-order averages do not vanish in this case. Starting from (2.2) one has,

$$\overline{\overline{u}}_1(z,t) = << u_0(x) >> + << \Delta u^{(1)}(x,t) >> . \tag{3.2}$$

The mean field $\overline{\overline{u}}_0(z)$ may be obtained immediately by taking into account that

$$u_0(x) = \hat{T}(s)\; u_0(z)\,,$$
$$<< u_0(x) >> \; = << \hat{T}(s) >> \; u_0(z)\,, \tag{3.3}$$

where the symbol

$$\hat{T}(s) = \exp[s^{(1)}(t)\;\partial / \partial z\,] \tag{3.4}$$

indicates the translational operator. From (3.4) and (3.1) follows that,

$$<< \hat{T}(s) >> \;\equiv\; \hat{D}(z;t) = \exp[\,2^{-1} << [s^{(1)}(t)]^2 >> \;\partial^2 / \partial z^2\,]\,, \tag{3.5}$$

where

$$<<[\sigma^{(1)}(t)]^2>>=2\sigma_s^2(t) = 2\,\overline{\overline{D}}\;t\,, \qquad \overline{\overline{D}} = \left\langle \overline{Y}^2 \middle| 1 \right\rangle \left\langle \overline{Y} \middle| u_0' \right\rangle^{-2} \;\sigma^2\,, \tag{3.6}$$

and $\hat{D}(z,t)$ denotes the "diffusion operator". From (3.3) and (3.5) immediately follows,

$$\frac{\partial \overline{\overline{u}}_0(z,t)}{\partial t} = \overline{\overline{D}}\;\frac{\partial^2 \overline{\overline{u}}_0(z,t)}{\partial z^2}\,, \qquad \overline{\overline{D}} = \frac{1}{2}\,\frac{\partial <<[\sigma^{(1)}(t)]^2>>}{\partial t}\,. \tag{3.7}$$

Relations (3.7) show that the mean field $\overline{\overline{u}}_0(z,t)$ is governed by the diffusion equation, with the diffusion coefficient $\overline{\overline{D}}$ expressed in terms of the average magnitude of the phase displacements. Namely, from (3.6) one has that $\overline{\overline{D}} \propto \sigma^2$. Note, that the "diffusion time" t_D, which characterizes the rate of the diffusive spreading of the field $\overline{\overline{u}}_0(z,t)$, may be evaluated by the relation $t_D \cong l_G^2\,/\,\overline{\overline{D}}$, where l_G indicates the size of the nucleus of the free excitation.

Let us turn to the additional field $\overline{\overline{\Delta u}}^{(1)}$ described by the relation

$$\overline{\overline{\Delta u}}^{(1)}(z,t) = \sum_{\alpha}{}' \int_{-\infty}^{\infty} dx\, Y_\alpha^{+}(x) \int_0^t d\tau\, \exp[\lambda_\alpha (\tau - t)] << f(x,\tau)\,\hat{T}(\sigma^{(1)}(\tau))>>\; Y_\alpha(z) \tag{3.8}$$

which follows directly from Eqs. (2.5) and (2.4). After evaluation of the required average in (3.8) one gets,

$$\overline{\overline{\Delta u}}^{(1)}(z,t) = 2\sigma^2 \left\langle \overline{Y} \middle| u_0' \right\rangle^{-1} \hat{D}(z;t)\,\frac{d}{dz}\,\sum_{\alpha}{}' b_\alpha(t)\,\lambda_\alpha^{-1} \left\langle \overline{Y} Y_\alpha \middle| 1 \right\rangle Y_\alpha(z)\,, \tag{3.9}$$

where $b_\alpha(t) = 1 - \exp(-\lambda_\alpha\, t)$. The factor $b_\alpha(t)$ in (3.9) indicates a significant increase of the field $\overline{\overline{\Delta u}}^{(1)}$ at the initial moments *t*. We remind that the translational mode $\overline{Y}$ is omitted in sum (3.9). Thus, the eigenvalue $\overline{\lambda} = 0$ is absent in the explicit expression of $b_a(t)$. It is well known (see, e.g., in Ref. 14) that the parameter $\overline{l} = 0$ indicates the lowest eigenvalue of the operator $\hat{L}$ in the considered case of the *K*-excitation. Hence, all eigenvalues l_a in Eq. (3.9) are positive, i.e., one has that $l_a > 0$. Now one may conclude that the factor $b_a(t)$ describes the "relaxation" (increase) of the averaged field $\overline{\overline{\Delta u}}^{(1)}$ to a "steady state" which is reached at the moments $t >> t_R = l_b^{-1}$. The parameter l_b, which indicates the "bottom eigenvalue", may be evaluated approximately from the relation,

