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Viscous Dissipation Effects on MHD Natural Convection Flow over a Sphere in the Presence of Heat Generation

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Abstract. In this paper, the viscous dissipation effects on magnetohydrodynamic natural convection flow over a sphere in the presence of heat generation have been described. The governing boundary layer equations are first transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations are then solved numerically using finite-difference method together with Keller-box scheme. The numerical results of the surface shear stress in terms of skin friction coefficient and the rate of heat transfer in terms of local Nusselt number, velocity as well as temperature profiles are shown graphically and tabular form for a selection of parameters set consisting of heat generation parameter Q, magnetic parameter M, viscous dissipation parameter N and the Prandlt number Pr.

Keywords: viscous dissipation, magnetohydrodynamics, heat generation, natural convection.

Nomenclature

a	radius of the sphere	P	fluid pressure
C_p	specific heat at constant pressure	Q	heat generation parameter
C_{fX}	local skin friction coefficient	q_w	surface heat flux
f	dimensionless stream function	T	temperature of the fluid
g	acceleration due to gravity	T_w	temperature at the surface
Gr	local Grashof number	T_∞	temperature of the ambient fluid
M	magnetic parameter	U	velocity component in the X-direction
N	viscous dissipation parameter	V	velocity component in the Y-direction
Nu_X	local Nusselt number coefficient	X	measured from the leading edge
Pr	Prandtl number	Y	distance normal to the surface

Greek symbols

β	coefficient of thermal expansion	ξ	the dimensionless coordinate
β_0	magnetic field strength	η	the pseudo-similarity variable
ν	kinematic viscosity	ψ	stream functions
μ	viscosity of the fluid	σ_0	the electrical conduction
θ	dimensionless temperature	κ	thermal conductivity of the fluid
ρ	density of the fluid		

Subscripts

w	wall conditions	∞	ambient temperature

1 Introduction

The study of the flow of electrically conducting fluid in the presence of magnetic field is important from the technical point of view and such types of problems have received much attention by many researchers. The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to the surface. The surface is maintained at a uniform temperature T_w, which may either exceed the ambient temperature T_∞ or may be less then T_∞. When $T_w > T_\infty$, an upward flow is established along the surface due to free convection, where as for $T_w < T_\infty$, there is a down flow. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motion. The velocity is very small so that the magnetic force, which is proportional to the magnitude of the longitudinal velocity and acts in the opposite direction is also very small. Additionally, a magnetic field of strength β_0 acts normal to the surface. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from the free stream pressure gradient. The stress work effects in laminar flat plate natural convection flow have been studied by Ackroyd [1]. However, the influence and importance of viscous stress work effects in laminar flows have been examined by Gebhart [2] and Gebhart and Mollendorf [3]. In both of the investigations, special flows over semi-infinite flat surfaces parallel to the direction of body force were considered. Gebhart [2] considered flows generated by the plate surface temperatures, which vary as powers of ξ (the distance along the plate surface from the leading edge). Gebhart and Mollendorf [3] considered flows generated by plate surface temperatures, which vary exponentially in ξ. The effect of laminar free convection from a sphere with blowing and suction has been investigated by Huang and Chen [4]. The problem of magneto hydrodynamic free convection in a strong cross-field was studied by Kuiken [5]. Also the effect of magnetic field on the free convection heat transfer has been studied by Sparrow and Cess [6]. MHD free convection flow of visco-elastic fluid past an infinite porous plate was investigated by Chowdhury and Islam [7]. The problem of magnetohydrodynamic free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux have been investigated by Raptis and Kafousias [8]. Also the effect of free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field has been discussed by Elbashbeshy [9]. Hossain [10] introduced the viscous and Joule heating effects on MHD-free convection flow with variable plate temperature. The heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation have been investigated by Vajravelu and Hadjinolauo [11]. In this study they considered that the volumetric rate of heat generation, q^m [w $\cdot$ m^{-3}], should be $q^m = Q_0(T - T_\infty)$, for $T \geq T_\infty$ and equal to zero for $T < T_\infty$, (Q_0 is the heat generation/absorption constant). The above relation is valid as an approximation of the state of some exothermic process and having T_∞ when they used $q^m = Q_0(T - T_\infty)$. The conjugate effects of conduction and natural convection heat transfer along a thin vertical plate with non-uniform internal heat generation has been studied by Mendez et al. [12]. Also the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation or absorption was considered by Molla et al. [13]. Magnetohydrodynamic natural convection flows on a sphere in presence of heat generation has been investigated by Molla et al. [14]. The problems of free convection boundary layer flow over or on bodies of various shapes, where discussed by many mathematicians, versed engineers and researchers. The free convection boundary layer flow on an isothermal sphere and on an isothermal horizontal circular cylinder in a micropolar fluid were considered by Nazar et al. [15]. The effect of pressure stress work and viscous dissipation in some natural convection flows have been shown by Joshi and Gebhart [16]. To the best of our knowledge, viscous dissipation effects on magnetohydrodynamics free convection flow from an isothermal sphere in presence of heat generation has not yet been studied and the present work demonstrate the issue.

