Magnetohydrodynamic Natural Convection Flow on a Sphere in Presence of Heat Generation

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Abstract. The present work describes the effect of magnetohydrodynamic natural convection flow on a sphere in presence of heat generation. The governing boundary layer equations are first transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations are then solved numerically using the Keller-box method. Here we have focused our attention on the evolution of the surface shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity distribution as well as temperature distribution for a selection of parameter sets consisting of heat generation parameter $Q(= 0.0, 0.5, 1.0, 2.0)$ and the magnetic parameter $M(= 0.0, 0.2, 0.5, 0.8, 1.0)$. Numerical solutions have been considered for Prandtl number $Pr(= 0.7, 1.0, 2.0)$.

Keywords: modelling, magnetohydrodynamic, natural convection, heat generation, sphere.

Nomenclature

a	Radius of the sphere
C_{fx}	Local skin friction coefficient
C_p	Specific heat at constant pressure
f	Dimensionless stream function
Gr	Grashof number
g	Acceleration due to gravity
k	Thermal conductivity
M	Magnetic parameter

Nu_x	Local Nusselt number
Pr	Prandtl number
Q	Heat generation parameter
q_w	Heat flux at the surface
r	Radial distance from the symmetric axis to the surface
T	Temperature of the fluid in the boundary layer
T_∞	Temperature of the ambient fluid
T_w	Temperature at the surface
u, v	Dimensionless velocity components along x, y directions
$\bar{u}, \bar{v}$	Dimensional velocity components along $\bar{x}, \bar{y}$ directions
x, y	Axis in the direction along and normal to the surface respectively

Greek symbols

ψ	Stream function
τ_w	Shearing stress
ρ	Density of the fluid
μ	Viscosity of the fluid
ν	Kinematic viscosity
θ	Dimensionless temperature function
β	Coefficient of thermal expansion
β_0	Strength of magnetic field
σ_0	Electric conduction

Subscripts

w	Wall conditions
∞	Ambient temperature

1 Introduction

A study of the flow of electrically conducting fluid in presence of magnetic field is important from the technical point of view and such types of problems have received much attention by many researchers. The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to the surface. The surface is maintained at a uniform temperature T_w which may either exceed the ambient temperature T_∞ or may be less then T_∞. When $T_w > T_\infty$, an upward flow is established along the surface due to free convection;

while when $T_w < T_\infty$, there is a down flow. Additionally, a magnetic field of strength β_0 acts normal to the surface. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motion. And near the leading edge the velocity is very small so that the magnetic force, which is proportional to the magnitude of the longitudinal velocity and acts in the opposite direction, is also very small. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from the free stream pressure gradient. Kuiken [1] studied the problem of magnetohydrodynamic free convection in a strong cross field. Also the effect of magnetic field on free convection heat transfer has studied by Sparrow and Cess [2]. MHD free convection flow of visco-elastic fluid past an infinite porous plate was investigated by Chowdhury and Islam [3]. Raptis and Kafoussias [4] have investigated the problem of magnetohydrodynamic free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Elbashbeshy [5] also discussed the effect of free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field. But Hossain [6] introduced the viscous and joule heating effects on MHD-free convection flow with variable plate temperature. Moreover, Hossian *et al.* [7–9] discussed the both forced and free convection boundary layer flow of an electrically conducting fluid in presence of magnetic field.

However, the study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution; consequently, the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. In fact, the literature is replete with examples dealing with the heat transfer in laminar flow of viscous fluids. Vajravelu and Hadjinolaou [10] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. In this study they considered that the volumetric rate of heat generation, q^m [w. m^{-3}], should be $q^m = Q_0(T - T_\infty)$, for $T \geq T_\infty$ and equal to zero for $T < T_\infty$, where Q_0 is the heat generation /

absorption constant. The above relation is valid as an approximation of the state of some exothermic process and having T_∞ as the onset temperature. When the inlet temperature are not less than T_∞ they used $q^m = Q_0(T - T_\infty)$. Hossain *et al.* [11] also discussed the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation / absorption. Also the effects of the conjugate conduction-natural convection heat transfer along a thin vertical plate with non-uniform heat generation have studied by Mendez and Trevino [12].

