

On the Discounted Penalty Function for Claims Having Mixed Exponential Distribution

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Abstract. It is considered the classical risk model with mixed exponential claim sizes. Using known results it is obtained the explicit expression of the Gerber-Shiu discounted penalty function

$$\psi(x, \delta) = E(e^{-\delta T} \mathbf{1}(T < \infty)),$$

by some infinite series. Here $\delta > 0$ is the force of interest, x – the initial reserve and T – ruin time.

The dependance of the discounted penalty function on the main parameters $x, \theta, \lambda, \delta, \alpha, \sigma, \nu$ is presented in diagrams, where $\lambda > 0$ is the parameter of Poisson process, $\theta > 0$ is the safety loading coefficient, $0 \leq \alpha \leq 1$ and $\sigma, \nu > 0$ are the parameters of the mixed exponential distribution.

Keywords: classical risk model, time to ruin, Gerber-Shiu discounted penalty function, mixed exponential distribution.

1 Introduction and main result

In 1957, E. Sparre Andersen [1] proposed a mathematical model, which was applied to the risk business of an insurance company. This model can be characterized in the following way. Suppose, that the premium rate of an insurance company is c . Claims occur at the times $0 < T_1 < T_2 < \dots$. The i -th claim arriving at time T_i causes the claim severity Y_i . Then the capital of the company

at a given time t is defined by

$$U(t) = x + ct - \sum_{n=1}^{N(t)} Y_n, \quad \text{where } N(t) = \max\{k: T_k \leq t\}$$

and x is an initial reserve at $T_0 = 0$. The time T when the capital $U(t)$ falls the first time below zero is called ruin time:

$$T = \inf \{t > 0: U(t) < 0\}.$$

The function

$$\psi(x) = P(T < \infty)$$

is called the probability of ruin. In described model, we suppose that Y, Y_1, Y_2, \dots is the sequence of i.i.d. random variables. $T_1, T_2 - T_1, T_3 - T_2, \dots$ is another sequence of i.i.d. random variables. In addition, sequences of r.v. Y, Y_1, Y_2, \dots and $T_1, T_2 - T_1, T_3 - T_2, \dots$ are mutually independent. If T_1 has the exponential distribution with positive parameter $\lambda > 0$, then $N(t)$ is the Poisson process with the same parameter λ . Usually such model is called the *classical* or *Lundberg's* model.

In 1998, H. Gerber and E. S. Shiu [2] proposed, instead of the probability of ruin $\psi(x) = P(T < \infty)$ in the classical risk model, to analyze the discounted penalty function

$$\psi(x, \delta) = E(e^{-\delta T} \mathbf{1}(T < \infty)), \quad \delta > 0,$$

which describes the expectation of the present (current) value of future bankruptcy. Here δ is the force of interest and T is the ruin time. In this case the penalty at the moment T is accepted to be unitary. It is clear, that

$$\psi(x, 0) = P(T < \infty) = \psi(x).$$

Therefore, $\psi(x, \delta)$ is more general than $\psi(x)$.

In the work we analyze the classical model with mixed exponential claims. Our purpose is to find the explicit expression of the Gerber-Shiu penalty function $\psi(x, \delta)$ in this case.

In 2000, a lot of fundamental results about the properties of $\psi(x, \delta)$ were presented by X. S. Lin and G. E. Willmot [3]. For example, it is known that the Laplace transform of the Gerber-Shiu penalty function satisfies the defective renewal equation and that $\psi(x, \delta)$ can be expressed by the tail of compound geometric distribution. We formulate one of these assertions below.

