Discrete Values of the Coefficient of Damping under Conditions of Explicit Acoustic Nonlinearity

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Abstract. Qualitative analysis of hypersound generation is described by the inhomogeneous Burgers equation in the case of the non-harmonic and arbitrary light field. A qualitative possibility of the appearance of discrete values of the coefficient of extinction of the sound wave and the possibility of generation of the same sound signal by different light fields is shown.

Keywords: evolution equation, integrable system, Burgers equation, acoustics, nonlinearity.

1 Introduction

In nonlinear optics, in media strongly influenced by dispersion the method of slowly changing amplitudes is widely applied [1]. In studies of the stimulated Brillouin scattering and the process of hypersound excitation, this method allows a significant simplification of the initially related systems of Maxwell and hydrodynamics equations [2–4]. As even for the highest laser hypersound frequencies $f \sim 2nc_0/\lambda \sim 10^{11}\,Hz$, where c_0 is sound velocity, n is the refraction index, and λ stands for the length of the light wave, dispersion is generally inessential, in the cases when acoustic non-linearity appears, the highest acoustic harmonics should be included into the description. These polyharmonics, which are neglected in the method of slowly changing amplitudes, lead to sound wave damping, which can exceed the common extinction and significantly alter the dynamics of the process.

It is well known that studying the stimulated scattering and the process of hypersound generation by laser radiation non-linear acoustic effects emerge [5–7]. When the excitation imposed, e.g., in form of harmonious oscillation is freely expanding, its distortion takes place, resulting in the formation of a saw-like wave at the distances $x \sim \lambda/2\pi\varepsilon M$ where λ is the wave length, M is the Mach number, and ε is the nonlinear parameter. For the nonlinear effects to appear at a greater distance sound absorption must be rather low, i.e. the acoustic Reynolds number must be large.

The theoretical studies usually come to a solution of a system of Maxwell equations and equation of hydrodynamics, which in turn generally come to a study of the inhomogeneous Burgers equation (BE) which describes hypersound generation in the given light fields.

The present work offers a qualitative analysis of hypersound generation described by an inhomogeneous BE in the cases of (a) harmonious, (b) periodical, (c) arbitrary light field. A qualitative peculiarity of appearance of the discrete values of the coefficient of sound wave extinction and the possibility of generating the same sound signal by different light fields is shown.

2 The initial equations

The equations corresponding to the complex amplitudes of the pumping wave and the Stokes wave E_s and E_p have been known long since and in the case of backward dissipation are expressed by equations (see, e.g., [5–7]):

$$\frac{dE_p}{dx} + k_\omega E_p = -i\,\frac{\omega_p}{4c}\,Y\beta\tilde{p}\,E_s\,e^{-i\Delta x}\,, \tag{1}$$

$$-\frac{dE_s}{dx} + k_\omega E_s = -i\,\frac{\omega_s}{4c}\,Y\beta\tilde{p}^*E_p\,e^{i\Delta x}\,, \tag{2}$$

where k_ω is the coefficient of light extinction, β is the compression of the substance, Y is the coefficient of the nonlinear optical-acoustic relation, $\tilde{p}$ is the complex amplitude of the sound pressure wave, and ω_s, ω_p stand for the frequency of the Stokes wave and the pumping wave.

To derive a simplified nonlinear acoustic equation, we will proceed from the wave equation for the acoustic field

$$\frac{\partial^2 p'}{\partial \tau^2} - c_0^2 \nabla^2 p' - \frac{b}{\rho_0}\,\frac{\partial}{\partial \tau}\nabla^2 p' = -\frac{c_0^2}{8\pi} Y \nabla^2 (E^2) + L_2(p'^2)\,, \tag{3}$$

where b is the dissipative coefficient, and $L_2(p'^2)$ is a symbolic representation of the nonlinear members quadratic regarding p' [6].