$$l_b \cong \overline{R'} = \min\{R'(u_m),\ R'(u_M)\}\,, \qquad (3.10)$$

as one may conclude from the explicit expression of the operator $\hat{L}$ (2.6) by taking into account that the "potential" *U(z)* approaches the fixed values $R'(u_m)$ and $R'(u_M)$ in the limiting cases $z \to \pm\infty$.

Summarizing we note that the additional field of the "steady state", i.e. the field $\overline{\overline{\Delta u}}^{(1)}$ given at the moments $t >> t_R$, is described by the relation

$$\overline{\overline{\Delta u}}^{(1)}(z,t) = \frac{2s^2}{\left\langle \overline{Y} \middle| u_0' \right\rangle}\ \hat{D}(z;t)\ \frac{d}{dz} \sum_a{}' \ l_a^{-1} \left\langle \overline{Y} Y_a \middle| 1 \right\rangle Y_a(z)\,, \qquad (3.11)$$

which follows directly from (3.9) by substitution of $b_a(t) = 1$. Now, from Eqs. (3.11), (3.5), and (3.6) follows that the additional field $\overline{\overline{\Delta u}}^{(1)}$ also obeys the diffusion equation (3.7). Hence, the total field of the "steady state", i.e., the field $\overline{\overline{u}}_1(z,t)$ at the moments $t >> t_R$, is governed by Eq. (3.7). Consequently, the averaged profile of the randomly walking *K*-excitation is fully determined by the diffusive spreading. It is interesting to note that a similar diffusion equation which describes the particular case of the randomly walking K-dV solitons (solitary excitations of Korteweg-de Vries equation) has been derived in Ref. 18.

From (3.7) the field $\overline{\overline{u}}_1(z,t)$ may be obtained immediately in the well known way

$$\overline{\overline{u}}_1(z,t) = [4\pi\, l_G^2\, \Delta t / \tau_D]^{-1/2} \int_{-\infty}^{\infty} dx \exp[-\frac{(z-x)^2}{4\, l_G^2\, \Delta t / \tau_D}]\; u_{00}(x)\,. \qquad (3.12)$$

where $\Delta t = t - t_0$ and the "initial field" $u_{00}(x)$ describes the mean field at any "initial" moment t_0 which is defined by the relation $t_0 >> \tau_R$. The relation $\overline{\overline{D}} = l_G^2 / \tau_D$, discussed above, has been employed in the derivation of (3.12). Evidently, the field $u_{00}(x)$ is defined by the relation,

$$u_{00}(x) \equiv \overline{\overline{u}}_1(x,t_0) = \overline{\overline{u}}_0(x,t_0) + \overline{\overline{\Delta u}}^{(1)}(x,t_0)\,. \qquad (3.13)$$

Let us evaluate the needed field $u_{00}(x)$. The averaged field $\overline{\overline{u}}_1(z,t)$ will be described below by the assumption $\tau_R << \tau_D$ which means that the diffusive spreading of the considered field is slow enough if compared with the rate of relaxation to the steady state discussed above. Now, the needed field $u_{00}(x)$ may be simply evaluated by use of Eqs. (3.13) and (3.11) , if the initial time t_0 is chosen in accordance with the relation $\tau_R << t_0 << \tau_D$. Indeed, the condition $t_0 << \tau_D$ implies that at the moment t_0 the diffusive spread of the averaged field is small enough. Thus, neglecting the diffusive spread of the fields $\overline{\overline{u}}_0(x,t_0)$ and $\overline{\overline{\Delta u}}^{(1)}(x,t_0)$, one substitutes $\hat{D}(x,t) \equiv 1$ into (3.11), and $\overline{\overline{u}}_0(x,t_0) = u_0(x)$ into (3.13) to obtain,

$$u_{00}(x) \approx u_0(x) + \Delta u_0(x)\,,$$

$$\Delta u_0(x) = \overline{\overline{\Delta u}}^{(1)}(x,t_0) = 2\sigma^2 \langle \overline{Y} | u_0' \rangle^{-1} \frac{d}{dx} \sum_{\alpha}{}' \lambda_\alpha^{-1} \langle \overline{Y} Y_\alpha | 1 \rangle Y_\alpha(x)\,. \qquad (3.14)$$