Theorem 1. [3] (see also [4,5]) Assume that claim sizes Y_1, Y_2, \dots in the classical model have absolutely continuous distribution Y with a d.f. $H(y)$ and a mean EY . Let the premium rate be $c = \lambda EY(1 + \theta)$, with $\theta > 0$. Then

$$\psi(x, \delta) = \sum_{n=1}^{\infty} (1 - \phi)\phi^n \bar{F}^{*n}(x), \tag{1}$$

where

$$\bar{F}(x) = \frac{\int_0^{\infty} e^{-\rho y} \bar{H}(x+y) dy}{\int_0^{\infty} e^{-\rho y} \bar{H}(y) dy}, \tag{2}$$

$$\phi = \frac{\int_0^{\infty} e^{-\rho y} \bar{H}(y) dy}{(1 + \theta)EY}, \tag{3}$$

and ρ is the unique non-negative root of the Lundberg equation

$$\lambda \int_0^{\infty} e^{-\rho y} H(y) dy = \lambda + \delta - c\rho. \tag{4}$$

In 2003, S. D. Drekić and G. E. Willmot [6] obtained the expression for $\psi(x, \delta)$ [6] (see also [3, 7] and [8]) under exponential claim sizes. They have demonstrated, that

$$\psi(x, \delta) = \phi e^{-\mu x(1-\phi)},$$

where

$$\phi = \frac{\mu}{(1 + \theta)(\mu + \rho)} \quad \text{and} \quad \rho = \frac{\lambda + \delta - c\mu + \sqrt{(\lambda + \delta + c\mu)^2 - 4c\lambda\mu}}{2c}.$$

Using this equation, in mentioned paper they obtained the expression for the density function of time to ruin.

In 1999, X. S. Lin and G. E. Willmot [9] considered function $\psi(x, \delta)$ in the classical risk model with the individual claim Y having the mixed exponential distribution. Unfortunately, the proposed expression depends on the roots of the Lundberg equation and it is rather difficult to calculate the values of $\psi(x, \delta)$ for the concrete parameters $x, \delta, \lambda, \nu, \sigma, \alpha$.

In 2005, using the double Laplace transform, J. M. A. Garcia [8] obtained the expression for the density function of time to ruin in the case when claim Y has mixed exponential distribution and when Y has Erlang(2) distribution.

In this work, we establish the explicit expression for $\psi(x, \delta)$ in case of mixed exponential claim sizes Y_1, Y_2, \dots . More precisely, we examine the case, when for all $y \geq 0$

$$P(Y \leq y) = H(y) = \alpha(1 - e^{-\sigma y}) + (1 - \alpha)(1 - e^{-\nu y}), \tag{5}$$

where $\nu, \sigma > 0, 0 \leq \alpha \leq 1$. In Section 3, we present several graphs of discounted penalty function. We can see from these graphs how $\psi(x, \delta)$ depends on initial capital x , interest rate δ , security loading θ , intensity of the Poisson process λ and parameters of individual claim distribution σ, ν, α .

The next statement is the main result of this paper.

Theorem 2. *Let individual claims in the classical model has d.f. $H(y)$ defined by [5]. Let, further, the parameter of Poisson process be $\lambda > 0$ and the relative security loading $\theta > 0$. Then*

$$\begin{aligned} \psi(x, \delta) = & \sum_{n=1}^{\infty} \frac{(1 - \phi)\phi^n}{(a + b)^n} \left[b^n e^{-\nu x} \sum_{j=0}^{n-1} \frac{(x\nu)^j}{j!} + a^n e^{-\sigma x} \sum_{j=0}^{n-1} \frac{(x\sigma)^j}{j!} \right. \\ & \left. + \sum_{k=1}^{n-1} \binom{n}{k} (a\sigma)^k (b\nu)^{n-k} \left(\frac{(-1)^{k-1}}{(k-1)!} \mathcal{V}_1 + \frac{(-1)^{n-k-1}}{(n-k-1)!} \mathcal{V}_2 \right) \right], \end{aligned} \tag{6}$$

where

$$\begin{aligned} \mathcal{V}_1 = & \frac{e^{-\sigma x}}{(n - k - 1)!} \sum_{i=0}^{k-1} \left(\binom{k-1}{i} \frac{(n - k + i - 1)! (\sigma - \nu)^{k-n-i}}{\sigma^{k-i}} \right. \\ & \left. \times \sum_{j=0}^{k-1-i} \frac{(x\sigma)^j (k-1-i)!}{j!} \right), \end{aligned} \tag{7}$$

