Robust optimal H_∞ control for irregular buildings with AMD via LMI approach*

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Abstract. In view of the numerous uncertainties of seismic disturbances and structural parameters, the irregular building structure is uncertain. In this paper, a new robust optimal H_∞ controller for irregular buildings is designed to guarantee the robust stability and performance of the closed-loop system in the presence of parameter uncertainties. Such a control method can provide a convenient design procedure for active controllers to facilitate practical implementations of control systems through the use of a quadratic performance index and an efficient solution procedure based on linear matrix inequality (LMI). To verify the effectiveness of the control method, a MDOF (multiple-degree-of-freedom) eccentric building structure with two active mass damper (AMD) systems on the orthogonal direction of the top storey subjected to bi-directional ground motions is analyzed. In the simulation, the active control forces of the AMD systems are designed by the robust optimal H_∞ control algorithm, and the structural system uncertainties are assumed in the system and input matrices. The simulation results obtained from the proposed control method are compared with those obtained from traditional H_∞ control method, which shows preliminarily that the performance of robust optimal H_∞ controllers is remarkable and robust. Therefore, the robust optimal H_∞ control method is quite promising for practical implementations of active control systems on seismically excited irregular buildings.

Keywords: robust H_∞ control, linear matrix inequality, quadratic performance index, irregular buildings, AMD systems, bi-directional ground motions.

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1 Introduction

As tall building and long-span bridges become lighter and more flexible, they are more susceptible to the effects of vibration. A critical aspect in the design of civil engineering structures is the reduction of response quantities such as velocities, deflections and accelerations induced by environmental dynamic loadings. Structural control methods are the most recent strategies for this purpose, which can be classified as active, semi-active, passive, and hybrid control methods [1, 2]. Although passive control is widely used in practice, there have been intensive researches in the area of active control of structures in the recent past because of achieving higher control of response [3]. Further, control algorithms developed for active control have been directly useful for developing other recent control strategies like semi-active control.

In research studies and practical applications, various active control algorithms have been investigated in designing controllers, such as linear quadratic regulator (LQR) [4, 5], linear quadratic Gaussian (LQG) [6], sliding mode control (SMC) [7, 8], and H_∞ control [4, 9]. Due to modeling errors, variation of materials properties, component nonlinearities, and changing load environments, the system description for these structural systems inevitably contains uncertainties of different nature and level [10]. These uncertainties can affect both the stability and performance of a control system. Robust control is concerned with maintaining performance with uncertainty in the dynamical system. Robust control means having a controller that maintains stability and performance specifications in spite of uncertainties. To accommodate such possible degradation of stability and performance, methods such as robust H_∞ control are often used. H_∞ control strategy is particularly useful in designing the robust controller because these robustness criteria in a way can be interpreted as the H_∞ norm of a transfer function to be smaller than a given value. The results from both numerical simulations and experimental tests indicate that H_∞ control is effective [9]. Calise et al. [11] and Wang et al. [12] proposed two robust H_∞ controllers and presented a numerically efficient methodology by solving algebraic Riccati equations. But Raccati equation approach is difficult to find their feasible solutions and minimization of H_∞-norm bound. Gahinet and Apkarian [13] proposed a new methodology based on the solution of linear matrix inequalities (LMI) which can be directly derived from the bounded real lemma and termed as the LMI-based H_∞ control. The theorem of the LMI-based solution method is more straightforward and no restriction is required. Besides, this solution procedure is quite efficient in terms of computation. In [14–17], the LMI-based solution approach had been used to design robust H_∞ control for a given H_∞-norm bound.

New types of devices have been invented in order to implement these active control schemes in practical applications. Active mass damper (AMD) systems have been a popular area of research in recent decades and significant progress has been made in this area over these years [18–21]. In previous studies, many researchers assumed that a controlled structure is a planar structure built on a fixed base. However, it is generally recognized that a real building is actually asymmetric to some degree even with a nominally symmetric plan. The asymmetric characteristic of the building creates simultaneous lateral and torsional vibration known as torsion coupling (TC) when under seismic disturbances [22,23].

In this paper, a robust optimal H_∞ control algorithm for two AMD systems on the orthogonal direction of the top story is proposed for the vibration control of irregular buildings subjected to bi-lateral ground acceleration. The new robust optimal H_∞ control takes the robustness criteria into account and uses the LMI-based solution procedure. In addition, the comparisons with the numerical results using the traditional H_∞ control are also made for demonstration of its control performance.

2 Equations of motion of structural systems

An n degree-of-freedom TC shear building equipped with two AMD systems on the orthogonal direction of the top storey and subjected to bi-lateral ground acceleration, $\ddot{x}_g(t)$ and $\ddot{y}_g(t)$, is shown in Fig. 1. The position coordinates of the two AMD systems along the x and y directions are l_{x1}, l_{x2}, l_{y1}, l_{y2}, respectively. In order to introduce a certain degree of asymmetry into the model, two-way floor eccentricities between the center of mass (denoted by M in Fig. 1) and the center of stiffness (denoted by S in Fig. 1) along the x and y directions (e_x and e_y) are considered. The corresponding control forces are represented as u_{a1} and u_{a2}. Under the following two assumptions: (i) the mass of each storey is idealized as a concentrated mass at the floor levels; and (ii) the center of mass and stiffness of each storey is located in the same point; the matrix equation of motion can be expressed as

$$\mathbf{M}\ddot{\mathbf{v}} + \mathbf{C}\dot{\mathbf{v}} + \mathbf{K}\mathbf{v} = -\mathbf{M}\mathbf{E}\mathbf{w}(t) + \mathbf{B}_s\mathbf{u}(t),$$

in which $\mathbf{v} = \{x, y, \theta, \Delta x_a, \Delta y_a\}^{\mathrm{T}}$ is an $(3n+2)$ vector; x, y, θ are the translations along the x- and y-directions and rotation along the z-axis, respectively; Δx_a, Δy_a are the translations along the x- and y- directions of the two AMD systems; $u = \{u_{a1}, u_{a2}\}^{\mathrm{T}}$ is a (2×1) dimensional vector consisting of two control forces; $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are $(3n+2) \times (3n+2)$ mass and stiffness matrices, respectively; $\mathbf{B}_s$ is a $(3n+2) \times 2$ matrix denoting the location of the two AMD control systems; $\mathbf{E}$ is a $(3n+2) \times 2$ dimensional matrix denoting the influence of the earthquake excitation; $\mathbf{w}(t) = \{\ddot{x}_g(t), \ddot{y}_g(t)\}^{\mathrm{T}}$.

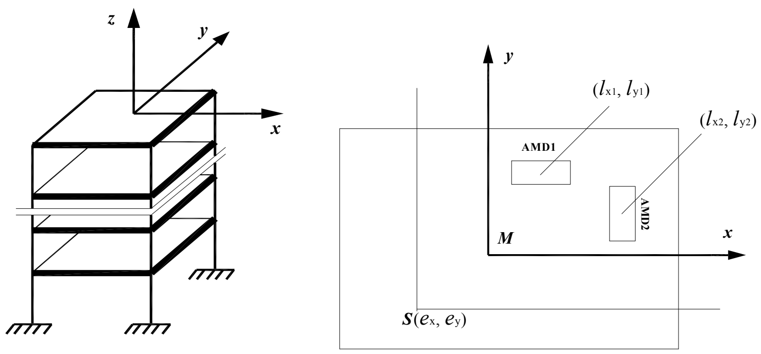


Fig. 1. A model of an eccentric building with AMD systems.

