Unsteady three dimensional flow of couple stress fluid over a stretching surface with chemical reaction

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Abstract. The unsteady three-dimensional flow of couple stress fluid over a stretched surface is investigated. Analysis has been performed in the presence of mass transfer and chemical reaction. Nonlinear flow analysis is computed by a homotopic approach. Plots are presented and analyzed for the various parameters of interest. A comparative study with existing solutions in a limiting sense is made.

Keywords: mass transfer effects, nonlinear analysis, couple stress fluid.

1 Introduction

The fluid flows with chemical reaction have attracted the attention of engineers and scientists in the recent times. Such flows have key importance in many processes including drying evaporation at the surface of a water body, energy transfer in a wet cooling tower, flow in a desert cooler, generating electric power, food processing, groves of fruit trees, crops damage because of freezing etc. There is always a molecular diffusion of species in the presence of a chemical reaction within or at the boundary during several practical diffusive operations. There are two types of reactions namely homogeneous and heterogenous. A homogeneous reaction takes place uniformly in the entire given phase whereas a heterogeneous reaction exists in a restricted region or within the boundary of a phase. The smog formation is an important example representing a first-order homogeneous chemical reaction. Several researchers in view of such facts are engaged for the discussion of flows with chemical reactions. For-instance Kandasamy et al. presented group analysis for Soret and Dufour effects on free convective heat and mass transfer with thermophoresis and chemical reaction over a porous stretching surface in the presence of heat source/sink. Pal and Talukdar [1] presented the combined effects of Joule heating and chemical reaction on unsteady magnetohydrodynamic mixed convection with viscous dissipation over a vertical plate in the presence of porous media and thermal radiation.

Effects of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micropolar fluid in a rotating frame of reference are analyzed by Das [2]. Joneidi et al. [3] presented analytical treatment of MHD free convection flow over a stretching sheet with chemical reaction. Recently Hayat and Nawaz [4] discussed unsteady stagnation point flow of viscous fluid due to rotating disk in presence of chemical reaction.

Understanding and modelling the flows of non-Newtonian fluids are of both fundamental and practical significance in the industrial and engineering applications. The rheological characteristics of such fluids are important in the flows of nuclear fuel slurries, lubrication with heavy oils and greases, paper coating, plasma and mercury, fossil fuels, polymers etc. Further, these fluids have a nonlinear relationship between the shear stress and shear rate. The associated equations of non-Newtonian fluids are very complex and higher order than the governing equations in viscous fluid. However during the past few decades there has been ample literature on various aspects of non-Newtonian fluids which is documented in the books [5–7]. At present many recent researchers are also engaged in the analysis of rheological characteristics of non-Newtonian fluids (see some studies [8–17]).

The boundary layer flow over a stretched surface has an increasing interest of the researchers due to its applications viz in the polymer processing of a chemical engineering plants, continuous casting, cable coating, glass blowing and spinning of synthetic fibers. Crane [18] initiated the study of two-dimensional boundary layer flow induced by the stretching of a flat elastic sheet. Later, extensive research works on various aspects of stretched flows have been presented. Some recent contributions on the topic can be seen through the investigations [19–24]. In view of flow diversity in nature, all the non-Newtonian fluids cannot be described by a single constitutive relationship between shear stress and shear rate. Among these the stress tensor for couple stress fluid is not symmetric and hence the flow behavior cannot be predicted by the Naviers-stokes theory. The fluid consisting of rigid randomly oriented particles suspended in a viscous medium such as blood, lubricants containing small amount of polymer additive, electro rheological fluids and synthetic fluids are examples of couple stress fluids. Devakar et al. [25] examined the Stokes' problems for an incompressible couple stress fluid. Srivastava [26] discussed flow of a couple stress fluid representing blood through stenotic vessels with a peripheral layer. Flow and heat transfer of couple stress fluid in a porous channel with expanding and contracting walls has been observed by Srinivasacharya et al. [27]. Squeeze film characteristics of finite journal bearings: couple stress fluid model has been examined by Lin [28]. Lin and Hung [29] also studied the combined effects of couple stresses and fluid inertia on the squeeze film characteristics between a long cylinder and an infinite plate. Jian et al. [30] examined nonlinear dynamic analysis of a hybrid squeeze-film damper-mounted rigid rotor lubricated with couple stress fluid and active control. Alyaquot and Elsharkawy [31] investigated the optimal film shape for two-dimensional slider bearings lubricated with couple stress fluids.