Natural convection boundary layer flow over a sphere of a viscous incompressible electrically conducting fluid in the presence of magnetic field and heat generation with the effects of viscous dissipation has been investigated. The governing boundary layer equations are reduced to locally non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are then solved numerically using implicit finite difference method together with the Keller box scheme. Here we have focused our attention on the evaluation of the surface shear stress in terms of skin friction coefficient, the rate of heat transfer in terms of local Nusselt number, velocity as well as temperature profiles for a selection of parameters set consisting of heat generation parameter Q, viscous dissipation parameter N, the magnetic parameter M and the Prandtl number Pr. Numerical results have been shown graphically as well as in tabular form.

2 Formulation of the problems

Natural convection boundary layer flow over a sphere of an electrically conducting and steady two-dimensional viscous incompressible fluid in the presence of strong magnetic field and heat generation is considered. It is assumed that the surface temperature of the sphere is T_w. Where $T_w > T_\infty$, here T_∞ being the ambient temperature of the fluid.

Under the usual Boussinesq and boundary layer approximation, the basic equations are

$$\frac{\partial}{\partial X}(rU) + \frac{\partial}{\partial X}(rV) = 0, \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \nu\frac{\partial^2 U}{\partial Y^2} + g\beta(T - T_\infty)\sin\frac{X}{a} - \frac{\sigma_0\beta_0^2}{\rho}U, \tag{2}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial Y^2} + \frac{\nu}{\rho C_p}\left(\frac{\partial U}{\partial Y}\right)^2 + \frac{Q_0}{\rho C_p}(T - T_\infty). \tag{3}$$

The boundary conditions for the equations (2) to (3) are

$$\begin{aligned} &U = V = 0, \quad T = T_w \quad \text{on} \quad Y = 0, \\ &U \to 0, \qquad\quad T \to T_\infty \quad \text{at} \quad Y \to \infty, \end{aligned} \tag{4}$$

where

$$r(X) = a\sin\frac{X}{a}, \tag{5}$$

$r = r(X),\ r$ is the radial distance from the symmetrical axis to the surface of the sphere, g is the acceleration due to gravity, β is the coefficient of thermal expansion, ν is the kinematics viscosity, T is the local temperature, C_p is the specific heat at constant pressure. The amount of heat generated or absorbed per unit volume is $Q_0(T - T_\infty),\ Q_0$ being a constant, which may take either positive or negative. The source term represents the heat generation when $Q_0 > 0$ and the heat absorption when $Q_0 < 0,\ \rho$ is the density, σ_0 is the electrical conduction and Pr is the Prandtl number.

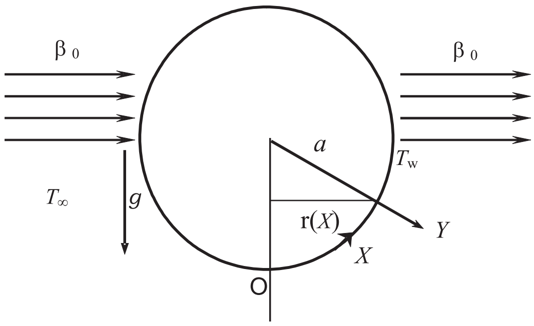


Fig. 1. Physical model and coordinate system.

