Dufour and Soret Effects on Mixed Convection Flow Past a Vertical Porous Flat Plate with Variable Suction

M. S. Alam[1]**, M. M. Rahman**[2]

[1]Department of Mathematics, Dhaka University of Engineering and Technology (DUET)
Gazipur-1700, Bangladesh
msalam631@yahoo.com

[2]Department of Mathematics, University of Dhaka
Dhaka-1000, Bangladesh
mansurdu@yahoo.com

Received: 12.12.2005 **Published online:** 27.02.2006

Abstract. In this paper the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate embedded in a porous medium have been studied numerically. The governing non-linear partial differential equations have been transformed by a similarity transformation into a system of ordinary differential equations, which are solved numerically by applying Nachtsheim-Swigert shooting iteration technique together with sixth order Runge-Kutta integration scheme. For fluids of medium molecular weight (H_2, air), profiles of the dimensionless velocity, temperature and concentration distributions are shown graphically for various values of suction parameter f_w, Dufour number Du and Soret number Sr. Finally, numerical values of physical quantities, such as the local skin-friction coefficient, the local Nusselt number and the local Sherwood number are presented in tabular form.

Keywords: mixed convection, porous medium, variable suction, Dufour effect, Soret effect.

Nomenclature

C	Concentration
c_p	Specific heat at constant pressure
c_s	Concentration susceptibility
Da	Local Darcy number
D_m	Mass diffusivity
Du	Dufour number
f_w	Dimensionless suction velocity

g	Acceleration due to gravity
g_c	Temperature buoyancy parameter
g_s	Mass buoyancy parameter
Gr	Local Grashof number
Gm	Local modified Grashof number
K	Darcy permeability
k_T	Thermal diffusion ratio
Nu	Nusselt number
Pr	Prandtl number
Re	Local Reynods number
Sc	Schmidt number
Sh	Sherwood number
Sr	Soret number
T	Temperature
T_m	Mean fluid temperature
U_∞	Uniform velocity
u, v	Darcian velocities in the x and y-direction respectively
x, y	Cartesian coordinates along the plate and normal to it, respectively

Greek symbols

α	Thermal diffusivity
β	Coefficient of thermal expansion
β^*	Coefficient of concentration expansion
σ	Electrical conductivity
ρ	Density of the fluid
ν	Kinematic viscosity
θ	Dimensionless temperature
ϕ	Dimensionless concentration
ω	Condition at wall
∞	Condition at infinity

1 Introduction

Combined heat and mass transfer by free convection in a porous medium has attracted considerable attention in the last several decades, due to its many important engineering and geophysical applications. A comprehensive reviews on this area have been made by many researchers some of them are Nield and Bejan [1], Ingham and Pop [2, 3], Bejan and Khair [4] and Trevisan and Bejan [5].

But in the preceding papers, the Dufour and Soret effects were neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. However, exceptions are observed therein. The thermal-diffusion (Soret) effect, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air) the diffusion-thermo (Dufour) effect was found to be of order of considerable magnitude such that it cannot be ignored (Eckert and Drake [6]). In view of the importance of above-mentioned effects, Kafoussias and Williams [7] studied thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Recently, Anghel *et al.* [8] investigated the Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. Very recently, Postelnicu [9] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Therefore, the objective of the present paper is to investigate the Dufour and Soret effects on steady mixed convection flow past a semi-infinite vertical porous flat plate in a porous medium with variable suction.

2 Formulation of the problem and similarity analysis

We consider the mixed free-forced convective and mass transfer flow of a viscous incompressible fluid over an isothermal semi-infinite vertical porous flat plate embedded in a porous medium. The flow is assumed to be in the x-direction, which is taken along the vertical plate in the upward direction, and the y-axis is taken to be normal to the plate. The surface of the plate is maintained at a uniform constant temperature T_w and a uniform constant concentration C_w, of a foreign fluid, which are higher than the corresponding values T_∞ and C_∞, respectively, sufficiently far away from the flat surface. It is also assumed that the free stream velocity U_∞, parallel to the vertical plate, is constant. The flow configuration and the coordinates system are shown in Fig. 1.

