Effect of radiation with temperature dependent viscosity and thermal conductivity on unsteady a stretching sheet through porous media

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Abstract. A numerical model is developed to study the effect of thermal radiation on unsteady boundary layer flow with temperature dependent viscosity and thermal conductivity due to a stretching sheet in porous media. The Rosseland diffusion approximation is used to describe the radiative heat flux in the energy equation. The governing equations reduced to similarity boundary layer equations using suitable transformations and then solved using the Runge–Kutta numerical integration, procedure in conjunction with shooting technique. A parametric study illustrating the influence of the radiation R, variable viscosity ε, Darcy number Da, porous media inertia coefficient γ, thermal conductivity κ and unsteady A parameters on skin friction and Nusselt number.

Keywords: radiation, unsteady flow, porous media, stretching sheet, temperature dependent viscosity and thermal conductivity.

Nomenclature

A	dimensionless parameter measure of the unsteadiness
B	dimensionless parameter
C	inertia coefficient
C_{fx}	local skin friction
C_P	specific heat
Da	Darcy number
f	dimensionless velocity
h	local heat transfer coefficient
K	permeability of the porous media
χ	mean absorption coefficient
k	thermal conductivity of fluid
k_∞	dynamic viscosity of ambient fluid
Nu_x	local Nusselt number ($Nu_x = hx/k$)
Pr	Prandtl number
q_w	wall heat flux
R	radiation parameter
Re_x	local Reynolds number
T	temperature
t	time
T_∞	ambient temperature
T_w	wall temperature
u	velocity component in x-direction
x	horizontal co-ordinate
y	vertical co-ordinate

Greek symbols

β	thermal expansion coefficient	ε	viscosity variation parameter
γ	inertia resistance parameter	κ	thermal conductivity variation
η	dimensionless co-ordinate	θ	dimensionless temperature
δ	positive constant	ψ	stream function
μ	absolute viscosity	ρ	density of the fluid
μ_∞	dynamic viscosity of the ambient fluid	σ	Stefan–Boltzman constant
		σ_s	scattering coefficient
ν	kinematic viscosity	σ_0	electrical conductivity

Subscripts

w	at the wall	∞	condition far away from the surface

Superscripts

$'$ differentiation with respect to η

1 Introduction

The heat transfer from a stretching surface is of interest in many practical applications. Such situations arise in the manufacturing process of plastic and rubber sheets where it often necessary to blow a gaseous medium through the unsolidified material. Extrusion processes, glass blowing, continuous coating and spinning of fibers also involve the flow due to a stretching surface.

The momentum boundary layer for linear stretching of sheet was first studied by Crane [1]. The temperature flied in the flow over stretching surface subject to a uniform heat flux was studied by Grubka and Bobba [2], while Elbashbeshy [3] considered the case of stretching surface with a variable surface heat flux. Elbashebeshy and Bazid [4] have presented similarity solutions of the boundary layer equations, which describe the unsteady flow and heat transfer over a stretching sheet. Sharidan et al. [5] investigated the unsteady flow and heat transfer over a stretching sheet in viscous and incompressible fluid.

Pop and Tsung [6] studied unsteady flow past a stretching sheet. Nazar et al. [7] have studied unsteady boundary layer flow in the region of the stagnation point on a stretching sheet. Yürüsoy [8] investigated unsteady boundary layer flow of power-law fluid on stretching sheet surface. Chen [9] have studied effect of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet. Dandapat et al. [10] investigated the effects of variable fluid properties and thermocapillarity on the flow of a thin film on an unsteady stretching sheet. Chiam [11] considered the effect of a variable thermal conductivity on the flow and heat transfer from a linearly stretching sheet.

Salem [12] investigated variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet. Seddeek et al. [13]

studied the effects of variable viscosity and thermal conductivity on an unsteady two-dimensional laminar flow of a viscous incompressible conducting fluid past a semi-infinite vertical porous moving plate taking into account the effect of a magnetic field in the presence of variable suction. Odda and Farhan [14] have considered the effects of variable viscosity and variable thermal conductivity on heat transfer from a stretching sheet. The fluid viscosity and the thermal conductivity are assumed to vary as inverse linear functions of temperature. Hossain et al. [15] have studied natural convection with variable viscosity and thermal conductivity from a vertical wavy cone. Hossain et al. [16] considered a steady two-dimensional laminar forced flow and heat transfer of a viscous incompressible fluid having temperature dependent viscosity and thermal conductivity past a wedge with a uniform surface heat flux. In formulating the equations governing the flow both the viscosity and the thermal conductivity of the fluid are considered to be linear function of temperature see [16].

Due to its numerous applications in a wide variety of industrial processes as well as in many natural circumstances, the subject of convective flow in porous media has attracted considerable attention in the last few decades. Examples of such technological applications are geothermal extraction, storage of nuclear waste material, ground water flows, thermal insulation engineering, food processing, fibrous insulation, soil pollution and packed-bed reactors to name just a few.