The mass matrix $\mathbf{M}$ can be expressed as following:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_s & \mathbf{0} \\ \mathbf{M}_c^{\mathrm{T}} & \mathbf{M}_{\mathrm{AMD}} \end{bmatrix}, \qquad \mathbf{M}_s = \begin{bmatrix} \mathbf{M}_{xx} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{yy} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{\theta\theta} \end{bmatrix},$$

$$\mathbf{M}_{xx} = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix}, \qquad \mathbf{M}_{yy} = \mathbf{M}_{xx},$$

$$\mathbf{M}_{\theta\theta} = \begin{bmatrix} J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_n \end{bmatrix}, \qquad \mathbf{M}_{\mathrm{AMD}} = \begin{bmatrix} m_{a1} & \\ & m_{a2} \end{bmatrix},$$

$$\mathbf{M}_c^{\mathrm{T}} = \begin{bmatrix} 0 & \cdots & m_{a1} & 0 & \cdots & 0 & 0 & \cdots & m_{a1}l_{y1} + m_{a2}l_{y2} \\ 0 & \cdots & 0 & 0 & \cdots & m_{a2} & 0 & \cdots & m_{a1}l_{x1} + m_{a2}l_{x2} \end{bmatrix},$$

where m_i is the ith story mass; $J_i = mr^2$ = floor mass polar moment of inertia about z-axis; r = radius of gyration of the floor.

The stiffness matrix $\mathbf{K}$ can be expressed as following:

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_s & \mathbf{K}_c \\ \mathbf{0} & \mathbf{K}_{\mathrm{AMD}} \end{bmatrix}, \qquad \mathbf{K}_s = \begin{bmatrix} \mathbf{K}_{xx} & \mathbf{0} & \mathbf{K}_{x\theta} \\ \mathbf{0} & \mathbf{K}_{yy} & \mathbf{K}_{y\theta} \\ \mathbf{K}_{\theta x} & \mathbf{K}_{\theta y} & \mathbf{K}_{\theta\theta} \end{bmatrix},$$

$$\mathbf{K}_{xx} = \begin{bmatrix} k_{x_1x_1} & k_{x_1x_2} & \cdots & & \\ k_{x_2x_1} & k_{x_2x_2} & k_{x_2x_3} & \cdots & \\ & & \cdots & \cdots & \\ & & k_{x_{n-1}x_{n-2}} & k_{x_{n-1}x_{n-1}} & k_{x_{n-1}x_n} \\ & & & k_{x_nx_{n-1}} & k_{x_nx_n} + k_{a1} \end{bmatrix},$$

$$\mathbf{K}_{yy} = \begin{bmatrix} k_{y_1y_1} & k_{y_1y_2} & \cdots & & \\ k_{y_2y_1} & k_{y_2y_2} & k_{y_2y_3} & \cdots & \\ & & \cdots & \cdots & \\ & & k_{y_{n-1}y_{n-2}} & k_{y_{n-1}y_{n-1}} & k_{y_{n-1}y_n} \\ & & & k_{y_ny_{n-1}} & k_{y_ny_n} + k_{a2} \end{bmatrix},$$

$$\mathbf{K}_{\theta\theta} = \begin{bmatrix} k_{\theta_1\theta_1} & k_{\theta_1\theta_2} & \cdots & & \\ k_{\theta_2\theta_1} & k_{\theta_2\theta_2} & k_{\theta_2\theta_3} & \cdots & \\ & & \cdots & \cdots & \\ & & k_{\theta_{n-1}\theta_{n-2}} & k_{\theta_{n-1}\theta_{n-1}} & k_{\theta_{n-1}\theta_n} \\ & & & k_{\theta_n\theta_{n-1}} & k_{\theta_n\theta_n} + k_{a1}l_{y1}^2 + k_{a2}l_{x2}^2 \end{bmatrix},$$

$$\mathbf{K}_{x\theta} = \mathbf{K}_{\theta x}^{\mathrm{T}} = \mathbf{K}_{xx} e_y, \qquad \mathbf{K}_{y\theta} = \mathbf{K}_{\theta y}^{\mathrm{T}} = \mathbf{K}_{yy} e_x,$$

$$\mathbf{K}_c^{\mathrm{T}} = \begin{bmatrix} 0 & \cdots & -k_{a1} & 0 & \cdots & 0 & 0 & \cdots & -k_{a1} l_{y1} \\ 0 & \cdots & 0 & 0 & \cdots & -m_{a2} & 0 & \cdots & -k_{a2} l_{x2} \end{bmatrix};$$

where $k_{x_i x_i}$, $k_{y_i y_i}$, $k_{\theta_i \theta_i}$ are story stiffness in x, y, and θ directions, respectively; k_{a1} and k_{a2} are stiffness of the two AMD systems.

The structural damping matrix $\mathbf{C}$ is assumed to be proportional to the mass and stiffness matrices as [24]

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K},$$
$$\alpha = \frac{2\omega_i \omega_j (\zeta_i \omega_j - \zeta_j \omega_i)}{\omega_j^2 - \omega_i^2}, \qquad \beta = \frac{2(\zeta_j \omega_j - \zeta_i \omega_i)}{\omega_j^2 - \omega_i^2},$$

in which α and β are the proportional coefficients; ω_i and ω_j are the structural modal frequencies of modes i and j, respectively; and ζ_i and ζ_j are the structural damping ratios for modes i and j.

The excitation influence matrix $\mathbf{E}$ is given by

$$\mathbf{E} = \begin{bmatrix} 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}.$$

The control location matrix $\mathbf{B}_s$ is given by

$$\mathbf{B}_s = \begin{bmatrix} 0 & \cdots & -1 & 0 & \cdots & 0 & 0 & \cdots & l_{y1} & 1 & 0 \\ 0 & \cdots & 0 & 0 & \cdots & -1 & 0 & \cdots & l_{x2} & 0 & 1 \end{bmatrix}.$$

3 Robust optimal H_∞ control via LMI

Active control systems utilize actuators to apply the external control forces to the structure. In this study, the desired control forces are determined and optimized through the following methods Robust optimal H_∞ control via LMI.

