

Control of Vibrations due to Moving Loads on Suspension Bridges

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Abstract. The flexibility and low damping of the long span suspended cables in suspension bridges makes them prone to vibrations due to wind and moving loads which affect the dynamic responses of the suspended cables and the bridge deck. This paper investigates the control of vibrations of a suspension bridge due to a vertical load moving on the bridge deck with a constant speed. A vertical cable between the bridge deck and the suspended cables is used to install a hydraulic actuator able to generate an active control force on the bridge deck. Two control schemes are proposed to generate the control force needed to reduce the vertical vibrations in the suspended cables and in the bridge deck. The proposed controllers, whose design is based on Lyapunov theory, guarantee the asymptotic stability of the system. The MATLAB software is used to simulate the performance of the controlled system. The simulation results indicate that the proposed controllers work well. In addition, the performance of the system with the proposed controllers is compared to the performance of the system controlled with a velocity feedback controller.

Keywords: suspension bridges, moving loads, vibration control.

1 Introduction

Long steel suspended cables such as the ones used in suspension bridges and electric transmission lines are prone to vibration induced by wind and vertical loads [1] and [2]. Suspended cables supporting bridges (see Fig. 1) are tensioned due to the weight of the bridge deck, the traffic loading, and their own weights. When the suspended cables are subjected to any disturbance due to wind or vertical loads, and due to the coupling between the bridge deck and the suspended

cables, the system behaves nonlinearly due to the flexibility of the cables [3]. Active control is a viable technology for enhancing structural functionality and safety of systems such as suspended cables supporting bridges [4].

Several researchers have investigated the control of structures such as buildings, bridges and cables. The paper by Ni *et al.* [5] gives a very good review of the research status of active/semiactive vibration control of cable supported bridges.

An active control to the girder stability problem due to wind loading of a very long suspension bridge is proposed in [6]; the active control is based on movable flaps attached to the bridge girder. Using wind tunnel experiments on a bridge section, it is shown that flaps can be used effectively to control bridge girder vibrations. In [2] and [7], an active vibration control of long span suspension bridge flutter using separated flaps is used to increase effectively the critical wind speed of the bridges as well as to reduce the mean square of girder response to turbulence buffeting. An active aerodynamic control method of suppressing flutter of a very long-span bridge is proposed in [8]. In this method, the control system consists of additional control surfaces attached to the bridge deck; their torsional movement, commanded via feedback control law, is used to generate stabilizing aerodynamic forces. A method of suppression of flutter in long-span bridges based on the concept of eccentric mass is proposed in [9]; an auxiliary mass is placed on the windward side of a bridge deck to shift the center of gravity, and thus, the aerodynamic moment acting on the deck is reduced, resulting in an increase in the flutter wind speed. Active Control of Flutter of Bridges is also investigated in [10, 11] and [12].

Recently, the bridge vibration controls due to high wind speeds has been investigated in [13]; the authors proposed a movable passive control facility which can effectively increase the maximum speed limit for bridge service in hurricane evacuations and simultaneously increase the flutter critical wind speed. The dynamic response of a suspension bridge due to a vertical load moving with a constant speed on the bridge deck is studied in [14]; control mechanisms are suggested to generate control forces to control the non-linear vibrations in the bridge deck and the suspended cables.

Control of nonlinear vibrations due to the interactions of moving vehicles and bridges structure has been tackled by many researchers [15–20]. Tsao *et*

al. proposed a system model representation for general multiple moving lumped-parameter systems interacting with a distributed-parameter systems and applied it to the vehicle-bridge interaction problem. The main aspect of this work is that the form of the linear parameter varying system developed allows us to consider the analysis and control design using the theoretical results in this field. In [19], Karoumi uses finite element method to model and analyze the cable-stayed bridges under the action of moving vehicles. In [17], the effect of using semi-active control strategy in vehicle suspensions on the coupled vibrations of a vehicle traversing a bridge is examined and a various designs of suspension systems for bridge-friendly vehicle are proposed. Recently, an intensive analysis and experimental work has been done to evaluate the load bearing capacity of the historic suspension bridges so that traffic loads are managed to ensure their continued safe operation [21, 22].

Benchmark problems in structural control have provided a mean for researchers and designers to assess the merits of various control strategies on a single problem with a common set of performance criteria. The first generation of benchmark problems for bridges was based on the cable-stayed bridge in Cape Girardeau, Missouri, USA. The first phase of the benchmark problem considers the simplest case of excitation which is a uniform excitation in the longitudinal direction of the bridge [23, 24]. This problem has been tackled by many researchers and different controllers have been proposed to solve this problem [25–28]. In the second phase of the benchmark cable-stayed bridge problem, the complexity of the excitation is increased. Multiple support excitation with different angles and times of arrival for each support is considered. Additionally, an alternate model is developed to study the robust stability and performance of the control system under realistic conditions. Here the mass of the bridge is incremented due to snow and rain loads [29, 30]. Different controllers have been proposed to solve such a problem that can be found in [31–34]. This paper shows the design of two control schemes to control the nonlinear vibrations in the suspended cable and the bridge deck due to vertical load moving on the bridge deck with a constant speed. Numerical example is used to show the effectiveness of the proposed controllers. Practical implementation will be the focus of one of our future research. The first control scheme is an optimal state feedback controller while the second control

scheme is a robust state feedback controller whose design is based on design of optimal controllers. In order to control the nonlinear vibrations in the suspended cable and the bridge deck, one may install a vertical cable between the bridge deck and the suspended cable to install a hydraulic actuator able to generate an active feedback control force.

This paper investigates the control of vibrations due to moving loads on suspension bridges. In order to control the nonlinear vibrations in the bridge deck, one may install a vertical cable between the bridge deck and the suspended cable to install a hydraulic actuator able to generate an active feedback control force. The design of the control force to greatly reduce the vertical vibrations in the suspended cables and the vertical vibrations in the bridge deck is discussed in this paper.

The rest of the paper is organized as follows. The dynamic model of a suspension bridge interacting with a moving load is presented in Section 2. A nonlinear controller to reduce the vibrations of the system is proposed in Section 3. A linear controller to reduce the vibrations of the system is presented in Section 4. Simulation results of the proposed control schemes are presented and discussed in Section 5. Finally, the conclusion is given in Section 6.

