Analysis of a duopoly game with heterogeneous players participating in carbon emission trading*

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Received: 16 January 2013 / **Revised:** 7 August 2013 / **Published online:** 25 November 2013

Abstract. In this paper, a price competition model with two heterogeneous players participating in carbon emission trading is formulated. The stable conditions of the equilibrium points of this system are discussed. Numerical simulations are used to show bifurcation diagrams, strange attractors, and sensitive dependence on initial conditions. We observe that the speed of adjustment of bounded rational player may change the stability of the Nash equilibrium and cause the system to behave chaotically. In addition, we find that the price of emission permits plays an important role in the duopoly game. The chaotic behavior of the system has been stabilized on the Nash equilibrium point by applying delay feedback control method.

Keywords: carbon emission trading, price competition, heterogeneous players, Nash equilibrium, chaos.

1 Introduction

Carbon emission trading is one of international flexible mechanisms of the Kyoto Protocol on climate change to reduce greenhouse gas emission. It refers to a mechanism that government distributes the total amount of emission permits of carbon dioxide to enterprises who involve in the emission trading scheme and allows enterprises to trade emission permits at the same time. This mechanism requires enterprises, which hold emission permits to emit carbon dioxide that is proportional to the amount in these emission permits they hold. Therefore, if an enterprise's carbon dioxide emission is less than expected, the enterprise can sell the remaining permits to get some profits. What's more, when an enterprise emits excessive carbon dioxide, it must purchase the additional permits to avoid fines and sanctions from the government. In this mechanism, enterprises, which can abate their emission at low costs have an incentive to reduce more carbon emission increasing,

*This research was supported by the National Natural Science Foundation of China (No. 71101071), the National Basic Research Program of China (No. 2012CB955802), the Jiangsu Provincial Education Philosophy and Social Fund (No. 2012SJD630086) and Science Foundation of NUAA (Nos. NR2013006, NR2012063).

since they can sell their surplus emission permits to enterprises with high abatement costs for profits. Carbon emission trading is proved to be cost-effective in reducing carbon dioxide emission and be applied in practice.

The vast majority of existing literatures studied carbon emission trading from macro perspective, such as the ways of initial allocation of emission permits, the impacts of carbon emission trading on an area or a sector and the competition of carbon emission trading between countries. Krysiak et al. [1] studied the optimal size of a permits market in terms of optimal number of trading zones. Goulder et al. [2] examined the implications of two types of emissions allowance allocation: auctioned allowances and freely allocated allowances. Lee et al. [3] analyzed the impacts of combining a carbon tax and emission trading on different industry sectors. Bernard et al. [4] discussed a strategic competition between Russia and some developing countries in international markets for carbon emission permits and assessed the impacts of this competition on the pricing of emission permits. Jaehn et al. [5] considered the causes why prices of allowances varied dramatically and found that market power, the combination of information asymmetry and price interdependencies played an important role in explaining the emissions trading paradox. So far, there is almost no literature studying the impacts of carbon emission trading on enterprises from micro-level.

With the assumption of a perfect market and rational behavior by all players, the price of emission permits will equal to the marginal abatement costs. However, this cannot always be achieved in reality for the existence of oligopoly market. The oligopoly is a special market controlled by a small number of enterprises, in which the actions of these enterprises affect supply and price of homogeneous or heterogeneous products. It is this characteristic of interdependence that makes an enterprise has to take into account of the actions and reactions of its rivals when a decision is made. In the repeated oligopoly games, all enterprises maximize their profits by selecting an output or setting a price. When participating in the carbon emission trading scheme, enterprises engage in oligopolistic competition for emission permits as well as products, so they might be able to exert market power in the permits market to their advantages or use their market power in the permits market to gain power in the product market.

There exists large number of literatures that deal with quantity and price competitions of homogeneous or heterogeneous products in static oligopoly, for example, the works by Dixit [6], Singh et al. [7] and Wauthy [8]. The findings of these literatures become a cornerstone of the oligopoly theory. Recently, the dynamics of the oligopoly game has been studied in [9–14]. Puu [9] firstly found abundant complex dynamics arising in the Cournot duopoly case, such as strange attractors with fractal dimension. Agiza et al. [10] studied the chaotic dynamics in nonlinear duopoly game, where players had heterogeneous expectations: bounded rational and adaptive expectations. Elsadany [11] formulated a bounded rationality duopoly game with delay and studied its dynamical behaviors. The analysis showed that enterprises using delayed bounded rationality have a higher chance of reaching a Nash equilibrium point. Fanti et al. [12] analyzed the dynamics of a nonlinear Cournot duopoly with managerial delegation and homogeneous players and observed some phenomena, which would not occur under profit maximization. Chen et al. [13] studied the complex dynamic process of the triopoly games in

Chinese 3G telecoms market by using a Bertrand model with bounded rationality, and there were also a variety of complicated dynamics in the system. Fanti et al. [14] studied the dynamics of a Bertrand duopoly with differentiated products by introducing opportune microeconomic foundations. The results demonstrated that an increase in either the degree of substitutability or complementarity between products of different varieties was a source of complexity in a duopoly with price competition. In this study we consider that enterprises are involving in carbon emission trading i.e. we analyze a duopoly game with two heterogeneous players participating in carbon emission trading.