Now consider the following uncertain structural system:

$$(\mathbf{M} + \Delta\mathbf{M})\ddot{\mathbf{v}} + (\mathbf{C} + \Delta\mathbf{C})\dot{\mathbf{v}} + (\mathbf{K} + \Delta\mathbf{K})\mathbf{v} = (\mathbf{M} + \Delta\mathbf{M})\mathbf{E}\mathbf{w}(t) + \mathbf{B}_s \mathbf{u}(t). \quad (1)$$

where $\Delta\mathbf{M}$, $\Delta\mathbf{K}$, $\Delta\mathbf{C}$, and $\Delta\mathbf{B}_s$ are corresponding perturbations. The uncertainty $\Delta\mathbf{M}$ is assumed to satisfy the following bound:

$$\left\|\Delta\mathbf{M}\mathbf{M}^{-1}\right\| \leqslant \|\boldsymbol{\delta}\| < 1. \quad (2)$$

Notice that the condition in Eq. (2) ensures that $\mathbf{M} + \Delta\mathbf{M}$ is non-singular. For simplicity, we consider $(\mathbf{I} + \boldsymbol{\delta})(\mathbf{I} + \boldsymbol{\delta}') = \mathbf{I}$, and then Eq. (1) can be rearranged in matrix form as

$$\ddot{\mathbf{v}} + (\mathbf{I} + \boldsymbol{\delta}')\mathbf{M}^{-1}(\mathbf{C} + \Delta\mathbf{C})\dot{\mathbf{v}} + (\mathbf{I} + \boldsymbol{\delta}')\mathbf{M}^{-1}(\mathbf{K} + \Delta\mathbf{K})\mathbf{v}$$
$$= -\mathbf{E}\mathbf{w}(t) + (\mathbf{I} + \boldsymbol{\delta}')\mathbf{M}^{-1}(\mathbf{B}_s + \Delta\mathbf{B}_s)\mathbf{u}(t). \quad (3)$$

In control theory, Eq. (3) can be conveniently rewritten in state-space form as

$$\dot{\mathbf{Z}}(t) = (\mathbf{A} + \Delta\mathbf{A})\mathbf{Z}(t) + (\mathbf{B} + \Delta\mathbf{B})\mathbf{u}(t) + \mathbf{H}\mathbf{w}(t), \tag{4}$$

where

$$\begin{aligned}
\mathbf{Z}(t) &= \begin{bmatrix} \mathbf{v} & \dot{\mathbf{v}}(t) \end{bmatrix}, & \mathbf{H} &= \begin{bmatrix} \mathbf{0} & -\mathbf{E} \end{bmatrix}, \\
\mathbf{A} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, & \Delta\mathbf{A} &= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\mathbf{M}^{-1}\boldsymbol{\delta}_K & -\mathbf{M}^{-1}\boldsymbol{\delta}_C \end{bmatrix}, \\
\mathbf{B} &= \begin{bmatrix} \mathbf{0} & \mathbf{M}^{-1}\mathbf{B}_s \end{bmatrix}, & \Delta\mathbf{B} &= \begin{bmatrix} \mathbf{0} & \mathbf{M}^{-1}\boldsymbol{\delta}_{B_s} \end{bmatrix},
\end{aligned}$$

$$\boldsymbol{\delta}_K = (\mathbf{I}+\boldsymbol{\delta}')\Delta\mathbf{K}+\boldsymbol{\delta}'\mathbf{K}, \qquad \boldsymbol{\delta}_C = (\mathbf{I}+\boldsymbol{\delta}')\Delta\mathbf{C}+\boldsymbol{\delta}'\mathbf{C}, \qquad \boldsymbol{\delta}_{B_s} = (\mathbf{I}+\boldsymbol{\delta}')\Delta\mathbf{B}_s+\boldsymbol{\delta}' B_s.$$

The “ontrolled output” defined in the following:

$$\mathbf{Y}_s = \boldsymbol{\Gamma}\mathbf{Z}(t) = \mathbf{C}_d\mathbf{v} + \mathbf{C}_v\dot{\mathbf{v}}$$

is used to identify response quantities that should be reduced for example, displacement and floor accelerations.

According to the condition in Eq. (2) and Eq. (4), we have

$$\boldsymbol{\delta}_K = \mathbf{L}_k\mathbf{F}_k\mathbf{E}_k, \qquad \boldsymbol{\delta}_C = \mathbf{L}_c\mathbf{F}_c\mathbf{E}_c, \tag{5}$$

where $\|\mathbf{F}_k\| \leqslant 1$, $\|\mathbf{F}_c\| \leqslant 1$, $\mathbf{L}_k$, $\mathbf{E}_k$, $\mathbf{L}_c$, $\mathbf{E}_c$ are known constant matrices, which characterize how the uncertain parameters in $\mathbf{F}_k$, $\mathbf{F}_c$ enter the nominal damping, stiffness and control location matrices $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{B}$, respectively. The uncertainties in structural system (1) satisfying (2), (5) are said to be admissible. Considering the condition in Eq. (5), we can rewritten $\Delta\mathbf{A}$ and $\Delta\mathbf{B}$ as

$$\Delta\mathbf{A} = \mathbf{D}\mathbf{F}(t)\mathbf{E}_1, \qquad \Delta\mathbf{B} = \mathbf{D}\mathbf{F}(t)\mathbf{E}_2, \tag{6}$$

where

$$\begin{aligned}
\mathbf{D} &= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{L}_K & -\mathbf{M}^{-1}\mathbf{L}_C \end{bmatrix}, & \mathbf{E}_1 &= \begin{bmatrix} \mathbf{E}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_c \end{bmatrix}, \\
\mathbf{F} &= \begin{bmatrix} \mathbf{F}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_k \end{bmatrix}, & \mathbf{E}_2 &= \begin{bmatrix} \mathbf{0} \\ (\boldsymbol{\delta}_c\mathbf{E}_c^{-1})^{-1}\boldsymbol{\delta}_{\mathbf{B}_s} \end{bmatrix},
\end{aligned}$$

For the linear time-invariant system in Eq. (4), the performance index J is given by

$$J = \int_0^\infty \bigl(\mathbf{Z}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{Z}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t)\bigr)\,\mathrm{d}t, \tag{7}$$

where $\mathbf{Q} \geqslant 0$ and $\mathbf{R} > 0$ are weighting matrices.

Definition 1. Considering system (4) with (6), (7), and $\mathbf{u}(t) = -\mathbf{K}_u\mathbf{Z}(t)$, the following conditions are satisfied:

(i) The closed-loop system (4) is asymptotically stable.

(ii) A minimization of the performance index J in Eq. (7) results in the optimal controller for the closed-loop system (4).

(iii) With zero initial condition (i.e. $\mathbf{Z}(0) = \mathbf{0}$), the signals $\mathbf{w}(t)$ and $\mathbf{Y}_s$ are bounded by

$$\int_0^\infty \mathbf{Y}_s^{\mathrm{T}}\mathbf{Y}_s\,\mathrm{d}t \leqslant \gamma^2 \int_0^\infty \mathbf{w}^{\mathrm{T}}\mathbf{w}\,\mathrm{d}t, \quad \text{i.e. } \|\mathbf{Y}_s\|_2^2 \leqslant \gamma^2\|\mathbf{w}\|_2^2$$

for all $\mathbf{w} \in L_2[0,\infty)$, $\mathbf{w} \neq 0$, for a constant $\gamma > 0$. In this condition, the system (4) with (6), (7) is said to be stable with disturbance attenuation γ, and control law $\mathbf{u}(t) = -\mathbf{K}_u\mathbf{Z}(t)$ is said to be a robust optimal H_∞ control for system (4) with (6), (7). The parameter γ is said to be the H_∞-norm bound for the robust optimal H_∞ control.