Under the boundary-layer and Darcy-Boussinesq approximations, the basic boundary-layer equations are given by:

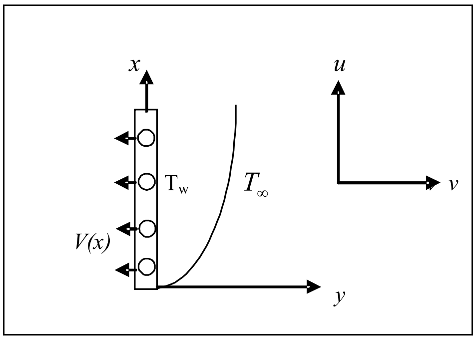

Fig. 1. Physical model and coordinate system.

continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\nu u}{K}, \tag{2}$$

energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

concentration equation

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where u and v are the fluid velocity components along the x- and y-axes (which are parallel and normal to the plate respectively), ν is the kinematic viscosity, g is the gravitational force due to acceleration, ρ is the density, β is the coefficient of volume expansion, β^* is the volumetric coefficient of expansion with concentration, K is the Darcy permeability, α is the thermal diffusivity, D_m is the coefficient of mass diffusivity, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature, k_T is the thermal diffusion ratio and c_s is the concentration susceptibility.

The boundary conditions for the present problem are given by:

$$\begin{cases} u = 0, \quad v = \pm V(x), \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \\ u \to U_\infty, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty. \end{cases} \tag{5}$$

We now introduce the following dimensionless variables:

$$\begin{cases} \eta = y\sqrt{\dfrac{U_\infty}{\nu x}}, \\ \psi = \sqrt{\nu x U_\infty}\, f(\eta), \\ \theta(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty}, \\ \phi(\eta) = \dfrac{C - C_\infty}{C_w - C_\infty}, \end{cases} \tag{6}$$

where ψ is the stream function that satisfies the continuity equation (1) and defined in the usual manner such that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature of the fluid in the boundary layer region, $\phi(\eta)$ is the dimensionless species concentration of the fluid in the boundary layer region and η is the similarity variable.

Introducing the relation (6) into the equations (2)–(4) we obtain the following local similarity equations:

$$f''' + \frac{1}{2} f f'' + g_s\theta + g_c\phi - \frac{1}{DaRe} f' = 0, \tag{7}$$

$$\theta'' + \frac{1}{2} Pr f \theta' + PrDu\phi'' = 0, \tag{8}$$

$$\phi'' + \frac{1}{2} Scf\phi' + ScSr\theta'' = 0, \tag{9}$$

where $Gr = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$ is the local Grashof number, $Gm = \frac{g\beta^*(C_w - C_\infty)x^3}{\nu^2}$ is the local modified Grashof number, $g_s = \frac{Gr}{Re^2}$ is the temperature buoyancy parameter, $g_c = \frac{Gm}{Re^2}$ is the mass buoyancy parameter, $Da = \frac{K}{x^2}$ is the local Darcy number, $Re = \frac{U_\infty x}{\nu}$ is the local Reynolds number, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $S_c = \frac{\nu}{D_m}$ is the Schmidt number, $Sr = \frac{D_m k_T (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)}$ is the Soret number and $Du = \frac{D_m k_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)}$ is the Dufour number.

The boundary conditions are now transformed to:

$$\begin{cases} f = f_w, \quad f' = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0, \\ f' = 1, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad \eta \to \infty, \end{cases} \tag{10}$$

where $f_w = (2xv(x)/\nu)Re^{-\frac{1}{2}}$ is the dimensionless suction velocity and prime denotes differentiation with respect to the variable η.

The set of equations (7)–(9) under the boundary conditions (10) have been solved numerically by applying the Nachtsheim-Swigert [10] (for detailed discussion of the method see Alam [11] and Ferdows *et al.* [12]) shooting iteration technique together with Runge-Kutta sixth-order integration scheme. From the process of numerical computation, the local skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$, are also worked out and their numerical values are presented in a tabular form.

3 Results and discussion

Numerical calculations have been carried out for different values of f_w, Sr, Du and for fixed values of Pr, Sc, g_s, g_c, Da and Re. The value of Prandtl number (Pr) is taken to be 0.71 which corresponds to air and the value of Schmidt number (Sc) is chosen to represent hydrogen at 25° C and 1 atm. The values of Dufour number (Du) and Soret number (Sr) are chosen in such a way that their product is constant provided that the mean temperature T_m is kept constant as well. The dimensionless parameter g_s takes the value 1, which corresponds to the combined free-forced convection regime, and the corresponding parameter gc takes the value 0.1 for low concentration.