It is noted that unsteady three dimensional boundary layer flow of a couple stress fluid has not been reported so far. Thus the purpose of current investigation is to examine the mass transfer effect on unsteady three dimensional flow of couple stress fluid over a stretching surface with chemical reaction. Mathematical formulation has been presented

in Section 2. Section 3 contains the solution of problem by employing homotopy analysis method HAM [32–35]. Sections 4 and 5 present the convergence and discussion of the solutions. Sections 6 contains the final conclusions

2 Mathematical formulation

Consider the three-dimensional unsteady flow of an incompressible couple stress fluid over a surface at $z = 0$. The motion in fluid is created by a stretching surface. In usual notations, the continuity, momentum and concentration equations for the boundary layer flow can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} - \nu'\frac{\partial^4 u}{\partial z^4}, \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2} - \nu'\frac{\partial^4 v}{\partial z^4}, \tag{3}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2} - K(t)C, \tag{4}$$

subject to the boundary conditions

$$u = \frac{ax}{1 - ct}, \quad v = \frac{by}{1 - ct}, \quad w = 0, \quad C = C_w \quad \text{at } z = 0,$$

$$u \to 0, \quad v \to 0, \quad \frac{\partial u}{\partial z} \to 0, \quad \frac{\partial v}{\partial z} \to 0, \quad C \to C_\infty \quad \text{as } z \to \infty. \tag{5}$$

Here u, v and w are the velocities in the x, y and z directions, respectively, $\nu = \mu/\rho$ the kinematic viscosity, $\nu' = n/\rho$ the couple stress viscosity, ρ the density, C the concentration of species, D the coefficients of diffusing species, $K(t) = K_1/(1 - ct)$ the reaction rate, C_w the concentration at the surface, C_∞ the concentration far away from the sheet, T_w the surface temperature and T_∞ the temperature far away from the surface.

The following transformations

$$\eta = \sqrt{\frac{a}{\nu(1 - ct)}}\, z, \quad u = \frac{ax}{1 - ct} f'(\eta),$$

$$v = \frac{ay}{1 - ct} g'(\eta), \quad w = -\sqrt{\frac{a\nu}{1 - ct}}\{f(\eta) + g(\eta)\}, \tag{6}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad C_w - C_\infty = \frac{ex}{1 - ct}$$

identically satisfy the continuity equation. However Eqs. (2)–(6) take the following forms:

$$f''' - f'' + (f+g)f'' - A\left(f' + \frac{\eta}{2}f''\right) - Kf^v = 0, \tag{7}$$

$$g''' - g'^2 + (f+g)g'' - A\left(g' + \frac{\eta}{2}g''\right) - Kg^v = 0, \tag{8}$$

$$\phi'' + Sc(f+g)\phi' - Sc\,A\left(\phi + \frac{\eta}{2}\phi'\right) - Sc\,\gamma\phi - Sc\,\phi f' = 0, \tag{9}$$

$$\begin{aligned} f(0) = 0, \quad g(0) = 0, \quad f'(0) = 1, \quad g'(0) = c, \quad \phi(0) = 1, \\ f'(\infty) = 0, \quad g'(\infty) = 0, \quad f''(\infty) = 0, \quad g''(\infty) = 0, \quad \phi(\infty) \to 0, \end{aligned} \tag{10}$$

in which $A = c/a$ is the time dependent parameter, $K = \nu'a/\nu^2(1-ct)$ the couple stress parameter, prime for the differentiation with respect to η and the constants $a > 0$ and $b > 0$. The stretching ratio c, Schmidt number Sc, chemical reaction parameter γ are as follows:

$$c = \frac{b}{a}, \quad Sc = \frac{\nu}{D}, \quad \gamma = \frac{K_1}{a}. \tag{11}$$

We point out that the two-dimensional ($g = 0$) case has been recovered for $c = 0$. For $c = 1$, we obtain axisymmetric case, i.e., ($f = g$) and for ($A = 0$) the system of Eqs. (7)–(9) reduce to the steady situation.