In the sequel, we denote by W^T the transpose of a matrix or a vector W . We use $W > 0$ ($W < 0$) to denote a positive- (negative-) definite matrix W . Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2 Dynamic model of the suspension bridge system

The basic equations of motion of the suspended cables (see Fig. 1) are defined in [35, 36] and [37]. According to the displacements directions defined in Fig. 2, the general equations of motion are:

$$\begin{aligned}
\frac{\partial}{\partial s} \left[(T_o + \tau) \frac{\partial(x + \bar{U})}{\partial s} \right] &= m \frac{\partial^2 \bar{U}}{\partial t^2}, \\
\frac{\partial}{\partial s} \left[(T_o + \tau) \frac{\partial(y + \bar{V})}{\partial s} \right] &= -mg + m \frac{\partial^2 \bar{V}}{\partial t^2} + c \frac{\partial \bar{V}}{\partial t} + F_v(s, t), \\
\frac{\partial}{\partial s} \left[(T_o + \tau) \frac{\partial \bar{W}}{\partial s} \right] &= m \frac{\partial^2 \bar{W}}{\partial t^2} + c \frac{\partial \bar{W}}{\partial t} + F_w(s, t),
\end{aligned} \tag{1}$$

where s is the spatial coordinate along the cable curved length; t is the time; $x(s)$ is the horizontal coordinate along the cable span; $y(s)$ is the cable static profile; $\bar{U}(s, t)$ is the displacement in the tangential direction of the cable; $\bar{V}(s, t)$ is the displacement in the vertical direction of the cable; $\bar{W}(s, t)$ is the displacement in the transversal direction of the cable; T_o is the static tension in the cable; τ is the additional dynamic tension in the cable; g is the gravitational acceleration; c is the damping coefficient in the cable; m is the mass of the cable per unit length; $F_v(s, t)$ is the external loading per unit length in the vertical direction; $F_w(s, t)$ is the external loading per unit length in the transverse direction.

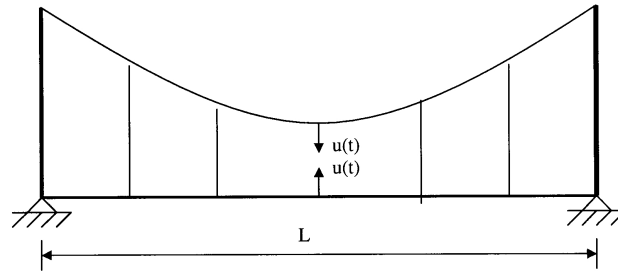


Fig. 1. Suspension bridge.

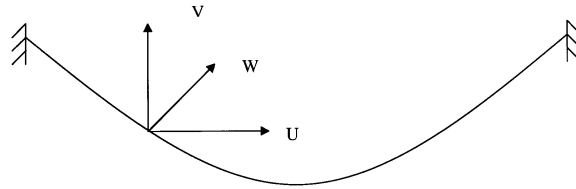


Fig. 2. Displacements directions of suspended cables.

The nonlinear strain-displacement relationship during the deformation of the cable is given by:

$$\frac{\tau}{EA} = \frac{ds' - ds}{ds}, \quad (2)$$

where E is the modulus of elasticity, and A is the cross section area of the cable.

The deformed cable segment, ds' , and the un-deformed cable segment, ds are defined as,

$$\begin{aligned} ds'^2 &= (dx + \partial\bar{U})^2 + (dy + \partial\bar{V})^2 + (\partial\bar{W})^2, \\ ds^2 &= dx^2 + dy^2. \end{aligned} \quad (3)$$

Equations (1) can be simplified [36] based on the assumption of a small curvature regime and condensing the longitudinal displacement \bar{U} in the case of zero longitudinal loading which leads to:

$$\begin{aligned} (1 + \alpha e)T_oL\bar{V}''(s, t) + \alpha\beta T_o e + LF_v(s, t) &= mL\ddot{\bar{V}}(s, t) + cL\dot{\bar{V}}(s, t), \\ (1 + \alpha e)T_oL\bar{W}''(s, t) + LF_w(s, t) &= mL\ddot{\bar{W}}(s, t) + cL\dot{\bar{W}}(s, t), \\ e &= -\beta\frac{\bar{V}(s, t)}{L} + \frac{1}{2}[\bar{V}'^2(s, t) + \bar{W}'^2(s, t)]. \end{aligned} \quad (4)$$

In equations (4), the prime indicates differentiation with respect to s (the spatial coordinate along the cable curved length) and the dot indicates differentiation with respect to time t . Also, L is the length of the suspended cables, and the parameters α and β are defined such as,

$$\alpha = \frac{EA}{T_o} \quad \text{and} \quad \beta = \frac{mgL}{T_o}. \quad (5)$$

When the suspended cables are supporting a bridge deck, the equations of motion become:

$$\begin{aligned} (1 + \alpha e)T_oL\bar{V}'' + \alpha\beta T_o e + LK_c(z - \bar{V}) + T_o u(t) &= mL\ddot{\bar{V}} + cL\dot{\bar{V}}, \\ (1 + \alpha e)T_oL\bar{W}'' + LF_w(s, t) &= mL\ddot{\bar{W}} + cL\dot{\bar{W}}, \\ EI\frac{\partial^4 z}{\partial x^4} + m_b\frac{\partial^2 z}{\partial t^2} + c_b\frac{\partial z}{\partial t} &= -K_c(z - \bar{V}) - u(t)\delta(x - x_p) + P\delta(x - x_p), \\ e &= -\beta\frac{\bar{V}}{L} + \frac{1}{2}[\bar{V}'^2 + \bar{W}'^2]. \end{aligned} \quad (6)$$

The term $K_c(z - \bar{V})$ in equations (6) was used to represent the vertical load $F_v(s, t)$ in equations (4) and it represents the distributed vertical force in the vertical hangers. Also, $z(x, t)$ is the vertical displacement of the bridge deck; K_c is the stiffness of the vertical cables which hang the bridge deck; m_b is the mass of the bridge deck; c_b is the damping coefficient of the bridge deck; EI is

the flexural rigidity of the bridge deck; P is the magnitude of the moving load; x_p is the location of the moving load at any time t from the left support; δ is the Dirac delta function which is used to introduce the concentrated moving load on the differential equation; $u(t)$ is the active control force.