Here, three qualitatively different cases are possible:

(a) E_p and E_s correspond to the stationary states described by harmonious waves. In this case, usually accepted is (see, e.g., [8, 9])

$$E^2 = \frac{1}{2} E_p E_s^* e^{i(\Omega t - qx + \Delta x)} + \frac{1}{2} E_p^* E_s e^{-i(\Omega t - qx + \Delta x)}\,, \tag{4}$$

where $q = \Omega / c_0,\ \Delta = k_p - k_s - q$. Passing to the reference frame of the wave moving with the velocity of sound and turning from the complex amplitudes of laser waves to the real amplitudes and phases $E_p = A_p e^{i\varphi_p}$ and $E_s = A_s e^{i\varphi_s}$ we obtain (see, e.g., [6]):

$$\begin{aligned} &\frac{\partial p'}{\partial x} - \frac{\varepsilon}{c_0^3 \rho_0} p' \frac{\partial p'}{\partial \tau} - \frac{b}{2 c_0^3 \rho_0} \frac{\partial^2 p'}{\partial \tau^2} \\ &\qquad = \frac{Yq}{16\pi}\Big(1 - \frac{\Delta}{q}\Big)^2 A_p A_s \sin\Big(\Omega\tau + \Phi - \frac{\pi}{2}\Big)\,, \end{aligned} \tag{5}$$

where ε stands for the acoustic nonlinearity parameter, and $\Phi = \Delta x + \varphi_p - \varphi_s + \pi/2$;

(b) E_p and E_s correspond to non-harmonic states described by periodical but non-harmonic waves. In this case, instead of (4) we will have

$$E^2 = \frac{1}{2} E_p E_c^* F(t) + \frac{1}{2} E_p^* E_s F^*(t)\,, \tag{6}$$

where $F(t)$ is a periodic but not harmonic function.

Turning to the wave reference frame, instead of equation (5) we obtain

$$\frac{\partial p'}{\partial x} - \frac{\varepsilon}{c_0^3 \rho_0} p' \frac{\partial p'}{\partial \tau} - \frac{b}{2 c_0^3 \rho_0} \frac{\partial^2 p'}{\partial \tau^2} = \frac{Y}{16\pi} f(\tau)\,, \tag{7}$$

where $f(\tau) = \phi(\tau + T)$ is the function, periodic with respect to τ, with the period T, but anharmonic;

(c) E_p and E_s correspond to the non-stationary states described by the non-periodical and anharmonic waves. In this case, instead of equations (5) and (7), we will have

$$\frac{\partial p'}{\partial x} - \frac{\varepsilon}{c_0^3\rho_0}p'\frac{\partial p'}{\partial \tau} - \frac{b}{2c_0^3\rho_0}\frac{\partial^2 p'}{\partial \tau^2} = \frac{Y}{16\pi}f(x,\tau)\,, \tag{8}$$

where $f(x,\tau)$ is a non-periodic and anharmonic function.

Depending on the case (a)–(c), the initial equations (1) an (2) undergo related changes.

(a) In this case we have

$$\frac{dA_p}{dx} + k_\omega A_p = -\frac{\omega_p}{4c}Y\beta p A_s \sin\Phi\,, \tag{9}$$

$$\frac{dA_s}{dx} - k_\omega A_s = -\frac{\omega_s}{4c}Y\beta p A_p \sin\Phi\,, \tag{10}$$

$$\frac{d\Phi}{dx} - \Delta + \frac{Y\beta}{4c}p\Big(\omega_s\frac{A_s}{A_p} + \omega_p\frac{A_p}{A_s}\Big)\cos\Phi = 0\,. \tag{11}$$

(b) and (c). In these cases

$$\frac{dA_p}{dx} + k_\omega A_p = -\frac{\omega_p}{4c}Y\beta p f(x,\tau)\,, \tag{12}$$

$$\frac{dA_s}{dx} - k_\omega A_s = -\frac{\omega_s}{4c}Y\beta p f(x,\tau)\,, \tag{13}$$

$$\frac{d\Phi}{dx} - \Delta + \frac{Y\beta}{4c}p f'_x(x,\tau) = 0\,, \tag{14}$$

the difference between (b) and (c) consists only in the properties of the function $f(x,\tau)$: in the case (b) it is periodical with respect to τ and does not depend on x, while in the case (c) it is arbitrary.

To make the system of equations (9)–(11) or (12)–(14) complete, a relation between the sound field parameter p' and the real sound pressure amplitude p should be added.