Chamkha [17] has studied unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching sheet immersed in a porous medium. Hassanien et al. [18] investigated variable viscosity and thermal conductivity effects on combined heat and mass transfer in mixed convection over a UHF/UMF wedge in porous media in the entire regime. Umavathi et al. [19] studied the effect of thermal radiation on mixed convection flow of two immiscible fluids in a vertical porous stratum. El-Amin et al. [20] considered the effect of thermal radiation on Forchheimer natural convection over vertical flat plate in a fluid saturated porous medium. El-Kabeir et al. [21] have investigated the effect of thermal radiation and a transverse magnetic field with surface mass transfer in free convection on a vertical stretching surface with suction and blowing. The effect of thermal radiation on free convection flow with variable viscosity and uniform suction velocity along a uniformly heated vertical porous plate embedded in a porous medium in the presence of a uniform transverse magnetic field is analyzed by Modather and El-Kabeir [22].

In the present work I propose to extend the work of Sharidan et al. [5] and study the effect of thermal radiation on unsteady boundary layer flow with temperature dependent viscosity and thermal conductivity due to a stretching sheet through porous media. The governing equations reduced to similarity boundary layer equations using suitable transformations and then solved using the Runge–Kutta numerical integration, procedure in conjunction with shooting technique. Numerical result are presented in terms of local skin friction coefficient and rate of heat transfer for various values of the variable viscosity ε, thermal conductivity κ, radiation R and unsteady A parameters against Darcy number Da, porous media inertia coefficient γ. The effect of variation in ε, κ, A, Da, R and γ on the dimensionless velocity, viscosity, thermal conductivity and temperature distribution are also depicted graphically.

2 Mathematical formulation

We consider the unsteady two-dimensional laminar flow of a viscous incompressible micropolar fluid with temperature dependent viscosity and thermal conductivity past a semi-infinite stretching sheet in the region $y > 0$, as shown in Fig. 1.

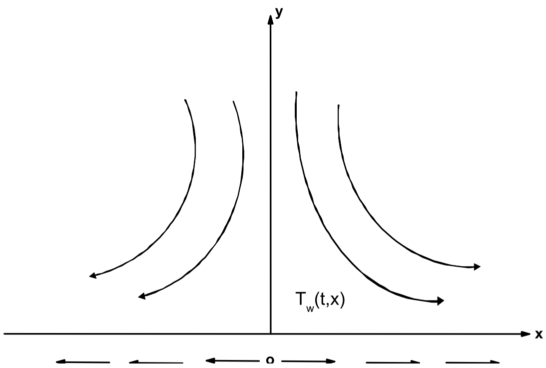

Fig. 1. Physical model and coordinate system.

Keeping the origin fixed, two equal and opposite forces are suddenly applied along the x-axis, which results in stretching of the sheet and hence, flow is generated. At the same time, the wall temperature $T_w(x,t)$ of the sheet is suddenly raised from T_∞ to $T_w(t,x)$ ($> T_\infty$). Under these assumptions, the basic unsteady boundary layer equations governing the flow and heat transfer due to the stretching sheet are given by

mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \tag{1}$$

momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\mu}{K}u - Cu^2; \tag{2}$$

energy:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y} - q^r\right). \tag{3}$$

In the above equations t is the time u and v are the components of fluid velocity in the x and y directions respectively, ρ – the density of ambient fluid, T is the fluid temperature in the boundary layer region. μ and k are respectively the dynamic viscosity and the thermal conductivity, following are given as below:

$$\mu = \mu_\infty\left[1 + \alpha_1\frac{T - T_\infty}{T_0 - T_\infty}\right] \tag{4a}$$

and

$$k = k_\infty \left[1 + \alpha_2 \frac{T - T_\infty}{T_0 - T_\infty} \right]. \tag{4b}$$

In (4) μ_∞ is the viscosity and k_∞ is the thermal conductivity of the ambient fluid, T_∞ is the temperature of the ambient fluid, T_0 is some reference temperature and α_1, α_2 are constant. Clearly $\alpha_1 = 0$ and $\alpha_2 = 0$ represent that the dynamical viscosity and the thermal conductivity be uniform.

Solutions of the above equations have to satisfy the following boundary conditions:

$$\begin{aligned} &t < 0: \quad u = v = 0, \quad T = T_\infty \quad \text{for any } x, y, \\ &t \geq 0: \quad u = u_w(t,x), \quad v = 0, \quad T = T_w(t,x) \quad \text{(VWT)}, \\ &\qquad\quad u \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty. \end{aligned} \tag{5a}$$

We assume now that the velocity of the sheet $u_w(t,x)$ and the sheet temperature $T_w(t,x)$ have the following form:

$$u_w(t,x) = cx(1 - \delta t)^{-1}, \quad T_w(t,x) = T_\infty + \frac{c}{2\nu x^2}(1 - \delta t)^{-3/2}. \tag{5b}$$

Where c is the stretching rate being a positive constant, δ is a positive constant, which measures the unsteadiness.