3 Series solutions

3.1 0th-order deformation problems

For the development of the homotopy solutions, the functions $f(\eta)$, $g(\eta)$ and $\phi(\eta)$ in the set of base functions

$$\left\{\eta^k \exp(-n\eta) \mid k \geq 0,\ n \geq 0\right\} \tag{12}$$

can be introduced as follows:

$$f(\eta) = a_{0,0}^0 + \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{m,n}^k \eta^k \exp(-n\eta), \tag{13}$$

$$g(\eta) = A_{0,0}^0 + \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A_{m,n}^k \eta^k \exp(-n\eta), \tag{14}$$

$$\phi(\eta) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} c_{m,n}^k \eta^k \exp(-n\eta), \tag{15}$$

with the following initial guesses and auxiliary linear operators

$$f_0(\eta) = 1 - \exp(-\eta), \quad g_0(\eta) = c\big(1 - \exp(-\eta)\big), \quad \phi_0(\eta) = \exp(-\eta), \tag{16}$$

$$\mathcal{L}_f = \frac{\mathrm{d}^3 f}{\mathrm{d}\eta^3} - \frac{\mathrm{d}f}{\mathrm{d}\eta}, \quad \mathcal{L}_g = \frac{\mathrm{d}^3 g}{\mathrm{d}\eta^3} - \frac{\mathrm{d}g}{\mathrm{d}\eta}, \quad \mathcal{L}_\phi = \frac{\mathrm{d}^2 \phi}{\mathrm{d}\eta^2} - \phi. \tag{17}$$

Obviously the linear operators have the following properties

$$\mathcal{L}_f\big[C_1 + C_2\exp(\eta) + C_3\exp(-\eta)\big] = 0, \tag{18}$$

$$\mathcal{L}_g\big[C_4 + C_5\exp(\eta) + C_6\exp(-\eta)\big] = 0, \tag{19}$$

$$\mathcal{L}_\phi\big[C_7\exp(\eta) + C_8\exp(-\eta)\big] = 0, \tag{20}$$

where $C_1 - C_8$ are the constants and $a_{m,n}^k$, $A_{m,n}^k$ and $c_{m,n}^k$ are the coefficients. The problems at the 0th order can be expressed as

$$(1-p)\mathcal{L}_f\big[\bar{f}(\eta,p) - f_0(\eta)\big] = p\hbar_f\mathcal{N}_f\big[\bar{f}(\eta,p), \bar{g}(\eta,p)\big], \tag{21}$$

$$(1-p)\mathcal{L}_g\big[\bar{g}(\eta,p) - g_0(\eta)\big] = p\hbar_g\mathcal{N}_g\big[\bar{f}(\eta,p), \bar{g}(\eta,p)\big], \tag{22}$$

$$(1-p)\mathcal{L}_\phi\big[\bar{\phi}(\eta,p) - \phi_0(\eta)\big] = p\hbar_\phi\mathcal{N}_\phi\big[\bar{f}(\eta,p), \bar{\phi}(\eta,p)\big], \tag{23}$$

$$\bar{f}(0,p) = 0, \quad \bar{f}'(0,p) = 1, \quad \bar{g}(0,p) = 0, \quad \bar{g}'(0,p) = c,$$
$$\bar{f}'(\infty,p) = 0, \quad \bar{f}''(\infty,p) = 0, \quad \bar{g}'(\infty,p) = 0, \quad \bar{g}''(\infty,p) = 0, \tag{24}$$
$$\bar{\phi}(0;p) = 1, \quad \bar{\phi}(\infty;p) = 0,$$

$$\mathcal{N}_f\big[\bar{f}(\eta,p), \bar{g}(\eta,p)\big] = \frac{\partial^3\bar{f}}{\partial\eta^3} - \left(\frac{\partial\bar{f}}{\partial\eta}\right)^2 + \{\bar{f}(\eta,p) + \bar{g}(\eta,p)\}\frac{\partial^2\bar{f}}{\partial\eta^2}$$
$$- K\bar{f}^v(\eta,p) - A\left(\frac{\partial\bar{f}}{\partial\eta} + \frac{\eta}{2}\frac{\partial^2\bar{f}}{\partial\eta^2}\right), \tag{25}$$