$$\mathcal{V}_2 = \frac{e^{-\nu x}}{(k-1)!} \sum_{i=0}^{n-k-1} \left(\binom{n-k-1}{i} \frac{(i+k-1)!(\nu-\sigma)^{-i-k}}{\nu^{n-k-i}} \times \sum_{j=0}^{n-k-1-i} \frac{(x\nu)^j (n-k-1-i)!}{j!} \right), \quad (8)$$

$$a = \alpha(\rho - \nu), \quad b = (1 - \alpha)(\rho - \sigma),$$

$$\phi = \frac{\sigma\nu(\alpha\nu + (1 - \alpha)\sigma + \rho)}{(1 + \theta)(\alpha\nu + \sigma(1 - \alpha))(\rho + \sigma)(\rho + \nu)},$$

$$\rho = \frac{1}{6} \sqrt[3]{E + 12\sqrt{F}} - \frac{2C - \frac{2}{3}B^2}{\sqrt[3]{E + 12\sqrt{F}}} - \frac{B}{3},$$

$$E = 36BC - 108D - 8B^3,$$

$$F = 12C^2 - 3B^2C^2 - 54BCD + 81D^2 + 12B^3D,$$

$$B = (\nu + \sigma) - \frac{\lambda + \delta}{c},$$

$$C = \nu\sigma - \frac{(\lambda + \delta)(\sigma + \nu)}{c} + \frac{\lambda}{c}(\alpha\sigma + (1 - \alpha)\nu),$$

$$D = -\frac{\delta\nu\sigma}{c},$$

$$c = \frac{\lambda(\alpha\nu + (1 - \alpha)\sigma)}{\nu\sigma}(1 + \theta).$$

2 Proof of Theorem 2

In this section, applying the equality (1) we will prove equality (6). Let

$$\bar{H}(y) = 1 - H(y) = \alpha e^{-\sigma y} + (1 - \alpha)e^{-\nu y}, \quad y \geq 0.$$

The expectation of the claim Y is

$$EY = \frac{\alpha}{\sigma} + \frac{1 - \alpha}{\nu} = \frac{\alpha\nu + (1 - \alpha)\sigma}{\sigma\nu}.$$

The proof of (6) we split into three steps.

Step 1. Firstly we will find the quantity ϕ . From (3) we have that

$$\begin{aligned}\phi &= \frac{\sigma\nu}{(1+\theta)(\alpha\nu+(1-\alpha)\sigma)} \int_0^{\infty} e^{-\rho y} (\alpha e^{-\sigma y} + (1-\alpha)e^{-\nu y}) dy \\ &= \frac{\sigma\nu(\alpha\nu+(1-\alpha)\sigma+\rho)}{(1+\theta)(\alpha\nu+\sigma(1-\alpha))(\rho+\sigma)(\rho+\nu)},\end{aligned}\tag{9}$$

where ρ is non-negative root of the Lundberg equation (4):

$$\frac{\lambda\alpha\sigma}{\rho+\sigma} + \frac{\lambda(1-\alpha)\nu}{\rho+\nu} = \lambda + \delta - c\rho,\tag{10}$$

with

$$c = \frac{\lambda(\alpha\nu+(1-\alpha)\sigma)}{\nu\sigma}(1+\theta).$$

Equation (10) is equivalent to

$$c\rho^3 - (\lambda + \delta - c(\nu + \sigma))\rho^2 + (\lambda(\alpha\sigma + (1 - \alpha)\nu) - (\lambda + \delta)(\sigma + \nu) + c\nu\sigma)\rho - \delta\nu\sigma = 0.$$

Let

$$\begin{aligned}B &= (\nu + \sigma) - \frac{\lambda + \delta}{c}, \quad D = -\delta\nu\sigma, \\ C &= \nu\sigma - \frac{(\lambda + \delta)(\sigma + \nu)}{c} + \frac{\lambda}{c}(\alpha\sigma + (1 - \alpha)\nu).\end{aligned}$$

The last equality implies

$$\rho^3 + B\rho^2 + C\rho + D = 0.\tag{11}$$

From the graph in Fig. 1, we note, that the equation (7) has the unique non-negative root.