The displacement functions $\bar{W}(s, t)$, $\bar{V}(s, t)$ and $z(x, t)$ are considered to be the contribution of the first modes of vibrations. Therefore they are assumed as follows:

$$\begin{aligned}\bar{W}(s, t) &= \psi(s)W(t), \\ \bar{V}(s, t) &= \phi(s)V(t), \\ z(x, t) &= \eta(x)\mathcal{B}(t),\end{aligned}\tag{7}$$

where $W(t)$ is the transverse displacement of the suspended cable; $V(t)$ is the vertical displacement of the suspended cable; $\mathcal{B}(t)$ is the vertical displacement of the bridge deck; $\phi(s)$ and $\psi(s)$ are the first mode shapes in the vertical and transversal directions, respectively; $\eta(s)$ is the first mode shape for the bridge deck.

The modes $\psi(s)$, $\phi(s)$ can be determined using linear theory of cables and to satisfy the boundary conditions [35] which provide:

$$\psi(s) = \sin \frac{\pi s}{L},\tag{8}$$

$$\phi(s) = K_o \left(1 - \tan \frac{\bar{\mu}\pi}{2} \sin \frac{\bar{\mu}\pi s}{L} - \cos \frac{\bar{\mu}\pi s}{L} \right),\tag{9}$$

where K_o is a constant chosen to make $\phi(\frac{L}{2}) = 1$ and $\bar{\mu}$ is a constant which fits with the boundary conditions.

For a two hinged bridge deck, the mode shape $\eta(x)$ can be assumed as:

$$\eta(x) = \sin \frac{\pi x}{L}.\tag{10}$$

Substituting equations (7)–(10) into equations (6) and applying an integral transformation one obtains, respectively, the equations of motion of the suspended cable in the vertical and the transversal directions and the vertical motion of the

bridge deck as follows:

$$\begin{aligned}
\ddot{V} + 2\zeta\omega_v\dot{V} + \omega_v^2V + c_1V^2 + c_2W^2 + c_3V^3 + c_4VW^2 \\
= d_1V + d_2\mathcal{B} + \kappa_1u(t) + f_v(t), \\
\ddot{W} + 2\zeta\omega_w\dot{W} + \omega_w^2W + c_5VW + c_6V^2W + c_7W^3 = f_w(t), \\
\ddot{\mathcal{B}} + 2\zeta_b\omega_b\dot{\mathcal{B}} + \omega_b^2\mathcal{B} = d_3V + d_4\mathcal{B} + \kappa_2u(t) + P^* \sin(\bar{v}t),
\end{aligned} \tag{11}$$

where ω_v is the natural frequency of the cable in the vertical direction, and it is defined in Appendix; ω_w is the natural frequency of the cable in the transversal direction, and it is defined in Appendix; ω_b is the natural frequency of the bridge deck, and it is defined in Appendix; ζ is the damping ratio in the suspended cable; ζ_b is the damping ratio in the bridge deck; $P^* = \frac{2P}{m_bL}$, where P is the magnitude of the moving load; \bar{x} is the location of the control force $u(t)$ with respect to the origin of x-axis; \bar{v} is the speed of the moving load. The scalars $c_1, c_2, c_3, \dots, c_7, d_1, d_2, d_3, d_4, \kappa_1$ and κ_2 are constants which are defined in Appendix.

The forces $f_v(t)$, and $f_w(t)$ are such,

$$f_v(t) = \frac{-\int_0^L \phi F_v(x, t) dx}{m \int_0^L \phi^2 dx}, \quad f_w(t) = \frac{-\int_0^L \psi F_w(x, t) dx}{m \int_0^L \psi^2 dx}. \tag{12}$$

Define the following state variables:

$$\begin{aligned}
x_1(t) = V(t), \quad x_2(t) = \dot{V}(t), \quad x_3(t) = \mathcal{B}(t), \\
x_4(t) = \dot{\mathcal{B}}(t), \quad x_5(t) = W(t), \quad x_6(t) = \dot{W}(t).
\end{aligned} \tag{13}$$

Hence, the equations of a suspension bridge interacting with a moving load can be written in state-space form as follows:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Bu(t) + g_x(\mathbf{x}(t)) + d(t), \tag{14}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\omega_v^2 + d_1 & -2\zeta\omega_v & d_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ d_3 & 0 & -\omega_b^2 + d_4 & -2\zeta_b\omega_b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -\omega_w^2 & -2\zeta\omega_w \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \kappa_1 \\ 0 \\ \kappa_2 \\ 0 \\ 0 \end{bmatrix}, \quad d(t) = \begin{bmatrix} 0 \\ f_v(t) \\ 0 \\ P^* \sin(\bar{v} t) \\ 0 \\ f_w(t) \end{bmatrix},$$

$$g_x(\mathbf{x}) = \begin{bmatrix} 0 \\ -c_1 x_1^2 - c_2 x_5^2 - c_3 x_1^3 - c_4 x_1 x_5^2 \\ 0 \\ 0 \\ -x_1 - c_5 x_1 x_5 - c_6 x_1^2 x_5 - c_7 x_5^3 \end{bmatrix}.$$

Let

$$g(\mathbf{x}, t) = g_x(\mathbf{x}) + d(t). \quad (15)$$

Hence the equations of the system in (14) can be written as,

$$\dot{\mathbf{x}} = A\mathbf{x} + B u + g(\mathbf{x}, t). \quad (16)$$

Simulations results of the uncontrolled system show that it is a stable system. Also, the simulations indicate that the response of the system oscillates. Therefore, the objective of this paper is to design control schemes to improve the stability of the system by reducing the oscillations.

Remark 1. *The simulation results indicate that the nonlinear function $g(\mathbf{x}, t)$ in (15) is uniformly bounded and hence it can be assumed that the nonlinear term $g(\mathbf{x}, t)$ satisfies the following cone-bounding constraint,*

$$\|g(\mathbf{x}, t)\| \leq \mu \|\mathbf{x}(t)\|, \quad (17)$$

where μ is a positive scalar.

Remark 2. *It can be checked that the pair (A, B) in (16) is controllable. Hence the poles of the closed loop system can be selected such that the response of the linear part of the system (i.e., $g(\mathbf{x}, t) = 0$) is as desired.*

3 Design of the first control scheme

In this section, a nonlinear controller is used to control the suspension bridge system described by (16). The control law is divided into a linear part and a nonlinear part. The linear part of the controller is designed by using the pole placement technique. The nonlinear part of the controller is designed to guarantee the asymptotic stability of the closed loop system.