To transform the above equations into non-dimensional, the following dimensionless variables are introduced:

$$\xi = \frac{X}{a}, \quad \eta = Gr^{1/4}\frac{Y}{a}, \quad u = \frac{a}{\nu}Gr^{-1/2}U, \quad v = \frac{a}{\nu}Gr^{-1/4}V, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{6}$$

where $Gr = g\beta(T_w - T_\infty)a^3/\nu^2$ is the Grashof number and θ is the non-dimensional temperature, then equation (5) becomes

$$r = a\sin\xi. \tag{7}$$

Using the above values, the equations (1) to (3) take the following form:

$$\frac{\partial}{\partial\xi}(ru) + \frac{\partial}{\partial\eta}(rv) = 0, \tag{8}$$

$$u\frac{\partial u}{\partial\xi} + v\frac{\partial u}{\partial\eta} = \frac{\partial^2 u}{\partial\eta^2} + \theta\sin\xi - Mu, \tag{9}$$

$$u\frac{\partial\theta}{\partial\xi} + v\frac{\partial\theta}{\partial\eta} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2} + N\left(\frac{\partial u}{\partial\eta}\right)^2 + Q\theta, \tag{10}$$

where, $M = \sigma_0\beta^2 a^2/\rho\nu Gr^{1/2}$ is the magnetic parameter and $Q = Q_0 a^2/\nu\rho C_p Gr^{1/2}$ is the heat generation parameter, $N = Gr/a^2 C_p(T_w - Y_\infty)$, is the viscous dissipation parameter. The boundary conditions (4) take the form

$$\begin{aligned} &u = v = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0,\\ &u \to 0, \qquad \theta \to 0 \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{11}$$

To solve equations (9) and (10) subject to the boundary conditions (11), we assume the following variables u and v is given by

$$u = \frac{1}{r}\frac{\partial\psi}{\partial\eta} \quad \text{and} \quad v = -\frac{1}{r}\frac{\partial\psi}{\partial\xi}, \tag{12}$$

where $\psi(\xi,\eta) = \xi r(\xi) f(\xi,\eta)$, $\psi(\xi,\eta)$ is a non-dimensional stream function,

$$\begin{aligned} &\frac{\partial^2 u}{\partial\eta^2} = \xi\frac{\partial^3 f}{\partial\eta^3}, \quad \frac{\partial u}{\partial\xi} = \frac{\partial f}{\partial\eta} + \xi\frac{\partial^2 f}{\partial\xi\partial\eta},\\ &v = -\left[\left(1 + \xi\frac{\cos\xi}{\sin\xi}\right)f(\xi,\eta) + \xi\frac{\partial f}{\partial\xi}\right]. \end{aligned} \tag{13}$$

Using the above transformed values in equations (9) and (10) and simplifying, we have the following:

$$\begin{aligned} &\frac{\partial^3 f}{\partial\eta^3} + \left(1 + \frac{\xi}{\sin\xi}\cos\xi\right)f\frac{\partial^2 f}{\partial\eta^2} - \left(\frac{\partial f}{\partial\eta}\right)^2 + \frac{\theta}{\xi}\sin\xi - M\frac{\partial f}{\partial\eta}\\ &\qquad = \xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta\partial\xi} - \frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right), \end{aligned} \tag{14}$$

$$\begin{aligned} &\frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2} + \left(1 + \frac{\xi}{\sin\xi}\cos\xi\right)f\frac{\partial\theta}{\partial\eta} + Q\theta + N\xi^2\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2\\ &\qquad = \xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi} - \frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right). \end{aligned} \tag{15}$$

The corresponding boundary conditions are

$$\begin{aligned} f = \frac{\partial f}{\partial \eta} = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0, \\ \frac{\partial f}{\partial \eta} \to 0, \qquad \theta \to 0 \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{16}$$

For the lower stagnation point of the sphere i.e. $\xi \approx 0$, equation (14) and (15) reduce to the following ordinary differential equations:

$$\frac{d^3 f}{d\eta^3} + 2f\frac{d^2 f}{d\eta^2} - \left(\frac{df}{d\eta}\right)^2 + \theta - M\frac{df}{d\eta} = 0, \tag{17}$$

$$\frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} + 2f\frac{\partial \theta}{\partial \eta} + Q\theta = 0 \tag{18}$$

with the boundary conditions

$$\begin{aligned} f = \frac{\partial f}{\partial \eta} = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0, \\ \frac{\partial f}{\partial \eta} \to 0, \qquad \theta \to 0 \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{19}$$