(a) $$p = \frac{2}{\pi}\int_0^\pi p'(x,\tau)\sin(\Omega\tau + \Phi - \pi/2)\,d(\Omega\tau)\,. \tag{15}$$

This correlation reflects the fact that equations (9)–(11) contain only the amplitude of the first harmonic p of the sound field, whereas the behavior of the field p' itself is defined by the interaction of an infinite number of harmonics, described by equation (5).

$$\text{(b)} \quad p = \frac{4}{T} \int_0^{T/2} p'(x,\tau) f(x,\tau)\, d\tau\,, \tag{16}$$

Finally, in the case (c) we have

$$\text{(c)} \quad p = \lim_{T\to+\infty} \frac{2}{T} \int_0^{T} p'(x,\tau) f(x,\tau)\, dt\,. \tag{17}$$

It should be noted that even in the simplest situation (a) when the light waves are spreading to meet each other, the evolutionary equations are rather intricate, and their analysis usually needs further simplifications.

3 Nonlinearly sound generation in a given field of laser radiation

The main peculiarity of the obtained system of equations as compared to a usually applied system obtained by the method of slowly changing amplitudes consists in the nonlinearity of hypersound generation equation (5). Let us consider the process of intensive excitation of hypersound in the field of two contrary light waves. Suppose that the optical dispersion of the medium allows a synchronous excitation of the sound of the frequency $\Omega = \omega_p - \omega_s = 2n\omega_p c_0/c$. Turning to the dimensionless variables

$$t = x/x_0\,, \quad x = -\tau/\tau_0\,, \quad \phi = p'/p_0\,, \tag{18}$$

where $x_0 = c_0/\Omega$, $\tau_0 = \varepsilon p_0 x_0 / c_0^3 \rho_0$ are space and time scale variables, the nonlinear evolutionary equation (5) and, correspondingly, equations (7) and (8) are reduced to the form of the inhomogeneous BE.

(a) In this case, we shall also assume that $\Delta = 0, \Phi = \pi/2$, the light amplitudes A_p and A_s are constant. Then equation (5) has the form

$$\frac{\partial\phi}{\partial t} + \phi\frac{\partial\phi}{\partial x} - \alpha\frac{\partial^2\phi}{\partial x^2} = -A\sin x\,, \quad \alpha = \frac{bc_0^3\rho_0}{2\varepsilon^2 p_0^2 x_0} = \frac{1}{\mathrm{Re}}\,. \tag{19}$$

The value α is a criterion of nonlinearity manifestation and the value of dimensionless amplitude $A = \varepsilon Y \beta q^2 A_p A_s / 16\pi\alpha^2$ can serve as a criterion of external influence. The case $A \ll 1$ corresponds to linear sound excitation; at $A \gg 1$ an effective generation harmonics takes place.

(b) In this case we shall obtain a non-homogeneous BE with a periodical non-homogeneous term:

$$\frac{\partial\phi}{\partial t} + \phi\frac{\partial\phi}{\partial x} - \alpha\frac{\partial^2\phi}{\partial x^2} = -\beta f(x)\,, \tag{20}$$

where the dimensionless constant β has the value of the governing parameter of the field $f(x)$ intensity.

(c) Finally, in the case when the intensity depends also on time, we shall obtain the BE with an arbitrary inhomogeneous term

$$\frac{\partial\phi}{\partial t} + \phi\frac{\partial\phi}{\partial x} - \alpha\frac{\partial^2\phi}{\partial x^2} = -\beta f(x,t)\,. \tag{21}$$

Thus, we may consider the case (c) as a generalization on the cases (a) and (b).

The nonlinear equation (21) can be written in the linear form as a result of Hopf-Cole substitution

$$\phi(x,t) = -2\alpha\partial_x \ln\, w(x,t)\,, \tag{22}$$

where $w(x,t)$ is a new, unknown function. Thus, we obtain a linear equation in which

$$\frac{\partial w}{\partial t} - \alpha\frac{\partial^2 w}{\partial x^2} - \frac{\beta}{2}F(x,t)w = 0\,, \tag{23}$$

$$F(x,t) = \int f(x,t)dx + C(t)\,, \tag{24}$$

and $C(t)$ is an arbitrary function t which depends on the initial conditions. Below we will consider the initial conditions for $\phi(x,t)$ chosen so as to ensure $C(t) \equiv 0$.

Consider the solutions of equation (23) in three different cases.