Consequently, the equation (11), which is equivalent to (10), also has the unique non-negative root. Applying Kardan's formula we find this root

$$\rho = \frac{1}{6} \sqrt[3]{E + 12\sqrt{F}} - \frac{2C - \frac{2}{3}B^2}{\sqrt[3]{E + 12\sqrt{F}}} - \frac{B}{3},$$

where

$$\begin{aligned}E &= 36BC - 108D - 8B^3, \\ F &= 12C^2 - 3B^2C^2 - 54BCD + 81D^2 + 12B^3D.\end{aligned}$$

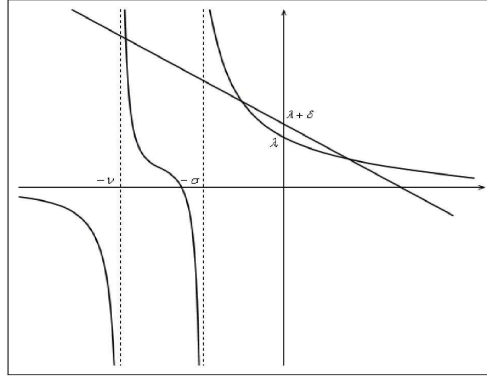


Fig. 1. Roots of equation (10).

Step 2. Now the distribution function $F(x)$ will be found. Applying (2) we get

$$\bar{F}(x) = \frac{\alpha(\rho + \nu)e^{-\sigma x} + (1 - \alpha)(\rho + \sigma)e^{-\nu x}}{\alpha(\rho + \nu) + (1 - \alpha)(\rho + \sigma)}.$$

Let

$$a = \alpha(\rho + \nu), \quad b = (1 - \alpha)(\rho + \sigma).$$

Then we can rewrite $\bar{F}(x)$ as

$$\bar{F}(x) = \frac{ae^{-\sigma x} + be^{-\nu x}}{a + b},$$

and consequently,

$$F(x) = \frac{a(1 - e^{-\sigma x}) + b(1 - e^{-\nu x})}{a + b}.$$

Step 3. In this part, the expression for $\bar{F}^{*n}(x)$ will be obtained. In our case the density function of the distribution function $F(x)$ is

$$p(x) = F'(x) = \frac{a\sigma e^{-\sigma x} + b\nu e^{-\nu x}}{a + b},$$

and the characteristic function

$$\varphi(t) = \int_0^{\infty} e^{itx} p(x) dx = \frac{1}{a + b} \left(\frac{a\sigma}{\sigma - it} + \frac{b\nu}{\nu - it} \right).$$

The characteristic function of $F^{*n}(x)$ is

$$\begin{aligned} \hat{\varphi}(t) &= \frac{1}{(a+b)^n} \left(\frac{a\sigma}{\sigma-it} + \frac{b\nu}{\nu-it} \right)^n \\ &= \frac{1}{(a+b)^n} \sum_{k=0}^n \binom{n}{k} \left(\frac{a\sigma}{\sigma-it} \right)^k \left(\frac{b\nu}{\nu-it} \right)^{n-k}. \end{aligned}$$

Applying the inversion formula, we get that the density function of the distribution function $F^{*n}(x)$

$$\begin{aligned} \hat{p}(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} \hat{\varphi}(t) dt \\ &= \frac{1}{(a+b)^n} \sum_{k=0}^n \binom{n}{k} (a\sigma)^k (b\nu)^{n-k} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-itx}}{(\sigma-it)^k (\nu-it)^{n-k}} dt. \end{aligned} \tag{12}$$

To obtain the expression of $\hat{p}(x)$ we have to calculate the integral

$$\mathcal{J} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-itx}}{(\sigma-it)^k (\nu-it)^{n-k}} dt = \frac{1}{2\pi i} \lim_{R \rightarrow \infty} \int_{\mathcal{L}_R} \frac{e^{-sx}}{(\sigma-s)^k (\nu-s)^{n-k}} ds,$$

where the integration contour $\mathcal{L}_R = \{it : t \in [-R, R]\}$. Adding segments l_1, l_2, l_3 to the contour \mathcal{L}_R we get the closed contour γ_R (see Fig. 2).