The problems of free convection boundary layer flow over or on bodies of various shapes discussed by many mathematicians, versed engineers and researchers. Amongst them are Nazar *et al.* [13,14], Huang and Chen [15]. Nazar *et al.* [13,14] considered the free convection boundary layer on an isothermal sphere and on an isothermal horizontal circular cylinder in a micropolar fluid. The effect of laminar free convection from a sphere with blowing and suction studied by Huang and Chen [15]. However, Cheng [16] studied the mixed convection both a horizontal cylinder and a sphere in a fluid-saturated porous medium. On the other hand the analysis of mixed forced and free convection about a sphere discussed by Chen and Mucoglu [17].Very recently, Hossain *et al.* studied [18–20] the conjugate effect of heat and mass transfer in natural convection flow from on isothermal sphere with chemical reaction, temperature dependent thermal conductivity and radiation effect respectively. To our best of knowledge, heat generation effect on magnetohydrodynamic free convection flow from an isothermal sphere has not been studied yet and the present work demonstrates the issue.

The present work considers the natural convection boundary layer flow on a sphere of an electrically conducting and steady viscous incompressible fluid in presence of strong magnetic field and heat generation. The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference scheme together with the Keller box technique [21] and [22]. Here we have focused our attention on the evolution of the surface shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity distribution as well as temperature distribution for a selection of parameters sets consisting of heat generation parameter $Q(=0.0, 0.5, 1.0, 2.0)$ and the magnetic parameter $M(=0.0, 0.2, 0.5, 0.8, 1.0)$. Numerical solutions have been considered for various Prandtl number $Pr = 0.7, 1.0$ and 2.0.

2 Formulation of the problem

The steady two dimensional laminar natural convection flow on a sphere of radius a, which is immersed in a viscous incompressible and electrically conducting fluid of ambient temperature T_∞ in presence of uniform transverse magnetic field of strength β_0 is considered. It is assumed that the surface temperature of the sphere is T_w, where $T_w > T_\infty$. The coordinates x and y measure the distance along the surface of the sphere from the stagnation point and the distance normal to the surface of the sphere respectively. The flow configuration and the coordinates system are shown in Fig. 1.

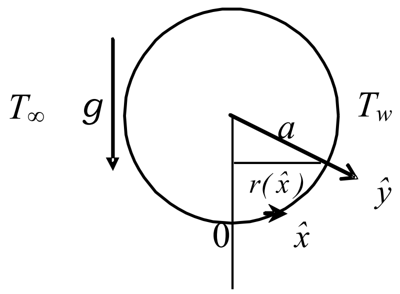

Fig. 1. Physical model and co-ordinate system.

Under the Boussinesq and boundary layer approximations, the governing equations for mass continuity, momentum and energy take the following forms:

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\bar{u}) + \frac{\partial}{\partial \bar{y}}(\bar{r}\bar{v}) = 0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T - T_\infty)\sin\left(\frac{x}{a}\right) - \frac{\sigma_0\beta_0^2}{\rho}\bar{u}, \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty). \tag{3}$$

The boundary conditions for the equations (1) to (3) are

$$\bar{u} = \bar{v} = 0, \quad T = T_w \quad \text{on} \quad \bar{y} = 0, \tag{4a}$$

$$\bar{u} \to 0, \quad T \to T_\infty \quad \text{at} \quad \bar{y} \to \infty. \tag{4b}$$

Where $\bar{r}(\bar{x}) = a\sin(x/a)$ is the radial distance from the symmetrical axis to the surface of the sphere, $(\bar{x}, \bar{y})$ are the dimensional coordinate along and normal to the tangent of the surface of the sphere and $(\bar{u}, \bar{v})$ are the velocity components along $(\bar{x}, \bar{y})$ directions, g is the acceleration due to gravity, β is the coefficient of thermal expansion, $\nu = \mu/\rho$ is the kinematic viscosity, T is the fluid temperature, ρ is the density, σ_0 is the electrical conduction, and $Pr = \mu C_p/k$ is the Prandtl number, k is the thermal conductivity and C_p the specific heat at constant pressure. The amount of heat generated or absorbed per unit volume is $Q_0(T - T_\infty)$, Q_0 being a constant, which may take either positive or negative. The source term represents the heat generation when $Q_0 > 0$ and the heat absorption when $Q_0 < 0$. To make the above equations dimensionless, we introduce the new variables follows as

