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A survey on stationary problems, Green's functions and spectrum of Sturm–Liouville problem with nonlocal boundary conditions*

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This paper is dedicated to Professor M. Sapagovas 75th Anniversary

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Abstract. In this paper, we present a survey of recent results on the Green's functions and on spectrum for stationary problems with nonlocal boundary conditions. Results of Lithuanian mathematicians in the field of differential and numerical problems with nonlocal boundary conditions are described.

Keywords: stationary problem, nonlocal boundary conditions, Green's function, Sturm–Liouville problem, spectrum.

1 Introduction

In the theory of differential equations, the basic concepts have been formulated studying the problems of classical mathematical physics. However, the modern problems motivate to formulate and investigate the new ones, for example, a class of nonlocal problems. Nonlocal conditions arise when we cannot measure data directly at the boundary. In this case, the problem is formulated, where the value of the solution and/or a derivative is linked to a few points or the whole interval.

A review on differential equations with more general boundary conditions (BC) involving also Stieltjes measures has been written by Whyburn [232]. In 1963, Cannon [26] formulated new problem with BCs, which are now called *nonlocal*. The term "nonlocal boundary value problem" most likely to have been used by Beals in 1964 [12, 13]. He investigated elliptic type differential equations with nonclassical BCs. A parabolic problem with integral boundary condition

$$\int_{x_1(t)}^{x_3(t)} g(x,t)u(x,t)\,\mathrm{d}t = E(t) \qquad (1)$$

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was investigated by Kamynin in 1964 [109]. Samarskii and Bitsadze investigated nonlocal problem for general elliptic equation and, in the case of rectangular space domain and Laplace operator, applying methods of the theory of integral equations. They proved the existence and uniqueness of solution in 1969 [19]. Now some one-dimensional nonlocal boundary conditions (NBC)

$$u(0) = \gamma_0 u(\xi_0) \quad \text{or} \quad u(1) = \gamma_1 u(\xi_1) \tag{2}$$

are called as *Bitsadze–Samarskii type* NBCs. Differential equations (for example, ordinary, elliptic, parabolic etc.) with various types of nonlocal conditions were investigated by many scientists. Two-points NBCs we can find in [90]. Il'in [78] and Moiseev [80, 81] investigated *multi-points* NBCs (the study of multi-point BCs was initiated by Picone [159])

$$u(0) = \sum_{i=1}^{m} \gamma_i u(\xi_i), \qquad u(1) = \sum_{i=1}^{m} \delta_i u(\xi_i). \tag{3}$$

Ionkin [87] considered parabolic problem with

$$u(0,t) = \nu(t), \qquad \int_0^1 u(1,t) = \mu(t). \tag{4}$$

In [91], he investigate stability FDS for a parabolic equation with NBCs

$$u(0,t) = 0, \qquad u_x(1,t) = u_x(0,t) \tag{5}$$

(*Samarskii–Ionkin type* NBCs).

Day investigated the heat equation subject to linear thermoelasticity with integral BCs [43, 44]

$$u(0,t) = \gamma_0 \int_{-1}^{1} \alpha_0(x) u(x,t)\,\mathrm{d}x, \qquad u(1,t) = \gamma_1 \int_{-1}^{1} \alpha_1(x) u(x,t)\,\mathrm{d}x. \tag{6}$$

Henderson proved existence theorems for BVPs for nth-order nonlinear difference equations [72]. A survey of numerical methods for the one-dimensional parabolic equation subject to nonlocal conditions was done by Dehghan [45]. Webb [229, 230], Infante [85, 86] investigated quasilinear problems with Bitsadze–Samarskii type NBCs and had proved results about the properties of positive solutions. We can find more results on the existence and multiplicity of solutions of nonlocal BVP involving the second-order ordinary differential equations in surveys of Ma [125] and Ntouyas [144]. In 2011, special issue for Nonlocal Boundary Conditions (27 articles) was published in the journal *Boundary Value Problems* [52]. Articles in this special issue deal with BVPs with NCs for ordinary, discrete, impulsive, neutral, parabolic, fractional and time scales equations. Existence, nonexistence, multiplicity, asymptotic behavior and approximation of solutions are investigated by using several methods as fixed point theorems, fixed point index, variational methods, iterative techniques, bifurcation theory, lower and upper solutions.

One of the objectives of this study is to describe the results of Lithuanian mathematicians in the field of differential and numerical problems with NBCs. Prof. M. Sapagovas was not only a pioneer in the study of such problems, but also the founder of the scientific school in Vilnius. The first problem with NBCs came from the applications, and it was the investigation of a mercury droplet in electric contact, given the droplet volume [174–176, 178–180]. Difference scheme for two-dimensional elliptic problem with an integral condition was constructed in [177]. Scientific supervisor of Sapagovas (Kiev, 1963–1965) Prof. V. Makarov also began investigating problems with NBCs [127–129]. Sapagovas and his doctoral student (Vilnius, 1982–1985) Čiegis investigated elliptic and parabolic problems with integral and Bitsadze–Samarskii type NBCs and finite-difference schemes for them [31, 181, 182]. They published some new results about numerical solutions for problems with NBCs in [32, 183, 190, 191].