The purpose of this paper is to study the dynamics of a price competition model, which contains two enterprises using heterogeneous expectations rules while participating in carbon emission trading, and analyze the impacts of carbon emission trading on the system. The paper is organized as follow. In Section 2, a price competition model with heterogeneous players participating in carbon emission trading is briefly formulated. In Section 3, we analyze the dynamics of duopoly game model. Explicit parametric conditions of the existence and local stability of equilibrium points will be given. In Section 4, dynamical behaviors under some change of control parameters of the game are presented by numerical simulations. Sensitive dependence on initial conditions, Lyapunov exponents and strange attractors are calculated numerically and the impacts of carbon emission trading on the duopoly game are discussed. In Section 5, delay feedback control method is applied to control chaos of the system. Finally, a conclusion is shown in Section 6.

2 Model

While participating in carbon emission trading, enterprises take not only the production costs into consideration, but also the carbon emission costs. In general, in order to reduce carbon emission, the emission permits that each enterprise obtains from government initially cannot satisfy its need. For the insufficient emission permits, enterprise may either buy emission permits from permits market or abate its carbon emission. Thus the carbon emission costs consist of abatement costs and tradable permits costs. Let C_e represents the abatement costs, e represents the amount of carbon abatement, E represents the amount of carbon emission, y represents the emission permits obtained initially from government and p_c represents the price of emission permit in the permits market, so the carbon emission costs of each enterprise are

$$C_t = C_e + p_c(E - e - y).$$

Here $E - e - y$ denotes the tradable emission permits obtained from the permits market, a positive (negative) term $E - e - y$ means that enterprise purchases (sells) emission permits.

We assume that there are two enterprises, which choose different prices for their products in an oligopoly market and the permits market is independent of the product market and strong i.e. the permits market can supply enough emission permits for enterprises who want to purchase or sell emission permits. It meets the following demands:

1. The price of each enterprise during period $t = 0, 1, 2, \dots$ is represented by $p_i(t)$, $i = 1, 2$, and the quantity each enterprise sells Q_i, a linear demand function, is determined by the following equations [15]: $Q_i = 1/(1-d^2)[a(1-d) - p_i + dp_j]$, $i \neq j$, where $a > 0$ and $0 \leqslant d \leqslant 1$. The parameter d denotes the degree of product differentiation or product substitution. The production cost function is $C_{qi} = c_i Q_i$, $i = 1, 2$, where c_i ($c_i > 0$) is marginal production cost of ith enterprise.
2. We assume the amount of carbon dioxide each enterprise produces is linear with his quantity Q_i i.e. $E_i = k_i Q_i$, $i = 1, 2$, where k_i ($k_i > 0$) denotes the emission coefficient for Q_i. The marginal abatement costs of each enterprise are β_i, and we set $e_i = \varepsilon_i E_i$ ($\varepsilon_i \in [0, 1]$), so the abatement costs are $C_{ei} = \beta_i \varepsilon_i k_i Q_i$.
3. Based on output, emission permits that each enterprise initially allocated are free of charge, that is y_0, then the tradable emission permits each enterprise need is $E - e - y = k_i(1 - \varepsilon_i)Q_i - y_0$, $i = 1, 2$.

Therefore, the profit of ith enterprise in a single period is give by

$$\pi_i(p_1, p_2) = (p_i - c_i)Q_i - \beta_i \varepsilon_i k_i Q_i - p_c\big[k_i(1 - \varepsilon_i)Q_i - y_0\big], \quad i = 1, 2. \tag{1}$$

Then the marginal profit of the ith enterprise in one period is

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{1 - d^2}\big[a(1-d) + c_i + \beta_i \varepsilon_i k_i + p_c k_i(1-\varepsilon_i) - 2p_i + dp_j\big], \quad i, j = 1, 2,\ i \neq j. \tag{2}$$

We assume $c_i' = c_i + \beta_i \varepsilon_i k_i + p_c k_i(1 - \varepsilon_i) > 0$, which are the total variable costs of enterprises, and then Eq. (2) becomes

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{1 - d^2}\big[a(1 - d) + c_i' - 2p_i + dp_j\big], \quad i, j = 1, 2,\ i \neq j. \tag{3}$$

This optimization problem has unique solution in the form

$$p_i = \frac{1}{2}\big[a(1 - d) + c_i' + dp_j\big], \tag{4}$$

which are called reaction function of ith enterprise to jth enterprise.