Before we obtain the main result, the following lemmas from [25, 26] are used.

Lemma 1. (*See* [25].) *Consider the uncertain structural system in Eq.* (4) *without excitation and the performance index J in Eq.* (7). *If there exists a positive definite matrix* $\mathbf{P}$ *such that*

$$(\mathbf{A} - \mathbf{B}\mathbf{K}_u + \Delta\mathbf{A}_1)^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{B}\mathbf{K}_u + \Delta\mathbf{A}_1) + \mathbf{Q} + \mathbf{B}\mathbf{K}_u^{\mathrm{T}}\mathbf{R}\mathbf{B}\mathbf{K}_u \tag{8}$$

is satisfied, then, a optimal full-state controller $\mathbf{u}$ *that stabilizes the linear system, Eq.* (4), *is given by* $\mathbf{u}(t) = -\mathbf{K}_u\mathbf{Z}(t)$. *In Eq.* (8), $\Delta\mathbf{A}_1 = \mathbf{D}\mathbf{F}(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u)$.

Lemma 2. (*See* [26].) *Let* $\mathbf{X}_C$, $\mathbf{Y}_C$, $\mathbf{Z}_C$ *be real matrices of appropriate dimensions. Then, for any scalar* $\eta > 0$,

$$\mathbf{X}_C^{\mathrm{T}}\mathbf{Y}_C + \mathbf{Y}_C^{\mathrm{T}}\mathbf{X}_C \leqslant \eta\mathbf{X}_C^{\mathrm{T}}\mathbf{X}_C + \eta^{-1}\mathbf{Y}_C^{\mathrm{T}}\mathbf{Y}_C.$$

Lemma 3. (*See Schur complement of* [26].) *For a given matrix* $\mathbf{S} = \left[\begin{smallmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ * & \mathbf{S}_{22} \end{smallmatrix}\right]$ *with* $\mathbf{S}_{11} = \mathbf{S}_{11}^{\mathrm{T}}$, $\mathbf{S}_{22} = \mathbf{S}_{22}^{\mathrm{T}}$, *then the following conditions are equivalent*:

(i) $\mathbf{S} < 0$;

(ii) $\mathbf{S}_{22} < 0$, $\mathbf{S}_{11} - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\mathbf{S}_{12}^{\mathrm{T}} < 0$.

Now the robust optimal H_∞ control will be designed from the following result.

Theorem 1. *Consider the linear time-invariant system in Eq.* (4) *and the performance index J in Eq.* (7). *For a given scalar* $\gamma > 0$, *if there exists a positive definite matrix* $\mathbf{P} = \mathbf{P}^{\mathrm{T}} > 0$ *such that*

$$\begin{aligned}&(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K}_u - \Delta\mathbf{B}\mathbf{K}_u)^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K}_u - \Delta\mathbf{B}\mathbf{K}_u)\\ &\quad + \mathbf{Q} + \mathbf{K}_u^{\mathrm{T}}\mathbf{R}\mathbf{K}_u + \gamma^{-2}\mathbf{P}\mathbf{H}\mathbf{K}^{\mathrm{T}}\mathbf{P} + \mathbf{\Gamma}^{\mathrm{T}}\mathbf{\Gamma} < 0\end{aligned} \tag{9}$$

is satisfied. Then, the closed-loop system (4) *is robust and optimal, and a robust optimal* H_∞ *full-state controller* $\mathbf{u}$ *that stabilizes the linear system, Eq.* (4)*, is given by* $\mathbf{u}(t) = -\mathbf{K}_u\mathbf{Z}(t)$.

Proof. Define the Lyapunov function as

$$V\big(\mathbf{Z}(t)\big) = \mathbf{Z}(t)^{\mathrm{T}}\mathbf{P}\mathbf{Z}(t), \tag{10}$$

where $\mathbf{P} = \mathbf{P}^{\mathrm{T}} > 0$. The time derivative of the Lyapunov function in (10), along the trajectories of (4) without excitation and $\mathbf{u}(t) = -\mathbf{K}_u\mathbf{Z}(t)$ is given by

$$\begin{aligned}\dot V\big(\mathbf{Z}(t)\big) &= \mathbf{Z}(\dot t)^{\mathrm{T}}\mathbf{P}\mathbf{Z}(t) + \mathbf{Z}(t)\mathbf{P}\mathbf{Z}(\dot t)^{\mathrm{T}} \\ &= \mathbf{Z}(t)^{\mathrm{T}}\big[(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K}_u - \Delta\mathbf{B}\mathbf{K}_u)^{\mathrm{T}}\mathbf{P} \\ &\quad + \mathbf{P}(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K}_u - \Delta\mathbf{B}\mathbf{K}_u)\big]\mathbf{Z}(t).\end{aligned} \tag{11}$$

From (11) with (9) and $\mathbf{w}(t) = 0$, we have

$$\dot V\big(\mathbf{Z}(t)\big) < -\mathbf{Z}(t)^{\mathrm{T}}\big[\mathbf{Q} + \mathbf{K}_u^{\mathrm{T}}\mathbf{R}\mathbf{K}_u + \gamma^{-2}\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} + \mathbf{\Gamma}^{\mathrm{T}}\mathbf{\Gamma}\big]\mathbf{Z}(t).$$

Hence the closed system (4) with $\mathbf{u}(t) = -\mathbf{K}_u\mathbf{Z}(t)$ and $\mathbf{w}(t) = 0$ is asymptotically stable with performance index γ, $\gamma > 0$.

According to Lemma 1, a minimization of the performance index J in Eq. (7) results in the optimal controller for the closed-loop system (4).

According to the bounded real lemma [27], for the linear time-invariant system, such as equation (4), the controller is designed such that the ratio of the L_2 norm of the control output (i.e. $\mathbf{Y}_s$) to the L_2 norm of the disturbance (i.e.w), with zero initial conditions, is smaller than γ. That is

$$\|\mathbf{Y}_s\|_2 < \gamma\|\mathbf{w}\|, \tag{12}$$

in which γ is the disturbance attenuation which is a measure of performance, and the L_2 norm is defined by

$$\|\mathbf{Y}_s\|_2^2 = \int_0^\infty \mathbf{Y}_s^{\mathrm{T}}\mathbf{Y}_s\,\mathrm{d}t, \qquad \|\mathbf{w}\|_2^2 = \int_0^\infty \mathbf{w}^{\mathrm{T}}\mathbf{w}\,\mathrm{d}t. \tag{13}$$

From (12) and (13), we have

$$\int_0^\infty \big(\mathbf{Y}_s^{\mathrm{T}}\mathbf{Y}_s - \gamma^2\mathbf{w}^{\mathrm{T}}\mathbf{w}\big)\,\mathrm{d}t < 0. \tag{14}$$

