Dufour and Soret Effects on Unsteady MHD Free Convection and Mass Transfer Flow past a Vertical Porous Plate in a Porous Medium

M. S. Alam[1], M. M. Rahman[2], M. A. Samad[3]

[1]Department of Mathematics, Dhaka University of Engineering and Technology
Gazipur-1700, Bangladesh
[2,3]Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh
[2]mansurdu@yahoo.com

Received: 01.03.2006 **Revised:** 05.06.2006 **Published online:** 01.09.2006

Abstract. The Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past an infinite vertical porous plate embedded in a porous medium have been studied numerically. The non-linear partial differential equations, governing the problem under consideration, have been transformed by a similarity transformation into a system of ordinary differential equations, which is solved numerically by applying Nachtsheim-Swigert shooting iteration technique together with sixth order Runge-Kutta integration scheme. The effects of various parameters entering into the problem have been examined on the flow field for a hydrogen-air mixture as a non-chemical reacting fluid pair. Finally, the numerical values of local Nusselt number and local Sherwood number are also presented in tabular form.

Keywords: free convection, unsteady flow, MHD, Dufour effect, Soret effect, numerical solution.

1 Introduction

In recent years, the problems of free convective and heat transfer flows through a porous medium under the influence of a magnetic field have been attracted the attention of a number of researchers because of their possible applications in many branches of science and technology, such as its applications in transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. On the other hand, flow through a porous medium have numerous engineering and geophysical applications, for example, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs. In view of these applications, many researchers have studied MHD free convective heat and mass transfer flow in a porous medium; some of them are Raptis and Kafoussias [1], Sattar [2] and Kim [3].

Due to the importance of Soret (thermal-diffusion) and Dufour (diffusion-thermo) effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported results for these flows of whom the names are Eckert and Drake [4], Dursunkaya and Worek [5], Anghel *et al.* [6], Postelnicu [7] are worth mentioning. Very recently, Alam and Rahman [8] studied the Dufour and Soret effects on steady MHD free convective heat and mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium. As a complementary study to that of Postelnicu [7] and Alam and Rahman [8], we propose to study the above-mentioned Dufour and Soret effects on unsteady free convection and mass transfer flow past an impulsively started infinite vertical porous flat plate in a porous medium under the influence of transversely applied magnetic field.

2 Mathematical Analysis

An unsteady two-dimensional flow of an incompressible and electrically conducting viscous fluid, along an infinite vertical porous flat plate embedded in a porous medium is considered. The x-axis is taken on the infinite plate, and parallel to the free-stream velocity which is vertical and the y-axis is taken normal to the plate. A magnetic field B_0 of uniform strength is applied transversely to the direction of the flow. Initially the plate and the fluid are at same temperature T_∞ in a stationary condition with concentration level C_∞ at all points. For $t > 0$, the plate starts moving impulsively in its own plane with a velocity U_0, its temperature is raised to T_w and the concentration level at the plate is raised to C_w. The flow configuration and coordinate system are shown in the following Fig. 1.

The fluid is assumed to have constant properties except that the influence of the density variations with temperature and concentration, which are considered

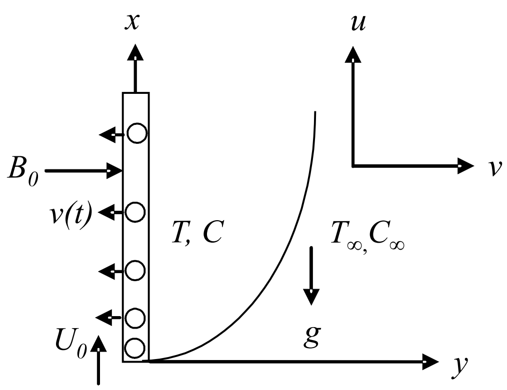

Fig. 1. Flow configuration and coordinate system.