Sturm–Liouville Problem (SLP) is very important for investigation of existence and uniqueness of solutions for classical stationary problems. This problem is very complicated because problem with NBCs are not self-adjoint and spectrum for such problems may be not positive (or real). Ionkin and Valikova investigated SLP with NBCs [92]

$$u(0) = u(1), \qquad u_x(1) = 0. \tag{7}$$

Gulin et al. [56,60] considered two-dimentional parabolic equation with Samarskii–Ionkin type NBCs in one spatial direction. They investigated a spectrum for one-dimensional SLPs and proved stability for FDS. Gulin, Morozova and Udovichenko investigated parabolic problems with two parameters in NBCs [54, 55, 57, 59, 61–66]

$$u(0,t) = \alpha u(1,t), \qquad u_x(1,t) = \gamma u_x(0,t). \tag{8}$$

Gulin and Mokin have similar results for NBCs [58, 138]

$$u(0,t) = 0, \qquad u_x(1,t) = u_x(0,t) + \alpha u(1,t). \tag{9}$$

Stability of a family of difference schemes for the Samarskii–Ionkin problem with variable coefficient is obtained in [139].

Sapagovas with co-authors began to investigate eigenvalues for Bitsadze–Samarskii type

$$u(0) = 0, \quad u(1) = \gamma u(\xi), \quad 0 < \xi < 1, \tag{10}$$

and integral type NBCs [41, 184, 185, 197]

$$u(0) = \gamma_0 \int_0^1 \alpha_0(x) u(x)\,\mathrm{d}x, \qquad u(1) = \gamma_1 \int_0^1 \alpha_1(x) u(x)\,\mathrm{d}x. \tag{11}$$

They showed that there exists eigenvalues, which do not depend on parameters γ_0 or γ_1 in boundary conditions (see Eqs. (2) and (6)) and complex eigenvalues may exist. Sapagovas with co-authors investigated the spectrum of discrete SLP, too. These results can be applied to prove stability of Finite-Difference Schemes (FDS) for nonstationary problems and convergence of iterative methods. Numerical methods were proposed for parabolic and iterative methods for solving two-dimensional elliptic equation with Bitsadze–Samarskii or integral type NBCs: Alternating Direction Method (ADM) for a two-dimensional parabolic equation with a NBC [194, co-authors: Kairytė, Štikonienė, Štikonas], FDS of increased order of accuracy for the Poisson equation with NCs [188], FDS for two-dimensional elliptic equation with a NC [101, co-authors: Jakubėlienė, Čiupaila], a fourth-order ADM for difference schemes with NC [198, co-author: Štikonienė] ADM for the Poisson equation with variable weight coefficients in an integral condition [196, co-authors: Štikonienė, Štikonas]; ADM for a mildly nonlinear elliptic equation with integral type NCs [199, co-author: Štikonienė], FDS for nonlinear elliptic equation with NC [40, co-authors: Čiupaila, Štikonienė]. Spectral analysis was applied for two- and three-layer FDS for parabolic equations with NBCs: FDS for one-dimensional differential operator with integral type NCs [171, co-author: Sajavičius], [195, co-authors: Ivanauskas, Meškauskas], [189]. Stability analysis was done for FDS in the case of one- and two-dimensional parabolic equation with NBCs [187], [93, co-authors: Ivanauskas, Meškauskas]. In [38] and in doctoral dissertation of Tumanova (scientific supervisor Čiegis) [223], the one-dimensional parabolic equation with three types of integral NBCs was approximated by the implicit Euler FDS and stability analysis was done. Some new results published by doctoral students (scientific supervisor Sapagovas): Jesevičiūtė (Jokšienė) investigate stability of the FDS for parabolic equations subject to integral conditions with applications for thermoelasticity [106, 107], Jakubėlienė investigates two-dimensional problems with double integral in BC [100, 102, 193], Jachimavičienė investigates pseudo-parabolic equation with NBCs and FDS [96, 97, 99]. The stability of explicit three-layer FDS for pseudo-parabolic problems were investigated in papers [98, co-authors: Jachimavičienė, Štikonienė, Štikonas], [39, Čiegis, Tumanova]. Analogues of results for hyperbolic equation with NBCs are in [94, co-authors: Ivanauskas, Novickij]. A computational experiment for stability analysis of FDS with NBCs is decribed in [192, co-authors: Čiupaila, Jokšienė, Meškauskas]. Sapagovas wrote a textbook on differential problems with NBCs and numerical methods for such problems [186]. Many of the results above are included in this handbook.

In this survey, we present recent results on stationary problems (Section 2), Green's functions (Section 3) and spectrum of SLP with NBCs (Section 4) and selected problems with NBCs (Section 5). Some of them were obtained by Sapagovas scientists or scientific school, which he created.