In practical application, the physical quantities of principal interest are skin-friction coefficient and the rate of heat transfer which can be written in non-dimensional form as

$$C_{fX} = \frac{Gr^{-3/4}a^2}{\mu\nu}\tau_w \quad \text{and} \quad Nu_X = \frac{aGr^{-1/4}}{\kappa(T_w - T_\infty)}, \tag{20}$$

where $\tau_w = \mu(\frac{\partial U}{\partial Y})_{Y=0}$ and $q_w = -\kappa(\frac{\partial T}{\partial Y})_{Y=0}$, κ being the thermal conductivity of the fluid. Using the new variables (6), we have

$$C_{fX} = \xi\left(\frac{\partial^2 f}{\partial \eta^2}\right)_{\eta=0}, \tag{21}$$

$$Nu_X = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0}. \tag{22}$$

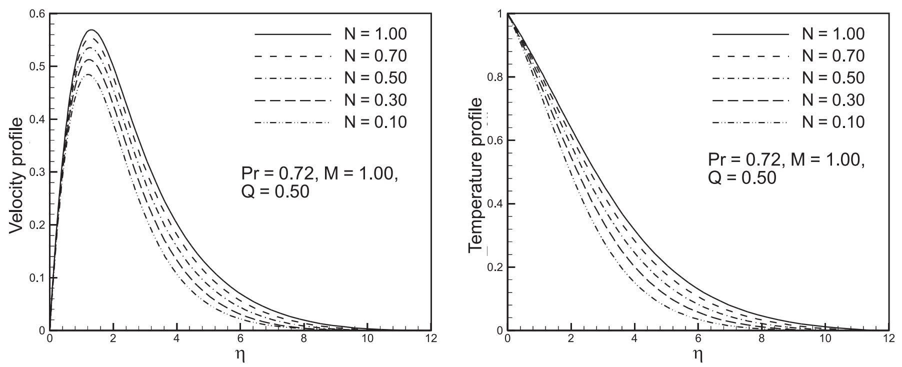
3 Method of solution

Solutions of the local non-similar partial differential equations (14) to (15) subjected to the boundary conditions (16) are obtained by using the implicit finite difference method, which has been described in details by Cebeci and Bradshaw [17] and used by Hossain et al. [18].

4 Results and discussion

The effects of viscous dissipation on magnetohydrodynamic natural convection flow over a sphere in the presence of heat generation have been investigated. The results are obtained in terms of the local skin-friction and the local rate of heat transfer, for different values of the aforementioned physical parameters and these are shown in tabular form in Table 1, Table 2 and graphically in Figs. 6, 7. The velocity and temperature distributions obtained by the finite difference method for various values of the governing parameters, are displayed in Figs. 2–5. The aim of these figures are to display how the profiles vary in ξ, the scaled stream wise coordinate.

From Fig. 2(a), it is observed that velocity increases as the values of viscous dissipation parameter N increase. Near the surface of the sphere velocity increases significantly along η and becomes maximum and then decreases slowly and finally approaches to zero, the asymptotic value. The maximum values of the velocity are $0.48550,\ 0.51328,$ $0.53567,\ 0.55394$ and 0.56863 for $N = 0.10,\ 0.30,\ 0.50, 0.70$ and 1.00 respectively which occur at $\eta = 1.23788$ for first, second and third maximum values, at $\eta = 1.30254$ for fourth and fifth maximum values. Here it is observed that the velocity increase by $17.12255\,\%$ as N increases from 0.10 to 1.00. From Fig. 2(b), it is seen that when the values of viscous dissipation parameter N increase, the temperature also increases.



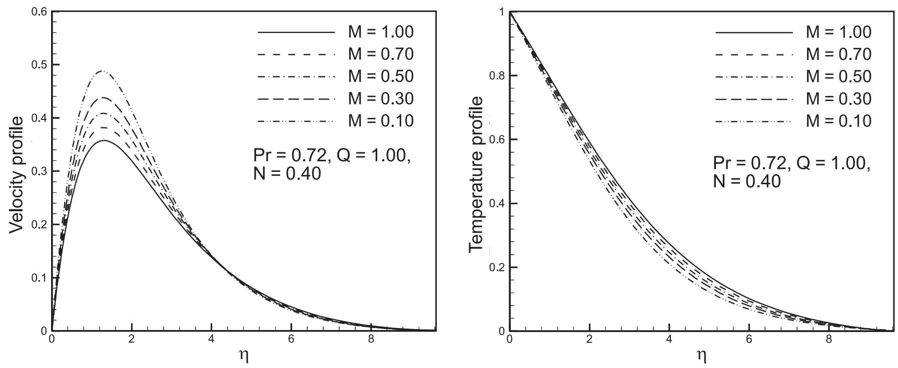
(a) (b)

Fig. 2. (a) Velocity and (b) temperature profiles for different values of viscous dissipation parameter N with others fixed parameters.