only in the body force term. Under the above assumptions, the physical variables are functions of y and t only. Assuming that the Boussinesq and boundary-layer approximation hold and using the Darcy-Forchheimer model, the basic equations, which govern the problem, are given by (see Alam and Rahman [8]):

$$\frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned}\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} &+ g\beta(T - T_\infty) \\ &+ g\beta^*(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{K}u - \frac{b}{K}u^2,\end{aligned} \tag{2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D_M\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where u, v are the Darcian velocity components in the x- and y- directions respectively, t is the time, ν is the kinematic viscosity, g is the acceleration due to gravity, ρ is the density, β is the coefficient of volume expansion, β^* is the volumetric coefficient of expansion with concentration, K is the Darcy permeability, b is the empirical constant, B_0 is the magnetic induction, T and T_∞ are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream, respectively, while C and C_∞ are the corresponding concentrations. Also, σ is the electric conductivity, α is the thermal diffusivity, D_m is the coefficient of mass diffusivity, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature, k_T is the thermal diffusion ratio and c_s is the concentration susceptibility.

Initially ($t = 0$) the fluid and the plate are at rest. Thus the no slip boundary conditions at the surface of the plate for the above problem for $t > 0$ are:

$$u = U_0, \quad v = v(t), \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \tag{5a}$$

$$u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as} \quad y \to \infty. \tag{5b}$$

The last term on the right-hand side of the energy equation (3) and concentration equation (4) signifies the Dufour or diffusion-thermo effect and the Soret or thermal-diffusion effect, respectively.

Now in order to obtain a local similarity solution (in time) of the problem under consideration, we introduce a time dependent length scale δ as

$$\delta = \delta(t). \tag{6}$$

In terms of this length scale, a convenient solution of the equation (1) is considered to be in the following form:

$$v = v(t) = -v_0 \frac{\nu}{\delta}, \tag{7}$$

where $v_0 > 0$ is the suction parameter.

We now introduce the following dimensionless variables:

$$\begin{aligned} \eta &= \frac{y}{\delta}, \\ u &= U_0 f(\eta), \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \\ \phi(\eta) &= \frac{C - C_\infty}{C_w - C_\infty}. \end{aligned} \tag{8}$$

Then introducing the relations (6)–(8) into the equations (2)–(3) respectively, we obtain (by using the analysis of Sattar and Hossain [9], see also Hasimoto [10]), the following dimensionless ordinary differential equations.

$$f'' + (2\eta + v_0) f' + Gr\,\theta + Gm\,\phi - Mf - \frac{1}{Da} f - \frac{Re\,Fs}{Da} f^2 = 0, \tag{9}$$

$$\theta'' + Pr(2\eta + v_0)\theta' + Pr\,Du\,\phi'' = 0, \tag{10}$$

$$\phi'' + Sc(2\eta + v_0)\phi' + Sc\,Sr\,\theta'' = 0, \tag{11}$$

where primes denote differentiation with respect to η and $Da = \frac{K}{\delta^2}$ is the local Darcy number, $Fs = \frac{b}{\delta}$ is the local Forchheimer number, $Re = \frac{U_0 \delta}{\nu}$ is the local Reynolds number, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Sc = \frac{\nu}{D_m}$ is the Schmidt number, $M = \frac{\sigma B_0^2 \delta^2}{\nu \rho}$ is the magnetic field parameter, $Sr = \frac{D_m k_T (T_w - T_\infty)}{T_m \nu (C_w - C_\infty)}$ is the Soret number, $Du = \frac{D_m k_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)}$ is the Dufour number, $Gr = \frac{g\beta (T_w - T_\infty)\delta^2}{\nu U_0}$ is the local Grashof number and $Gm = \frac{g\beta^* (C_w - C_\infty)\delta^2}{\nu U_0}$ is the local modified Grashof number.

The corresponding boundary conditions for $t > 0$ are obtained as:

$$f = 1, \quad \theta = 1, \quad \phi = 1, \quad \text{at} \quad \eta = 0, \tag{12a}$$

$$f = 0, \quad \theta = 0, \quad \phi = 0, \quad \text{as} \quad \eta \to \infty. \tag{12b}$$

The equations (9)–(11) are locally similar in time but not explicitly time dependent. The above system together with the boundary equations (12) have been solved numerically for various values of the parameters entering into the problem. From the process of numerical computation, the local Nusselt number and the local Sherwood number, which are respectively proportional to $-\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in tabular form.