The quantity q^r on the right-hand side of equation (3) represents the radiative heat flux in the y direction. In order to reduce the complexity of the problem and to provide a means of comparison with future studies, a more detailed representation for radiative heat flux, for optically thick radiation limit, is considered in the present analysis. Thus the radiative heat flux term in the energy equation is simplified by utilizing the Roesseland diffusion approximation (Sparrow and Cess [23]) for an optically thick boundary layer as follows:

$$q^r = -\frac{4\sigma}{3\chi(a_0 + \sigma_s)} \frac{\partial T^4}{\partial y}. \tag{5c}$$

Where σ is the Stefan–Boltzman constant, χ is the mean absorption coefficient, a_0 is the Rosseland mean absorption coefficient and σ_s is the scattering coefficient. This approximation is valid at point optically far from the bounding surface, and is good only for intensive absorption, that is for an optically thick boundary layer. If temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, then the Taylor series for T^4 about T_∞ , after neglecting higher order terms, is given by:

$$T^4 \approx 4TT_\infty^3 - 3T_\infty^4.$$

We introduce now the following new variables

$$\begin{aligned} &\psi = \sqrt{\frac{c\nu_\infty}{1 - \delta t}}\, x f(\eta), \quad \eta = \sqrt{\frac{c}{\nu_\infty(1 - \delta t)}}\, y, \\ &T = T_\infty + \frac{c}{2\nu_\infty x^2}(1 - \delta t)^{-3/2}\theta(\eta). \end{aligned} \tag{6}$$

Where ψ is the stream function satisfying the continuity equation, $f(\eta)$ is the dimensionless stream function, η is pseudo-similarity variable, $\theta(\eta)$ is the dimensionless temperature of the fluid in the boundary layer region, $\nu_\infty = \mu_\infty/\rho$ is the free stream kinematic viscosity.

Substituting the transformation given in (6) into (2), (3) one obtains the following non-similar system of equations governing the flow:

$$[1+\varepsilon\theta]f''' + \left(f+\varepsilon\theta'\right)f'' - (1+\gamma)f'^2 - A\left(f' + \frac{1}{2}\eta f''\right) - \frac{1+\varepsilon\theta}{Da}f' = 0, \quad (7)$$

$$\frac{1}{Pr}\left\{\left(1+\frac{4}{3}R\right)(1+\kappa\theta)\theta'' + \kappa\theta'^2\right\} + f\theta' + 2\theta f' - \frac{A}{2}\left(3\theta + \eta\theta'\right) = 0. \quad (8)$$

The corresponding boundary conditions transform to:

$$\begin{aligned} &f(0)=0, \quad f'(0)=1, \quad \theta(0)=1, \\ &f'(\infty)=0, \quad \theta(\infty)=0. \end{aligned} \quad (9)$$

Where

$$A = \frac{\delta}{c}, \quad Pr = \frac{\mu_\infty c_p}{k}, \quad \varepsilon = \alpha_1\frac{T-T_\infty}{T_0-T_\infty}, \quad \kappa = \alpha_2\frac{T-T_\infty}{T_0-T_\infty}. \quad (10)$$

Where ε is the viscosity variation, κ is the thermal conductivity parameters, A is a non-dimensional constant which measures the flow and heat transfer unsteadiness and Pr the Prandtl number.

The quantities of physical interested, namely, the local skin friction C_f and the rate of heat transfer in terms of local Nusselt number Nu_x are prescribed by:

$$C_f = \frac{\tau_w}{(1/2)\rho u_\infty^2}, \quad (11)$$

$$Nu_x = \frac{q_w x}{k_\infty(T_w - T_\infty)}. \quad (12)$$

Where τ_w is the skin friction and q_w is the heat transfer from the sheet are given by:

$$\tau_w = \mu_\infty\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu_\infty f''(0), \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},$$

$$C_f Re_x^{1/2} = f''(0), \quad Nu_x/Re_x^{1/2} = -\theta'(0),$$

where $Re_x = u_w x/\nu_\infty$ is the local Reynolds number.

3 Results and discussion

Equations (7), (8) with the boundary conditions (9) are solved numerically, employing the sixth order implicit Runge–Kutta–Butcher initial value problem solver along with

Nachtsheim–Swigert iteration technique. Here, solutions are obtained, up to the third level of truncation.

Calculations were carried out for the value of Prandtl number 1.0, the viscosity variation parameter ε ranged from 0 to 1.0, the thermal conductivity variation parameter κ ranged from 0.0 to 1.0, unsteady parameter A ranged from 0.0 to 2.0, radiation parameter R ranged from 0.0 to 4.0, Darcy number Da range from 1.0 to ∞ and porous media inertia coefficient γ range from 0.0 to 1.0.