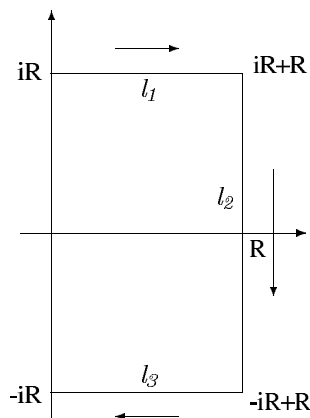


Fig. 2. The contour γ_R .

It is clear, that:

$$\begin{aligned} \left| \int_{l_1} \frac{e^{-sx} ds}{(s-\sigma)^k (s-\nu)^{n-k}} \right| &\leq \int_0^R \frac{e^{-ux} du}{(\sqrt{(u-\sigma)^2+R^2})^k (\sqrt{(u-\nu)^2+R^2})^{n-k}} \\ &< \frac{1}{R^n} \int_0^R e^{-ux} du < \frac{1}{xR^n}. \end{aligned}$$

Analogously,

$$\left| \int_{l_3} \frac{e^{-sx}}{(s-\sigma)^k (s-\nu)^{n-k}} ds \right| < \frac{1}{xR^n}$$

and

$$\left| \int_{l_2} \frac{e^{-sx}}{(s-\sigma)^k (s-\nu)^{n-k}} ds \right| < \frac{2e^{-Rx}}{R^{n-1}}.$$

Hence, according to residues' theorem,

$$\begin{aligned} \mathcal{J} &= \frac{1}{2\pi i} \lim_{R \rightarrow \infty} \int_{\gamma_R} \frac{e^{-sx}}{(\sigma-s)^k (\nu-s)^{n-k}} ds \\ &= (-1)^{n+1} \left(\operatorname{Res}_{s=\sigma} \frac{e^{-sx}}{(s-\sigma)^k (s-\nu)^{n-k}} + \operatorname{Res}_{s=\nu} \frac{e^{-sx}}{(s-\sigma)^k (s-\nu)^{n-k}} \right). \end{aligned}$$

We remark that $s = \sigma$ is the k th order pole, and $s = \nu$ is the $(n-k)$ -th order pole, so we get

$$\operatorname{Res}_{s=\sigma} \frac{e^{-sx}}{(s-\sigma)^k (s-\nu)^{n-k}} = \frac{1}{(k-1)!} \lim_{s \rightarrow \sigma} \left(\frac{e^{-sx}}{(s-\nu)^{n-k}} \right)^{(k-1)}$$

and

$$\operatorname{Res}_{s=\nu} \frac{e^{-sx}}{(s-\sigma)^k (s-\nu)^{n-k}} = \frac{1}{(n-k-1)!} \lim_{s \rightarrow \nu} \left(\frac{e^{-sx}}{(s-\sigma)^k} \right)^{(n-k-1)},$$

for $k = 1, \dots, n-1$. As

$$(e^{-zx}(z-d)^{-m})^{(l)} = (-1)^l e^{-zx} \sum_{i=0}^l \binom{l}{i} x^{l-i} \frac{(m+i-1)!}{(m-1)!} (z-d)^{-(m+i)}$$

we have

$$\begin{aligned} & \frac{1}{(k-1)!} \lim_{s \rightarrow \sigma} \left(\frac{e^{-sx}}{(s-\nu)^{(n-k)}} \right)^{(k-1)} \\ &= \frac{(-1)^{k-1} e^{-\sigma x}}{(k-1)!} \sum_{i=0}^{k-1} \binom{k-1}{i} x^{k-1-i} \frac{(n-k+i-1)!}{(n-k-1)!} (\sigma-\nu)^{k-n-i} \end{aligned}$$