According to Eq. (10) and Eq. (14), we have

$$\begin{aligned}&\int_0^\infty \big(\mathbf{Y}_s^{\mathrm{T}}\mathbf{Y}_s - \gamma^2\mathbf{w}^{\mathrm{T}}\mathbf{w} + \dot V\mathbf{Z}(t)\big)\,\mathrm{d}t \\ &\quad = \int_0^\infty \big(\mathbf{Y}_s^{\mathrm{T}}\mathbf{Y}_s - \gamma^2\mathbf{w}^{\mathrm{T}}\mathbf{w}\big)\,\mathrm{d}t + V\mathbf{Z}(\infty) - V\mathbf{Z}(0).\end{aligned} \tag{15}$$

From (14) with (15), $V\mathbf{Z}(\infty) = 0$ and $\mathbf{Z}(0) = 0$, we have

$$\mathbf{Y}_s^{\mathrm{T}}\mathbf{Y}_s - \gamma^2\mathbf{w}^{\mathrm{T}}\mathbf{w} + \dot{V}\mathbf{Z}(t) < 0. \tag{16}$$

According to Eq. (10), Eq. (16) can be written as

$$\begin{aligned} &\mathbf{Y}_s^{\mathrm{T}}\mathbf{Y}_s - \gamma^2\mathbf{w}^{\mathrm{T}}\mathbf{w} + \dot{V}\mathbf{Z}(t) \\ &\quad = \mathbf{Z}(t)^{\mathrm{T}}\big[(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K}_u - \Delta\mathbf{B}\mathbf{K}_u)^{\mathrm{T}}\mathbf{P} \\ &\qquad + \mathbf{P}(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K}_u - \Delta\mathbf{B}\mathbf{K}_u)\big]\mathbf{Z}(t) \\ &\qquad + \mathbf{w}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{Z}(t) + \mathbf{Z}(t)^{\mathrm{T}}\mathbf{P}\mathbf{H}\mathbf{w} + \mathbf{Z}(t)^{\mathrm{T}}\mathbf{\Gamma}^{\mathrm{T}}\mathbf{\Gamma}\mathbf{Z}(t) - \gamma^2\mathbf{w}^{\mathrm{T}}\mathbf{w} < 0. \end{aligned} \tag{17}$$

By Lemma 2, the following inequality also holds:

$$\mathbf{Z}(t)^{\mathrm{T}}\mathbf{P}\mathbf{H}\mathbf{w} + \mathbf{w}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{Z}(t) \leqslant \gamma^{-2}\mathbf{Z}(t)^{\mathrm{T}}\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{Z}(t) + \gamma^2\mathbf{w}^{\mathrm{T}}\mathbf{w}. \tag{18}$$

From (17) with (18), we have

$$\begin{aligned} &\mathbf{Z}(t)^{\mathrm{T}}\big[(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K}_u - \Delta\mathbf{B}\mathbf{K}_u)^{\mathrm{T}}\mathbf{P} \\ &\quad + \mathbf{P}(\mathbf{A} + \Delta\mathbf{A} - \mathbf{B}\mathbf{K}_u - \Delta\mathbf{B}\mathbf{K}_u)\mathbf{Z}(t) \\ &\quad + \gamma^{-2}\mathbf{Z}(t)^{\mathrm{T}}\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{Z}(t) + \mathbf{Z}(t)^{\mathrm{T}}\mathbf{\Gamma}^{\mathrm{T}}\mathbf{\Gamma}\mathbf{Z}(t) < 0. \end{aligned} \tag{19}$$

From Eq. (9), we can get Eq. (19). Hence the closed system (4) with the robust optimal H_∞ full-state controller $\mathbf{u}(t) = -\mathbf{K}_u\mathbf{Z}(t)$ is asymptotically stable with performance index γ, $\gamma > 0$. □

Theorem 2. *Consider the uncertain structural system in Eq.* (4) *and the performance index J in Eq.* (7)*. For a given scalar $\gamma > 0$, $\epsilon > 0$, if there exists a positive definite matrix $\mathbf{N}$ and a matrix $\mathbf{Y}$ such that*

$$\begin{bmatrix} \mathbf{\Pi}_1 & \mathbf{N} & \mathbf{Y}^{\mathrm{T}} & \mathbf{H} & \mathbf{N}\mathbf{\Gamma}^{\mathrm{T}} & \mathbf{\Pi}_2 \\ * & -\mathbf{Q}^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\mathbf{R}^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\gamma^2\mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\mathbf{I} & 0 \\ * & * & * & * & * & -\epsilon^{-1}\mathbf{I} \end{bmatrix} < 0 \tag{20}$$

is satisfied. Then, the closed-loop system (4) *is robust and optimal, and a full-state controller $\mathbf{u}$ that stabilizes the linear system, Eq.* (4)*, is given by*

$$\mathbf{u}(t) = -\mathbf{K}_u\mathbf{Z}(t) = -\mathbf{Y}\mathbf{N}^{-1}\mathbf{Z}(t),$$

where $\mathbf{\Pi}_1 = \mathbf{N}\mathbf{A}^{\mathrm{T}} - \mathbf{Y}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{A}\mathbf{N} - \mathbf{B}\mathbf{Y} - \epsilon^{-1}\mathbf{D}\mathbf{D}^{\mathrm{T}}$, $\mathbf{\Pi}_2 = (\mathbf{E}_1\mathbf{N} - \mathbf{E}_2\mathbf{Y})^{\mathrm{T}}$, the notation $*$ is used to represent a matrix which is inferred from symmetry.

Proof. According to Lemma 2, there exists

$$\begin{aligned}
&(\Delta\mathbf{A} - \Delta\mathbf{B}\mathbf{K}_u)^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\Delta\mathbf{A} - \Delta\mathbf{B}\mathbf{K}_u) \\
&\quad = (\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u)^{\mathrm{T}}\mathbf{F}^{\mathrm{T}}\mathbf{D}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{D}\mathbf{F}(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u) \\
&\quad \leqslant \epsilon(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u)^{\mathrm{T}}\mathbf{F}^{\mathrm{T}}\mathbf{F}(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u) + \epsilon^{-1}\mathbf{P}\mathbf{D}\mathbf{D}^{\mathrm{T}}\mathbf{P} \\
&\quad \leqslant \epsilon(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u)^{\mathrm{T}}(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u) + \epsilon^{-1}\mathbf{P}\mathbf{D}\mathbf{D}^{\mathrm{T}}\mathbf{P}.
\end{aligned} \tag{21}$$

From (8) with (21), we have

$$\begin{aligned}
&(\mathbf{A} - \mathbf{B}\mathbf{K}_u)^{\mathrm{T}}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{B}\mathbf{K}_u) + \mathbf{Q} + \mathbf{K}_u^{\mathrm{T}}\mathbf{R}\mathbf{K}_u + \gamma^{-2}\mathbf{P}\mathbf{H}\mathbf{H}^{\mathrm{T}}\mathbf{P} \\
&\quad + \boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{\Gamma} + \epsilon(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u)^{\mathrm{T}}(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u) + \epsilon^{-1}\mathbf{P}\mathbf{D}\mathbf{D}^{\mathrm{T}}\mathbf{P} < 0.
\end{aligned} \tag{22}$$