$$\mathcal{N}_g\big[\bar{f}(\eta,p), \bar{g}(\eta,p)\big] = \frac{\partial^3\bar{g}}{\partial\eta^3} - \left(\frac{\partial\bar{g}}{\partial\eta}\right)^2 + \{\bar{f}(\eta,p) + \bar{g}(\eta,p)\}\frac{\partial^2\bar{g}}{\partial\eta^2}$$
$$- A\left(\frac{\partial\bar{g}}{\partial\eta} + \frac{\eta}{2}\frac{\partial^2\bar{g}}{\partial\eta^2}\right) - K\bar{g}^v(\eta,p), \tag{26}$$

$$\mathcal{N}_\phi\big[\bar{f}(\eta;p), \bar{g}(\eta,p), \bar{\phi}(\eta;p)\big] = \frac{\partial^2\bar{\phi}(\eta;p)}{\partial\eta^2} + Sc\big(\bar{f}(\eta,p) + \bar{g}(\eta,p)\big)\frac{\partial\bar{\phi}(\eta;p)}{\partial\eta}$$
$$- ScA\big(\bar{\phi}(\eta;p) + \frac{\eta}{2}\bar{\phi}'(\eta;p)\big)$$
$$- Sc\gamma\bar{\phi}(\eta;p) - Sc\bar{\phi}(\eta;p)\bar{f}'(\eta,p), \tag{27}$$

where $\hbar_f$, $\hbar_g$, $\hbar_\phi$ the auxiliary non-zero parameters and $p \in [0,1]$ is an embedding parameter. When $p = 0$ and $p = 1$, we obtain

$$\bar{f}(\eta,0) = f_0(\eta), \quad \bar{f}(\eta,1) = f(\eta), \tag{28}$$
$$\bar{g}(\eta,0) = g_0(\eta), \quad \bar{g}(\eta,1) = g(\eta), \tag{29}$$
$$\bar{\phi}(\eta;0) = \phi_0(\eta), \quad \bar{\phi}(\eta;1) = \phi(\eta). \tag{30}$$

Through Taylor series expansion one obtains

$$\bar{f}(\eta, p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m, \tag{31}$$

$$\bar{g}(\eta, p) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta) p^m, \tag{32}$$

$$\bar{\phi}(\eta; p) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta) p^m, \tag{33}$$

with

$$f_m(\eta) = \frac{1}{m!} \frac{\partial^m \bar{f}(\eta, p)}{\partial p^m} \bigg|_{p=0}, \tag{34}$$

$$g_m(\eta) = \frac{1}{m!} \frac{\partial^m \bar{g}(\eta, p)}{\partial p^m} \bigg|_{p=0}, \tag{35}$$

$$\phi_m(\eta) = \frac{1}{m!} \frac{\partial^m \bar{\phi}(\eta; p)}{\partial p^m} \bigg|_{p=0}. \tag{36}$$

The convergence of series $(31) - (33)$ depends upon the auxiliary parameters $\hbar_f$, $\hbar_g$ and $\hbar_\phi$. The values of $\hbar_f$, $\hbar_g$ and $\hbar_\phi$ have been selected in the manner that the series (31)–(33) converge at $p = 1$. Hence

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{37}$$

$$g(\eta) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta), \tag{38}$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta). \tag{39}$$

3.2 mth order deformation problems

Here the problems are given by

$$\mathcal{L}_f \big[f_m(\eta) - \chi_m f_{m-1}(\eta) \big] = \hbar_f \mathcal{R}_{f,m}(\eta), \tag{40}$$

$$\mathcal{L}_g \big[g_m(\eta) - \chi_m g_{m-1}(\eta) \big] = \hbar_g \mathcal{R}_{g,m}(\eta), \tag{41}$$

$$\mathcal{L}_\phi \big[\phi_m(\eta) - \chi_m \phi_{m-1}(\eta) \big] = \hbar_\phi \mathcal{R}_m^\phi(\eta), \tag{42}$$

$$\begin{aligned} f_m(0) = f_m'(0) = f_m'(\infty) = f_m''(\infty) = g_m(0) \\ = g_m'(0) = g_m'(\infty) = g_m''(\infty) = 0, \\ \phi_m(0) = 0, \quad \phi_m(\infty) = 0, \end{aligned} \tag{43}$$

$$\mathcal{R}_m^f(\eta) = f_{m-1}''' - A\left(f_{m-1}' + \frac{\eta}{2}f_{m-1}''\right) - Kf_{m-1}^v$$
$$+ \sum_{k=0}^{m-1}\left[(f_{m-1-k} + g_{m-1-k})f_k'' - f_{m-1-k}'f_k'\right], \tag{44}$$