2 Stationary problem with nonlocal boundary conditions and characteristic curve

One of the directions of Sapagovas scientific school is the development of a general theory for problems with NBCs. The various linear NBCs (1)–(11) can be written as linear functional (for classical BC, see [42]). The first results in this direction were obtained in [35]. In this paper, the second-order linear stationary equation with nonlocal boundary conditions was investigated

$$lu := -\bigl(p(x)u'\bigr)' + q(x)u = f, \quad x \in (0,1), \tag{12}$$

$$u(0) = \gamma_0\langle k_0, u\rangle + f_0, \quad \gamma_0 \geqslant 0, \tag{13}$$

$$u(1) = \gamma_1\langle k_1, u\rangle + f_1, \quad \gamma_1 \geqslant 0, \tag{14}$$

where k_0 and k_1 are linear functionals:

$$\langle k_0, u\rangle := \alpha_0 u(a_0) + \int_0^1 \beta_0(x)u(x)\,\mathrm{d}x, \tag{15}$$

$$\langle k_1, u\rangle := \alpha_1 u(a_1) + \int_0^1 \beta_1(x)u(x)\,\mathrm{d}x. \tag{16}$$

Note that given expressions are examples of functionals and the results were proved in a general case. For a parabolic equation (in the case $\alpha_0 = \alpha_1 = 0$), Ekolin [49] proved the convergence of the forward and backward Euler methods. The integrals in the nonlocal boundary conditions were approximated by the trapezoidal rule. The convergence of these methods is proved under the assumption that

$$\gamma_0\bigl(|\beta_0|, 1\bigr) < 1, \qquad \gamma_1\bigl(|\beta_1|, 1\bigr) < 1, \tag{17}$$

where $(\cdot,\cdot)$ denotes the standard inner product $(f,g) = \int_0^1 f(x)g(x)\,\mathrm{d}x$. For the Crank–Nicolson method, the convergence is proved under the assumption that

$$\gamma_0\|\beta_0\| + \gamma_1\|\beta_1\| < \frac{\sqrt{3}}{2}, \tag{18}$$

where $\|f\| := \sqrt{(f,f)}$. Using Galerkin method, Fairweather and Lopez-Marcos [51] solved semilinear parabolic problem with integral boundary conditions under the assumptions

$$\gamma_0\|\beta_0\| < 1, \qquad \gamma_1\|\beta_1\| < 1. \tag{19}$$

In papers [35, 36], analogous results for FDS

$$LU := -\delta(P\delta U) + QU = F \quad \text{in } \omega^h, \tag{20}$$

$$U|_{i=0} = \gamma_0\langle K_0, U\rangle + f_0, \quad \gamma_0 \geqslant 0, \tag{21}$$

$$U|_{i=n} = \gamma_1\langle K_1, U\rangle + f_1, \quad \gamma_1 \geqslant 0 \tag{22}$$

and for implicit FDS for parabolic equation were established. In papers [35, 36] and in Suboč doctoral dissertation (scientific supervisor Čiegis) [220], the dependence of solution on parameters γ_0 and γ_1 was investigated. The main result of these papers is that the solution for differential problem with NBCs (15)–(16) or solution for FDS with NBC (21)–(22) exists and is unique for all $\gamma_0 \geqslant 0$ and $\gamma_1 \geqslant 0$ except points on hyperbola or line(s). So, sufficient and necessary conditions for existence of unique solution for stationary problem were found. In [36], the stability region in the plane of parameters was found and the stability of FDS stationary problem and implicit FDS for a parabolic equation with NBCs were proved. These results used for investigation of FDS for nonlinear parabolic equation [34].

In [38], the one dimensional parabolic equation with three types of integral nonlocal boundary conditions is approximated by the implicit Euler finite difference scheme. Stability analysis is done in the maximum norm and it is proved that the radius of the stability region depends on the signs of coefficients in the nonlocal boundary condition.

The analysis of stationary problem with one classical boundary condition ($\gamma_0 = 0$) and another NBC [197,214,218] and investigation of auxiliary stationary problems [107,187, 198] shows that restrictions $\gamma_0 \geqslant 0$ and $\gamma_1 \geqslant 0$ are not necessary, and, in general case, we can take $\gamma_0, \gamma_1 \in \mathbb{R}$. The main research tools in [35,36] were the maximum principle and comparison theorems. Existence and uniqueness of a solution for the stationary problem is equivalent to existence of zero eigenvalue. So, we formulate SLP with two nonlocal boundary conditions (as functional conditions):

$$\mathcal{L}u := -(p(x)u')' + q(x)u = 0, \tag{23}$$

$$\langle k_0, u\rangle = \gamma_0\langle n_0, u\rangle, \quad \gamma_0 \in \mathbb{R}, \tag{24}$$

$$\langle k_1, u\rangle = \gamma_1\langle n_1, u\rangle, \quad \gamma_1 \in \mathbb{R}. \tag{25}$$

We can write many problems with NBC in this form, where $\langle k_i, u\rangle$ is a classical part and $\langle n_i, u\rangle$, $i = 0, 1$, is a nonlocal part of boundary conditions. For example, the functionals n_i, $i = 0, 1$, can describe the multi-point or integral NBCs and the functional k_i, $i = 0, 1$, can describe the local (classical) boundary conditions. If $\gamma_0 = \gamma_1 = 0$, then problem (23)–(25) becomes classical.