Tables 1–5 displays the results for the values of the skin friction factor and Nusselt number to show the effect of thermal radiation on unsteady boundary layer flow with temperature dependent viscosity and thermal conductivity over a stretching sheet in porous media. In order to know the accuracy or the method used, computed values of $f''(0)$ and $\theta'(0)$ were obtained for ($\varepsilon = \kappa = \gamma = Da = R = 0$) and compared with obtained by Grubka and Bobba [2], Elbashbeshy and Bazid [4] and Sharidan et al. [5] in Table 1 and good agreement has been obtained with their results.

The effect of temperature dependent viscosity and thermal conductivity with Darcy number on local skin friction coefficient and Nusselt number is shown in Table 2, It can be seen from this table that the viscosity variation parameter has a significant effect on the local Nusselt number. As the viscosity variation parameter decreases, the thermal boundary layer thickness decreases and, thus, the rate of the heat transfer increases. The viscosity variation parameter also has a noticeable effect on the local skin friction coefficient, decreasing the viscosity within the boundary layer leads to an increase in the velocity within the layer and, thus, increases the local skin friction coefficient. Darcy number parameter also has a noticeable effect on the local skin friction coefficient, increasing Darcy number within the boundary layer leads to an increase in the velocity within the layer and thus increases the local skin friction coefficient, on the other hand as it increases the thermal boundary layer thickness decreases and thus the rate of the heat transfer increases. Moreover, the effect of thermal conductivity variation on the local skin friction coefficient and heat transfer rate is observed that as thermal conductivity variation parameter increases, there is a decrease in the local skin friction coefficient while the local Nusselt number has the opposite behavior.

Table 1. Values of $f''(0)$ and $\theta'(0)$ with $Pr = 1.0$ and $Da = R = \gamma = \kappa = \varepsilon = 0$.

A	$-f''(0)$	$-\theta'(0)$	
0.0	1.000008	0.999445	Present result
		1.00000	Grubka and Bobba [2]
		0.99999	Sharidan et al. [5]
		0.99999	Elbashbeshy and Bazid [4]
0.8	1.261043	0.471188	Present result
	1.261042	0.471190	Sharidan et al. [5]
	1.3321	0.6348	Elbashbeshy and Bazid [4]
1.2	1.377725	0.788172	Present result
	1.377722	0.788173	Sharidan et al. [5]
	1.4691	0.9491	Elbashbeshy and Bazid [4]

Table 2. Values of $f''(0)$ and $\theta'(0)$ for various values of κ, Da and ε with $Pr = 1.0$, $A = 0.8$, $\gamma = 1.0$ and $R = 2.0$.

κ	Da	$\varepsilon = 0.0$		$\varepsilon = 0.5$		$\varepsilon = 1.0$	
		$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
0.0	1	1.804849	0.367644	1.541395	0.353886	1.394407	0.344499
	2	1.659984	0.361331	1.380994	0.338475	1.222417	0.326509
	5	1.566511	0.353554	1.275778	0.327903	1.108013	0.313702
	20	1.517562	0.349134	1.220073	0.321742	1.046844	0.306106
	∞	1.500882	0.347568	1.200984	0.319529	1.025561	0.305376
0.5	1	1.804849	0.326887	1.546014	0.314645	1.401093	0.309576
	2	1.659976	0.318430	1.384603	0.303685	1.227816	0.294586
	5	1.566487	0.312146	1.278745	0.295288	1.112327	0.284380
	20	1.517522	0.308537	1.222667	0.290361	1.050510	0.278287
	∞	1.500834	0.307251	1.203444	0.288587	1.028926	0.278635
1.0	1	1.804849	0.298869	1.547278	0.304209	1.405831	0.284980
	2	1.659976	0.291880	1.387654	0.282823	1.231707	0.271561
	5	1.566487	0.286653	1.280874	0.271903	1.115450	0.262953
	20	1.517522	0.283639	1.224530	0.267738	1.053168	0.257783
	∞	1.500834	0.282562	1.205210	0.266233	1.035148	0.264350

Table 3. Values of $f''(0)$ and $\theta'(0)$ for various values of γ, Da and A with $Pr = 1.0$, $R = 0.5$, $\varepsilon = 0.5$ and $\kappa = 0.5$.