$$\mathcal{R}_m^g(\eta) = g_{m-1}''' - A\left(g_{m-1}' + \frac{\eta}{2}g_{m-1}''\right) - Kg_{m-1}^v$$
$$+ \sum_{k=0}^{m-1}\left[(f_{m-1-k} + g_{m-1-k})g_k'' - g_{m-1-k}'g_k'\right], \tag{45}$$

$$\mathcal{R}_m^\phi(\eta) = \phi_{m-1}'' - Sc\,\gamma\phi_{m-1} - Sc\,A\left(\phi_{m-1} + \frac{\eta}{2}\phi_{m-1}'\right)$$
$$+ Sc\sum_{k=0}^{m-1}\left[(f_{m-1-k} + g_{m-1-k})\phi_k' - \phi_{m-1-k}f_k'\right], \tag{46}$$

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases} \tag{47}$$

The equations (40)–(42) have been solved one after the other in the order $m = 1, 2, 3, \ldots$ by employing Mathematica.

4 Convergence of the series solutions

Here our desire is to ensure the convergence of the obtained series solutions. Thus Figs. 1 and 2 have been plotted for the admissible values of $\hbar_f$, $\hbar_g$ and $\hbar_\phi$ regarding convergence of the solutions (37)–(39). Ultimate the admissible values have been noticed in the ranges $-1.25 \le \hbar_f, \hbar_g \le -0.25$ and $-1 \le \hbar_\phi \le -0.5$. Further $\hbar_f = -0.7 = \hbar_g = \hbar_\phi$ one has the better solution.
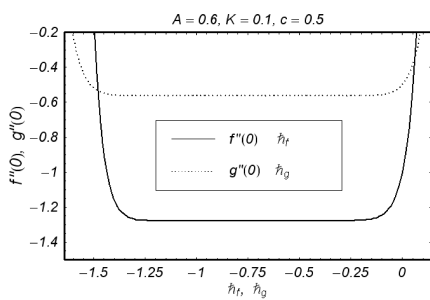

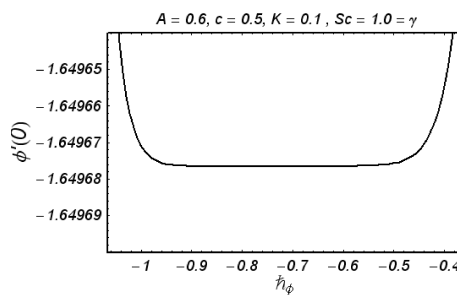Fig. 1. $\hbar$-curves of f and g. Fig. 2. $\hbar$-curve of ϕ.

Table 1 is presented to find that how much order of approximations are necessary for a convergent solution. It is noticed that 15th order approximations are sufficient for the velocity fields whereas 25th order of approximation are required for the concentration

field. Table 2 provides a comparative study for viscous flow. It is found that HAM solution in a limiting case of present study has a good agreement with the exact and HPM solutions provided in [27].

Table 1. Convergence of the HAM solutions for different order of approximations when $c = 0.5$, $A = 0.6$, $K = 0.5$, $Sc = \gamma = 1$.

Order of approximation	$-f''(0)$	$-g''(0)$	$-\phi'(0)$
1	1.21933333	0.55133333	1.5658333
2	1.26218363	0.56192793	1.6216753
5	1.27716191	0.56371964	1.6491773
10	1.27749261	0.56359020	1.6496750
15	1.27749261	0.56358705	1.6496764
20	1.27749258	0.56358694	1.6496764
25	1.27749257	0.56358692	1.6496764
30	1.27749257	0.56358692	1.6496764
40	1.27749257	0.56358692	1.6496764
50	1.27749257	0.56358692	1.6496764

Table 2. Illustrating the variation of $-f''(0)$ and $-g''(0)$ with c when $A = 0 = K$ using HAM, HPM [27] and exact solution [27].