Let the matrix A_c be such that

$$A_c = A - BK \quad (18)$$

and let the symmetric positive definite matrix P_1 be the solution of the following Lyapunov equation,

$$A_c^T P_1 + P_1 A_c = -Q_1, \quad (19)$$

where $Q_1 = Q_1^T > 0$.

Theorem 1. *The control law given by (20)–(22) when applied to the suspension bridge system (16) guarantees the asymptotic stability of the system.*

$$u = u_L + u_N \quad (20)$$

with

$$u_L = -K\mathbf{x} \quad (21)$$

and

$$u_N = -\rho_1 \text{sign}(B^T P_1 \mathbf{x}). \quad (22)$$

Proof. Using (16), (20) and (21), it follows that

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B(-K\mathbf{x} + u_N) + g(\mathbf{x}, t) \\ &= (A - BK)\mathbf{x} + Bu_N + g(\mathbf{x}, t) \\ &= A_c\mathbf{x} + Bu_N + g(\mathbf{x}, t). \end{aligned} \quad (23)$$

Consider the following Lyapunov function candidate,

$$V_1 = \mathbf{x}^T P_1 \mathbf{x}. \quad (24)$$

Note that $V_1 > 0$ for $\mathbf{x} \neq 0$ and $V_1 = 0$ for $\mathbf{x} = 0$. Taking the derivative of V_1 with respect to time and using equations (23), (22), (19) and (17), it follows that

$$\begin{aligned}
 \dot{V} &= \dot{\mathbf{x}}^T P_1 \mathbf{x} + \mathbf{x}^T P_1 \dot{\mathbf{x}} \\
 &= (A_c \mathbf{x} + B u_N + g(\mathbf{x}, t))^T P_1 \mathbf{x} + \mathbf{x}^T P_1 (A_c \mathbf{x} + B u_N + g(\mathbf{x}, t)) \\
 &= \mathbf{x}^T (A_c^T P_1 + P_1 A_c) \mathbf{x} + 2 \mathbf{x}^T P_1 B u_N + 2g(\mathbf{x}, t)^T P_1 \mathbf{x} \\
 &= -\mathbf{x}^T Q_1 \mathbf{x} + 2 \mathbf{x}^T P_1 B u_N + 2g(\mathbf{x}, t)^T P_1 \mathbf{x} \\
 &\leq -\lambda_{\min}(Q_1) \|\mathbf{x}\|^2 + 2\mu \lambda_{\max}(P_1) \|\mathbf{x}\|^2 - 2\rho_1 \mathbf{x}^T P_1 B \frac{B^T P_1 \mathbf{x}}{|B^T P_1 \mathbf{x}|} \\
 &= -\lambda_{\min}(Q_1) \|\mathbf{x}\|^2 + 2\mu \lambda_{\max}(P_1) \|\mathbf{x}\|^2 - 2\rho_1 |B^T P_1 \mathbf{x}| \\
 &\leq -(\lambda_{\min}(Q_1) - 2\mu \lambda_{\max}(P_1)) \|\mathbf{x}\|^2.
 \end{aligned} \tag{25}$$

Therefore, it can be concluded that $\dot{V} < 0$ if the matrices P_1 and Q_1 are selected such that the condition $\lambda_{\min}(Q_1) - 2\mu \lambda_{\max}(P_1) > 0$. Hence the control scheme given by (20)–(22) guarantees the asymptotic stability of the closed loop system. \square

4 Design of the second control scheme

In this section, a linear controller is designed to control the suspension bridge system described by (16). Again, the control law is divided into two parts. The first part of the controller is designed by using the pole placement technique as in the previous section. The second part of the controller is designed to guarantee the asymptotic stability of the closed loop system.

The matrix A_c is such that

$$A_c = A - BK. \tag{26}$$

Let the symmetric positive definite matrix P_2 be the solution of the following Lyapunov equation,

$$A_c^T P_2 + P_2 A_c = -Q_2, \tag{27}$$

where $Q_2 = Q_2^T > 0$.

Let the design parameter γ be such that

$$\gamma \geq \frac{\lambda_{\max}(P_2)}{\lambda_{\min}(P_2 B B^T P_2)} \mu. \tag{28}$$

Theorem 2. *The control law given by (29)–(31) when applied to the suspended cables system (16) guarantees the asymptotic stability of the system.*

$$u = u_{L_1} + u_{L_2} \quad (29)$$

with

$$u_{L_1} = -K\mathbf{x} \quad (30)$$

and

$$u_{L_2} = -\gamma B^T P_2 \mathbf{x}. \quad (31)$$

Proof. Using (16), (29) and (30), it follows that

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B(-K\mathbf{x} + u_{L_2}) + g(\mathbf{x}, t) \\ &= (A - BK)\mathbf{x} + Bu_{L_2} + g(\mathbf{x}, t) \\ &= A_c\mathbf{x} + Bu_{L_2} + g(\mathbf{x}, t). \end{aligned} \quad (32)$$

Consider the following Lyapunov function candidate,

$$V_2 = \mathbf{x}^T P_2 \mathbf{x}. \quad (33)$$

Note that $V_2 > 0$ for $\mathbf{x} \neq 0$ and $V_2 = 0$ for $\mathbf{x} = 0$. Taking the derivative of V_2 with respect to time and using (32), (31) and (27), it follows that

$$\begin{aligned} \dot{V}_2 &= \dot{\mathbf{x}}^T P_2 \mathbf{x} + \mathbf{x}^T P_2 \dot{\mathbf{x}} \\ &= (A_c\mathbf{x} + Bu_{L_2} + g(\mathbf{x}, t))^T P_2 \mathbf{x} + \mathbf{x}^T P_2 (A_c\mathbf{x} + Bu_{L_2} + g(\mathbf{x}, t)) \\ &= \mathbf{x}^T (A_c^T P_2 + P_2 A_c)\mathbf{x} + 2g(\mathbf{x}, t)^T P_2 \mathbf{x} + 2\mathbf{x}^T P_2 B u_{L_2} \\ &= -\mathbf{x}^T Q_2 \mathbf{x} + 2g(\mathbf{x}, t)^T P_2 \mathbf{x} + 2\mathbf{x}^T P_2 B u_{L_2} \\ &\leq -\mathbf{x}^T Q_2 \mathbf{x} + 2\mu \|P_2 \mathbf{x}\| \|\mathbf{x}\| - 2\gamma \mathbf{x}^T P_2 B B^T P_2 \mathbf{x} \\ &\leq -\mathbf{x}^T Q_2 \mathbf{x} + 2\mu \lambda_{\max}(P_2) \|\mathbf{x}\|^2 - 2\gamma \lambda_{\min}(P_2 B B^T P) \|\mathbf{x}\|^2 \\ &= -\mathbf{x}^T Q_2 \mathbf{x} + 2(\mu \lambda_{\max}(P_2) - \gamma \lambda_{\min}(P_2 B B^T P)) \|\mathbf{x}\|^2 \\ &\leq -\mathbf{x}^T Q_2 \mathbf{x}. \end{aligned} \quad (34)$$

The choice of γ guarantees that $(\mu \lambda_{\max}(P_2) - \gamma \lambda_{\min}(P_2 B B^T P)) \leq 0$.