$$\begin{aligned} &x = \frac{\bar{x}}{a}, \quad y = Gr^{1/4}\frac{\bar{y}}{a}, \quad u = \frac{a}{\nu}Gr^{-1/2}\bar{u}, \quad v = \frac{a}{\nu}Gr^{-1/4}\bar{v}, \\ &\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\beta(T_w - T_\infty)a^3}{\nu^2}. \end{aligned} \tag{5}$$

Where Gr is the is the Grashof number and θ is the non dimensional temperature. Thus we have

$$r(x) = a\sin x. \tag{6}$$

Introducing the above dimensionless variables into equations (1) to (3), we have

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta\sin x - \frac{\sigma_0\beta^2 a^2}{\rho\nu Gr^{1/2}}u, \tag{8}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + \frac{Q_0}{\rho c_p} - \frac{a^2}{\nu Gr^{1/2}}\theta. \tag{9}$$

The boundary conditions associated with equations (7) to (9) are

$$u = v = 0, \quad \theta = 1 \quad \text{at} \quad y = 0, \tag{10a}$$

$$u \to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty. \tag{10b}$$

To solve the equations (7) to (9) subject to the boundary conditions (10), we assume the following variablesy $\psi = xr(x)f(x,y)$. Here ψ is the non-dimensional stream function, which is related to the velocity components in the usual way as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x}. \tag{11}$$

We may proceed to transform the conservation of momentum and energy equations (8) and (9) into the new co-ordinates. To facilitate the transformation, it is useful to have the velocity components explicitly expressed in terms of the new variables. Therefore we obtained

$$\frac{\partial^3 f}{\partial y^3} + \Big(1 + \frac{x}{\sin x}\cos x\Big) f \frac{\partial^2 f}{\partial y^2} - \Big(\frac{\partial f}{\partial y}\Big)^2 + \frac{\theta}{x}\sin x - M\frac{\partial f}{\partial y} = x\Big(\frac{\partial f}{\partial y}\,\frac{\partial^2 f}{\partial y \partial x} - \frac{\partial f}{\partial x}\,\frac{\partial^2 f}{\partial y^2}\Big), \tag{12}$$

$$\frac{1}{Pr}\,\frac{\partial^2 \theta}{\partial y^2} + \Big(1 + \frac{x}{\sin x}\cos x\Big) f \frac{\partial \theta}{\partial y} + Q\theta = x\Big(\frac{\partial f}{\partial y}\,\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\,\frac{\partial \theta}{\partial y}\Big), \tag{13}$$

where $M = \frac{\sigma_0 \beta_0^2 a^2}{\mu G r^{1/2}}$ is the magnetic parameter $Q = \frac{a^2 Q_0}{C_p \mu G r^{1/2}}$ is the heat generation parameter.

The corresponding boundary conditions for the present problem then turn into

$$u = v = 0, \quad \theta = 1 \quad \text{at} \quad y = 0, \tag{14a}$$

$$\frac{\partial f}{\partial y} \to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty. \tag{14b}$$

It has been seen that the lower stagnation point of the sphere i.e. $x \approx 0$, equations (12) and (13) reduce to the following ordinary differential equations:

$$\frac{d^3 f}{dy^3} + 2f\,\frac{d^2 f}{dy^2} - \Big(\frac{df}{dy}\Big)^2 + \theta - M\frac{df}{dy} = 0, \tag{15}$$

$$\frac{1}{Pr}\,\frac{d^2\theta}{dy^2} + 2f\,\frac{d\theta}{dy} + Q\theta = 0 \tag{16}$$

along with the boundary conditions

$$f = \frac{\partial f}{\partial y} = 0, \quad \theta = 1 \quad \text{at} \quad y = 0, \tag{17a}$$

$$\frac{\partial f}{\partial y} \to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty. \tag{17b}$$