Pre- and post-multiplying the matrix in (22) by $\mathbf{P}^{-1}$ and $\mathbf{P}^{-T}$ with $\mathbf{P}^{-T} = \mathbf{P}^{-1}$, respectively, we can obtain the following inequality:

$$\begin{aligned}
&\mathbf{P}^{-1}\mathbf{A}^{\mathrm{T}} - \mathbf{P}^{-1}\mathbf{K}_u^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{A}\mathbf{P}^{-1} - \mathbf{B}\mathbf{K}_u\mathbf{P}^{-1} + \mathbf{P}^{-1}\mathbf{Q}\mathbf{P}^{-1} \\
&\quad + \mathbf{P}^{-1}\mathbf{K}_u^{\mathrm{T}}\mathbf{R}\mathbf{K}_u\mathbf{P}^{-1} + \gamma^{-2}\mathbf{H}\mathbf{H}^{\mathrm{T}} + \mathbf{P}^{-1}\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{\Gamma}\mathbf{P}^{-1} \\
&\quad + \epsilon\mathbf{P}^{-1}(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u)^{\mathrm{T}}(\mathbf{E}_1 - \mathbf{E}_2\mathbf{K}_u)\mathbf{P}^{-1} + \epsilon^{-1}\mathbf{D}\mathbf{D}^{\mathrm{T}} < 0.
\end{aligned}$$

Define the new variable $\mathbf{P}^{-1} = \mathbf{N}$, $\mathbf{K}_u\mathbf{P}^{-1} = \mathbf{Y}$, we can obtain $\mathbf{K}_u = \mathbf{Y}\mathbf{N}^{-1}$ and the following inequality:

$$\begin{aligned}
&\mathbf{N}\mathbf{A}^{\mathrm{T}} - \mathbf{Y}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}} + \mathbf{A}\mathbf{N} - \mathbf{B}\mathbf{Y} + \mathbf{N}\mathbf{Q}\mathbf{N} + \mathbf{Y}^{\mathrm{T}}\mathbf{R}\mathbf{Y} + \gamma^{-2}\mathbf{H}\mathbf{H}^{\mathrm{T}} \\
&\quad + \mathbf{N}\boldsymbol{\Gamma}^{\mathrm{T}}\boldsymbol{\Gamma}\mathbf{N} + \epsilon(\mathbf{E}_1\mathbf{N} - \mathbf{E}_2\mathbf{Y})^{\mathrm{T}}(\mathbf{E}_1\mathbf{N} - \mathbf{E}_2\mathbf{Y}) - \epsilon^{-1}\mathbf{D}\mathbf{D}^{\mathrm{T}} < 0.
\end{aligned} \tag{23}$$

From (23) with Lemma 3, we have LMI (20). This proof can be completed. □

4 Numerical results

In this section, to verify the effectiveness of the control method, a three storey eccentric building structure with two active mass damper (AMD) systems on the orthogonal direction of the top storey subjected to bi-directional ground motions is analyzed. The mass and stiffness coefficient of each storey unit are $m_i = 500$ metric ton, $k_{xi} = 490000$ kN/m, $k_{yi} = 98000$ kN/m, and $k_{\theta i} = 25000000000$ kN/m, respectively. Floor mass polar moment of inertia is $J_i = 3 \cdot 10^5$ kg m^2. Two-way floor eccentricities between the center of mass and the center of stiffness along the x and y directions are $e_x = 3$ m and $e_y = 2$ m. The damper ratio of the structure is 0.01. The mass and stiffness coefficient of the two AMD systems are $m_a = 50$ metric ton and $k_a = 1970$ kN/m, respectively. As a dynamic effect, the east-west acceleration component and the north-south acceleration component of the 1940 EI Centro earthquake records scaled to a maximum ground acceleration of 0.18 g are used as the input excitation, and they are shown in Fig. 2.

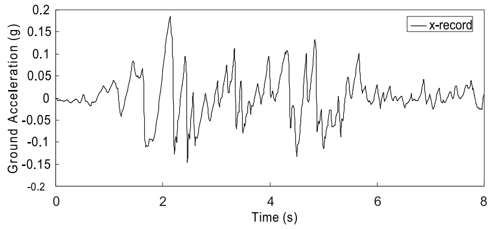


Fig. 2. The acceleration component of 1940 EI Centro earthquake.

Table 1. Maximum response quantities.

Cases	Storey	Displacement			Acceleration			Control force	
		x cm	y cm	θ 10^{-2} rad	$\ddot{x}$ cm s^{-2}	$\ddot{y}$ cm s^{-2}	$\ddot{\theta}$ 10^{-2} rad s^{-2}	AMD1 kN	AMD2 kN
Uncontrolled	1	2.46	6.07	0.13	546	417	23.54		
	2	4.30	10.31	0.24	796	519	36.28		
	3	5.25	12.31	0.30	889	677	41.34		
$H(0\%)$	1	1.62	4.41	0.09	417	333	15.86	311	256
	2	2.79	7.56	0.16	542	399	23.31		
	3	3.41	9.06	0.20	657	494	27.87		
$H(-20\%)$	1	2.28	3.51	0.13	455	251	40.1	344	197
	2	3.95	6.49	0.22	643	322	59.47		
	3	4.85	8.27	0.27	722	393	34.92		
$H(-50\%)$	1	2.59	5.97	0.12	397	295	18.05	50425	33710
	2	5.08	10.89	0.2	463	227	22.98		
	3	7.26	16.26	0.25	630	382	28.39		

According to [27], we design an H_∞ state feedback controller for the nominal system (i.e. the system has no parametric uncertainties). Using $\gamma = 31.62$ and $\mathbf{\Gamma} = \mathbf{I}_{22}$, we can get the following control gain:

$$\mathbf{K}_u = 10^8 \begin{bmatrix} 0.85 & -3.19 & 36.98 & 2.24 & 9.55 & -0.92 & 125.60 & 630 & 251.55 & -1.07 & 0.34 \\ 9.71 & 34.14 & 11.05 & 2.83 & 5.48 & 6.58 & 179.01 & 915.25 & 398.98 & 0.31 & -0.83 \end{bmatrix}$$
$$\times \begin{bmatrix} -0.1 & -0.24 & -0.16 & 0.09 & 0.19 & 0.04 & -0.12 & 0.10 & 1.36 & 0.34 & 0.28 \\ -0.00 & -0.02 & -0.05 & 0.17 & 0.32 & -0.22 & -0.2 & 0.11 & 2.07 & 0.29 & 0.57 \end{bmatrix}.$$

In the different cases, the maximum displacement and the maximum acceleration of each storey unit respect to the ground are presented in Table 1. In Table 1, the response quantities for the system without control devices are designated as "Uncontrolled", the response quantities for the nominal system (with AMD control devices) are designated as "$H(0\%)$". When we vary the stiffness and damper of all storey unit of the building by -20%, the response quantities for the system (with AMD control devices) are designated as "$H(-20\%)$". When we vary the stiffness and damper of all storey unit of the building by -50%, the response quantities for the system (with AMD control devices) are designated as "$H(-50\%)$".

From Table 1, we can see clearly that the structure system will be unstable as the system has bigger parametric uncertainties. To guarantee the robust stability performance

Table 2. Maximum response quantities.