3 Numerical solution

The set of nonlinear ordinary differential equations (9)–(11) with boundary conditions (12) have been solved numerically by applying Nachtsheim-Swigert [11] shooting iteration technique along with sixth order Runge-Kutta integration scheme. A step size of $\Delta\eta = 0.01$ was selected to be satisfactory for a convergence criterion of 10^{-6} in all cases. The value of η_∞ was found to each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, to each group of parameters v_0, Sr, Du, Da, M, Pr, Sc, Gr, Gm, Re and Fs determined when the value of the unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-6}.

In order to see the effects of step size ($\Delta\eta$) we ran the code for our model with three different step sizes as $\Delta\eta = 0.01$, $\Delta\eta = 0.004$, $\Delta\eta = 0.001$ and in each case we found very good agreement between them. Fig. 2 shows the velocity profiles for different step sizes.

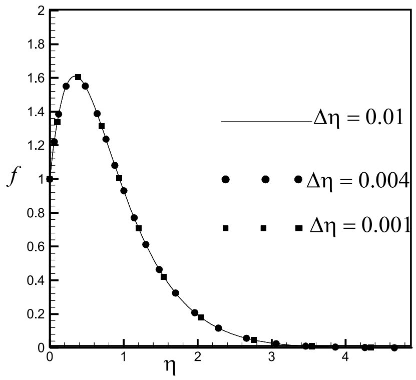

Fig. 2. Velocity profiles for different step sizes.

4 Results and discussion

Numerical calculations have been carried out for different values of v_0, Sr, Du, Da, M and for fixed values of Pr, Sc, Gr, Gm, Re and Fs. The value of Pr is taken to be 0.71 which corresponds to air and the value of Sc is chosen to represent hydrogen at 25^0 C and 1 atm. Due to free convection problem positive large values of $Gr = 12$ and $Gm = 6$ are chosen. The value of Re is kept 100 and Fs equal to 0.09. The values of Dufour number and Soret number are chosen in such a way that their product is constant provided that the mean temperature T_m is kept constant as well. However, the values of v_0, Da and M are chosen arbitrarily. The numerical results for the velocity, temperature and concentration profiles are displayed in Figs. 3–5.

The effects of suction parameter on the velocity field are shown in Fig. 3(a). It is seen from this figure that the velocity profiles decrease monotonically with an increase of suction parameter indicating the usual fact that suction stabilizes the boundary layer growth. The effects of suction parameter on the temperature and concentration field are displayed in Fig. 3(b) and Fig. 3(c) respectively. From Fig. 3(b) we see that the temperature decreases with an increase of suction parameter. From Fig. 3(c) we observe that the concentration increases with an increase

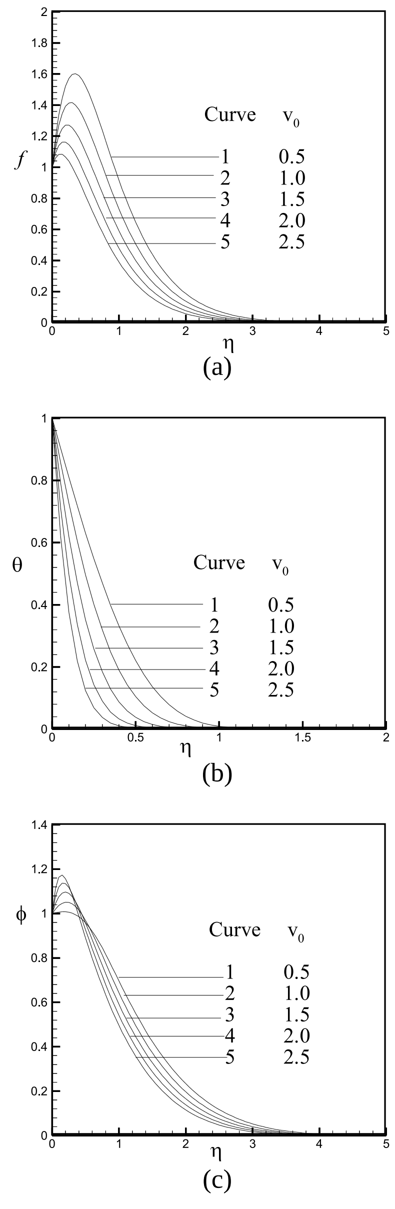

Fig. 3. (a) Velocity, (b) temperature, and (c) concentration profiles for different values of v_0 for $Sr = 2.0$, $Du = 0.03$, $Da = 0.5$ and $M = 0.3$.