However, in practical application, it is very important to calculate the values of the rate of heat transfer and surface shear stress in terms of Nusselt number and the skin friction coefficient respectively. These can be written in non-dimensional form as

$$C_f = \frac{Gr^{-3/4}a^2}{\mu\nu}\tau_w \quad \text{and} \quad Nu = \frac{aGr^{-1/4}}{k(T_w - T_\infty)}q_w. \tag{18}$$

Where $\tau_w = \mu(\frac{\partial \bar{u}}{\partial \bar{y}})_{\bar{y}=0}$ and $q_w = -k(\frac{\partial T}{\partial \bar{y}})_{\bar{y}=0}$, k being the thermal conductivity of the fluid. Using the new variables (6), we have

$$C_{fx} = x\Big(\frac{\partial^2 f}{\partial y^2}\Big)_{y=0}, \tag{19}$$

$$Nu_x = -\Big(\frac{\partial \theta}{\partial y}\Big)_{y=0}. \tag{20}$$

Also we discuss velocity distribution as well as temperature distribution for a selection of parameter sets consisting of heat generation parameter Q, magnetic parameter M with the Prandtl number Pr at $x = \pi/6$ of the surface of the sphere.

3 Results and discussion

Here we have investigated the effect of magnetohydrodynamic natural convection flow on a sphere in presence of heat generation parameter Q. The numerical solutions start at the lower stagnation point of the sphere i.e. at $x \approx 0$, with initial profile as given by equations (12) and (13) along with the boundary conditions (14) and proceed round the sphere up to the point $x \approx \pi/2$. Solutions are obtained for different values of Prandtl numbers $Pr = 0.7, 1.0, 3.0, 7.0$ for a wide range of values of magnetic parameter $M (= 0.0, 0.2, 0.5, 0.8, 1.0)$ with the heat generation parameter $Q = 0.0, 0.5, 1.0, 2.0$. Also the results for local rate of heat transfer and local skin friction coefficient have been obtained from equations (19) and (20) for fluids having Prandtl number $Pr = 0.7, 1.0, 2.0$ and 7.0 at different position of x for a wide range of values of magnetic parameter M with the heat generation parameter Q.

Numerical values of the rate of heat transfer in terms of local Nusselt number Nu_x, as given by equation (19), has been obtained by several authors in the absence of magnetic field and the numerical results are summarized in Table 1 for Prandtl number $Pr = 0.7$ and $Pr = 7.0$ for the surface of the sphere from lower stagnation point $x \approx 0$ to the point $x \approx \pi/2$.

Table 1 depicts the comparisons of the present numerical results of the Nusselt number Nu_x with those obtained by Nazar *et al.* [13] and Huang and Chen [15]. Here, the magnetic parameter M and heat generation parameter Q is ignored and Prandtl numbers $Pr = 0.7$ and 7.0 are chosen. The present results agreed well with the solutions of Nazar *et al.* [13] in the absence of micropolar parameter and of Huang and Chen [15] in the absence of suction and blowing.

Table 1. Comparisons of the present numerical results of Nu for the Prandtl numbers $Pr = 0.7, 7.0$ without effect of the magnetic and heat generation parameter with those obtained by Nazar *et al.* [13] and Huang and Chen [15]

	$Pr = 0.7$			$Pr = 7.0$		
x in degree	Nazar *et al.* [13]	Huang & Chen [15]	Present results	Nazar *et al.* [13]	Huang & Chen [15]	Present results
0	0.4576	0.4574	0.4576	0.9595	0.9581	0.9582
10	0.4565	0.4563	0.4564	0.9572	0.9559	0.9558
20	0.4533	0.4532	0.4532	0.9506	0.9496	0.9492
30	0.4480	0.4480	0.4479	0.9397	0.9389	0.9383
40	0.4405	0.4407	0.4404	0.9239	0.9239	0.9231
50	0.4308	0.4312	0.4307	0.9045	0.9045	0.9034
60	0.4189	0.4194	0.4188	0.8801	0.8805	0.8791
70	0.4046	0.4053	0.4045	0.8510	0.8518	0.8501
80	0.3879	0.3886	0.3877	0.8168	0.8182	0.8161
90	0.3684	0.3694	0.3683	0.7774	0.7792	0.7768