Therefore, it can be concluded that $\dot{V}_2 < 0$. Hence the control scheme given by (29)–(31) guarantees the asymptotic stability of the closed loop system. \square

5 Simulation results

The controllers designed in Sections 3 and 4 are simulated using the MATLAB software.

The example in Abdel-Rohman and Spencer [1] is used for simulation purposes. The example consists of a suspended cable of length $L = 200$ m, diameter $D = 10$ cm, mass $m_c = 62$ Kg/m, tension in the cable $T_o = 2 \times 10^6$ N, axial stiffness $EA = 1.57 \times 10^9$ N. The damping ratio of the cable is assumed to be $\zeta = 0.1\%$. The mass of the bridge deck is $m_b = 10000$ kg/m, the damping in the bridge is $\zeta_b = 0.01$. The vertical hangers stiffness is assumed to be $K_c = 10^6$ N/m, the flexural rigidity is taken to be $EI = 5 \times 10^{10}$ Nm². The natural frequencies are such that $\omega_w = 2.8$ rps, $\omega_v = 2.8$ rps, and $\omega_b = 0.552$ rps. The parameters $c_1 - c_7$ are such that $c_1 = 1.2196$, $c_2 = 0.41$, $c_3 = 0.578$, $c_4 = 0.56535$, $c_5 = 0.8015$, $c_6 = 0.5634$, $c_7 = 0.55$. The parameters $d_1 - d_4$ are such that $d_1 = -K_c/m$, $d_2 = K_c/m$, $d_3 = K_c/m_b$, $d_4 = -K_c/m_b$. The parameters κ_1 and κ_2 are such that $\kappa_1 = \frac{2}{mL}$, $\kappa_2 = \frac{-2}{m_bL}$. The magnitude of the moving load is $P = 100000$, and $P^* = \frac{2P}{m_bL}$. The speed of the moving load is $\bar{v} = 10$ m/s, and the location of the control force $u(t)$ is at $0.5L$.

Fig. 3 shows the vertical displacement of the suspended cable, $V(t)$ when no control is applied to the system; it can be seen that the response oscillates with

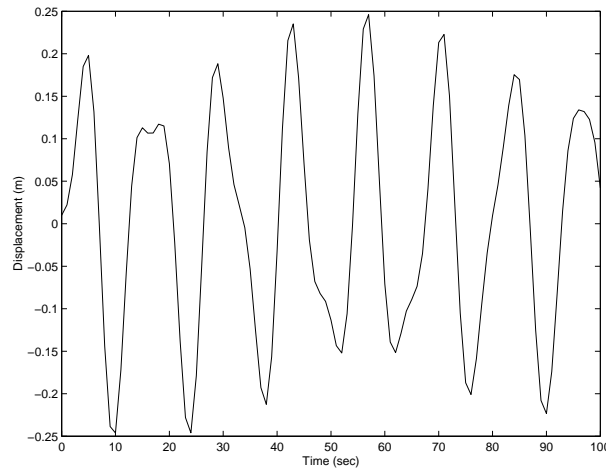


Fig. 3. The vertical displacement of the suspended cable, $V(t)$ with no control.

amplitude of about 0.5 m peak to peak. Fig. 4 shows the vertical displacement of the bridge deck, $\mathcal{B}(t)$ when no control is applied to the system; it can be seen that the response oscillates with amplitude of about 0.5 m peak to peak. Fig. 5 shows the transverse displacement of the suspended cable, $W(t)$ when no control is applied to the system; it can be seen that the response oscillates with amplitude

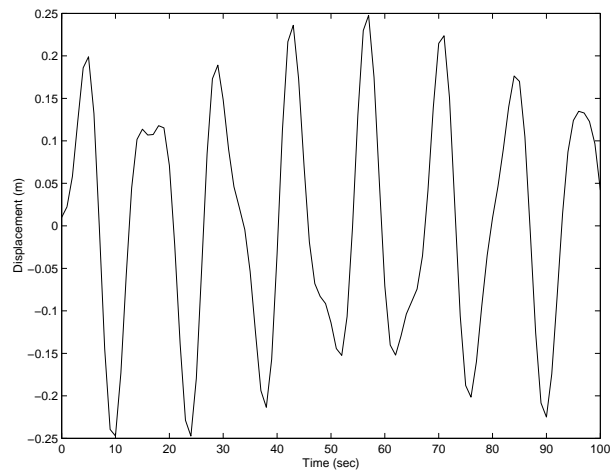


Fig. 4. The vertical displacement of the bridge deck, $\mathcal{B}(t)$ with no control.

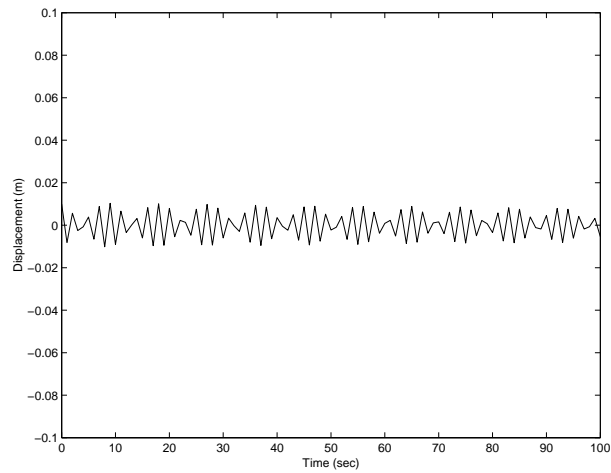


Fig. 5. The transverse displacement of the suspended cable, $W(t)$ with no control.

of about 0.02 m peak to peak. Therefore the objective of the proposed control schemes is to greatly reduce the oscillations of $V(t)$ and $\mathcal{B}(t)$.