Expectations play an important role in making decision. In this work, we consider two heterogeneous players, two enterprises think with different strategies for profit maximization. The first enterprise uses bounded rational expectations and the other is an adaptive player. For the bounded rational player, he does not have a complete knowledge of the product market, and determines his price of product on the basis of the expected marginal profit [16]. That is, if the marginal profit is positive (negative), he decides to increase (decrease) his price at next period. Thus, this dynamic adjustment mechanism is described by

$$p_1(t + 1) = p_1(t) + \alpha p_1(t)\frac{\partial \pi_1(p_i, p_j)}{\partial p_1}, \quad t = 0, 1, 2, \dots, \tag{5}$$

where α is a positive parameter, which represents the relative speed of adjustment of bounded rational player.

However, for the adaptive player, he computes his price with weights between last period's price and his reaction function. Hence, the dynamic equation of the adaptive player has the form

$$p_2(t+1) = (1-\nu)p_2(t) + \nu r_2\big(p_1(t)\big), \quad t = 0, 1, 2, \ldots, \tag{6}$$

where $\nu \in [0, 1]$ is a speed of adjustment of adaptive player.

With above assumptions, the dynamic price competition model with two heterogeneous players is formed from inserting Eq. (3) in Eq. (5) and Eq. (4) in Eq. (6), then combining Eqs. (5) and (6). Thus the dynamical system can be modeled as

$$\begin{aligned} p_1(t+1) &= p_1(t) + \frac{\alpha p_1(t)}{1-d^2}\big[a(1-d) + c_1' - 2p_1(t) + dp_2(t)\big], \\ p_2(t+1) &= (1-\nu)p_2(t) + \frac{\nu}{2}\big[a(1-d) + c_2' + dp_1(t)\big]. \end{aligned} \tag{7}$$

3 Equilibrium points and local stability

In order to study the dynamical behaviors of system (7), we define the equilibrium points of the dynamical duopoly game as a nonnegative fixed point of system (7) and discuss their stability. Equilibrium points are obtained by setting $p_i(t+1) = p_i(t)$, $i = 1, 2$, in system (7). There are two fixed points of system (7):

$$\begin{aligned} &E_0\left(0, \frac{1}{2}\big[a(1-d) + c_2'\big]\right), \\ &E_*\left(\frac{a(1-d)(d+2) + 2c_1' + dc_2'}{4-d^2}, \frac{a(1-d)(d+2) + dc_1' + 2c_2'}{4-d^2}\right). \end{aligned}$$

Obviously, E_0 is a boundary equilibrium, and the fixed point E_* is a Nash equilibrium and has economic meaning.

To investigate the local stability of the equilibrium points E_0 and E_*, we must estimate the Jacobian matrix of system (7) on the complex plane, which is given by

$$J(p_1, p_2) = \begin{bmatrix} 1 + \frac{\alpha}{1-d^2}[a(1-d) + c_1' - 4p_1 + dp_2] & \frac{\alpha d p_1}{1-d^2} \\ \frac{1}{2}\nu d & 1-\nu \end{bmatrix}. \tag{8}$$

Theorem. *The boundary equilibrium E_0 of system* (7) *is an unstable equilibrium point.*

Proof. In order to prove this result, we consider the eigenvalues of Jacobian matrix J at E_0, which take the form

$$J(E_0) = \begin{bmatrix} 1 + \frac{\alpha}{1-d^2}[a(1-d) + c_1' + \frac{a(1-d)d + dc_2'}{2}] & 0 \\ \frac{1}{2}\nu d & 1-\nu \end{bmatrix}, \tag{9}$$

whose eigenvalues are

$$\lambda_1 = 1 + \frac{\alpha}{1-d^2}\left[a(1-d) + c_1' + \frac{a(1-d)d + dc_2'}{2}\right], \qquad \lambda_2 = 1-\nu.$$

From the conditions that $a,c_i' > 0$, $0 \leqslant d \leqslant 1$, α are positive and $\nu \in [0,1]$, we have that $|\lambda_1| > 1$ and $|\lambda_2| \leqslant 1$. Thus E_0 is an unstable equilibrium point. $\square$

Therefore, the boundary equilibrium E_0 is a non-hyperbolic point if $\nu = 0$. E_0 is a saddle point if $0 < \nu \leqslant 1$.