Cases	Storey	Displacement x cm	y cm	θ 10^{-2} rad	Acceleration $\ddot{x}$ cm s^{-2}	$\ddot{y}$ cm s^{-2}	$\ddot{\theta}$ 10^{-2} rad s^{-2}	Control force AMD1 kN	AMD2 kN
	1	2.46	6.07	0.13	546	417	23.54		
Uncontrolled	2	4.30	10.31	0.24	796	519	36.28		
	3	5.25	12.31	0.30	889	677	41.34		
	1	1.63	4.11	0.08	409	323	15.47		
RH1(0%)	2	2.80	7.04	0.15	524	373	22.46	253	213
	3	3.38	8.55	0.19	621	484	25.86		
	1	1.93	3.14	0.11	439	302	21.01		
RH1(−20%)	2	3.36	5.90	0.18	615	296	29.78	276	138
	3	4.10	7.83	0.22	717	391	31.90		
	1	2.12	2.92	0.10	364	292	17.47		
RH1(−50%)	2	4.00	5.33	0.17	502	236	25.25	245	179
	3	5.03	7.15	0.21	641	400	29.78		
	1	1.30	3.98	0.07	349	320	21.42		
RH2(0%)	2	2.23	6.90	0.13	470	366	26.15	399	307
	3	2.71	8.29	0.17	573	460	24.47		
	1	1.78	3.06	0.09	410	282	22.38		
RH2(−20%)	2	3.11	5.75	0.16	576	306	31.51	482	208
	3	3.82	7.46	0.20	609	362	29.37		
	1	1.88	2.99	0.09	333	286	19.27		
RH2(−50%)	2	3.60	5.35	0.14	485	235	25.34	444	306
	3	4.63	6.75	0.17	630	375	32.55		

of the structural system in the presence of parameter uncertainties, we can design robust optimal H_∞ controller on the basis of Section 3. The uncertainties in the damping, stiffness matrices etc. are, respectively, modeled as

$$\Delta\mathbf{M} = 0.05\mathbf{M}, \qquad \Delta\mathbf{K} = 0.5\mathbf{K}, \qquad \Delta\mathbf{C} = 0.5\mathbf{C}, \qquad \Delta\mathbf{B}_s = 0.25\mathbf{B}_s.$$

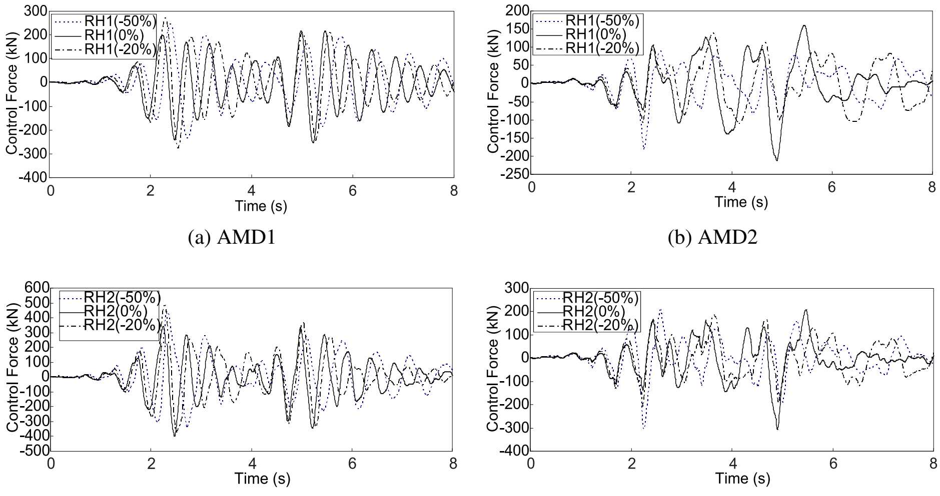
In order to have a comparison of the robust stability between the traditional H_∞ controller and the optimal H_∞ controller based on the almost same maximum response quantities, the following weighing matrices are used:

$$\begin{aligned}\mathbf{Q} = {}& 10^{-4}\,\mathrm{diag}\left[\,10^9\ \ 10^9\ \ 0.5\cdot 10^9\ \ 2\cdot 10^8\ \ 2\cdot 10^8\ \ 10^8\ \ 5\cdot 10^{10}\ \ 5\cdot 10^{10}\ \ 2.5\cdot 10^{10}\ \ 2\cdot 10^6\ \ 2\cdot 10^6\,\right]\\ &\times\left[\,5\cdot 10^7\ \ 5\cdot 10^7\ \ 5\cdot 10^7\ \ 5\cdot 10^7\ \ 5\cdot 10^7\ \ 5\cdot 10^7\ \ 3\cdot 10^7\ \ 3\cdot 10^7\ \ 3\cdot 10^7\ \ 10^6\ \ 10^6\,\right],\\ \mathbf{R} = {}& \mathrm{diag}\left[10^{-5}\ \ 10^{-5}\right] \quad \text{in Eq. (7).}\end{aligned}$$

Using $\gamma = 2.79\cdot 10^4$, $\epsilon = 1.38\cdot 10^6$ and $\mathbf{E}_1 = \mathbf{\Gamma} = \mathbf{I}_{22}$, (20) results in the following control gain:

$$\begin{aligned}\mathbf{K}_u = 10^5 &\begin{bmatrix} -0.1882 & -44 & 20.38 & 3.07 & 3.67 & -0.63 & 12.15 & 55.80 & 128.53 & 14.54 & -0.04\\ 0.49 & 0.31 & -5.83 & 2.92 & -23.97 & -49.68 & 18.23 & 83.99 & 198.39 & -0.09 & 15.12 \end{bmatrix}\\ &\times\begin{bmatrix} -0.91 & -2.25 & 4.42 & 0.07 & 0.13 & 0.10 & 0.62 & 1.27 & 19.38 & 7.59 & 0.08\\ 0.10 & 0.11 & 0.01 & 0.67 & -0.16 & 4.55 & 0.89 & 1.84 & 30.19 & 0.09 & 8.04 \end{bmatrix}.\end{aligned}$$

In the different cases, the maximum displacement and the maximum acceleration of each storey unit respect to the ground are presented in Table 2. The control force time



(a) AMD1 (b) AMD2

(c) AMD1 (d) AMD2

Fig. 3. Control force time histories of the structure with two AMD systems: (a), (b) in the “RH1” cases; (c), (d) in the “RH2” cases.

histories of AMD1 and AMD2 are presented in Fig. 3(a), (b). In Table 2 and Fig. 3(a), (b), the response quantities for the nominal system (with AMD control devices) are designated as “$\mathrm{RH1}(0\%)$”. When we vary the stiffness and damper of all storey unit of the building by -20%, the response quantities for the system (with AMD control devices) are designated as “$\mathrm{RH1}(-20\%)$”. When we vary the stiffness and damper of all storey unit of the building by -50%, the response quantities for the system (with AMD control devices) are designated as “$\mathrm{RH1}(-50\%)$”.