Fig. 6 shows the vertical displacement of the suspended cable, $V(t)$ when the first controller is applied to the system; it can be seen that the response oscillates with amplitude of about 0.05 m peak to peak. Fig. 7 shows the vertical displace-

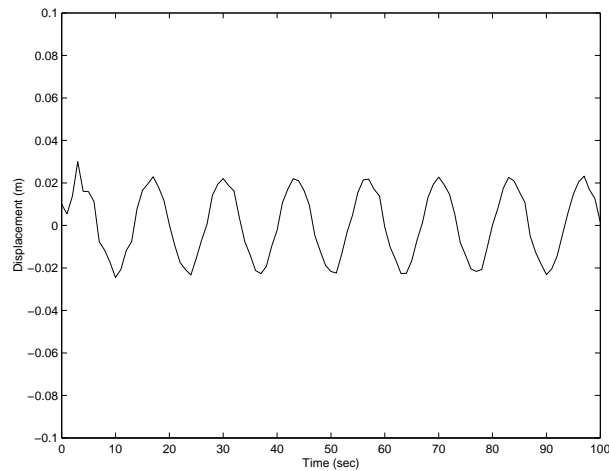


Fig. 6. The vertical displacement of the suspended cable, $V(t)$ when controller 1 is used.

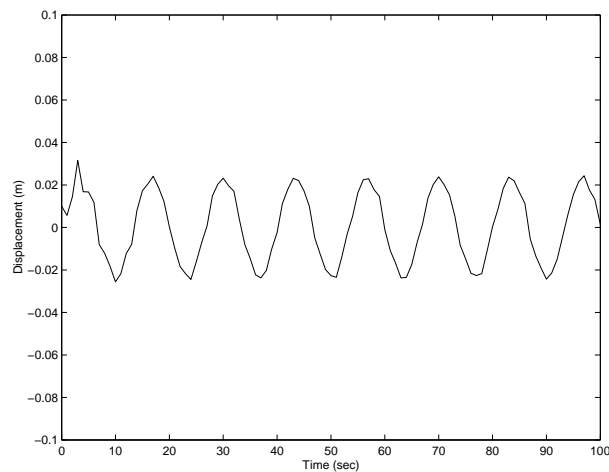


Fig. 7. The vertical displacement of the bridge deck, $\mathcal{B}(t)$ when controller 1 is used.

ment of the bridge deck, $\mathcal{B}(t)$ when the first controller is applied to the system; it can be seen that the response oscillates with amplitude of about 0.05 m peak to peak. Fig. 8 shows the transverse displacement of the suspended cable, $W(t)$ when the first controller is applied to the system; it can be seen that the response oscillates with amplitude of about 0.02 m peak to peak. Hence, it can be concluded that the first control scheme is able to greatly reduce the oscillations of $V(t)$ and $\mathcal{B}(t)$. The controller did not have much of an effect on the transverse displacement of the suspended cable, $W(t)$. The plot of controller 1 versus time is shown in Fig. 9; the range of the controller is about 0.8×10^5 .

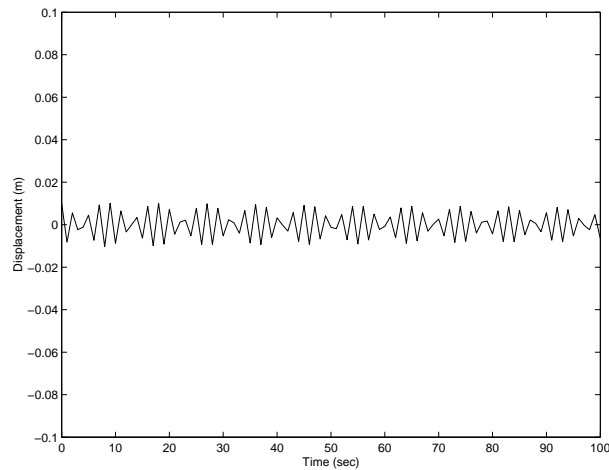


Fig. 8. The transverse displacement of the suspended cable, $W(t)$ when controller 1 is used.

Fig. 10 shows the vertical displacement of the suspended cable, $V(t)$ when the second controller is applied to the system; it can be seen that the response oscillates with amplitude of about 0.06 m peak to peak. Fig.11 shows the vertical displacement of the bridge deck, $\mathcal{B}(t)$ when the second controller is applied to the system; it can be seen that the response oscillates with amplitude of about 0.06 m peak to peak. Fig. 12 shows the transverse displacement of the suspended cable, $W(t)$ when the second controller is applied to the system; it can be seen that the response oscillates with amplitude of about 0.02 m peak to peak. Hence, it can be concluded that the second control scheme is able to greatly reduce the oscillations of $V(t)$ and $\mathcal{B}(t)$. The controller did not have much of an effect on the transverse

displacement of the suspended cable, $W(t)$. The plot of controller 2 versus time is shown in Fig. 13; the range of the controller is about 0.9×10^5 .

Therefore, the simulation results show that the proposed control schemes are able to greatly reduce the oscillations of the vertical displacement of the suspen-

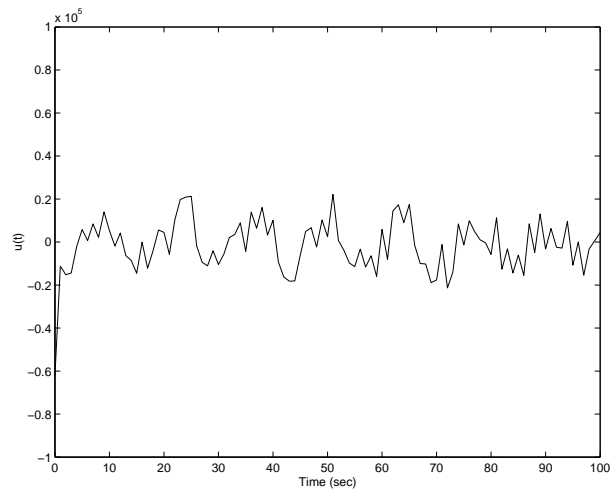


Fig. 9. The response of controller 1 versus time.