The numerical results for the velocity, temperature and concentration profiles are displayed in Figs. 2–7. The effects of suction parameter f_w on the velocity field are shown in Fig. 2. It is seen from this figure that the velocity profiles decrease monotonically with the increase of suction parameter indicating the usual fact that suction stabilizes the boundary layer growth. The effect of suction parameter (f_w) on the temperature and concentration field are displayed in Fig. 3 and Fig. 4 respectively and we see that both the temperature and concentration decreases with the increase of suction parameter. Sucking decelerated fluid particles through the porous wall reduce the growth of the fluid boundary layer as well as thermal and concentration boundary layers.

The effects of Soret and Dufour numbers on the velocity field are shown in Fig. 5. We observe that quantitatively when $\eta = 2.0$ and Sr decreases from 2.0

to 0.4 (or Du increases from 0.03 to 0.15) there is 3.64 % increase in the velocity value, whereas the corresponding increase is 3.38 %, when Sr decreases from 0.4 to 0.08.

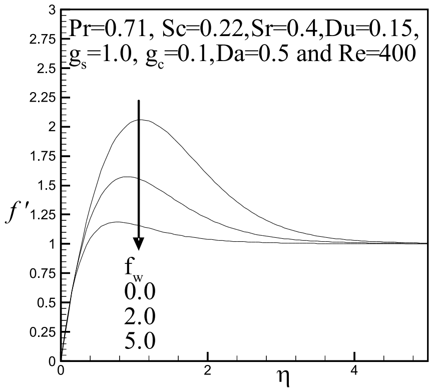

Fig. 2. Velocity profiles for different values of f_w.

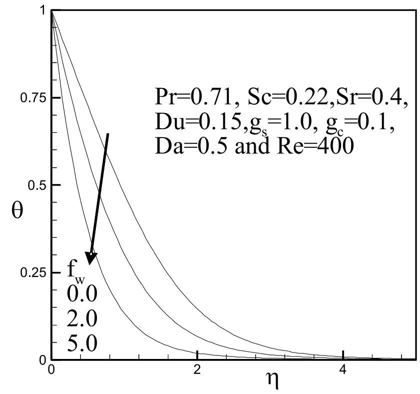

Fig. 3. Temperature profiles for different values of f_w.

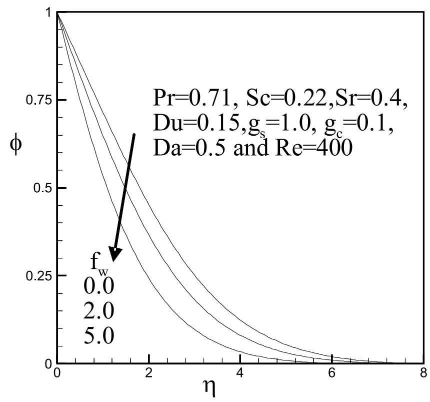

Fig. 4. Concentration profiles for different values of f_w.

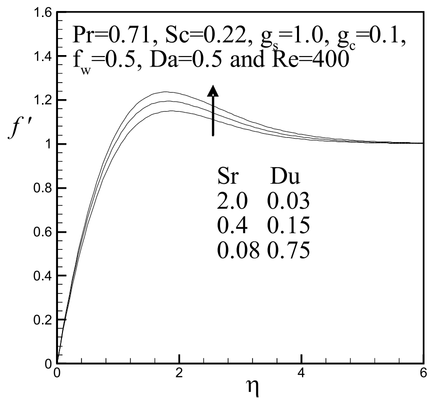

Fig. 5. Velocity profiles for different values of Du and Sr.

The effects of Soret and Dufour numbers on the temperature field are shown in Fig. 6. We observe that quantitatively when $\eta = 2.0$ and Sr decreases from 2.0 to 0.4 (or Du increases from 0.03 to 0.15) there is 8.14 % increase in the temperature value, whereas the corresponding increase is 15.49 %, when Sr decreases from 0.4 to 0.08.

The effects of Soret and Dufour numbers on the concentration field are shown in Fig. 7. We observe that quantitatively when $\eta = 2.0$ and Sr decreases from 2.0 to 0.4 (or Du increases from 0.03 to 0.15) there is $33.62\,\%$ decrease in the concentration value, whereas the corresponding decrease is $3.34\,\%$, when Sr decreases from 0.4 to 0.08.

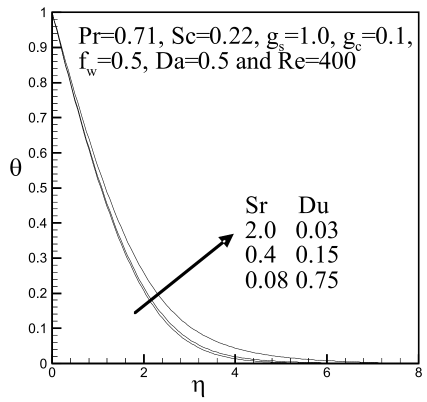

Fig. 6. Temperature profiles for different values of Du and Sr.