Numerical values of C_{fx} and Nu_x for different values of Prandtl number Pr while $M = 1.0$ and $Q = 1.0$ are depicted in Table 2. From Table 2, we found that the values of local skin friction coefficient C_{fx} increase at different position of x for Prandtl number $Pr = 0.7, 1.0$ and 2.0. The rate of increase of local skin friction coefficient C_{fx} is $3.81335\,\%$ as the Prandtl number Pr change from

Table 2. The values of C_{fx} and Nu_x while $M = 1.0$ and $Q = 1.0$ for different values of Prandtl number Pr

x	$Pr = 0.7$		$Pr = 1.0$		$Pr = 2.0$	
	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x
0.0	0.00000	−0.75118	0.00000	−0.99796	0.00000	−1.78159
$\pi/18$	0.16585	−0.75593	0.16731	−1.00384	0.17158	−1.79078
$\pi/9$	0.32912	−0.76901	0.33222	−1.02002	0.34112	−1.81608
$\pi/6$	0.48732	−0.79061	0.49241	−1.04678	0.50664	−1.85794
$\pi/4$	0.70962	−0.83968	0.71873	−1.10761	0.74297	−1.95330
$\pi/3$	0.90643	−0.91000	0.92121	−1.19498	0.95879	−2.09077
$\pi/2$	1.18795	−1.12109	1.22032	−1.45852	1.29766	−2.50950

0.7 to 2.0. Furthermore, it is seen that the numerical values of local heat transfer rate Nu_x decrease for increasing values Prandtl number Pr. As Pr increases, the maximum point moves towards the stagnation point and this suggest that a surface transfer more heat for small values of Prandtl number Pr. And the rate of decrease of local heat transfer rate Nu_x is $57.44695\,\%$ as the Prandtl number Pr change from 0.7 to 2.0.

Figs. 2(a)–(b) illustrate the variation of local skin friction coefficient C_{fx} and the rate of local heat transfer Nu_x with x for different values of heat generation parameter $Q(= 0.0, 0.5, 1.0, 2.0)$ as obtained by solving numerically equations (12) and (13) where $M = 0.5$. The Prandtl number Pr is taken equal to 0.7, which corresponds to the air. It is seen from Fig. 2(a) that the skin friction coefficient

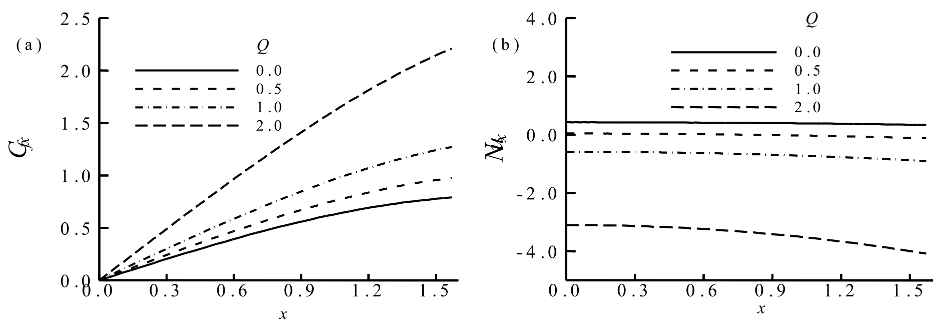

Fig. 2. Variation of Local skin-friction C_{fx} (a) and rate of heat transfer Nu_x (b) with x for different values Q while $M = 0.5$ and $Pr = 0.7$.

C_{fx} is influenced considerably and increases when the values of heat generation parameter Q increases at different position of x (in radian) with magnetic parameter $M = 0.5$ and Prandtl number $Pr = 0.7$. Thus we may conclude that when the body is at low temperature compared with fluid, say air, the skin friction is high. Moreover, Fig. 2(b) indicates that the rate of heat transfer Nu_x decreases owing to increase the values of heat generation parameter Q with magnetic parameter $M = 0.5$ and the Prandtl number $Pr = 0.7$. It can be seen that the analytical solution is an excellent agreement with the numerical solution and this is expected, since the heat generation mechanism will increase the fluid temperature near the surface. On the other hand, the presence of heat absorption ($Q < 0$) creates a layer of cold fluid adjacent to the heated surface and therefore the heat transfer rate from the surface increases. This means that if the body is at low temperature compared with fluid (say air) the rate of heat transfer is very slow.