c	$-f''(0)$			$-g''(0)$		
	HAM	HPM [27]	Exact [27]	HAM	HPM [27]	Exact [27]
0.0	1	1	1	0	0	0
0.1	1.020259	1.017027	1.020259	0.066847	0.073099	0.066847
0.2	1.039495	1.034587	1.039495	0.148736	0.158231	0.148736
0.3	1.057954	1.052470	1.057954	0.243359	0.254347	0.243359
0.4	1.075788	1.070529	1.075788	0.349208	0.360599	0.349208
0.5	1.093095	1.088662	1.093095	0.465204	0.476290	0.465204
0.6	1.109946	1.106797	1.109946	0.590528	0.600833	0.590528
0.7	1.126397	1.124882	1.126397	0.724531	0.733730	0.724531
0.8	1.142488	1.142879	1.142488	0.866682	0.874551	0.866682
0.9	1.158253	1.160762	1.158253	1.016538	1.022922	1.016538
1.0	1.173720	1.178511	1.173720	1.173720	1.178511	1.173720

5   Results and discussion

Computations for the effects of different parameters on velocity and concentration fields have been carried out and Figs. 3–11 have been displayed. The variations of K on f' and g' are presented in the Figs. 3 and 4. It is observed the f' and g' are decreasing functions of the couple stress parameter K. It is noted that couple stress parameter is dependent upon the couple stress viscosity n and this couple stress viscosity acts as a retarding agent which makes the fluid more denser resulting into a decrease in the velocity of the fluid. The variations of A on f' and g' are presented in the Figs. 5 and 6. These figures

elucidate that both the velocity components decrease with an increase in A. It is due to the fact that A is inversely proportional to the stretching coefficient a. Thus an increase in A decreases the stretching ratio. As a consequence the velocity decreases. Hence larger values of A accompany with positive value of couple stress parameter K will retards the flow. From physical point of view we can conclude that flow over the stretched surface can be controlled very accurately by an increase (or decrease) in the velocity of stretching.
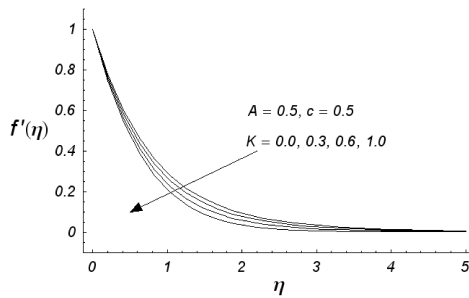
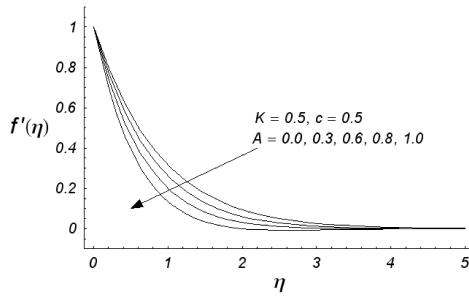Fig. 3. Influence of K on f'.
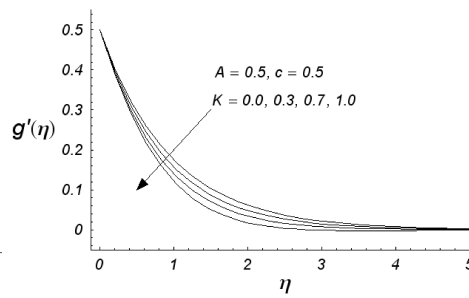
Fig. 4. Influence of K on f'.

Fig. 5. Influence of A on f'.
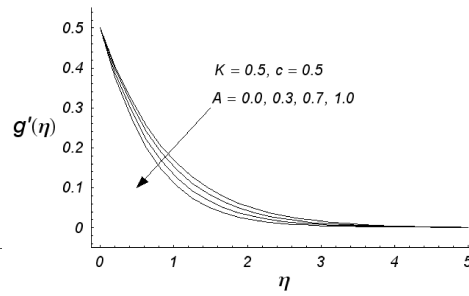
Fig. 6. Influence of A on g'.

The variations of A, K, γ and Sc on the concentration field ϕ are portrayed in the Figs. 7–11. Fig. 7 plots the influence of A on concentration field ϕ. It is observed that concentration field ϕ is an increasing function of A, Since stretching velocity can control the fluid motion making the system more susceptible to the concentration of species. The effect of K on ϕ has been shown in Fig. 8 which shows that ϕ is an increasing function of K. The concentration boundary layer also increases with an increase in K. Fig. 9 explains the variation of generative ($\gamma < 0$) chemical reaction on the concentration field ϕ. It is noted that generative ($\gamma < 0$) chemical reaction causes an increase in the concentration. Fig. 10 predicts the effects of destructive ($\gamma > 0$) chemical reactions on ϕ. It is noticed that ϕ decreases in case of destructive ($\gamma > 0$) chemical reaction. It is also noted that the magnitude of ϕ is larger in case of generative chemical reaction ($\gamma < 0$) in comparison to the case of destructive chemical reaction ($\gamma > 0$). Physically for generative case, chemical reaction γ takes place without creating much disturbance whereas disorder in the case of