Figs. 3(a) and 3(b) display results for the velocity and temperature profiles, based on equations (14) and (15) with the boundary conditions (16), for different values of magnetic parameter M ($M = 0.10,\ 0.30,\ 0.50,\ 0.70,\ 1.00$) plotted against η at $\xi = \pi/6$ having Prandtl number $Pr = 0.72$, $Q = 1.0$ and $N = 0.4$. It is observed that, as the magnetic parameter M increases, the velocity profile decreases between $0 \leq \eta \leq 4.1$ and then increases with very small difference and finally approaches to zero along η direction. The temperature profile increases with increasing magnetic parameter M. The maximum values of the velocity are recorded as $0.48763,\ 0.43810,\ 0.40870,\ 0.38191$ and 0.35758

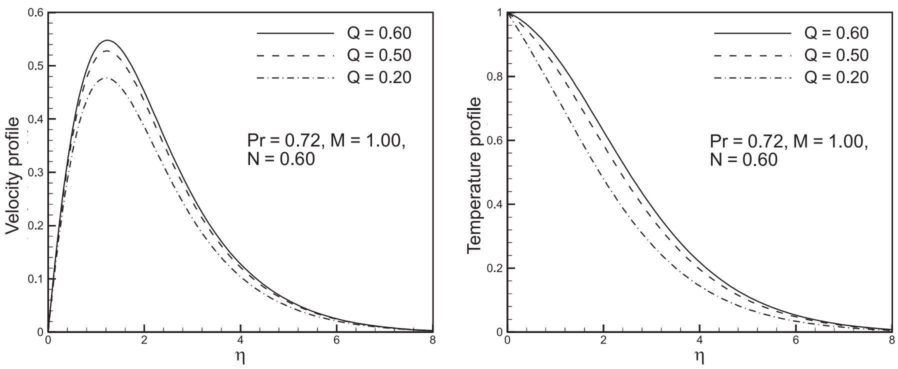
for $M = 0.10,\ 0.30,\ 0.50,\ 0.70$ and 1.00, respectively which occur at $\eta = 1.30254$ for 1st, 2nd, 3rd and 4th maximum values, at $\eta = 1.23788$ for 5th maximum value. It is found that the velocity decreases by 26.67% as the magnetic parameter M increases from 0.1 to 1.0.

From Fig. 4(a), velocity distribution increases as the values of heat generation parameter Q increase. The maximum values of the velocity are $0.47757,\ 0.52865$ and 0.55807 for $Q = 0.20,\ 0.50$ and 0.60 respectively which occur at $\eta = 1.17520$ for first maximum value, at $\eta = 1.123788$ for second and third maximum values. Here it is observed that the velocity increase by $16.85616\,\%$ as Q increases from 0.10 to 0.60. From Fig.4(b), it is seen that when the values of heat generation parameter Q increase, the temperature distributions also increase.



(a) (b)

Fig. 3. (a) Velocity and (b) temperature profiles for different values of magnetic parameter M with others fixed parameters.



(a) (b)

Fig. 4. (a) Velocity and (b) temperature profiles for different values of heat generation parameter Q with others fixed parameters.

Figs. 5(a) and 5(b) indicate the effects of the Prandtl number Pr with $M = 1.00$, $Q = 0.50$ and $N = 0.40$ on the velocity profiles and the temperature profiles. From Fig. 5(a) it is observed that the increasing values of Prandtl number Pr leads to the decrease in the velocity profiles. The maximum values of the velocity are $0.49156,\ 0.46167$, 0.43584 and 0.39056 for $Pr = 0.50,\ 0.72,\ 1.00$ and 1.74 respectively which occur at $\eta = 1.36929$ for first maximum value and $\eta = 1.30254$ for second, third maximum values and $\eta = 1.17520$ for last maximum value. Here it is depicted that the velocity decreases by $20.547\,\%$ as Pr increases from 0.50 to 1.74. Again from Fig. 5(b) it is observed that the temperature profiles decreases with the increasing values of Prandtl number Pr.