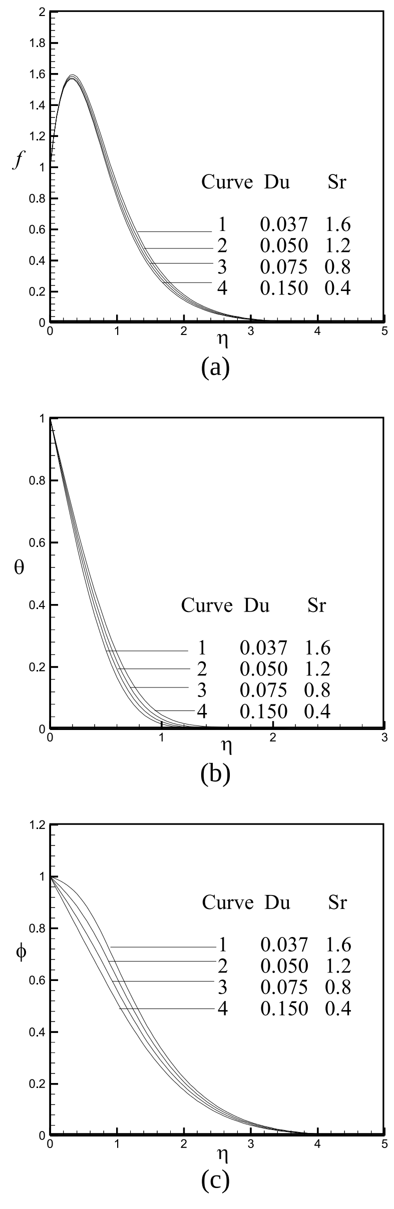

Fig. 4. (a) Velocity, (b) temperature, and (c) concentration profiles for different values of Du, Sr for v_0=0.5, $Da = 0.5$ and $M = 0.3$.

of suction parameter close to the wall (approx. $\eta \leq 0.50$) whereas for $\eta \geq 0.50$, the concentration decreases with an incraese of suction parameter. Sucking decelerated fluid particles through the porous wall reduce the growth of the fluid boundary layer as well as thermal and concentration boundary layers.

The effects of Soret and Dufour numbers on the velocity field are shown in Fig. 4(a). We observe that quantitatively when $\eta = 1.5$ and Sr decreases from 1.6 to 1.2 there is $6.25\,\%$ decrease in the velocity value, whereas the corresponding decrease is $5.87\,\%$, when Sr decreases from 0.8 to 0.4. The effects of Soret and Dufour numbers on the temperature field are shown in Fig. 4(b). We observe that quantitatively when $\eta = 0.80$ and Sr decreases from 1.6 to 1.2 there is $27.56\,\%$ increase in the temperature value, whereas the corresponding increase is $23\,\%$, when Sr decreases from 0.8 to 0.4. The effects of Soret and Dufour numbers on the concentration field are shown in Fig. 4(c). We observe that quantitatively when $\eta = 1.0$ and Sr decreases from 1.6 to 1.2 there is $7.64\,\%$ decrease in the concentration value, whereas the corresponding decrease is $8.90\,\%$, when Sr decreases from 0.8 to 0.4.