Now we investigate the local stability of Nash equilibrium point E_*. The Jacobian matrix J at E_* is

$$J(E_*) = \begin{bmatrix} 1 - \frac{2\alpha p_1^*}{1-d^2} & \frac{\alpha d p_1^*}{1-d^2} \\ \frac{1}{2}\nu d & 1-\nu \end{bmatrix}, \tag{10}$$

where

$$p_1^* = \frac{a(1-d)(d+2) + 2c_1' + dc_2'}{4-d^2}.$$

The characteristic equation of the matrix $J(E_*)$ has the form

$$f(\lambda) = \lambda^2 - \mathrm{Tr}(J)\lambda + \mathrm{Det}(J) = 0,$$

where $\mathrm{Tr}(J)$ is the trace and $\mathrm{Det}(J)$ is the determinant of the Jacobian matrix $J(E_*)$, which are given by

$$\mathrm{Tr}(J) = 2 - \nu - \frac{2\alpha p_1^*}{1-d^2}, \qquad \mathrm{Det}(J) = (1-\nu)\left(1 - \frac{2\alpha p_1^*}{1-d^2}\right) - \frac{\alpha\nu d^2 p_1^*}{2(1-d^2)}.$$

Since

$$\mathrm{Tr}^2(J) - 4\,\mathrm{Det}(J) = \left(\nu - \frac{2\alpha p_1^*}{1-d^2}\right)^2 + \frac{2\alpha\nu d^2 p_1^*}{1-d^2}.$$

It is clear that $\mathrm{Tr}^2(J) - 4\,\mathrm{Det}(J) > 0$, we deduce that the eigenvalues of Nash equilibrium are real.

The local stability conditions of Nash equilibrium are given by using Jury's conditions [17], which are the sufficient and necessary conditions for $|\lambda_i| < 1$, $i = 1,2$:

$$1 - \mathrm{Tr}(J) + \mathrm{Det}(J) = \frac{\alpha\nu p_1^*}{1-d^2}\left(2 - \frac{1}{2}d^2\right) > 0, \tag{11_1}$$

$$1 + \mathrm{Tr}(J) + \mathrm{Det}(J) = 2(2-\nu)\left(1 - \frac{\alpha p_1^*}{1-d^2}\right) - \frac{\alpha\nu d^2 p_1^*}{2(1-d^2)} > 0, \tag{11_2}$$

$$1 - \mathrm{Det}(J) = \frac{2\alpha p_1^*}{1-d^2}(1-\nu) + \nu + \frac{\alpha\nu d^2 p_1^*}{2(1-d^2)} > 0. \tag{11_3}$$

The first condition is always satisfied, whereas the other two conditions (11_2) and (11_3) define a bounded region of stability in the parameters space (α,ν). Then we can get the necessary and sufficient conditions of the stability region in the plane of the speeds of adjustment (α,ν), which is defined by two inequalities, i.e.

$$\begin{gathered} 2(2-\nu)\left(1 - \frac{\alpha p_1^*}{1-d^2}\right) - \frac{\alpha\nu d^2 p_1^*}{2(1-d^2)} > 0, \\ \frac{2\alpha p_1^*}{1-d^2}(1-\nu) + \nu + \frac{\alpha\nu d^2 p_1^*}{2(1-d^2)} > 0. \end{gathered} \tag{12}$$

For the values of (α, ν) inside the stability region defined by Eq. (12), the Nash equilibrium point E_* is a stable node (a sink point). But if α, ν have gone beyond this area, E_* will become unstable. Besides, from these results, we obtain information that the systematic parameters have effect on the local stability of Nash equilibrium E_*. The stability of the system in Nash equilibrium point is not only decided by α, ν, but also by other parameters presented in Eq. (12), namely, by the influence of every parameter in Eq. (12).

According to reference [18], the sufficient and necessary conditions that Nash equilibrium point is a saddle point are

$$f(-1) = 2(2-\nu)\left(1 - \frac{\alpha p_1^*}{1-d^2}\right) - \frac{\alpha\nu d^2 p_1^*}{2(1-d^2)} < 0.$$

The sufficient and necessary conditions that Nash equilibrium point is a source point are

$$f(-1) = 2(2-\nu)\left(1 - \frac{\alpha p_1^*}{1-d^2}\right) - \frac{\alpha\nu d^2 p_1^*}{2(1-d^2)} > 0,$$
$$\mathrm{Det}(J) - 1 = -\frac{2\alpha p_1^*}{1-d^2}(1-\nu) - \nu - \frac{\alpha\nu d^2 p_1^*}{2(1-d^2)} > 0.$$

The sufficient and necessary conditions that Nash equilibrium point is a non-hyperbolic point are

$$f(-1) = 2(2-\nu)\left(1 - \frac{\alpha p_1^*}{1-d^2}\right) - \frac{\alpha\nu d^2 p_1^*}{2(1-d^2)} = 0,$$
$$\mathrm{Tr}(J) = 2 - \nu - \frac{2\alpha p_1^*}{1-d^2} \neq 0, -2.$$

4 Numerical simulations

To understand the dynamic behaviors of system (7) better and briefly, in this section, we present various numerical simulations to show its complexity, including bifurcations diagrams, strange attractors, Lyapunov exponents, and sensitive dependence on initial conditions. In order to study the local stability properties of the equilibrium points, it is convenient to take the parameters' values as follows: $a = 3$, $d = 0.1$, $c_1 = 0.2$, $c_2 = 0.4$, $k_1 = 0.1$, $k_2 = 0.2$, $\beta_1 = 0.4$, $\beta_2 = 0.7$, $\varepsilon_1 = 0.3$, $\varepsilon_2 = 0.5$, $p_c = 1$.