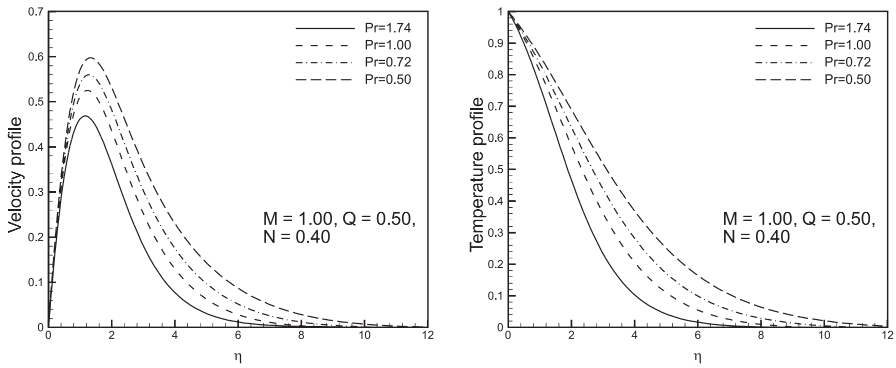
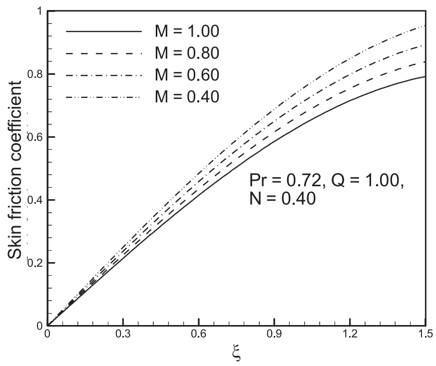


Fig. 5. (a) Velocity and (b) temperature profiles for different values of Prandtl number Pr with others fixed parameters.

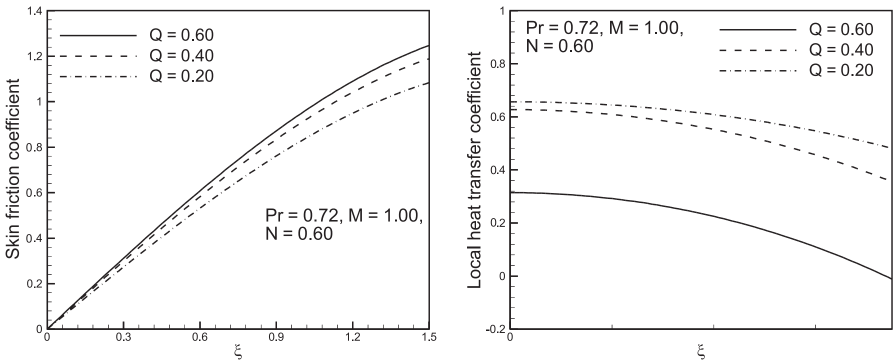
It can easily be seen that the effect of the magnetic parameter M leads to a decrease in the local skin friction coefficient C_{fX} and the local Nusselt number Nu_X in Fig. 6(a) and 6(b). This phenomenon can easily be understood from the fact that the magnetic parameter M increases the Lorentz force, which opposes the flow, therefore decreases the velocity gradient and hence the local skin friction coefficient C_{fX} decreases. Owing to increasing values of M in the presence of heat generation, the fluid temperature with in the boundary layer increases and the associate thermal boundary layer becomes thicker. For increasing fluid temperature, the temperature difference between fluid and surface decreases and the corresponding rate of heat transfer decreases. Also it is observed that $x = 0.50615$, the skin friction coefficient C_{fX} and the local Nusselt number Nu_X decrease by $14.2335\,\%$ and $9.1810\,\%$, respectively, as M increases from 0.40 to 1.00.

The variation of the reduced local skin friction coefficient and the local rate of heat transfer for different values of the heat generation parameter Q ($Q = 0.20,\ 0.40,\ 0.60$) are illustrated in Figs. 7(a) and 7(b) while $M = 1.00$, $N = 0.60$ and Prandtl number $Pr = 0.72$. From the figures it can be seen that the increase of the heat generation parameter Q leads to an increase in the local skin-friction coefficient C_{fX} and a decrease in the local Nusselt number Nu_X. These are expected, since the heat generation mechanism creates a layer of hot fluid near the surface, and finally the resultant temperature of the fluid exceeds the surface temperature. For this reason the rate of heat transfer from the surface decreases. Owing to the enhanced temperature, the viscosity of the fluid increases and the corresponding local skin-friction coefficient increases. Moreover, it is seen that at $\xi = 0.50615$ the skin friction coefficient C_{fX} increases by $14.2927\,\%$ and the local Nusselt number Nu_X decreases by $53.2775\,\%$ respectively, as Q increases from 0.20 to 0.60.



(a) (b)

Fig. 6. (a) Skin friction coefficient and (b) local heat transfer coefficient for different values of magnetic parameter M with others fixed parameters.