The example of NBCs is

$$u(0) = \gamma_0 \int_0^1 \alpha(x)u(x)\,\mathrm{d}x, \qquad u(1) = \gamma_1 \int_0^1 \beta(x)u(x)\,\mathrm{d}x. \tag{26}$$

Sometimes instead of NBCs (26), the following conditions (one or both) are used for SLP:

$$\int_0^1 \alpha(x)u(x)\,\mathrm{d}x = 0, \qquad \int_0^1 \beta(x)u(x)\,\mathrm{d}x = 0. \tag{27}$$

Formally, we can say that such cases are realized for $\gamma_0 = \infty$ or $\gamma_1 = \infty$. More general problem will be if we consider NBCs

$$\int_0^1 \alpha(x)u(x)\,\mathrm{d}x = \tilde{\gamma}_0 u(0), \qquad \int_0^1 \beta(x)u(x)\,\mathrm{d}x = \tilde{\gamma}_1 u(1). \tag{28}$$

Now left-hand side of these BC is “classical” and right-hand side “nonlocal”.

The condition for existence of zero eigenvalue is

$$D(n_0, n_1)[\boldsymbol{u}]\gamma_0\gamma_1 - D(n_0, k_1)[\boldsymbol{u}]\gamma_0 - D(k_0, n_1)[\boldsymbol{u}]\gamma_1 + D(k_0, k_1)[\boldsymbol{u}] = 0, \qquad (29)$$

where $\boldsymbol{u} = [u_0, u_1]$ is any fundamental system of stationary equation (23) and

$$D(\boldsymbol{f})[\boldsymbol{w}] = D(f_1, f_2)[w_1, w_2] := \begin{vmatrix} \langle f_1, w_1\rangle & \langle f_2, w_1\rangle \\ \langle f_1, w_2\rangle & \langle f_2, w_2\rangle \end{vmatrix}.$$

We call the solution of equation (29) a *characteristic curve* for problem (23)–(25) and denote a set of it's points in plane $\mathbb{R}^2_{\gamma_0,\gamma_1}$ by the letter $\mathcal{C}$.

Let denote matrix

$$\mathbf{A} = \begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix} := \begin{pmatrix} D(n_0, n_1) & D(n_0, k_1) \\ D(k_0, n_1) & D(k_0, k_1) \end{pmatrix}. \qquad (30)$$

The two main results on characteristic curve are the following lemmas [215].

Lemma 1. *A characteristic curve for problem* (23)–(25) *in the plane* $\mathbb{R}^2$ *can be one of the following five types*:

(i) *If* $D(n_0, n_1) = D(k_0, k_1) = D(n_0, k_1) = D(k_0, n_1) = 0$*, then the curve is* whole plane;

(ii) *If* $D(n_0, n_1) = D(n_0, k_1) = D(k_0, n_1) = 0$*,* $D(k_0, k_1) \neq 0$*, then the curve is* empty set;

(iii) *If* $D(n_0, n_1) = 0$*,* $D(n_0, k_1) \neq 0$ *or* $D(n_0, n_1) = 0$*,* $D(k_0, n_1) \neq 0$*, then the curve is* line;

(iv) *If* $D(n_0, n_1) \neq 0$ *and* $\det \mathbf{A} = 0$*, then the curve is* union of vertical and horizontal lines;

(v) *If* $D(n_0, n_1) \neq 0$ *and* $\det \mathbf{A} \neq 0$*, then the curve is* hyperbola.

Lemma 2. *On torus, Characteristic Curve for problem* (23)–(25) *can be one of the following three types*:

(i) *If* $\mathbf{A} \in GL_2(\mathbb{R})$*, then the curve is homeomorphic to a circle, and this curve winds around the torus one time* (*one time in one direction and one in the other direction*);

(ii) *If* $\mathbf{O} \neq \mathbf{A} \notin GL_2(\mathbb{R})$*, then the curve is the union of two circles* (*strictly "latitudinal" and strictly "longitudinal"*) *with one common point*;

(iii) *Otherwise* (*i.e.,* $\mathbf{A} = \mathbf{O}$)*, the curve is whole torus.*

These lemmas are valid for the discrete SLP

$$\mathcal{L}^h U^h := -\delta\big(P^h \delta U^h\big) + Q^h U^h = 0 \quad \text{in } \omega^h, \qquad (31)$$

$$\big\langle K_0^h, U^h\big\rangle = \gamma_0\big\langle N_0^h, U^h\big\rangle, \qquad \big\langle K_1^h, U^h\big\rangle = \gamma_1\big\langle N_1^h, U^h\big\rangle, \qquad (32)$$

where $(\gamma_0, \gamma_1) \in \mathbb{T}^2$.