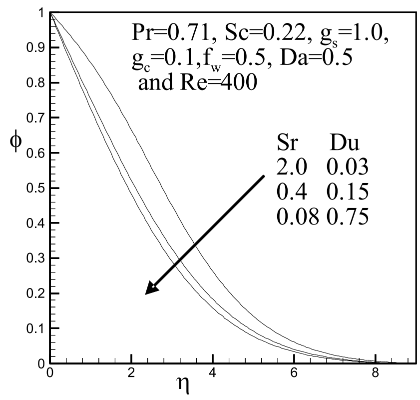

Fig. 7. Concentration profiles for different values of Du and Sr.

Finally, the effects of Dufour and Soret number on the local skin-friction coefficient and the rate of heat and mass transfer are shown in Table 1. The behavior of these parameters is self-evident from the Table 1 and hence any further discussion about them seem to be redundant.

Table 1. Numerical values of local skin-friction coefficient (Cf), local Nusselt number (Nu) and local Sherwood number (Sh) for $Pr = 0.71$, $Sc = 0.22$, $g_s = 1.0$, $g_c = 0.1$, $f_w = 0.5$, $Da = 0.5$ and $Re = 400$

Du	Sr	Cf	Nu	Sh
0.030	2.0	1.6795	0.5310	0.1292
0.037	1.6	1.6758	0.5299	0.1605
0.050	1.2	1.6724	0.5285	0.1921
0.060	1.0	1.6712	0.5275	0.2077
0.075	0.8	1.6707	0.5263	0.2233
0.120	0.5	1.6723	0.5230	0.2470
0.600	0.1	1.7218	0.4908	0.2817

4 Conclusions

In this paper we have studied numerically the Dufour and Soret effects on mixed convection flow past a semi-infinite vertical porous flat plate embedded in a porous medium for a hydrogen-air mixture as the non-chemical reacting fluid pair. From the present study we have found that wall suction stabilizes the velocity, thermal as well as concentration boundary layer growth. The presented analysis has also shown that the flow field is appreciably influenced by the Dufour and Soret effects. Therefore, we can conclude that for fluids with medium molecular weight (H_2, air), Dufour and Soret effects should not be neglected.

References

1. D. A. Nield, A. Bejan, *Convection in Porous Media*, 2nd edition, Springer, New York, 1999.

2. D. B. Ingham, I. Pop, *Transport Phenomena in Porous Media I,* Pergamon, Oxford, 1998.

3. D. B. Ingham, I. Pop, *Transport Phenomena in Porous Media II,* Pergamon, Oxford, 2002.

4. A. Bejan, K. R. Khair, Heat and mass transfer by natural convection in a porous medium, *Int. J. Heat Mass Transfer,* **28**, pp. 909–918, 1985.

5. O. V. Trevisan, A. Bejan, Combined heat and mass transfer by natural convection in a porous medium, *Adv. Heat Transfer,* **20**, pp. 315–352 1990.

6. E. R. G. Eckert, R. M. Drake, *Analysis of Heat and Mass Transfer,* McGraw-Hill, New York, 1972.

7. N. G. Kafoussias, E. M. Williams, Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity, *Int. J. Engng. Sci.,* **33**, pp. 1369–1384, 1995.

8. M. Anghel, H. S. Takhar, I. Pop, Dufour and Soret effects on free convection boundary-layer over a vertical surface embedded in a porous medium, *Studia Universitatis Babes-Bolyai. Mathematica,* **XLV**(4), pp. 11–21, 2000.

9. A. Postelnicu, Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects, *Int. J. Heat Mass Transfer,* **47**, pp. 1467–1472, 2004.

10. P. R. Nachtsheim, P. Swigert, *Satisfaction of the asymptotic boundary conditions in numerical solution of the system of nonlinear equations of boundary layer type,* NASA TND-3004, 1965.

11. M. S. Alam, *Thermal diffusion and diffusion-thermo effects on Magnetohydrodynamic heat and mass transfer,* M. Phil. Thesis, BUET, Bangladesh, 2004.

12. M. Ferdows, M. A. Sattar, M. N. A. Siddiki, Numerical Approach on Parameters of the Thermal Radiation Interaction with Convection in a Boundary Layer Flow at a Vertical Plate with Variable Suction, *Thammasat Int. J. Sci. Tech.,* **9**(3), pp. 19–28, 2004.