(a) (b)

Fig. 7. (a) Skin friction coefficient and (b) local heat transfer coefficient for different values of heat generation parameter Q with others fixed parameters.

In Table 1 are given the tabular values of the local skin friction coefficient C_{fX} and local Nusselt number Nu_X for different values of viscous dissipation parameter N while $Pr = 0.72$, $M = 1.00$ and $Q = 0.50$. Here we found that the values of local skin friction coefficient C_{fX} increase at different position of ξ for viscous dissipation parameter $N = 0.10,\ 0.50,\ 0.70,\ 1.00$. The the local skin friction coefficient C_{fX} is increase by $8.5699\,\%$ as the viscous dissipation parameter N changes from 0.10 to 1.00

and $\xi = 1.04720 = \pi/3$. Furthermore, it is seen that the numerical values of the local Nusselt number Nu_X increase for increasing values of viscous dissipation parameter N. The rate of local Nusselt number Nu_X is increased by $69.08\,\%$ at position $\xi = 1.0472 = \pi/3$ as the viscous dissipation parameter N changes from 0.10 to 1.00.

Numerical values of local heat transfer rate, Nu_X are calculated from equation (22) for the surface of the sphere from lower stagnation point to upper stagnation point. In order to verify the accuracy of the present work, the values of local Nusselt number Nu_X for $N = M = Q = 0.0$ having Prandtl number $Pr = 0.7,\ 7.0$ at different position of ξ (in degree) are compared with those reported by Nazar et al. [15] and Molla et al. [14] as presented in Table 2. The results are found to be in excellent agreement.

Table 1. Skin friction coefficient and rate of heat transfer against ξ for different values of viscous dissipation parameter N with other controlling parameters $Pr = 0.72$, $Q = 0.50$ and $M = 1.00$

	$N = 0.10$		$N = 0.50$		$N = 0.70$		$N = 1.00$	
ξ	C_{fX}	Nu_X	C_{fX}	Nu_X	C_{fX}	Nu_X	C_{fX}	Nu_X
0	0.00000	0.84401	0.00000	1.12528	0.00000	1.24688	0.00000	1.35948
$\pi/18$	0.16143	0.62483	0.17022	0.85247	0.17328	0.95091	0.17583	1.04193
$\pi/9$	0.31993	0.61026	0.33730	0.83433	0.34335	0.93127	0.34839	1.02090
$\pi/6$	0.47266	0.59539	0.49819	0.81517	0.50708	0.91027	0.51449	0.99819
$2\pi/9$	0.61686	0.57701	0.64992	0.79122	0.66143	0.88390	0.67101	0.96959
$5\pi/18$	0.74982	0.55426	0.78959	0.76142	0.80343	0.85106	0.81496	0.93391
$\pi/3$	0.86897	0.52667	0.91444	0.72523	0.93026	0.81114	0.94344	0.89051
$7\pi/18$	0.97186	0.49388	1.02182	0.68218	1.03921	0.76364	1.05369	0.83887
$4\pi/9$	1.05612	0.45551	1.10922	0.63180	1.12769	0.70806	1.14308	0.77844
$\pi/2$	1.11947	0.41114	1.17419	0.57358	1.19322	0.64382	1.20908	0.70860

Table 2. Comparisons of the present numerical results of Nu_X for the values of Prandtl number $Pr = 0.7$ and 7.0 without the effects of viscous dissipation, heat generation and magnetic field with those of obtained by Nazar et al. [15] and Molla et al. [14]

	$Pr = 0.7$			$Pr = 7.0$		
ξ in degree	Nazar et al. [15]	Molla et al. [14]	Present	Naza et al. [15]	Molla et al. [14]	Present
0	0.4576	0.4576	0.4529	0.9595	0.9582	0.9437
10	0.4565	0.4564	0.4516	0.9572	0.9558	0.9416
20	0.4533	0.4532	0.4485	0.9506	0.9492	0.9354
30	0.4480	0.4479	0.4444	0.9397	0.9383	0.9248
40	0.4405	0.4404	0.4367	0.9239	0.9231	0.9100
50	0.4308	0.4307	0.4282	0.9045	0.9034	0.8909
60	0.4189	0.4188	0.4134	0.8801	0.8791	0.8673
70	0.4046	0.4045	0.4012	0.8510	0.8501	0.8390
80	0.3879	0.3877	0.3836	0.8168	0.8161	0.8059
90	0.3684	0.3683	0.3641	0.7774	0.7768	0.7675