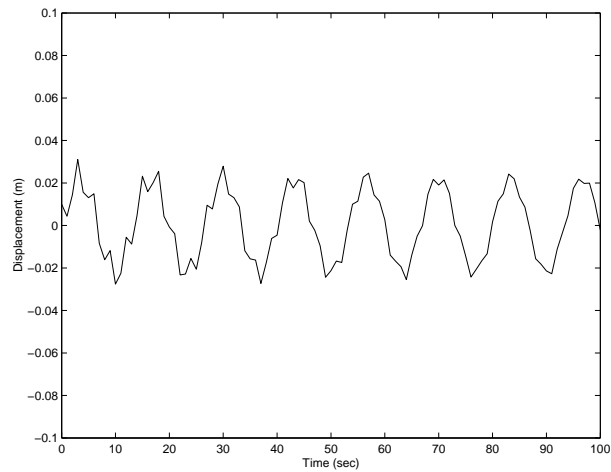


Fig. 10. The vertical displacement of the suspended cable, $V(t)$ when controller 2 is used.

ded cable and the vertical displacement of the bridge deck. It should be mentioned that the first controller gave slightly better results than the second controller.

For comparison purposes, a simple velocity feedback controller is designed for the suspension bridge system.

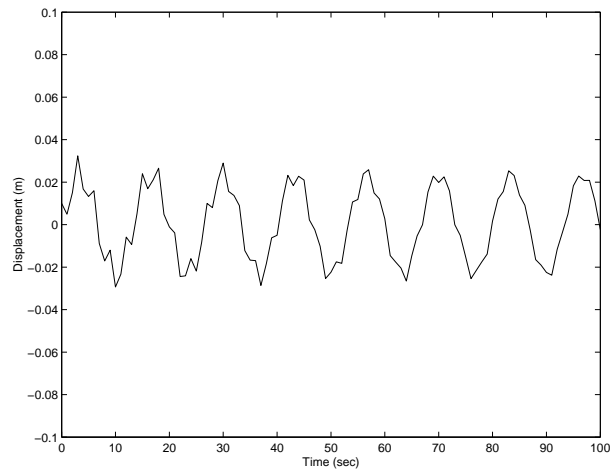


Fig. 11. The vertical displacement of the bridge deck, $B(t)$ when controller 2 is used.

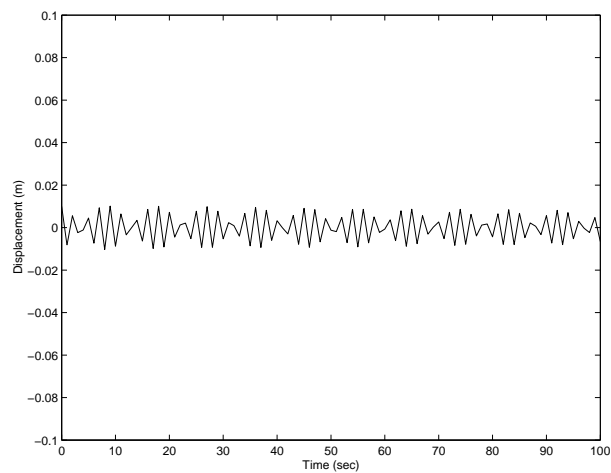


Fig. 12. The transverse displacement of the suspended cable, $W(t)$ when controller 2 is used.

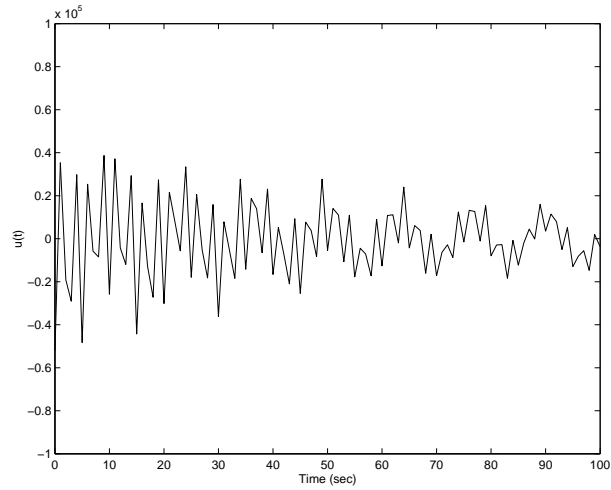


Fig. 13. The response of controller 2 versus time.

The controller is as follows:

$$u = -\alpha_1 \dot{V} - \alpha_2 \dot{B}, \quad (35)$$

where α_1 and α_2 are design parameters.

Fig. 14 shows the vertical displacement of the suspended cable, $V(t)$ when

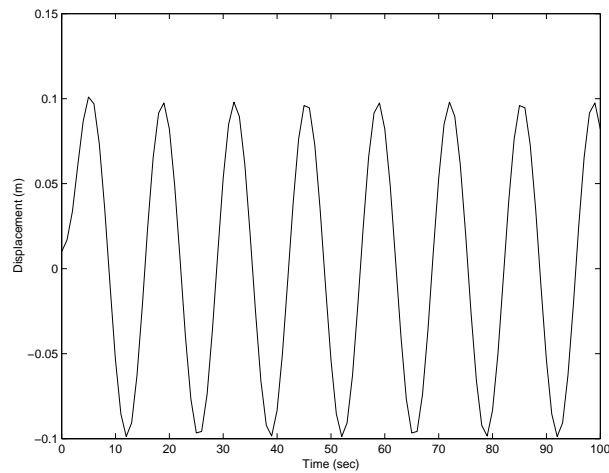


Fig. 14. The vertical displacement of the suspended cable, $V(t)$ when the velocity feedback controller is used.

the velocity feedback controller is applied to the system; it can be seen that the response oscillates with amplitude of about 0.2 m peak to peak. Fig. 15 shows the vertical displacement of the bridge deck, $\mathcal{B}(t)$ when the velocity feedback controller is applied to the system; it can be seen that the response oscillates with amplitude of about 0.2 m peak to peak. Fig. 16 shows the transverse displacement

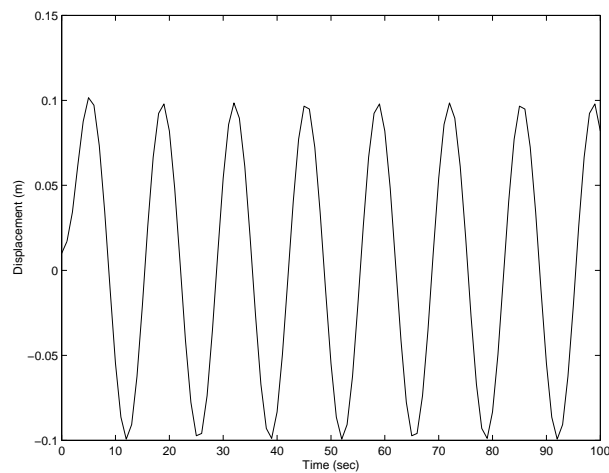


Fig. 15. The vertical displacement of the bridge deck, $\mathcal{B}(t)$ when the velocity feedback controller is used.