and

$$\begin{aligned} & \frac{1}{(n-k-1)!} \lim_{s \rightarrow \nu} \left(\frac{e^{-sx}}{(s-\sigma)^k} \right)^{(n-k-1)} \\ &= \frac{(-1)^{n-k-1} e^{-\nu x}}{(n-k-1)!} \sum_{i=0}^{n-k-1} \binom{n-k-1}{i} x^{n-k-1-i} \frac{(i+k-1)!}{(k-1)!} (\nu-\sigma)^{-i-k} \end{aligned}$$

for $k = 1, \dots, n-1$. If $k = 0$ we get

$$\begin{aligned} \frac{1}{2\pi i} \lim_{R \rightarrow \infty} \int_{\gamma_R} \frac{e^{-sx} ds}{(\nu-s)^n} &= (-1)^{n+1} \operatorname{Res}_{s=\nu} \frac{e^{-sx}}{(s-\nu)^n} \\ &= \lim_{s \rightarrow \nu} \frac{(-1)^{n+1}}{(n-1)!} (e^{-sx})^{(n-1)} = \frac{e^{-\nu x} x^{n-1}}{(n-1)!}. \end{aligned}$$

If $k = n$, similarly

$$\frac{1}{2\pi} \lim_{R \rightarrow \infty} \int_{\gamma_R} \frac{e^{-sx} ds}{(\sigma-s)^n} = \frac{e^{-\sigma x} x^{n-1}}{(n-1)!}.$$

Thus, obtained equalities and (12) imply

$$\begin{aligned} \hat{p}(x) &= \frac{1}{(a+b)^n} \left[(b\nu)^n \frac{e^{-\nu x} x^{n-1}}{(n-1)!} + (-1)^{n+1} \sum_{k=1}^{n-1} \binom{n}{k} (a\sigma)^k (b\nu)^{n-k} \right. \\ &\quad \left. \times \left(\frac{(-1)^{k-1}}{(k-1)!} \mathcal{U}_1 + \frac{(-1)^{n-k-1}}{(n-k-1)!} \mathcal{U}_2 \right) + (a\sigma)^n \frac{e^{-\sigma x} x^{n-1}}{(n-1)!} \right], \end{aligned}$$

where

$$\mathcal{U}_1 = e^{-\sigma x} \sum_{i=0}^{k-1} \binom{k-1}{i} x^{k-1-i} (\sigma-\nu)^{k-n-i} \frac{(n-k+i-1)!}{(n-k-1)!},$$

$$\mathcal{U}_2 = e^{-\nu x} \sum_{i=0}^{n-k-1} \binom{n-k-1}{i} x^{n-k-1-i} (\nu - \sigma)^{-i-k} \frac{(k+i-1)!}{(k-1)!}.$$

Therefore, the expression for the tail of the distribution function $F^{*n}(x)$ is

$$\begin{aligned} \bar{F}^{*n}(x) &= \int_x^\infty \hat{p}(y) dy = \frac{1}{(a+b)^n} \left[\frac{(b\nu)^n}{(n-1)!} \int_x^\infty e^{-\nu y} y^{n-1} dy \right. \\ &+ (-1)^{n+1} \sum_{k=1}^{n-1} \binom{n}{k} (a\sigma)^k (b\nu)^{n-k} \left(\frac{(-1)^{k-1}}{(k-1)!} \int_x^\infty \mathcal{U}_1 dy + \frac{(-1)^{n-k-1}}{(n-k-1)!} \int_x^\infty \mathcal{U}_2 dy \right) \\ &\left. + \frac{(a\sigma)^n}{(n-1)!} \int_x^\infty e^{-\sigma x} x^{n-1} dy \right]. \end{aligned}$$

Note, that

$$\int_x^\infty e^{-\mu y} y^m dy = \frac{e^{-\mu x}}{\mu^{m+1}} \sum_{j=0}^m \frac{(x\mu)^j m!}{j!}.$$