γ	Da	$A = 0.0$		$A = 0.5$		$A = 1.0$	
		$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
0.0	1	1.294566	0.241462	1.386068	0.380362	1.443195	0.551130
	2	1.085033	0.299643	1.207967	0.359428	1.276415	0.536405
	5	0.936731	0.352202	1.088573	0.343105	1.166437	0.525498
	20	0.853072	0.386685	1.024233	0.333378	1.107950	0.519249
	∞	0.823396	0.399757	1.001974	0.329841	1.087862	0.517024
0.5	1	1.376551	0.231743	1.462568	0.385577	1.516750	0.555174
	2	1.181227	0.284640	1.294557	0.366532	1.358575	0.541664
	5	1.046027	0.331382	1.183448	0.352014	1.255377	0.531833
	20	0.971168	0.361634	1.124213	0.343517	1.200931	0.526277
	∞	0.944881	0.373042	1.103838	0.340457	1.182310	0.524312
1.0	1	1.454084	0.222989	1.535320	0.390313	1.586932	0.558874
	2	1.270508	0.271545	1.375839	0.372824	1.436121	0.546393
	5	1.145616	0.313684	1.271487	0.359734	1.338559	0.537434
	20	1.077433	0.340655	1.216301	0.352179	1.287403	0.532423
	∞	1.053671	0.350781	1.197401	0.349479	1.269963	0.530644

Table 3 show the effect of unsteady parameter A and porous media inertia coefficient γ on local skin friction coefficient and Nusselt number, the results indicate also that as the unsteady parameter A increases the shear stress and Nusselt number decrease, on the other hand as porous media inertia coefficient γ increases the shear stress decreases and Nusselt number increases. The effect of the radiation on the local skin friction and the heat transfer coefficients for steady state ($A = 0$) and unsteady state ($A = 0.8$) is shown in Table 4 and 5 respectively. It can be seen from this Table 4 that an increase in the value of the radiation parameter R leads to a significant change in the values of the local skin friction coefficient and the local heat transfer coefficient. This can be attributed to the thermal radiation interaction enhancement, which increases the fluid velocity and, consequently, increases the velocity gradient at the wall. The opposite trend is observed for unsteady state ($A = 0.8$) in Table 5.

Table 4. Values of $f''(0)$ and $\theta'(0)$ for various values of γ, R and κ with $Pr = 1.0$, $A = 0.0$ (steady state), $\varepsilon = 1.0$ and $Da = 2.0$.

γ	R	$\kappa = 0.0$		$\kappa = 0.5$		$\kappa = 1.0$	
		$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
0.0	0.0	1.048243	1.052308	1.031088	0.678673	1.023109	0.500619
	0.5	1.025387	0.582366	1.009216	0.331936	1.002733	0.222608
	2.0	0.999373	0.182529	0.991010	0.071933	0.987438	0.021727
0.5	0.0	1.125377	1.005542	1.108849	0.650439	1.101048	0.480097
	0.5	1.103010	0.555882	1.087712	0.317078	1.081475	0.211968
	2.0	1.077983	0.171354	1.070110	0.065279	1.066718	0.016919
1.0	0.0	1.197590	0.965235	1.181595	0.625844	1.173956	0.462116
	0.5	1.175656	0.532811	1.161084	0.303988	1.155059	0.202560
	2.0	1.151475	0.161559	1.144004	0.059411	1.140763	0.012671

Table 5. Values of $f''(0)$ and $\theta'(0)$ for various values of R and Da with $Pr = 1.0$, $A = 0.8$, $\varepsilon = 1.0$, $\gamma = 1.0$ and $\kappa = 1.0$.

R	Da	$-f''(0)$	$-\theta'(0)$	R	Da	$-f''(0)$	$-\theta'(0)$
0.0	1	1.384226	0.357011	2.0	1	1.405831	0.284980
	2	1.213950	0.334307		2	1.231707	0.271561
	5	1.101175	0.317045		5	1.115450	0.262953
	20	1.041008	0.306966		20	1.053168	0.257783
	∞	1.020308	0.303344		∞	1.031693	0.255898
0.5	1	1.392282	0.340256	4.0	1	1.413726	0.246878
	2	1.220370	0.322886		2	1.238710	0.232856
	5	1.106307	0.309494		5	1.121091	0.226891
	20	1.045368	0.301605		20	1.057967	0.223262
	∞	1.024387	0.298756		∞	1.036180	0.221931

Figs. 2–5 display the dimensionless of velocity f' and temperature θ profiles for various values of Darcy number parameter Da, unsteady parameter A, the thermal conductivity variation parameter κ and porous media inertia coefficient γ respectively, while the other parameters are fixed. it can be seen that the velocity and temperature of the fluid increase as Darcy number parameter Da and the thermal conductivity variation parameter κ increase, while the opposite trend is observed for the effect of A and porous media inertia coefficient γ, i.e. increasing A and γ lead to decrease the velocity and temperature profiles, except for $A = 0.0$. In this case $A = 0.0$ (steady-flow case) the temperature profiles overshoot its value at the sheet surface, as can be seen from Figs. 3, 5. This behavior is in agreement with the results of Elbashbeshy and Bazid [4] and Sharidan et al. [5]. Moreover, the boundary-layer thickness decreases with an increase in A which in turn increases the skin friction coefficient and the Nusselt number. In addition, the effect of κ on the boundary-layer separation is not very pronounced compared to the effects of A.

Figs. 6–10 display the dimensionless of velocity f' temperature θ profiles for various values of the viscosity variation parameter ε, Darcy number parameter Da and radiation parameter R, respectively.