(a) Let the initial condition for $\phi(x,t)$ be $\phi(x,0) = 0$. In this case solution of equation (23), where $\beta F(x,t) = A\cos x$, can be found analytically:

$$\phi(x,t) = -2\alpha\frac{\partial}{\partial_x}\ln\left[\sum_{n=0}^{\infty} a_{2n} e^{-\frac{\lambda_{2n}(A)}{4}t} ce_{2n}\Big(-\frac{x}{2}, A\Big)\right]\,, \tag{25}$$

where

$$a_{2n} = \int_0^{2\pi} ce_o\Big(-\frac{x}{2}, A\Big)\, dx \Big/ \int_0^{2\pi} ce_{2n}^2\Big(-\frac{x}{2}, A\Big)\, dx\,, \tag{26}$$

and $ce_{2n}(z)$ is the Mathieu function (see, e.g., [10]).

An explicit expression of the solution (25) allows the following evolution of the wave profile. This is the solution for an ever-steeping wave. At $t \gg 1$ the wave profile becomes stable and does not depend on the value of the distance covered by the wave, $\lambda_0 - \lambda_n < 0$, and in the sum (25) only the first member will be essential, while the whole solution will convert to the stationary solution of the equation

$$\phi\frac{\partial \phi}{\partial x} - \alpha\frac{\partial^2 \phi}{\partial x^2} = A \sin x\,, \tag{27}$$

which can be represented also analytically:

$$\phi(x) = -2\alpha\frac{\partial}{\partial x}\ln\ ce_o\Big(-\frac{x}{2}, A\Big). \tag{28}$$

(b) Substitution of (22) into (20) gives the equation

$$\frac{\partial w}{\partial t} - \alpha\frac{\partial^2 w}{\partial x^2} - \frac{\beta}{2}F(x)w = 0\,. \tag{29}$$

In this case the solution evidently depends on the explicit form of the function $F(x)$. Nevertheless, we can show some common features of the solutions of equation (29). We will search for the solution of equation (29) in the form of a dumping wave

$$w(x,t) = e^{-\lambda t}v(x)\,. \tag{30}$$

Then the function $v(x)$ for $\alpha = 1$ must obey the equation

$$\frac{\partial^2 v}{\partial x^2} + \Big[\lambda + \frac{\beta}{2}F(x)\Big]v = 0\,, \tag{31}$$

which, under rather general assumptions concerning the properties of the function $F(x)$, can be solved numerically and in a series of cases may have an exact analytical solution.

As an example, let us consider the simplest case when $F(x) = 2cx$ and $c = const, \alpha = 1$. The solution of equation (31) is

$$v(x) = A(-z), \quad z = (\beta c)^{1/3}x + \lambda(\beta x)^{-2/3}\,, \tag{32}$$

where $A(x)$ is the Airy function [10]. The solution of the initial equation, in its turn, does not depend on t and has the form

$$\phi(x) = -2\alpha\frac{\partial}{\partial x}\ln A(-z), \tag{33}$$

where z has been defined in equation (32).

(c) Substitution of (22) into equation (21) gives

$$\frac{\partial w}{\partial t} - \alpha\frac{\partial^2 w}{\partial x^2} - \frac{\beta}{2}F(x,t)w = 0\,. \tag{34}$$

In the case when the function $F(x,t) = F(x-ut)$ is a spreading perturbation, we shall consider, as the solution of equation (34), the travelling wave

$$w(x,t) = w(x-ut) = w(\xi)\,, \quad \xi = x - ut\,, \tag{35}$$

corresponding to the wave spreading in the positive direction of the t axis. In this case the evolutionary equation (29) for $\alpha = 1$ turns into an equation of oscillatory type:

$$\frac{\partial^2 w}{\partial \xi^2} + u\frac{\partial w}{\partial \xi} + \frac{\beta}{2}F(\xi)w = 0\,. \tag{36}$$

As an example we may consider an explicit analytical solution of equation (36) for the case when

$$F(\xi) = 2e^{2u\xi}\,. \tag{37}$$

Upon substituting $w = \eta(\varsigma)/\varsigma$, $\varsigma = e^{u\xi}$, we obtain the following equation:

$$\frac{\partial^2 \eta}{\partial \xi^2} + \frac{\beta}{u^2}\eta = 0\,. \tag{38}$$

Assuming that $\beta > 0$, we obtain that the solution of the initial equation (21) has the form

$$\phi(x,t) = -2\frac{\partial}{\partial x}\ln w(x,t)\,, \tag{39}$$

where

$$w(x,t) = e^{-u\xi}\Big[C_1 \cos\Big(\frac{\sqrt{\beta}}{u}e^{u\xi}\Big) + C_2 \sin\Big(\frac{\sqrt{\beta}}{u}e^{u\xi}\Big)\Big] \tag{40}$$

and $\xi = x - ut$.