Using this expression, we get

$$\begin{aligned} \bar{F}^{*n}(x) &= \frac{1}{(a+b)^n} \left[b^n e^{-\nu x} \sum_{j=0}^{n-1} \frac{(x\nu)^j}{j!} \right. \\ &+ \sum_{k=1}^{n-1} \binom{n}{k} (a\sigma)^k (b\nu)^{n-k} \left(\frac{(-1)^{n+k}}{(k-1)!} \mathcal{V}_1 + \frac{(-1)^k}{(n-k-1)!} \mathcal{V}_2 \right) \\ &\left. + a^n e^{-\sigma x} \sum_{j=0}^{n-1} \frac{(x\sigma)^j}{j!} \right], \end{aligned} \tag{13}$$

where the quantities \mathcal{V}_1 and \mathcal{V}_2 are defined by (7) and (8).

The desired relation (6) follows from (9) and (13).

3 Graphs

In this section, we present several plots of the discounted penalty function $\psi(x, \delta)$. We examine the dependence of the function $\psi(x, \delta)$ on the main parameters such as the initial capital x (graphs: I, II), security loading θ (graphs: III, IV), intensity

λ (graphs: v, vi), force of interest δ (graphs: vii, viii) and parameters of claims distributions α, σ, ν (graphs: ix, x, xi, xii).

In the cases when the initial capital x (graphs i, ii), the safety loading θ (graphs iii, iv), the force of interest δ (graphs vii, viii) and the parameters ν, σ (graphs xi, xii) vary, we note the decreasing behavior of the function $\psi(\cdot)$. Looking more closely at the parameter settings in these examples, we may examine the known function in detail. While the fixed values of x and θ remain large (graphs ii, iii, viii), we observe that the value of future bankruptcy is less than in the cases when these parameters are small (graphs i, iv, vii).

Further, from graphs v and vi we note that function $\psi(\cdot)$ increases, when the claim intensity λ grows. Moreover, comparing these two graphs, we see that the value of future bankruptcy is visibly smaller when x is bigger (graph vi). The same tendency may be observed in the graphs ix and x. As we see, the increase of the parameter α causes the increase of the function $\psi(\cdot)$.

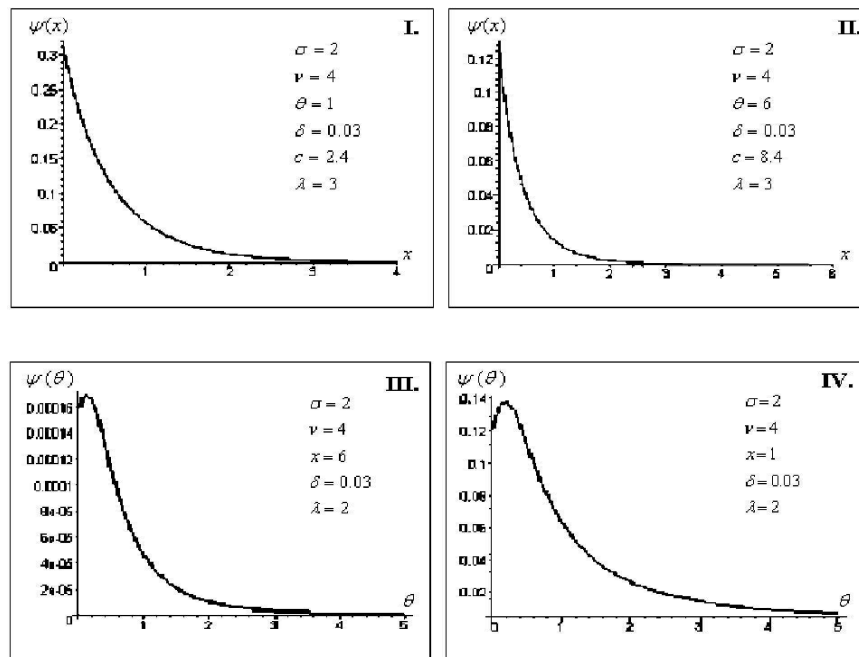


Fig. 3. $\psi(x, \delta)$ dependance on parameters $\delta, \sigma, \nu, x, \theta, \lambda$.