4.1 Period doubling bifurcations and chaotic behaviors

Figs. 1 and 2 show the bifurcation diagrams with respect to the parameter α while $\nu = 0.3, 0.8$, respectively. In all these figures the Nash equilibrium E_* is locally stable for small values of the parameter α. If α increases, the Nash equilibrium point E_* will become unstable, then period-doubling bifurcations appear and finally chaotic behaviors occur.

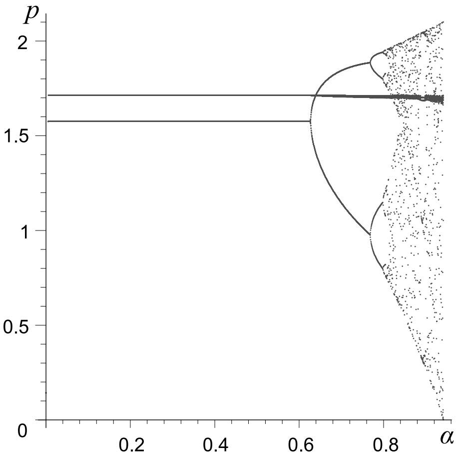

Fig. 1. Bifurcation diagram for $\nu = 0.3$.

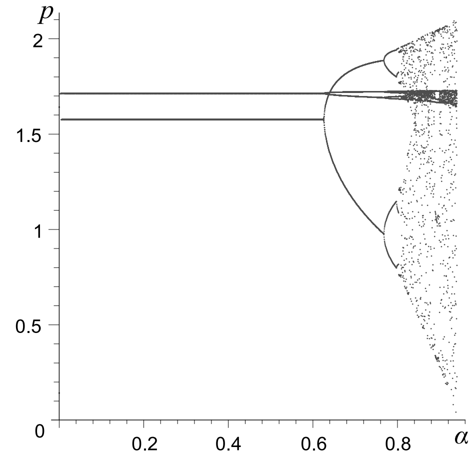

Fig. 2. Bifurcation diagram for $\nu = 0.8$.

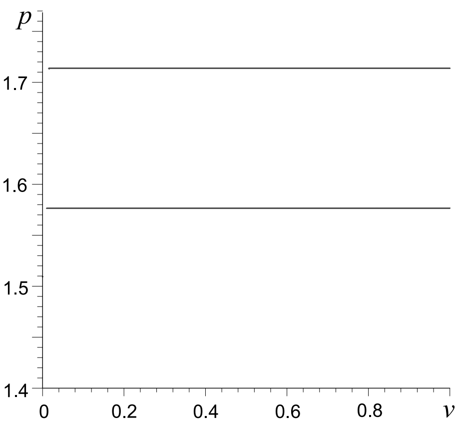

Fig. 3. Bifurcation diagram for $\alpha = 0.5$.

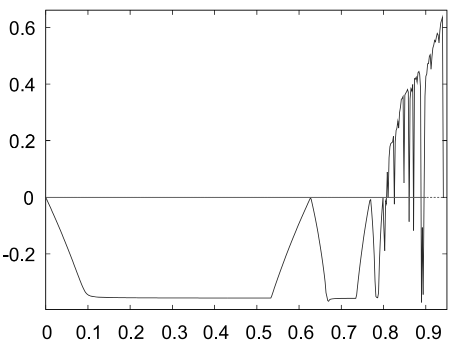

Fig. 4. Maximal Lyapunov exponent for $\nu = 0.3$.

Fig. 3 shows a one-parameter bifurcation diagram with respect to ν while $\alpha = 0.5$. From this figure, one observes that there is no period-doubling bifurcation for $\alpha = 0.5$, stable Nash equilibrium point for $0 < \nu < 1$.

In order to detect chaos, the maximal Lyapunov exponents corresponding to Fig. 1 are drawn in Fig. 4. This figure displays the related maximal Lyapunov exponents as a function of α. According to Fig. 4, one can easily find that the Lyapunov exponents are negative for different values of $\alpha \in (0, 0.6279)$, corresponding to a stable coexistence of the system. When $\alpha = 0.6279$, the value of maximal Lyapunov is zero. With the increase of α, the Nash equilibrium becomes unstable. Period doubling bifurcations appears and finally when the value of maximal Lyapunov is larger than zero, chaotic behaviors occur.

The phase portraits corresponding to Fig. 1 and 2 are shown in Fig. 5 for showing the strange attractors at different values of α.