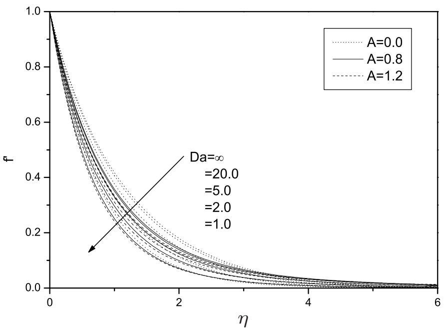

Fig. 2. Velocity profiles for various values of the unsteady parameter A and Darcy number Da.

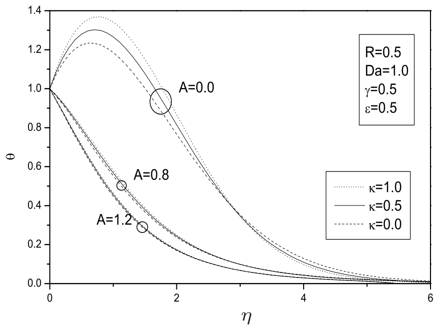

Fig. 3. Velocity profiles for various values of the unsteady A and thermal conductivity κ parameters.

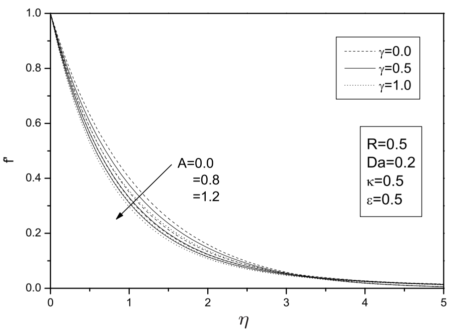

Fig. 4. Velocity profiles for various values of the unsteady A and permeability γ parameters.

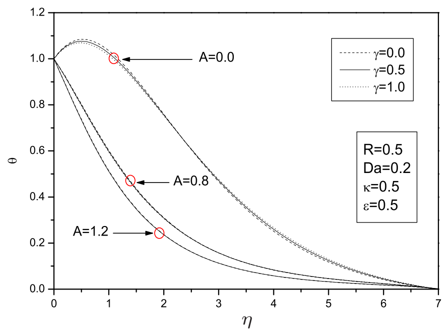

Fig. 5. Temperature profiles for various values of the unsteady A and permeability γ parameters.

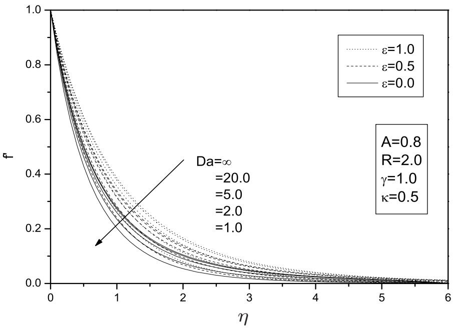

Fig. 6. Velocity profiles for various values of viscosity parameter ε and Darcy number Da.

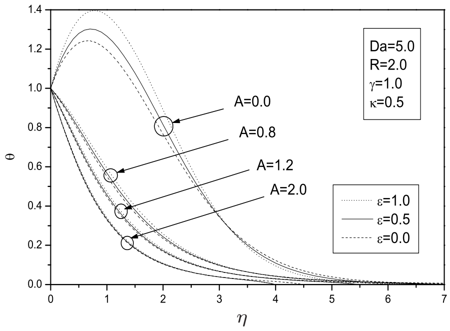

Fig. 7. Temperature profiles for various values of the unsteady A and viscosity ε parameters.

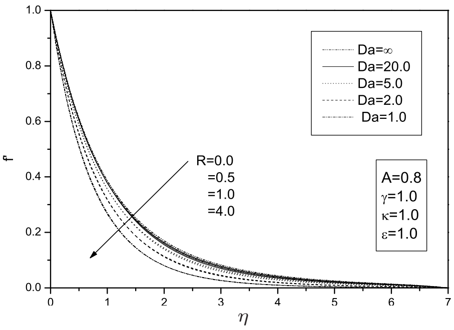

Fig. 8. Velocity profiles for various values of radiation parameter R and Darcy number Da.

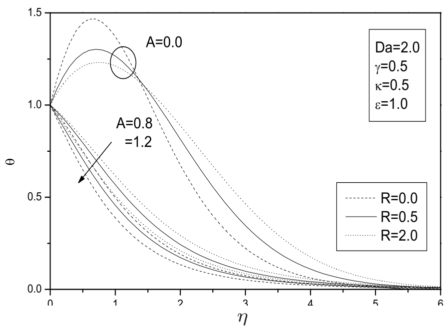

Fig. 9. Temperature profiles for various values of the unsteady A and radiation R parameters.