Figs. 3(a)–(b) display results for the velocity and temperature profiles, based on equations (12) and (13) with the boundary conditions (14), for different small values of generation parameter $Q(= 0.0, 0.5, 1.0, 2.0)$ plotted against y at $x = \pi/6$ having Prandtl number $Pr = 0.7$ and $M = 0.5$. It is seen from Fig. 3(a) that the velocity profile is influenced considerably and increases when the value of heat generation parameter Q increases. But near the surface of the sphere velocity increases significantly and then decreases slowly and finally approaches to zero. The maximum values of the velocity are $0.29811, 0.38816, 0.52165$ and 0.86885 for $Q = 0.0, 0.5, 1.0$, and 2.0 respectively which occur at $y = 1.05539$ for first

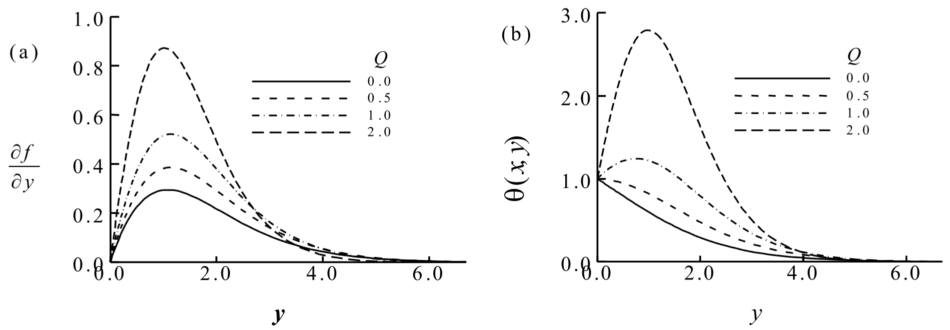

Fig. 3. Variation of velocity profiles (a) and dimensionless temperature (b) with y for various values of Q while $M = 0.5$ and $Pr = 0.7$.

maximum value, $y = 1.11440$ for second and third maximum values and at $y = 1.99806$ for last maximum value. Here it is observed that the velocity increase by $65.6891\,\%$ as Q increases from 0.0 to 2.0. Also from Fig. 3(b), we observed that when the value of heat generation parameter Q increases, the temperature distribution $q(x, y)$ also increases significantly. Here it is observed that for $Q = 0.0$ and 0.5, the maximum values of the temperature profiles are attained at the surface but for $Q = 1.0$ and 2.0 the maximum values of the temperature are 1.21952 and 2.69520 respectively which occur at $y = 0.73363$ and $y = 0.94233$. Thus the temperature profiles increase by $55.6440\,\%$ as Q increases from 0.0 to 2.0.

The variation of the reduced local skin friction coefficient C_{fx} and local rate of heat transfer Nu_x for different values of magnetic parameter $M (= 0.0, 0.2, 0.5, 0.8, 1.0)$ with x are illustrated in Figs. 4(a)–(b) both for $Q = 1.0$ and $Pr = 0.7$. From Fig. 4, it is observed that both local skin friction coefficient C_{fx} and local rate of heat transfer Nu_x decrease slightly as the values of magnetic parameter M increases at different position of x (in radian). Thus we can say that the magnetic field is limited to retardation though in presence of heat generation parameter Q.

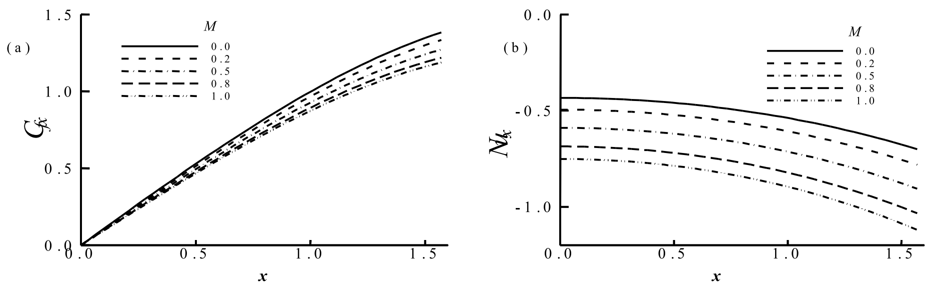

Fig. 4. Variation of Local skin-friction C_{fx} (a) and Local Nusselt number Nu_x (b) with x for some small values of M where $Q = 1.0$ and $Pr = 0.7$.