4 The discrete character of the coefficient of sound wave damping

Note one point which is of significance in our approach. As shown in the previous part, in the case of non-periodical but time-independent perturbation $f(x)$ in equation (20), the corresponding linearized equation (29) can be reduced to equation (31). The damping coefficient λ, generally speaking, can be arbitrary as, for example, in the case of solution (32). However, there are cases when the parameter λ can acquire only discrete, absolutely definite values depending on the parameter β. Indeed, equation (31) completely conforms to the stationary Schrödinger equation, which may have not only a continuous, but also a discrete spectrum.

For example, let the right part in the BE (20) be a linear function

$$f(x) = -4cx\,, \quad \textit{i.e.} \quad F(x) = -2cx^2\,. \tag{41}$$

This means that equation (31) has the form

$$\frac{\partial^2 v}{\partial x^2} + [\lambda - \beta c x^2]\, v = 0\,. \tag{42}$$

In the case when $\beta c < 0$,

$$v(x) = D_{-\frac{1}{2}-i\frac{p}{2}}\big[\pm(1+i)z\big]\,, \quad z = x\Big|\frac{\beta c}{2}\Big|^{1/4}, \quad \lambda = p\Big(\frac{\beta c}{2}\Big)^{1/2}, \tag{43}$$

where $D_\alpha\,(x)$ is the function of a parabolic cylinder [10], and for the parameter λ we have a continuous series of values, since the parameter p corresponding to the impulse of wave can change continuously.

However, in the case when $\beta c > 0$,

$$v(x) = H_n(z)\,, \quad z = \Big|\frac{\beta c}{2}\Big|^{1/4} x\,, \quad \lambda = \lambda_n = (2n+1)\Big(\frac{\beta c}{2}\Big)^{1/2}, \tag{44}$$

where $H_n(z)$ is the Hermite functions [10], and for the damping parameter λ we have a discrete set of values, since $n \in N = 0,\ 1,\ 2,\ \ldots$ changes only in a discrete way.

Note that the difference between the two adjacent values of λ_n in this case does not depend on n,

$$\Delta\lambda_n \equiv \lambda_{n+1} - \lambda_n = (2\beta c)^{1/2}\,, \tag{45}$$

and is determined only by the intensity of influence of the inhomogeneous term in the BE (20).

5 Analysis of the influence of different perturbations

Let us note one more important circumstance. The solutions of equation (5) and, correspondingly, (7) and (8) evidently depend on the initial conditions and on the form of the function in the right-hand part of the inhomogeneous BE. At that, it seems obvious that to different forms of the function $f(x,t)$ correspond different solutions of the BE $\phi(x,t)$.

However, this is not quite so. Cases can be shown when to qualitatively different kinds of the inhomogeneous function $f(x,t)$ correspond coinciding solutions.

As an example, consider the case (b) corresponding to equation (20). After substitution of (22) it acquires the form of equation (29). For a solution in the form of the damping wave (30), the spatial part $v(x)$ must obey equation (31). Suppose that $\lim_{x\to\pm\infty} F(x) = -\infty$ and $F(x)$ may be presented in the form of

$$-\frac{\beta}{2}F(x) = \varphi^2(x) - \varphi'(x)\,, \tag{46}$$

where $\varphi(x)$ is a certain function of the variable x. This can be done by solving the differential Riccati equation (46) relative to $\varphi(x)$. If the function $\varphi(x)$ is found, then the corresponding equation (31) with the new function

$$-\frac{\beta}{2}\tilde{F}(x) = \varphi^2(x) + \varphi'(x) \tag{47}$$

has the same solutions and λ eigenvalues as in the case (42), except the ground state

$$v_0(x) = C\,e^{-\int\varphi(x)dx}\,. \tag{48}$$

The reason for such an interesting behavior of $v(x)$ solutions in this case lies in the internal symmetry of equation (31). The $v(x)$ solutions for the perturbation function $F(x)$ and $\tilde{v}(x)$ solutions for the perturbation function $\tilde{F}(x)$are interconnected by the transformation

$$\tilde{v}(x) = \sqrt{\frac{2}{\lambda}}\Big(\frac{\partial}{\partial x} + \varphi(x)\Big)v(x)\,, \tag{49}$$

$$v(x) = \sqrt{\frac{2}{\lambda}}\Big(\frac{\partial}{\partial x} - \varphi(x)\Big)\tilde{v}(x)\,. \tag{50}$$