One may conclude now that the averaged field of *K*-excitation, which is governed by evolution equation (1.1), is described by Eqs. (3.12) and (3.14) in a sufficiently general case of *N*-system. We remind that the mean field given by Eqs. (3.12) and (3.14) describes the steady state that was achieved at the moments $t >> \tau_R$ after the “disturbing noise” has been “turned on”. It follows from Eq. (3.9) that the additional field of the “transition stage”, i.e., the field $\overline{\overline{\Delta u}}^{(1)}$ at the moments $t << \tau_R$ does not satisfy the diffusion equation (3.7). Thus, the averaged field at this stage is described in a rather complicated way.

Finally, we present the explicit expressions of the averaged field $\overline{\overline{u}}_1(z,t)$ which has been obtained with the help of Eqs. (3.12), (3.14), and the explicit characteristics *R*(*u*) and *P*(*u*) given in [18]. The characteristics *R*(*u*) and *P*(*u*) presented in [18] may be applied to the various *N*-systems that include the localized structures of the free electrons in Gunn diodes [14], the “overdamped” vortices in Josephson junctions [13], the structures of the order parameter in the liquid crystals [15], and also some simplified models which describe the pattern formation in reaction-diffusion media, biological systems [6 - 9] etc. Without going into details of the employed technique we present the final result which describes the needed field $\overline{\overline{u}}_1(z,t)$ at short ($t_R <<\ t << t_D$) and long ($t >> t_D$) times,

$$\overline{\overline{u}}_1(z,t) \approx u_o[z/L_G(z,t)], \quad L_G(z,t) = 1 + \tau_D^{-1}\,\Delta t\ \tanh z/z\ ,$$
$$(t_R <<\ t << t_D), \tag{3.15}$$

$$\overline{\overline{u}}_1(z,t) \approx \overline{F} + 2^{-1}\Delta\ \ \Phi(2^{-1}\sqrt{\tau_D / t}\ \ z)\,, \qquad (t >> t_D). \tag{3.16}$$

Here $\Phi(x)$ indicates the Fresnel integral. We note that the free *K*-excitation is described by the expression (see in Ref. 18),

$$u_0(z) = \overline{F} + \frac{\Delta}{\pi}\ \text{arctan}\sinh\ z\,, \tag{3.17}$$

where $\overline{F}$ and Δ indicate the free parameters of the excitation. From Eqs. (3.15) and (3.16) the infinite diffusive spreading of the averaged profile of *K*-excitations follows. One can see that at short times the mean field $\overline{\overline{u}}_1(z,t)$ is similar to that of the free excitation with the size of the nucleus linearly increasing in time. The rate of the increase of the nucleus size is proportional to the intensity of the disturbing noise σ^2, as follows from (3.15) and (3.6). It follows from (3.16) that the mean field at long times describes the kink-shaped excitation with the nucleus widely extended over the distances $\left| z \right| \leq \sqrt{t / t_D}$. In addition, the extreme values of the averaged field strictly followed those of the free excitation

It should be noted that the averaged field described by Eqs. (3.15) and (3.16) is very similar to that earlier obtained for the random walk of *K*-excitations influenced by the spatially homogeneous torque $f(t)$ [17]. Nevertheless, the diffusive spreading of *K*-excitation is more intensive in the case of the spatially homogeneous torque $f(t)$, if the intensities of the "disturbing noise" coincide in both cases discussed. Such difference comes from the rather different lengths of the spatial correlations which characterize the irregular torque f in the cases discussed. The increased diffusive spread of the "averaged excitation", which is the case of the spatially homogeneous torque $f(t)$, is a consequence of the extremely strong (rigid) spatial correlations that characterize a spatially homogeneous torque.

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