The effects of Darcy parameter and magnetic field parameter on the velocity field are shown in Fig. 5. This figure shows that velocity increases with the increase of Darcy number. For large Darcy number, porosity of the medium increases, hence fluid flows quickly. We also see that velocity profiles decrease with the increase of magnetic effect indicating that magnetic field tends to retard

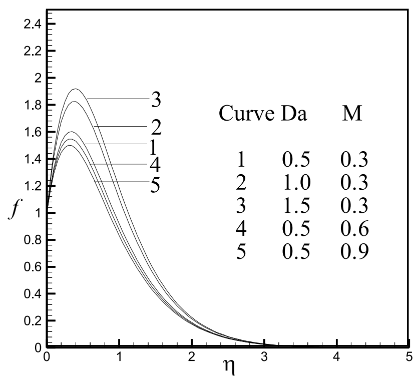

Fig. 5. Velocity profiles for different values of Da, M for $v_0 = 0.5$, $Sr = 2.0$ and $Du = 0.03$.

the motion of the fluid. Magnetic field may control the flow characteristics.

Finally, the effects of Soret and Dufour numbers on the local Nusselt number and local Sherwood number are shown in Table 1. The behavior of these parameters is self-evident from the Table 1 and hence they will not discuss any further due to brevity.

Table 1. Numerical values of Nu and Sh at $v_0 = 0.5$, $Da = 0.5$, $M = 0.3$

Sr	Du	Nu	Sh
2.0	0.03	1.934325	-0.086492
1.0	0.06	1.652241	0.315615
0.5	0.12	1.541984	0.468128
0.4	0.15	1.517881	0.496002
0.2	0.30	1.450355	0.549515
0.1	0.60	1.364561	0.575236

5 Conclusions

In this paper we have studied numerically the Dufour and Soret effects on an unsteady MHD free convection and mass transfer flow past an impulsively started infinite vertical porous plate embedded in a porous medium. From the present numerical study the following conclusions can be drawn:

- Velocity profiles increase with the increase of Darcy number.
- Magnetic field retards the motion of the fluid.
- Suction stabilizes the hydrodynamic, thermal as well as concentration boundary layers growth.
- For fluids with medium molecular weight (H_2, air), Dufour and Soret effects should not be neglected.

References

1. A. Raptis, N. G. Kafoussias, Magnetohydrodynamic free convection flow and mass transfer through porous medium bounded by an infinite vertical porous plate with constant heat flux, *Can. J. Phys.*, **60**(12), pp. 1725–1729, 1982.

2. M. A. Sattar, Unsteady hydromagnetic free convection flow with Hall current mass transfer and variable suction through a porous medium near an infinite vertical porous plate with constant heat flux, *Int. J. Energy Research*, **17**, pp. 1–5, 1993.

3. Y. J. Kim, Heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium, *Transport in Porous Media,* **56**(1), pp. 17–37, 2004.

4. E. R. G. Eckert, R. M. Drake, *Analysis of Heat and Mass Transfer,* McGraw-Hill, New York, 1972.

5. Z. Dursunkaya, W. M. Worek, Diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from vertical surface, *Int. J. Heat Mass Transfer,* **35**(8), pp. 2060–2065, 1992.

6. M. Anghel, H. S. Takhar, I. Pop, Dufour and Soret effects on free-convection boundary layer over a vertical surface embedded in a porous medium, *Studia Universitatis Babes-Bolyai, Mathematica,* **XLV**(4), pp. 11–21, 2000.

7. A. Postelnicu, Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects, *Int. J. Heat Mass Transfer,* **47**(6–7), pp. 1467–1472, 2004.

8. M. S. Alam, M. M. Rahman, Dufour and Soret effects on MHD free convective heat and mass transfer flow past a vertical flat plate embedded in a porous medium, *J. Naval Architecture and Marine Engng.,* **2**(1), pp. 55–65, 2005.

9. M. A. Sattar, M. M. Hossain, Unsteady hydromagnetic free convection flow with Hall current and mass transfer along an accelerated porous plate with time-dependent temperature and concentration, *Can. J. Phys.,* **70**, pp. 369–374, 1992.

10. H. Hashimoto, Boundary layer growth on a flat plate with suction or injection, *J. Phys. Soc. Japan*, **12**(1), pp. 68–72, 1957.

11. P. R. Nachtsheim, P. Swigert, *Satisfaction of the asymptotic boundary conditions in numerical solution of the system of nonlinear equations of boundary layer type*, NASA TND-3004, 1965.