5 Conclusions

The effects of viscous dissipation on natural convection flow over a sphere in the presence of magnetic field and heat generation with electrically conducting fluid have been investigated theoretically. The governing boundary layer equations of motion are transformed into a non-dimensional form and the resulting non-linear systems of partial differential equations are reduced to local non-similarity boundary layer equations, which are solved numerically by using implicit finite difference method together with the Keller-box scheme. From the present investigation the following conclusions may be drawn:

- With effect of magnetic parameter M in presence of heat generation, the local skin-friction coefficient C_{fX} and the local rate of heat transfer Nu_X decrease.
- An increase in values of M leads to decrease the velocity distribution but slightly increase the temperature distribution.
- For increasing values of heat generation parameter Q, the skin-friction coefficient increases but the Nusselt number decreases significantly within the boundary layer.
- With the effect of heat generation both the velocity and temperature distributions increase significantly the thickness of the thermal boundary layer.
- As viscous dissipation parameter N increases, both the velocity and the temperature distributions increase significantly.
- An increasing value of Prandtl number Pr leads to decrease in the velocity and the temperature distributions.

References

1. J. A. D. Ackroyd, Stress work effects in laminar flat-plate natural convection, *J. Fluid Mech.*, **62**(4), pp. 677–695, 1974.
2. B. Gebhart, Effects of viscous dissipation in natural convection, *J. Fluid Mech.*, **14**(2), pp. 225–232, 1962.
3. B. Gebhart, J. Mollendorf, Viscous dissipation in external natural convection flows, *J. Fluid Mech.*, **38**(1), pp. 97–107, 1969.
4. M. J. Huang, C. K. Chen, Laminar free convection from a sphere with blowing and suction, *J. Heat Transfer*, **109**, pp. 529–532, 1987.
5. H. K. Kuiken, Magnetohydrodynamic free convection in a strong cross field, *Journal of Fluid Mechanics*, **4**(1), pp. 21–38, 1970.
6. E. M. Sparrow, R. D. Cess, The effect of a magnetic field on free convection heat transfer, *Int. J. Heat Mass Transfer*, **3**(2), pp. 267–274, 1961.
7. M. K. Chowdhury, M. N. Islam, MHD free convection flow of visco- elastic fluid past an infinite porous plate, *Heat and Mass Transfer*, **36**(5), pp. 439–447, 2000.

8. A. Raptis, N. G. Kafousias, Magnetohydrodynamic free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux, *Canadian Journal of Physics*, **60**(12), pp. 1725–1729, 1982.

9. E. M. A. Elbashbeshy, Free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field, *Int. J. Engineering Science*, **38**(2), pp. 207–213, 2000.

10. M. A. Hossain, Viscous and Joule heating effects on MHD-free convection flow with variable plate temperature, *Int. J. Heat Mass Transfer*, **35**(12), pp. 3485-3487, 1992.

11. K. Vajravelu, A. Hadjinicolaou, Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation, *Int. Comm. Heat Mass Transfer*, **20**, pp. 417–430, 1993.

12. F. Mendez, C. Trevino, The conjugate conduction natural convection heat transfer along a thin vertical plate with non-uniform internal heat generation, *Int. J. Heat and Mass Transfer*, **43**(15), pp. 2739–2748, 2000.

13. M. M. Molla, M. A. Hossain, L. S. Yao, Natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation / absorption, *Int. J. Thermal Science*, **43**(2), pp. 157–163, 2004.

14. Md. M. Molla, M. A. Taher, Md. M. K. Chowdhury, Md. A. Hossain, Magnetohydrodynamic natural convection flow on a sphere in presence of heat generation, *Nonlinear Analysis: Modelling and Control*, **10**(4), pp. 349–363, 2005.

15. R. Nazar, N. Amin, T. Grosan, I. Pop, Free convection boundary layer on an isothermal sphere in a micropolar fluid, *Int. Comm. Heat Mass Transfer*, **29**(3), pp. 377–386, 2002.

16. Y. Joshi, B. Gebhart, Effect of pressure stress work and viscous dissipation in some natural convection flows, *Int. J. Heat Mass Transfer*, **24**(10), pp. 1377–1388, 1981.

17. T. Cebeci, P. Bradshaw, *Physical and Computational Aspects of Convective Heat Transfer*, Springer, N.Y., 1984.

18. M. A. Hossain, M. A. Alim, D. A. S. Rees, The effect of radiation on free convection from a porous vertical plate, *J. Heat and Mass Transfer*, **42**(1), pp. 181–191,1999.