Figs. 5(a)–(b) deal with the effect of magnetic parameter M on the velocity and temperature distributions against y at $x = \pi/6$ with the heat generation parameter $Q = 1.0$ and the Prandtl number $Pr = 0.7$. Here it is found from Fig. 5(a) that the velocity distribution decreases slightly as the magnetic parameter $M (= 0.0, 0.2, 0.5, 0.8, 1.0)$ increases in the region $y \in [0, 6]$ but near the surface of the sphere velocity increases and become maximum and then decreases and

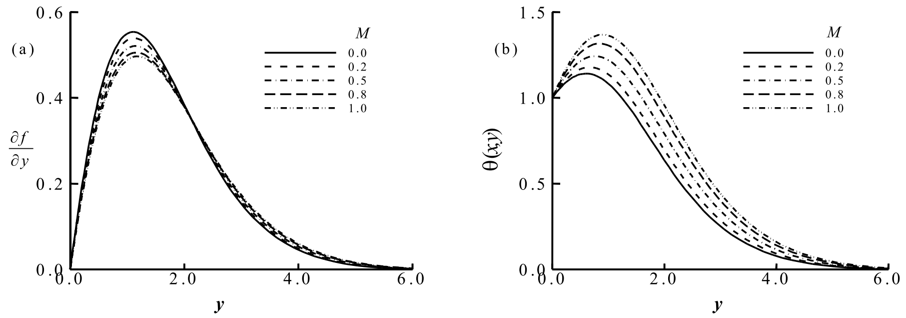

Fig. 5. Variation of velocity profiles (a) and dimensionless temperature (b) with y for some small values of M while $Q = 1.0$ and $Pr = 0.7$.

finally approaches to zero. The maximum values of the velocity are 0.55382, 0.53513, 0.52090 and 0.49694 for $M = 0.0, 0.2, 0.5, 0.8$ and 1.0 respectively which occur at $y = 1.11446$ for first and third maximum values and at $y = 1.17520$ for last two maximum values. Here we see that the velocity decreases by $10.27\,\%$ as M increases from 0.0 to 1.0. However Fig. 5(b) shows the distribution of the temperature profiles $q(x, y)$ with y for some small values of magnetic parameter $M(= 0.0, 0.2, 0.5, 0.8, 1.0)$. Clearly it is seen that the temperature distribution $q(x, y)$ increases owing to increasing the values of magnetic parameter M and the maximum moves closer to the wall. The local maximum values of the temperature profiles are $1.14148, 1.17917, 1.24350, 1.31649$ and 1.36874 for $M = 0.0, 0.2, 0.5, 0.8$ and 1.0 respectively which occurs at $y = 0.58973, 0.68459$, $0.73363, 0.83530$ and 0.88811. Here we found that the temperature profiles increase by $16.6035\,\%$ as M increases from 0.0 to 1.0.

4 Conclusions

Magnetohydrodynamics natural convection flow from an isothermal sphere with temperature dependent heat generation has been investigated here. Numerical results of the equations governing the flow are obtained by using implicit finite difference method together with the Keller-box method. From the present investigation, it may be drawn the following conclusions:

- An increase in the values of heat generation parameter Q leads to increase the local skin friction coefficient C_{fx} but decrease the local rate of heat transfer Nu_x.

- Both the velocity and temperature profiles increase significantly when the values of heat generation parameter Q increase.
- The local skin friction coefficient C_{fx} and the local rate of heat transfer Nu_x decrease slightly when the values of magnetic parameter M increase.
- For increased values of magnetic parameter M the velocity distribution decreases but the temperature distribution increases slightly.

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