As a simplest example of such interrelation by the function $\varphi(x)$, let us consider the case $\varphi(x) = x$. Then $F(x) = -2(x^2 - 1)/\beta$, and equation (31) for $v = v(x)$ acquires the form

$$\frac{\partial^2 v}{\partial x^2} + [\lambda + 1 - x^2]\, v = 0 \tag{51}$$

and according to (44)

$$v(x) = H_n(z)\,,\ z = \Big(\frac{1}{2}\Big)^{1/4} x\,,\ \lambda_n = \frac{2n - \sqrt{2} + 1}{\sqrt{2}}\,, \quad n \in \mathrm{N}\,. \tag{52}$$

At the same time, for the perturbation function $\tilde{F}(x) = -2(x^2 + 1)/\beta$, equation (31) acquires the form

$$\frac{\partial^2 \tilde{v}}{\partial x^2} + [\lambda - 1 - x^2]\, \tilde{v} = 0\,, \tag{53}$$

and according to (44)

$$\tilde{v}(x) = H_n(z)\,,\ z = \Big(\frac{1}{2}\Big)^{1/4} x\,,\ \tilde{\lambda}_n = \tilde{\lambda}_n = \frac{2n + \sqrt{2} + 1}{\sqrt{2}}\,, \quad n \in \mathrm{N}\,. \tag{54}$$

Note now that, as follows from (45), $\Delta\lambda = \tilde{\lambda}_n - \lambda_n = 2$. This means that

$$\tilde{\lambda}_n = \lambda_n + 2\,, \tag{55}$$

and the solutions $v_n(x)$ and $\tilde{v}_n(x)$ coincide, except $v_0(x)$:

$$v_0(x) = Ce^{-x^2/2}. \tag{56}$$

In more complicated cases when $\varphi(x)$ is not a linear function, the form of the perturbation function $F(x)$ and, respectively, of $\tilde{F}(x)$ is also more complicated. In some cases, as in the case of $\varphi(x) = x$, an analytical form of the solution is possible [11, 12].

6 The area of stationary solution

Analysis of three different cases allows qualitatively and in some cases also analytically to trace the process of hypersound generation.

In the cases (a) and (b) it is possible to evaluate the distance z_0 when the solution becomes stationary. For example, in case (a) for $A \ll 1$ the distance $z_0 \sim 1$, because at this distance the transitory processes fade out and the effect of the border conditions is not pronounced any longer. For $A \gg 1$, the distance of going into a stationary regime $z_0 \sim 1/\sqrt{A}$. The stationary form of the wave is determined by the solution (28) which for $A \gg 1$ has the form

$$\phi(x) = 2\sqrt{A}\cos\frac{x}{2}\ \mathrm{sign}\, x, \quad -\pi \le x \le \pi, \tag{57}$$

what corresponds to a distorted, not saw-like profile of the wave.

The hypersound intensity in the saturation regime is also related to the form of the asymptotic solution. Thus, integrating equation (27) by x from 0 to 2π, we obtain

$$-\frac{d\phi}{dx} + \frac{1}{2}\phi^2 = -A\cos x + \frac{1}{2}\,C\,, \tag{58}$$

where

$$C = \frac{1}{2\pi}\int\limits_0^{2\pi} \phi^2 dx\,. \tag{59}$$

Linearization of equation (58) gives the expression $C = -\lambda_0$, whence

$$I_s = |\lambda_0|\,\rho_0 c_0^5 \alpha^2 / b^2 \Omega^2\,. \tag{60}$$

For $A \ll 1$, $|\lambda| \approx A^2/2$ and $I_s = Y^2 I_p I_s \Omega^2 / 8c^2 \rho_0 c_0^3 \alpha^2$, which corresponds to a common linear case in the approximation of slowly changing amplitudes. In the other limited case $A \gg 1$, $|\lambda_0| \approx 2A$ and $I_s = Y c_0 (I_p I_s)^{1/2} / n\varepsilon c$, what corresponds to a deceleration of sound intensity increase at the expense of nonlinear damping.