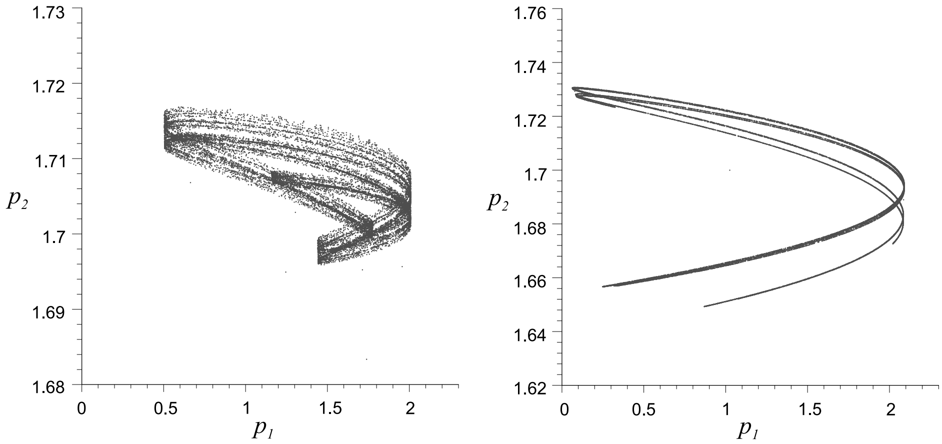

Fig. 5. Strange attractors for $\nu = 0.3, 0.8$.

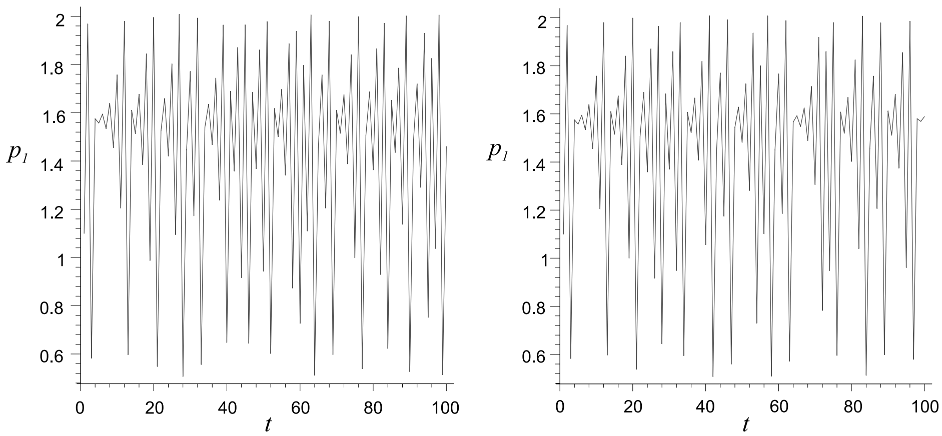

Fig. 6. Shows sensitive dependence on initial conditions, the two orbits of p_1-coordinates at the parameters' values $(\alpha, \nu) = (0.857, 0.3)$ and $(p_{10}, p_{20}) = (1.1, 1.3)$.

From these numerical simulations, we conclude that the speed of adjustment of bounded rational player may change the stability of the Nash equilibrium and cause the system to behave chaotically. But, with the speed of adjustment of adaptive player varying, the structure market of the duopoly game is stable and Nash equilibrium becomes asymptotically stable. Hence, the complex dynamics of the duopoly game depends on the parameter α.

To demonstrate the sensitivity to initial conditions of the system (7), we compute two orbits with initial points (p_{10}, p_{20}) and $(p_{10} + 0.00001, p_{20})$ at the parameters' values $\alpha = 0.857$, $\nu = 0.3$, respectively. The corresponding results are shown in Fig. 6. From these two figures, it is clear that the results are indistinguishable at the beginning, but after a number of iterations, the difference between them builds up rapidly.

4.2 The impacts of carbon emission trading on the duopoly game

When participating in carbon emission trading, enterprises have to consider its impacts on their products. Relatively, emission coefficient k_i and the marginal abatement costs $\beta_i$are determinated by enterprise's technology and equipment, which don't change in the short run. And the amount of carbon abatement depends on whether their marginal abatement costs are higher than the price of emission permits. If so, enterprises will buy additional permits; otherwise they choose to reduce their carbon emission. Therefore, it is only the price of emission permits that affects the duopoly game.

Inequalities (12) define the region of stability in the plane of the speed of adjustments (α, ν). We set all parameters in the initial values, and then we can get the region of stability of Nash equilibrium point, which is shown in Fig. 7(a). When all other parameters are fixed, the price of emission permits p_c varies to $p_c = 5$ from $p_c = 1$, we could see that the stable area decreases in the direction of α as shown in Fig.7(b). It proves that the system's stability will decrease by the increase of p_c. What's more with the rise of p_c, the Nash equilibrium point E_* increases from $E_*(1.58, 1.71)$ to $E_*(1.73, 1.92)$, which means the price of each enterprise increases with the addition of p_c.