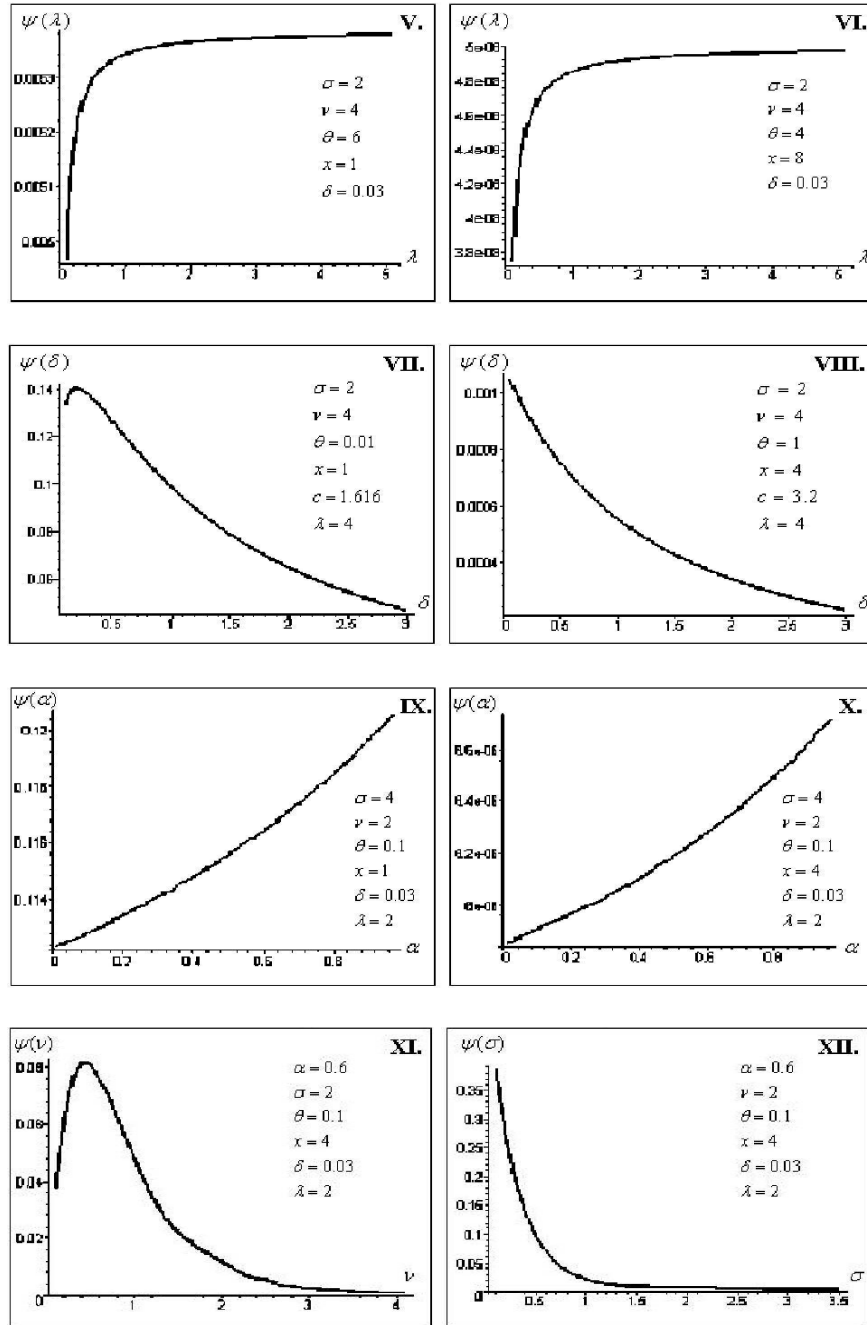


Fig. 4. $\psi(x, \delta)$ dependence on parameters $\delta, \alpha, \sigma, \nu, x, \theta, \lambda$.

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