7 Conclusions

Thus, the present work shows that the initial system of equations for complex amplitudes of the pumping wave E_p and Stokes wave E_s in the one-dimensional case may be reduced to a non-homogeneous Burgers equation. As a result of a proper selection of the space and time scales x_0 and τ_0, the non-homogeneous Burgers equation is reduced to a dimensionless type. Applying a nonlinear substitution of the variables, the non-homogeneous Burgers equation is reduced to a disturbed diffusion-type equation.

Three basic perturbations of the external field are considered: the periodical stationary spatial perturbation a) $F(x,t) \sim \cos x$; linearly increasing stationary spatial perturbation b) $F(x,t) \sim x$, and the travelling wave type perturbation c) $F(x,t) \sim f(x-ut)$.

It is shown that for a certain class of non-periodical and time-nonstationary perturbations, the coefficient λ of wave absorption can be only a discrete value. From the physical point of view, this is a result of the interaction between the external perturbation and the nonlinear dynamic system. In the mathematical approach, the non-homogeneous Burgers equation after the non-linear substitution of variables is reduced to a Schrödinger-type equation which, in turn, may have a discrete spectrum.

Another interesting peculiarity of the initial nonlinear system is related to the fact that different types of external perturbations may result in equivalent consequences, i.e. solutions of a corresponding non-homogeneous Burgers equation. On a concrete example of $F(x) \sim -x^2 + c$ and $F(x) \sim -x^2 - c$ it is shown that the corresponding solution of the Burgers equation and the discrete spectrum of the extinction coefficients λ coincide. In the general case two potentials $F_1(x) = \phi^2 + \phi'$ and $F_2(x) = \phi^2 - \phi'$, where $\phi = \phi(x)$ is a some potential-type function with asymptotic conditions $\phi(x) \to 0$ when $x \to \pm\infty$, have the same spectrum of the extinction coefficients λ.

The above analysis of three different types of perturbations allows a qualitative and in some cases also quantitative tracing of the hypersound generation process. The cases of weak $A \ll 1$ and strong $A \gg 1$ perturbations are considered separately. The asymptotic intensities of the corresponding nonlinear damped waves are determined.

The perturbations a)–c) discussed above certainly do not exhaust all the possible types of perturbations of a nonlinear system of interacting waves. However, a combination of the above examples and their generalization allow deriving qualitative and in some cases also quantitative information on hypersound generation possibilities in a nonlinear medium in the other cases not discussed in the present paper.

References

1. *Nonlinear Acoustics at the turn of the Millennium: ISNA 15, 15th International Symposium, Göttingen, Germany 1–4 September 1999* (AIP Conference Proceedings), American Institute of Physics, Berlin, 2000.
2. *Physical acoustics and optics: Molecular scattering of light, propagation of hypersound, metal optics,* Consultants Bureau, Berlin, 1975.
3. S. A. Akhmanov, R. B. Khokhlov, *The problems of nonlinear optics,* VINITI, Moscow, 1975 (in Russian).
4. M. F. Hamilton, D. T. Blackstock, *Nonlinear Acoustics: Theory and Applications,* Academic Press, NY, 1998.
5. V. S. Starukhanov, I. L. Fabelinsky, Generation of hypersound by the stimulated radiation, *Usp. Fiz. Nauk,* **98**(5), pp. 441–463, 1969 (in Russian).
6. O. V. Rudenko, S. Soluyan, *Theoretical Foundation on Nonlinear Acoustics,* Plenum Press, NY, 1977.
7. O. V. Rudenko, The principles of nonlinear acoustics, *JETP Lett.,* **20**, pp. 445–449, 1974.
8. V. E. Gusev, O. V. Rudenko, Non-stationary semi-one-dimensional acoustic flows, *Acoustic Journal,* **XXV**(6), pp. 875–881, 1979.
9. O. V. Rudenko, A. S. Chirkin, Theory of nonlinear interaction between monochromatic and noise waves in weakly dispersive media, *JETP,* **67**(5), pp. 1903–1911, 1974.
10. M. Abramowitz, I. A. Stegun, *Handbook of Mathematical Functions,* Dover, New York, 1970.
11. P. Miškinis, One more exact spectrum of the one-dimensional Schrödinger equation, *Lith. J. of Phys.,* **29**(2), pp. 156–158, 1989.
12. S. Grenda, P. Miškinis, Finding of the full exact spectra applying supersymmetric quantum mechanics, *Lith. J. of Phys.,* **30**(1), pp. 3–10, 1990.