destructive chemical reaction is much larger. Due to this fact molecular motion in the case of $(\gamma > 0)$ is quite larger which finally results into an increase in the mass transport phenomenon. Finally Fig. 11 plots the effects of Schmidt number Sc on the concentration field. This figure elucidates that concentration field ϕ decreases with an increase in Sc. From the definition of Schmidt number given in Eq. (11), it is clearly observed that Sc is inversely proportional to the diffusion coefficient D. Therefore larger values of Sc correspond to a decrease in the concentration field.
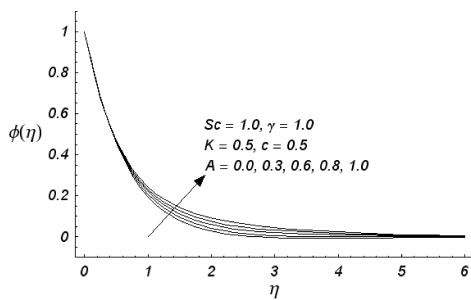
Fig. 7. Influence of A on ϕ.
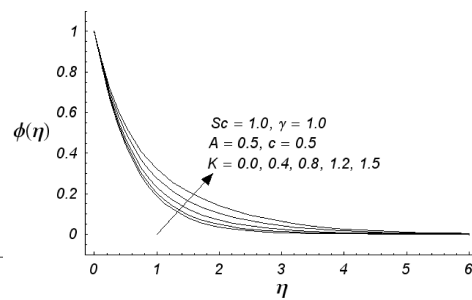
Fig. 8. Influence of K on ϕ.
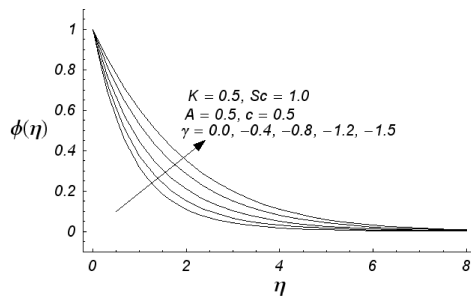
Fig. 9. Influence of $\gamma < 0$ on ϕ.
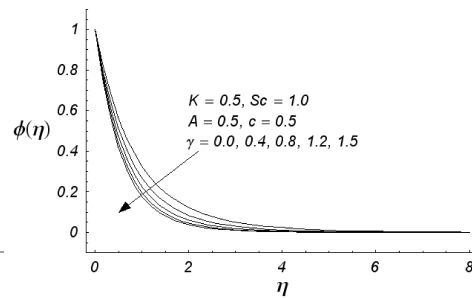
Fig. 10. Influence of $(\gamma > 0)$ on ϕ.
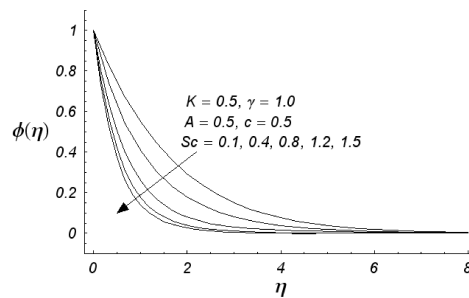
Fig. 11. Influence of Sc on ϕ.

6 Concluding remarks

Effects of mass transfer on the unsteady three-dimensional flow of couple stress fluid over a stretching sheet is investigated in the presence of chemical reaction. The main observations have been listed below.

- Velocity field f' and boundary layer thickness are decreasing functions of A and K.
- Variations of A and K on f' for two-dimensional, three dimensional and axisymmetric flow are qualitatively similar.
- The effects of A and K on f' and g' are similar.
- Influence of A and K on velocity and concentration fields are opposite.
- Concentration field ϕ is a decreasing function of Schmidt number Sc.
- The concentration field ϕ has opposite results for destructive $(\gamma > 0)$ and generative $(\gamma < 0)$ chemical reactions.

Acknowledgement

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