3 Green's functions for problems with nonlocal boundary conditions

Green's functions for problems with classical boundary conditions were described by Stakgold [212, 213], Duffy [47]. We can find Green's functions for problems with difference operator in the books of Samarskii and Nikolaev [172, 173] and the textbook of Bahvalov, Zhidkov and Kobel'kov [9]. The investigation of the semilinear problems with NBCs and the existence of positive solutions are based on the investigation of Green's function for the linear problems with NBCs. Green's functions for the second- and higher-order boundary problems with various NCs were constructed by Anderson [3, 4], Webb and Infante [82, 83, 85, 86, 226, 227], Ma and An [121, 124, 126], Sun [221], Truong, Ngoc and Long [222], Yang [236], Zhao [240, 241], Xie, Liu and Bai [233] and other scientists. In their works, authors considered the existence and multiplicity of solutions by applying various methods: lower and upper solution method, Leggett–Williams fixed-point theorem, Guo–Krasnoselskii fixed-point theorem, Leray–Schauder continuation principle, Avery–Peterson fixed-point theorem. Henderson and Ntouyas [77], Guo, Sun and Zhao [67], Liu and O'Regan [117] have considered the third- and higher-order differential equations with various NBCs and investigate the existence of solutions. Bai [10] proved the existence of one or two positive solutions for the nonlocal fourth-order BVP

$$\begin{aligned} &u^{(4)} + \beta u'' = \lambda f(t, u, u''), \\ &u(0) = u(1) = \int_0^1 p(s)u(s)\,\mathrm{d}s, \qquad u''(0) = u''(1) = \int_0^1 p(s)u(s)\,\mathrm{d}s. \end{aligned} \tag{33}$$

L. Kong and Q. Kong [113] considered nonlinear BVP with nonhomogeneous multi-point BCs. Hao, Liu, Wub and Sun [69] considered nth-order singular nonlocal BVP

$$u^{(n)} + \lambda a(t)f(t, u) = 0, \qquad u(0) = \cdots = u^{(n-2)}(0), \quad u(1) = \int_0^1 u(s)\,\mathrm{d}A(s) \tag{34}$$

with Riemann–Stieltjes integral in one BC and proved existence of positive solutions. Zhao et al. [239] investigated the second-order BVP with four-point BC. The existence results of multiple monotone and convex positive solutions for some fourth-order multi-point BVP was established by Liu, Weiguo and Chunfang in [118]. Ma [122] studied the existence and nonexistence of positive solutions of nonlinear periodic BVP and Green's function for corresponding linear problem. Henderson and Luca in [74, 75] presented some results for positive solutions of a system of nonlinear the second-order ODEs subject to multi-point BCs. Wang and An [224] investigated the existence, multiplicity, and nonexistence of positive solutions for nonhomogeneous m-point BVP with two parameters. Under conditions weaker than those used by Ma, Zhang and Ge [238] established various results on the existence and nonexistence of symmetric positive solutions to fourth-order BVP with integral BC. By using a fixed point theorem in a cone and the nonlocal third-order BVP Green's function, the existence of at least one positive solution for the third-order BVP with the integral BC was considered by Guo and Yang in [68]. A class of second-order three-point integral BVP at resonance was investigated in [116] by Liu and Ouyang. Jankowski established the existence of at least three nonnegative solutions of some nonlocal BVP to third-order differential equations with advanced arguments [103]. He considered nonlocal BVP for systems of second-order differential equations with dependence on the first-order derivatives and deviating arguments [104]. In these papers, nonlinear differential equations has nonlinearity in the right-hand side only (see (33)–(34) as examples). Green's functions were constructed for simple linear differential operator. They were used for investigation of properties of solutions nonlinear differential equations.

The investigations of Green's functions and relations between them there were one of the main task of the group of researchers in Vilnius university. Linear NBCs can be written in the form (24)–(25). The first results were obtained for the second-order differential problem in [165, 166]. These results were generalised in [216] for BVP

$$-\bigl(p(x)u'\bigr)' + q(x)u = f(x), \tag{35}$$

$$\langle L_1, u\rangle = g_1, \qquad \langle L_2, u\rangle = g_2. \tag{36}$$

In this paper, the second-order linear differential equation with two additional conditions was investigated and Green's function was constructed. The relation between two Green's functions for two such problems with different additional conditions was derived and general formulae were applied to a stationary problem with NBCs (24)–(25). Green's function for a problem with NBCs can be expressed per Green's function for a problem with classical BCs. The differential equation (35) can be approximated by the difference equation

$$\mathcal{L}u := a_i^2 u_{i+2} + a_i^1 u_{i+1} + a_i^0 u_i = f_i, \tag{37}$$

where $a^2, a^0 \neq 0$. Let S be a two-dimensional linear space of solutions for Eq. (35) or Eq. (37). Then the following lemma is valid for the differential problem [216] and discrete problem [168] with two additional conditions (36).