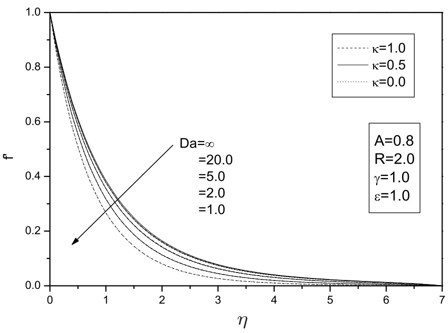

Fig. 10. Velocity profiles for various values of thermal conductivity parameter κ and Darcy number Da.

It is observed that the velocity and temperature across the boundary layer, increase with increasing viscosity variation parameter ε. These results show that increasing the viscosity variation is to accelerate the velocity and hence produces a decrease in the temperature. Further, the boundary-layer thickness increases with an increase in ε which in turn decreases the velocity gradient at the surface, while increases the rate of heat transfer $-\theta'(0)$ and hence produces a decrease in the skin friction coefficient and Nusselt number. On the other hand, as the value of radiation parameter R is increased, a significant decrease in the velocity and temperature distributions is observed.

Figs. 11–14 describes the behavior of the velocity and temperature fields for case of steady state flow ($A = 0.0$), it can be seen that, the velocity and temperature fields of the fluid increase as the Darcy number Da increases, while the velocity decreases as the thermal radiation parameter R increases and temperature has the opposite behavior, on the other hand as thermal conductivity parameter κ increases the velocity of the fluid decreases while the velocity increases.

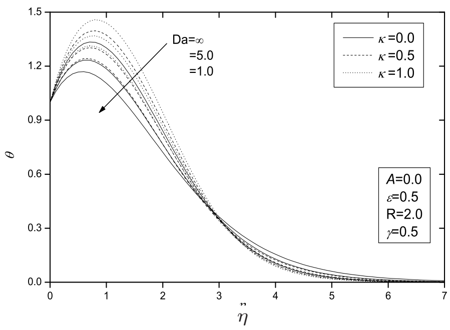

Fig. 11. Temperature profiles for various values of thermal conductivity parameter κ and Darcy number Da.

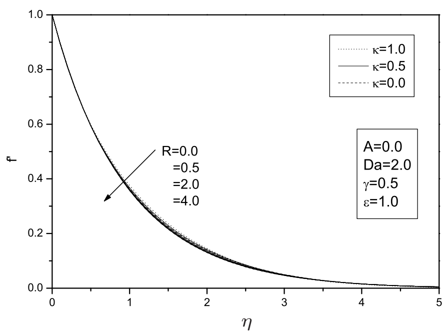

Fig. 12. Velocity profiles for various values of radiation R and thermal conductivity κ parameters.

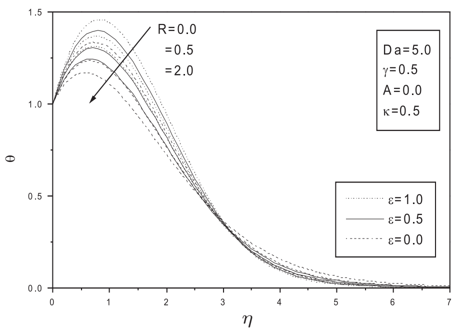

Fig. 13. Temperature profiles for various values of radiation R and viscosity ε parameters.

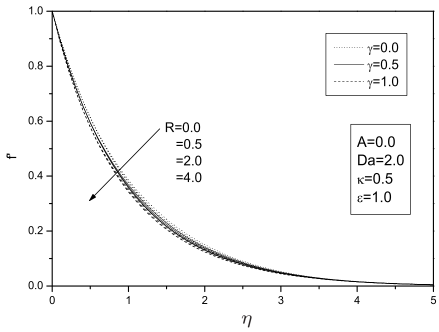

Fig. 14. Velocity profiles for various values of radiation R and permeability γ parameters.

4 Concluding remarks

In the present work we consider the effect of thermal radiation on unsteady boundary layer flow with temperature dependent viscosity and thermal conductivity due to a stretching sheet through porous media. The governing equations reduced to similarity boundary layer equations using suitable transformations and then solved using the Runge–Kutta numerical integration, procedure in conjunction with shooting technique. Numerical result are presented in terms of local skin friction coefficient and rate of heat transfer for various values of the variable viscosity ε, thermal conductivity κ and unsteady A parameters against Darcy number Da, porous media inertia coefficient γ. The effect of variation in $\varepsilon, \kappa, A, Da$ and γ on the dimensionless velocity, viscosity, thermal conductivity and temperature distribution are also depicted graphically. Nusselt number decrease. We notice the values of Nusselt number increases with increasing of radiation parameter R and thermal conductivity parameter κ, while the skin friction has the opposite behavior, on the other hand as porous media inertia coefficient γ increases the shear stress decreases and Nusselt number increases.