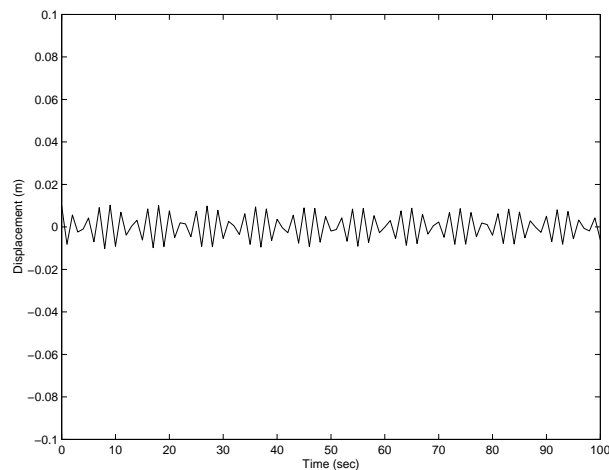


Fig. 16. The transverse displacement of the suspended cable, $W(t)$ when the velocity feedback controller is used.

of the suspended cable, $W(t)$ when the velocity feedback controller is applied to the system; it can be seen that the response oscillates with amplitude of about 0.02 m peak to peak. The plot of the velocity feedback controller versus time is shown in Fig. 17; the range of the controller is about 0.2×10^5 .

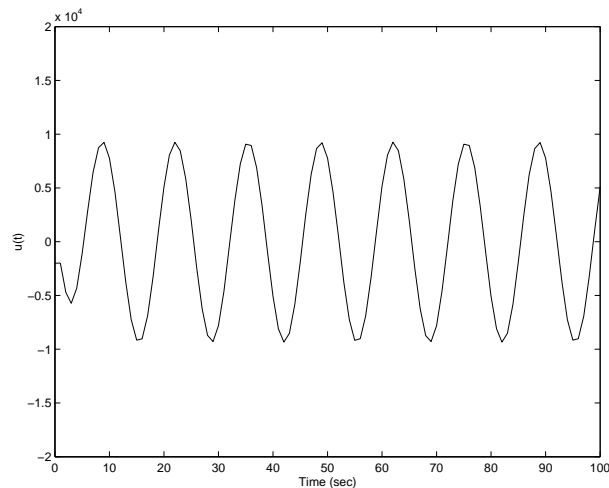


Fig. 17. The response of the velocity feedback controller versus time.

Therefore, it can be concluded that the velocity feedback controller is able to reduce the oscillations of the vertical displacement of the suspended cable, and the vertical displacement of the bridge deck. However, the reduction of the oscillations with the velocity feedback controller is less than the reduction of the oscillations when the proposed two control schemes are applied to the system. The range of the proposed controllers is a bit higher than the range of the velocity feedback controller.

6 Conclusion

The control of the nonlinear vibrations of suspension bridges due to moving loads is investigated in this paper. In order to control the vertical vibrations of the suspended cables and the bridge deck, a hydraulic actuator can be installed between the bridge deck and the suspended cables. This actuator is used to generate an active control force on the bridge deck. A linear and a nonlinear control

schemes are presented to generate the active control force. These controllers guarantee the asymptotic stability of the closed loop system. The performance of the controlled system is investigated through simulations using the MATLAB software. The simulation results indicate that the proposed control schemes work well. Moreover, simulation results indicate that the proposed controllers give better results than a velocity feedback controller.

Appendix

The natural frequency of the cable in the vertical direction is such:

$$\omega_v^2 = \frac{-H \int_0^L \phi \phi'' dx}{m \int_0^L \phi^2 dx}.$$

The natural frequency of the cable in the transversal direction is such:

$$\omega_w^2 = \frac{-H \int_0^L \psi \psi'' dx}{m \int_0^L \psi^2 dx}.$$

The natural frequency of the bridge deck is such:

$$\omega_b^2 = \frac{EI \int_0^L \eta \eta'''' ds}{m_b \int_0^L \eta^2 ds}.$$

The parameters $c_1, c_2, c_3, c_4, c_5, c_6$ and c_7 are such:

$$c_1 = \frac{1.5\beta EA \int_0^L \phi \phi'^2 dx}{mL \int_0^L \phi^2 dx}, \quad c_2 = \frac{\beta EA \int_0^L \phi \psi'^2 dx}{2mL \int_0^L \phi^2 dx},$$

$$c_3 = \frac{-1.5EA \int_0^L \phi \phi'^2 \phi'' dx}{m \int_0^L \phi^2 dx}, \quad c_4 = \frac{-EA \int_0^L \phi \phi' \psi' \psi'' dx + 0.5 \int_0^L \phi \phi'' \psi'^2 dx}{m \int_0^L \phi^2 dx},$$

$$c_5 = \frac{EA \int_0^L \phi \psi \psi'' dx}{mL \int_0^L \psi^2 dx}, \quad c_6 = \frac{-EA \int_0^L \phi' \phi'' \psi \psi' dx + 0.5 \int_0^L \phi'^2 \psi \psi'' dx}{m \int_0^L \psi^2 dx},$$

$$c_7 = \frac{-1.5EA \int_0^L \psi \psi'^2 \psi'' dx}{m \int_0^L \psi^2 dx}.$$

The parameters d_1, d_2, d_3 and d_4 are such:

$$d_1 = \frac{-K_c}{m}, \quad d_2 = \frac{K_c}{m}, \quad d_3 = \frac{K_c}{m_b}, \quad d_4 = \frac{-K_c}{m_b}.$$

The parameters κ_1 and κ_2 are such:

$$\kappa_1 = \frac{2}{mL}, \quad \kappa_2 = \frac{-2}{m_b L}.$$

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