Lemma 3. *Let $\{u^1, u^2\}$ be the basis of the linear space S. Then the following propositions are equivalent*:

(i) *The functionals L_1, L_2 are linearly independent*;

(ii) $D(\boldsymbol{L})[\boldsymbol{u}] \neq 0$.

Green's function for problem (37) with homogeneous conditions $\langle L_1, u\rangle = 0$, $\langle L_2, u\rangle = 0$, where the functionals L_1 and L_2 are linearly independent, is

$$G_{ij} = \frac{D(\boldsymbol{L}, \delta_i)[\boldsymbol{u}, G^c_{\cdot,j}]}{D(\boldsymbol{L})[\boldsymbol{u}]}.$$

For the theoretical investigation of problems with NBCs, the next result about the relations between Green's functions G^u_{ij} and G^v_{ij} of two nonhomogeneous discrete problems

$$\begin{cases} \mathcal{L}u = f, \\ \langle l_m, u\rangle = 0, \quad m = 1, 2, \end{cases} \qquad \begin{cases} \mathcal{L}v = f, \\ \langle L_m, v\rangle = 0, \quad m = 1, 2, \end{cases}$$

is useful [168].

Theorem 1. *If Green's function G^u exists and functionals L_1 and L_2 are linearly independent, then*

$$G^v_{ij} = \frac{D(\boldsymbol{L}, \delta_i)[\boldsymbol{u}, G^u_{\cdot,j}]}{D(\boldsymbol{L})[\boldsymbol{u}]}.$$

Let us investigate Green's function for the problem with nonlocal boundary conditions

$$\langle L_1, u\rangle := \langle \kappa_1, u\rangle - \gamma_1\langle \varkappa_1, u\rangle = 0, \tag{38}$$

$$\langle L_2, u\rangle := \langle \kappa_2, u\rangle - \gamma_2\langle \varkappa_2, u\rangle = 0. \tag{39}$$

If $\gamma_1, \gamma_2 = 0$, then problem (38)–(39) becomes classical. Suppose that there exists Green's function G^{cl}_{ij} for the classical case. If $\vartheta := D(\boldsymbol{L})[\boldsymbol{u}] \neq 0$, then Green's function exists for problem (37)–(39). The following formula is valid:

$$\vartheta = D(\kappa_1 \cdot \kappa_2)[\boldsymbol{u}] - \gamma_1 D(\varkappa_1 \cdot \kappa_2)[\boldsymbol{u}] - \gamma_2 D(\kappa_1 \cdot \varkappa_2)[\boldsymbol{u}] + \gamma_1\gamma_2 D(\varkappa_1 \cdot \varkappa_2)[\boldsymbol{u}]$$

($\theta = 0$ is equation of the characteristic curve) and

$$G_{ij} = G^{\mathrm{cl}}_{ij} + \gamma_1\big\langle \varkappa_1^k, G^{\mathrm{cl}}_{kj}\big\rangle \frac{D(\delta_i, L_2)}{\vartheta} + \gamma_2\big\langle \varkappa_2^k, G^{\mathrm{cl}}_{kj}\big\rangle \frac{D(L_1, \delta_i)}{\vartheta}.$$

Analogous results are valid in differential case [216] and for the third-order linear differential equation with three additional conditions [167]. In the case of mth-order differential equation with m additional conditions

$$u^{(m)} + a^{m-1}(x)u^{(m-1)} + \dots + a^1(x)u' + a^0(x)u = f(x),$$
$$\langle L_i, u\rangle = f_i, \quad i = 1, \dots, m,$$

or discrete mth-order problem

$$a_i^m u_{i+m} + \dots + a_i^2 u_{i+2} + a_i^1 u_{i+1} + a_i^0 u_i = f_i,$$
$$\langle L_1, u\rangle = g_1, \dots, \quad \langle L_m, u\rangle = g_m,$$

formulae for Green's functions were derived in [164, 217]. These results are summarized in Roman doctoral dissertation [163].

3.1 Generalized Green's functions

In the case $D[\boldsymbol{L}](\boldsymbol{u}) = 0$, Green's function do not exist. For investigation of solutions, we can construct generalized Green's function [14]. Most authors have developed a theory of generalized Green's matrix for systems of differential equations with two-points BCs. Brown used generalized Green's functions and generalized inverses for linear differential systems with Stieltjes BCs [23, 24]. Locker investigated the generalized Green's function for nth-order linear differential operator and k linearly independent boundary values as a kernel of the Moore–Penrose inverse that describes the minimum norm least squares solution for compatible BVP [119, 120].