Fig. 8 shows the bifurcation diagrams with respect to the parameter p_c while $\alpha = 0.4$ and $\nu = 0.3$. With the rise of price of emission permits p_c, the Nash equilibrium point E_* increases gradually, and is locally stable for small values of the parameter p_c. As p_c increases, the Nash equilibrium point E_* becomes unstable and one observes complex dynamic behavior such as cycles of high order and chaos.

Appearance of such phenomenon is because carbon emission trading increases the total variable costs of enterprises. When participating in carbon emission trading, carbon emission becomes a part of total costs of product so that the marginal costs of enterprises increase. To maximize profit, enterprises could use their market power in the product market to transfer their costs to customers, which makes the price of each enterprise add.

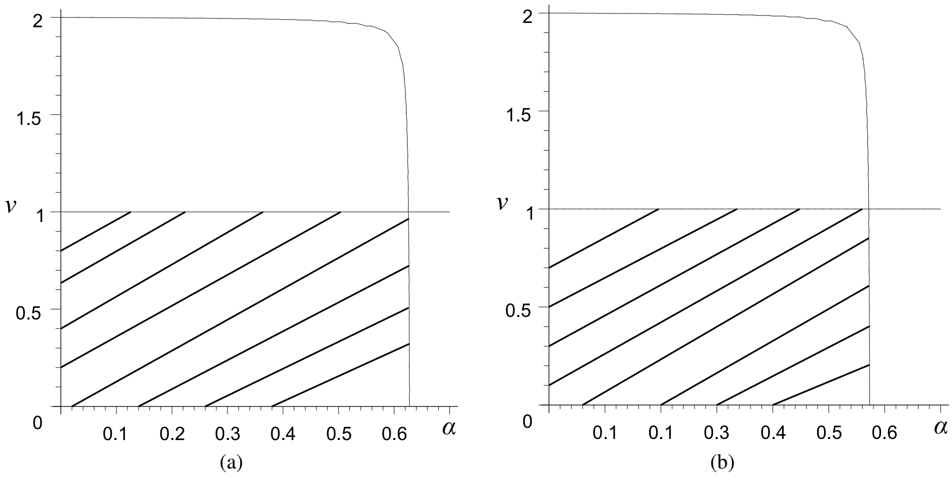

Fig. 7. Region of stability of Nash equilibrium.

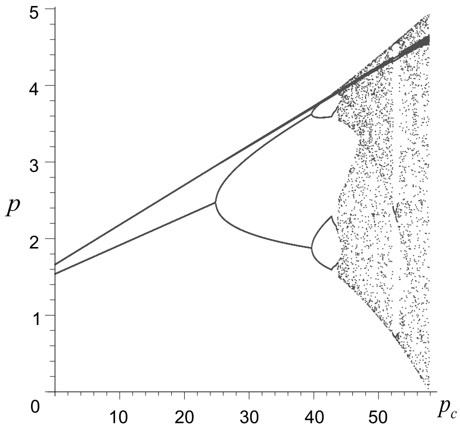

Fig. 8. Bifurcation diagram for $\alpha = 0.4$ and $\nu = 0.3$.

The higher the price of emission permits is, the higher the price of each enterprise will become. If the price of emission permits is high enough that crosses the boundary of stable region, the price of each enterprise would become unpredictable and get into chaos through period-doubling bifurcations.

It is obvious from the above figures and analysis that the price of emission permits plays an important role in the duopoly game: it not only influences the system's stability, but also has an effect on the Nash equilibrium point.

5 Chaos control

From last section, we know the market is irregular when chaos occurs, which is harmful for each enterprise. In the chaotic market, nobody could predict the development of market and a little adjustment of the initial price that each enterprise make might result in great variation of price. It is necessary for enterprises to control chaos.

To control chaos, a number of methods have been proposed. Delay feedback control method, which was proposed by Pyragas [19], is one of most effective methods for controlling chaos in oligopoly models. Elabbasy et al. [20] and Ding et al. [21] have applied the delay feedback control method to control chaos in two economic models. In this section, we apply this method to control chaotic behavior of the duopoly game with heterogeneous players participating in carbon emission trading. We modify the first equation of system (7) in the same way as Elabbasy et al. [20] did, so the controlled system is given by

$$\begin{aligned} p_1(t+1) &= p_1(t) + \frac{\alpha p_1(t)}{1-d^2}\bigl[a(1-d) + c_1' - 2p_1(t) + dp_2(t)\bigr] \\ &\quad + K\bigl(p_1(t+1-T) - p_2(t+1)\bigr), \\ p_2(t+1) &= (1-\nu)p_2(t) + \frac{\nu}{2}\bigl[a(1-d) + c_2' + dp_1(t)\bigr], \end{aligned} \tag{13}$$

where K is the controlling factor and T is the time delay.