References

1. L.J. Crane, Flow past a stretching plate, *Z. Angew. Math. Phys.*, **21**, pp. 645–647, 1970.
2. L.J. Grubka, K.M. Bobba, Heat transfer characteristic of a continuous stretching surface with variable temperature, *J. Heat Transf.*, **107**, pp. 248–250, 1985.
3. E.M.A. Elbashebeshy, Heat transfer over a stretching surface with variable surface heat flux, *J. Phys. D Appl. Phys.*, **31**, pp. 1951–1954, 1998.
4. E.M.A. Elbashebeshy, M.A.A. Bazid, Heat transfer over an unsteady stretching surface, *Heat Mass Transfer*, **41**, pp. 1–4, 2004.
5. S. Sharidan, T. Mahmood, I. Pop, Similarity solution for the unsteady boundary layer flow and heat transfer due to a stretching sheet, *Int. J. Appl. Mech. Eng.*, **11**(3), pp. 647–654, 2006.
6. I. Pop, T.-Y. Na, Unsteady flow past a stretching sheet, *Mech. Res. Commun.*, **23**(4), pp. 413–422, 1996.
7. R. Nazar, N. Amin, D. Filip, I. Pop, Unsteady boundary layer flow in the region of the stagnation point on a stretching sheet, *Int. J. Eng. Sci.*, **42**(11–12), pp. 1241–1253, 2004.
8. M. Yürüsoy, Unsteady boundary layer flow of power-law fluid on stretching sheet surface, *Int. J. Eng. Sci.*, **44**(5–6), pp. 325–332, 2006.
9. Ch.H. Chen, Effect of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet, *J. Non-Newton. Fluid*, **135**(2–3), pp. 128–135, 2006.
10. B.S. Dandapat, B. Santra, K. Vajravelu, The effects of variable fluid properties and thermocapillarity on the flow of a thin film on an unsteady stretching sheet, *Int. J. Heat Mass Tran.*, **50**(5–6), pp. 991–996, 2007.

11. T.C. Chiam, Heat transfer in a fluid with variable thermal conductivity over a linearly stretching sheet, *Acta Mech.*, **129**, pp. 63–72, 1998.

12. A.M. Salem, Variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet, *Phys. Lett. A*, **369**(4), pp. 315–322, 2007.

13. M.A. Seddeek, F.A. Salama, The effects of temperature dependent viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, *Comp. Mater. Sci.*, **40**(2), pp. 186–192, 2007.

14. S.N. Odda, A.M. Farhan, Chebyshev finite difference method for the effects of variable viscosity and variable thermal conductivity on heat transfer to a micro-polar fluid from a non-isothermal stretching sheet with suction and blowing, *Chaos Soliton. Fract.*, **30**(4), pp. 851–858, 2006.

15. M.A. Hossain, M.S. Munir, I. Pop, Natural convection with variable viscosity and thermal conductivity from a vertical wavy cone, *Int. J. Therm. Sci.*, **40**, pp. 437–443, 2001.

16. M.A. Hossain, M.S. Munir, D.A.S. Rees, Flow of viscous incompressible fluid with temperature dependent viscosity and thermal conductivity past a permeable wedge with uniform surface heat flux, *Int. J. Therm. Sci.*, **39**, pp. 635–644, 2000.

17. A.J. Chamkha, Unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching sheet immersed in a porous medium, *Int. Commun. Heat Mass*, **25**(6), pp. 899–906, 1998.

18. I.A. Hassanien, A.H. Essawy, N.M. Moursy, Variable viscosity and thermal conductivity effects on combined heat and mass transfer in mixed convection over a UHF/UMF wedge in porous media: the entire regime, *Appl. Math. Comput.*, **145**(2–3), pp. 667–682, 2003.

19. J.C. Umavathi, A.J. Chamkha, M.H. Manjula, A. Al-Mudhaf, Radiative heat transfer of a two-fluid flow in a vertical porous Stratum, *J. Fluid Mechanics Research*, **35**(6), pp. 510–543, 2008.

20. M.F. El-Amin, I. Abbas, R.S.R. Gorla, Effect of thermal radiation on natural convection in a porous medium, *Int. J. Fluid Mechanics Research*, **34**(2), pp. 129–144, 2007.

21. S.M.M. EL-Kabeir, M. Modather, M. Abdou, R.S.R. Gorla, Hydromagnetic combined convection from radiating vertical stretching sheet with surface mass transfer, *Int. J. Fluid Mechanics Research*, **35**(6), pp. 544–556, 2008.

22. M. Modather, M. Abdou, S.M.M. EL-Kabeir, Magnetohydrodynamics and radiative effect on free convection flow of fluid with variable viscosity from a vertical plate through a porous medium, *J. Porous Media*, **10**(5), pp. 503–514, 2007.

23. E.M. Sparrow, D.R. Cess, *Radiation Heat Transfer Augmented Edition*, *Hemisphere*, Publishing Corporation, Washington, D.C., 1978.