Now we will review recent articles related to generalized Green's function for differential equations and some applications. In [20], linear Fredholm's BVPs for systems of ODE with constant coefficients and a single delay were investigated, assuming that these solutions satisfy the initial and boundary conditions. A method of pseudoinverse by Moore–Penrose matrices led to an explicit and analytical form of a criterion for the existence of solutions. Some commonly used interpolation algorithms were analysed in [46]. Among all of the methods, biharmonic spline interpolation was considered, which is based on Green's function. In [135], a new class of generalized inverses on semigroups was introduced by means of Green's relations. The classical generalized inverses (group inverse, Drazin inverse and Moore–Penrose inverse) belong to this class. In [1], Abu-Saman reviews the mathematical considerations behind the generalized inverse of a matrix. Simple derivations for the determination of different types of generalized inverses of a matrix are presented. These include results of the generalized inverse of singular and rectangular matrices. It also includes applications of the generalized inverse to solution of a set of linearly dependent equations. In [115], Li and Zhang extend and generalize the standard spectral graph theory on undirected graphs to digraphs. In particular, they introduce and define a normalized digraph Laplacian for digraphs, and prove that its Moore–Penrose pseudoinverse is the discrete Green's function of the Diplacian matrix as an operator on digraphs. Xu and Yau proved an explicit formula of Chung–Yau's discrete Green's functions as well as hitting times of random walks on graphs [234]. Maroncelli and Rodríguez investigated least squares solutions for a linear nonhomogeneous BVP with impulses [132]. The analysis of least squares solution of minimal norm is directly related to generalized inverses and generalized Green's functions for current BVP. The explicit formula of least squares solution is given.

Paukštaitė investigated a generalized Green's function that describes the minimum norm least squares solution of every second-order discrete problem with two nonlocal conditions. In [149–151], properties of generalized Green's function that are analogous to properties of ordinary Green's function were proved. She investigated the nullity of discrete problem and presented its classifications. Null spaces of discrete problem and its adjoint problem were also analysed, explicit formulas of bases of null spaces were found. Moreover, the necessary and sufficient existence conditions of exact solutions were given.

4 Sturm–Liouville problem

An eigenvalue problem for the second-order ordinary differential operator with nonlocal condition was formulated and analysed by Ionkin [88, 89] and by Cahlon, Kulkarni, Shi [25]. In these works, eigenvalue problem is associated with the stability of difference schemes for one-dimensional parabolic equations. In Sapagovas article [185], the eigenvalue problem is associated with iterative methods for finite-difference schemes for problems with NBCs. Shkalikov [204] investigated the properties of eigenfunctions. We describe the results of Gulin with co-authors in Section 1. Note that the eigenvalue for problems with NBCs is the separate part of the general nonselfadjoint operator theory (see the article of Il'in [79] and the book of Mennicken and Möller [136]).

The structure of spectrum remains uncertain for problems with nonlocal boundary conditions and variable coefficients. However, even for the simplest operator $-\,\mathrm{d}^2/\,\mathrm{d}x^2$ interesting results on the spectrum were obtained. Čiupaila, Jesevičiūtė and Sapagovas [41] investigated eigenvalue problem

$$-u'' = \lambda u, \quad x \in (0,1), \tag{40}$$

$$u(0) = 0, \tag{41}$$

$$u(1) = \gamma \int\limits_0^1 u(x)\,\mathrm{d}x. \tag{42}$$

For this problem, real eigenvalues exist only in the following cases: if $\gamma < 2$, then all eigenvalues are positive; if $\gamma \geqslant 2$, then all eigenvalues are positive, except one negative eigenvalue ($\gamma > 2$) or zero eigenvalue ($\gamma = 2$). The same result is valid for NBCs

$$u(0) = \gamma_0 \int\limits_0^1 u(x)\,\mathrm{d}x, \qquad u(1) = \gamma_1 \int\limits_0^1 u(x)\,\mathrm{d}x$$

with $\gamma = \gamma_0 + \gamma_1$. Ionkin and Valikova [92] for NBCs (7) prove that all nonzero eigenvalues are not simple, i.e., for each such eigenvalue, there exist eigenfunction and generalized eigenfunction. Such (multiply) eigenvalues (see [41]) exist for NBCs

$$u(0) = 0, \qquad u(1) = \gamma \int\limits_{1/4}^{3/4} u(x)\,\mathrm{d}x, \tag{43}$$

too. In [197], Sapagovas and Štikonas investigate the eigenvalue problem with one Bitsadze–Samarskii type NBC

$$u(0) = 0, \qquad u(1) = \gamma u(\xi), \quad \xi \in (0,1). \tag{44}$$

They found that some eigenvalues do not depend on parameter γ and for some γ complex eigenvalues exist. More results about real eigenvalues for this problem are in [214].

SLP (40)–(41) with one of the two cases of integral type NBCs

$$u(1) = \gamma_0 \int\limits_0^\xi u(x)\,\mathrm{d}x, \quad u(1) = \gamma_1 \int\limits_\xi^1 u(x)\,\mathrm{d}x, \quad \gamma \in \mathbb{R},\ \xi \in [0,1], \tag{45}$$

were investigated in [156, 157]. All eigenvalues (countable set) of problem (40)–(41), (45b) are real and simple. If $\gamma = 1/(1-\xi^2)$, then the minimal eigenvalue is zero. If $\gamma > 1/(1-\xi^2)$, then there exist one negative eigenvalue. In the case of the boundary condition (45a), multiple and complex eigenvalues may exist for some values of parameter γ. SLP (40)–(41) with one of the three cases of two-point NBCs

$$u'(1) = \gamma u(\xi), \quad u'(1) = \gamma u'(\xi), \quad u(1) = \gamma u'(\xi), \quad \gamma \in \mathbb{R},\ \xi \in [0,1], \tag{46}$$

were investigated in [153]. In the case of boundary (46c), two negative eigenvalues may exist for some values of parameter γ. Properties of eigenfunctions for such problems were obtained in [154]. These results are part of doctoral dissertation of Pečiulytė [152]. The properties of negative eigenvalues and negative critical points of a characteristic function for such problems were investigated in [158]. Some results about all critical points are described in [155].