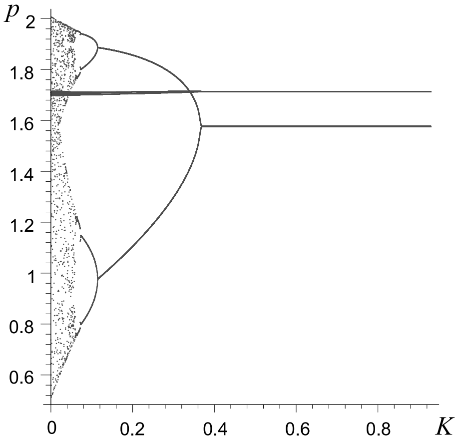

Fig. 9. Bifurcation diagram with respect to the controlling factor K.

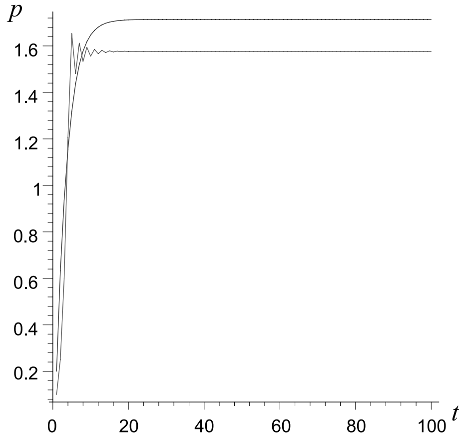

Fig. 10. Stabilization of Nash equilibrium with $K = 0.6$.

By choosing $T = 1$, the controlled system (13) becomes

$$\begin{aligned} p_1(t+1) &= p_1(t) + \frac{\alpha p_1(t)}{(1-d^2)(K+1)}\big[a(1-d) + c_1' - 2p_1(t) + dp_2(t)\big], \\ p_2(t+1) &= (1-\nu)p_2(t) + \frac{\nu}{2}\big[a(1-d) + c_2' + dp_1(t)\big]. \end{aligned} \tag{14}$$

And the Jacobian matrix of (14) takes the form

$$J(p_1, p_2) = \begin{bmatrix} 1 + \frac{\alpha}{(1-d^2)(K+1)}[a(1-d) + c_1' - 4p_1 + dp_2] & \frac{\alpha d p_1}{(1-d^2)(K+1)} \\ \frac{1}{2}\nu d & 1-\nu \end{bmatrix}. \tag{15}$$

We know the system (7) is chaotic for the initial parameter values and $(\alpha, \nu) = (0.857, 0.3)$. Substituting by the Nash equilibrium point into (15) and using the initial values of parameters the Jacobian matrix (15) has the form

$$J(p_1, p_2) = \begin{bmatrix} \frac{(-1.7415343+K)}{K+1} & \frac{0.1367737}{K+1} \\ 0.015 & 0.7 \end{bmatrix}. \tag{16}$$

By applying Jury condition (11) on the matrix (16) has eigenvalues with an absolute less than one when $K > 0.3702$. Hence, when $K > 0.3702$, the controlled system (14) is stable around the Nash equilibrium point.

Figure 9 shows the bifurcation diagram with respect to the controlling factor K when other parameters take the initial values and $(\alpha, \nu) = (0.857, 0.3)$. From this figure, we can see that with the control factor K increasing, the system gets rid of chaotic behaviors and is controlled to a stable state when $K > 0.37$. Shown in Fig. 10 is the stable behavior of the controlled system when $K = 0.6$ starts from initial values $(p_1(0), p_2(0)) = (0.1, 0.2)$.

6 Conclusions

In this paper, we analyze the dynamics of a price competition model, which contains two enterprises using heterogeneous expectations rules while participating in carbon emission trading. The stability conditions of the equilibrium points of this system are discussed. Through the discussion, we know that the stability of the system in Nash equilibrium point depends on all systematic parameters. Numerical simulations are used to show bifurcation diagrams, strange attractors, Lyapunov exponents, and sensitive dependence on initial conditions. We find the complex dynamics of the duopoly game depends on the parameter α (the speed of adjustment of bounded rational player), and the speed of adjustment of adaptive player ν has a stabilization effect on the system. The speed of adjustment of bounded rational player may change the stability of the Nash equilibrium and cause the system to behave chaotically. For the low speed of adjustment of bounded rational player, the system is stable. However, with the increase of the speed of adjustment of bounded rational player, the system will be unstable and moves towards chaos from double periodical bifurcation.

Moreover, we reveal that the price of emission permits plays an important role in the duopoly game. It not only influences the system's stability, but also has an effect on the Nash equilibrium point. The higher the price of emission permits is, the higher the price of each enterprise will become. When the price of emission permits is too high, then Nash equilibrium will become unstable and the system gets into chaos through period-doubling bifurcations. At last, we apply the delay feedback control method to control the system in chaos state on the Nash equilibrium point.

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