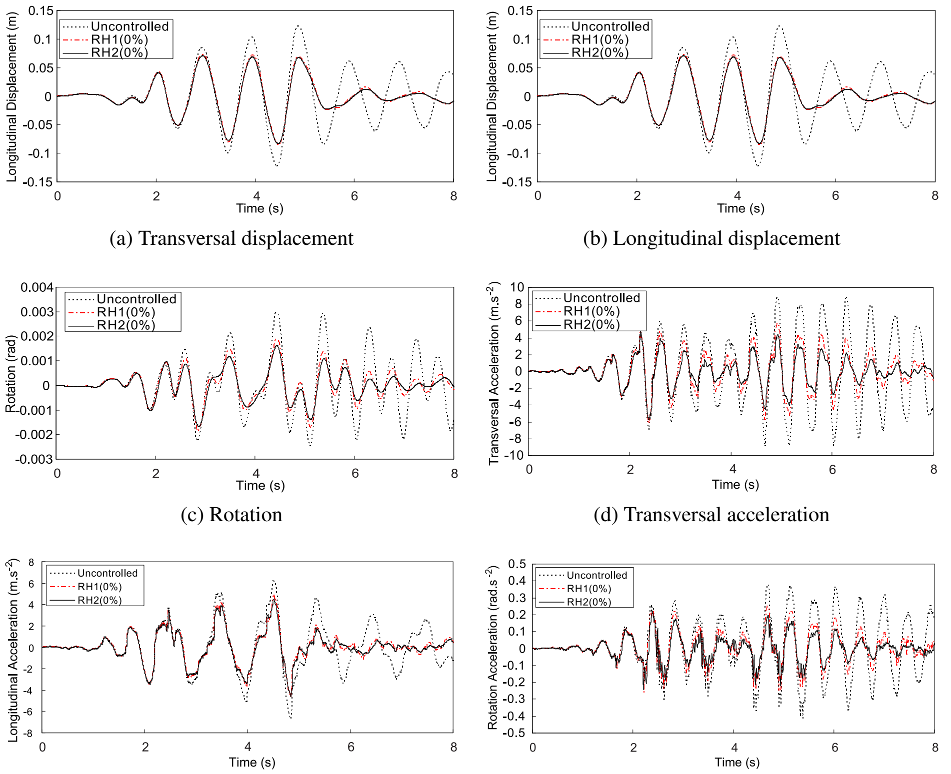
From Table 2 and Fig. 3(a), (b), we can see clearly that the new robust optimal H_∞ controller can be designed to guarantee the robust stability and performance of the closed-loop system in the presence of parameter uncertainties.

In order to have a better control effectiveness in comparison with the “RH1” cases, the following weighing matrices are used:

$$\begin{aligned}\mathbf{Q} &= 10^{-3}\,\mathrm{diag}\left[\,10^9\ \ 10^9\ \ 0.5\cdot 10^9\ \ 2\cdot 10^8\ \ 2\cdot 10^8\ \ 10^8\ \ 5\cdot 10^{10}\ \ 5\cdot 10^{10}\ \ 2.5\cdot 10^{10}\ \ 2\cdot 10^6\ \ 2\cdot 10^6\,\right],\\ &\quad \times\left[\,5\cdot 10^7\ \ 5\cdot 10^7\ \ 5\cdot 10^7\ \ 5\cdot 10^7\ \ 5\cdot 10^7\ \ 5\cdot 10^7\ \ 3\cdot 10^7\ \ 3\cdot 10^7\ \ 3\cdot 10^7\ \ 10^6\ \ 10^6\,\right],\\ \mathbf{R} &= \mathrm{diag}\left[10^{-8}\ \ 10^{-8}\right] \quad \text{in Eq. (7).}\end{aligned}$$

Using $\gamma = 2.79\cdot 10^4$, $\epsilon = 3.08\cdot 10^6$ and $\mathbf{E}_1 = \mathbf{\Gamma} = I_{22}$, (20) results in the following control gain:

$$\begin{aligned}\mathbf{K}_u = 10^6 &\begin{bmatrix} 14.15 & -27.03 & -53.28 & 0.55 & 2.19 & 5.79 & -2.14 & -0.43 & 228.70 & 8.84 & -1.39\\ 2.36 & 5.96 & 4.28 & 11.34 & 25.46 & -57.69 & -2.71 & -1.749 & 334.12 & -1.18 & 7.85 \end{bmatrix}\\ &\times\begin{bmatrix} 0.16 & -0.49 & 1.43 & -0.10 & -0.03 & -0.19 & -0.03 & -0.04 & 8.56 & 3.41 & -0.19\\ 0.09 & 0.16 & -0.03 & 0.47 & 0.70 & 1.72 & -0.05 & -0.07 & 13.04 & -0.15 & 3.33 \end{bmatrix}.\end{aligned}$$



(a) Transversal displacement

(b) Longitudinal displacement

(c) Rotation

(d) Transversal acceleration

(e) Longitudinal acceleration

(f) Rotation acceleration

Fig. 4. Comparisons of response quantities between the “RH1” case and the the “RH2” case.

In the different cases, the maximum displacement and the maximum acceleration of each storey unit respect to the ground are presented in Table 2. The control force time histories of AMD1 and AMD2 are presented in Fig. 3(c), (d). In Table 2 and Fig. 3(c), (d), the response quantities for the nominal system (with AMD control devices) are designated as “$\mathrm{RH2}(0\%)$”. When we vary the stiffness and damper of all storey unit of the building by -20%, the response quantities for the system (with AMD control devices) are designated as “$\mathrm{RH2}(-20\%)$”. When we vary the stiffness and damper of all storey unit of the building by -50%, the response quantities for the system (with AMD control devices) are designated as “$\mathrm{RH2}(-50\%)$”.

We can see clearly from Table 2 and Fig. 3(c), (d) that it can get a better control effectiveness through the adjustment of weighing matrices.

The comparisons of the response time histories of the structure between the “$\mathrm{RH1}(0\%)$ case and the “$\mathrm{RH2}(0\%)$” case are shown in Fig. 4.

We can see clearly from Fig. 4 that considering reasonable weighting matrices, the controller can acquire considerably optimal response quantities under the better performance of robust.

5 Conclusions

AMD systems have been a popular area of research in recent decades and significant progress has been made in this area over these years. In view of the numerous uncertainties of seismic disturbances and structural parameters, the building structure is uncertain. Because robust controllers can tolerate uncertainties, control of a building structures seismic response is an ideal application. In this paper, considering that a real building is actually asymmetric to some degree even with a nominally symmetric plan, a robust optimal H_∞ control algorithm for two AMD systems on the orthogonal direction of the top storey is proposed for the vibration control of irregular buildings subjected to bi-lateral ground acceleration. Simulation results show preliminary that such a control method can guarantee the robust stability and performance of the closed-loop system in the presence of parameter uncertainties. In addition, the comparisons with the numerical results using the traditional H_∞ control are also made for demonstration of its control performance. For practical implementations of active control systems on seismically excited irregular buildings, it is unavoidable to consider the time delay in control, sensor and actuator failures. So for the further study, it is important to design the robust controllers in the presence of the complicated environment.

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