In [218], the characteristic function method for investigation of the spectrum for the problem (40), (41), (44) were used. This paper presents some new results on a spectrum in a complex plane for the second-order stationary differential equation with one Bitsadze–Samarskii type NBC. Some new results on characteristic functions are proved. A definition of constant eigenvalues and the characteristic function is introduced for the SLP with general NBCs. This method of the characteristic function used for investigation of complex eigenvalues and their qualitative behaviour (dynamics) if parameters γ and ξ in NBC (45) are changed [205]. The dynamics of complex eigenvalues for analogues discrete problems is shown in [207] (NBC (45) are approximated by the trapezoidal rule or by Simpson's rule) and in [207] (two-points NBCs). In [206], for SLP (40), (41) with NBC

$$u(1) = \gamma \int_{\xi}^{1-\xi} u(x)\,\mathrm{d}x, \quad \gamma \in \mathbb{R},\ \xi \in [0, 1/2], \tag{47}$$

characteristic functions in the neighbourhood constant eigenvalue point and in the neighbourhood of the critical point of the second order were investigated. For NBC

$$u(\xi) = \gamma u(1-\xi), \quad \gamma \in \mathbb{R},\ \xi \in [0, 1], \tag{48}$$

analogous investigation was done in [208].

The paper of Bandyrskii, Lazurchak, Makarov and Sapagovas [11] deals with numerical methods for eigenvalue problem for the second-order ordinary differential operator with variable coefficient

$$u'' + \big(\lambda - q(x)\big)u = 0, \quad x \in (0, 1),$$

subject to NBC (43). FD method (functional-discrete method) is derived and analysed for calculating of eigenvalues, particularly complex eigenvalues. Jesevičiūtė and Sapagovas [107] investigated the stability of FDS for parabolic equation with integral type NBCs with variable weight functions

$$u(-l, t) = \gamma_0 \int_{-l}^{l} \alpha_0(x) u(x, t)\,\mathrm{d}x, \qquad u(l, t) = \gamma_1 \int_{-l}^{l} \alpha_1(x) u(x, t)\,\mathrm{d}x$$

and analysed discrete SLP in the case of various weights α_0 and α_1 (for approximation integrals the trapezoidal rule was used). In [105], an eigenvalue problem for a differential operator with nonlocal integral conditions, when variable coefficients arise in nonlocal integral conditions, was investigated and was found how eigenvalues depend on the parameters occurring in the nonlocal boundary conditions. Discrete SLP

$$-\frac{U_{i-1}-2U_i+U_{i+1}}{h^2}=\lambda U_i,\quad i=1\dots N-1, \tag{49}$$

$$\frac{U_0+U_N}{2}+\sum_{i=1}^{N-1}U_i=0,\qquad \frac{U_N}{2}+\sum_{i=1}^{N-1}U_i ih=0 \tag{50}$$

was analysed by Jachimavičienė, Jesevičiūtė and Sapagovas [99]. These results are part of doctoral dissertation of Jesevičiūtė (Jokšienė) [106]. Instead BC (50), Jachimavičienė obtained few new results [95, 96] about spectrum for NBCs

$$U_0=\gamma_0 h\left(\frac{U_0+U_N}{2}+\sum_{i=1}^{N-1}U_i\right)=0, \tag{51}$$

$$U_N=\gamma_1 h\left(\frac{U_0+U_N}{2}+\sum_{i=1}^{N-1}U_i\right)=0. \tag{52}$$

Jakubėlienė analysed an eigenvalue problem for the system

$$-u_i''=\lambda u_i,\quad i=1,\dots,N-1,$$

with boundary conditions

$$u_i(0)=0,\quad u_i(1)=\gamma_i h\sum_{k=1}^{N-1}\int_0^1 u_k(x)\,\mathrm{d}x,\quad i=1,\dots,N-1,\ Nh=1,$$

and analogous discrete SLP

$$-\frac{U_{i-1,j}-2U_{ij}+U_{i+1,j}}{h^2}=\lambda U_i,j,\quad i,j=1\dots N-1, \tag{53}$$

$$U_{i0}=0,\quad u_{iN}=\gamma_i h^2\sum_{k=1}^{N-1}\left(\frac{U_{kN}}{2}+\sum_{j=1}^{N-1}U_{kj}\right) \tag{54}$$

in [100, 102]. The spectrum and eigenfunctions for SLP (49), (51)–(52) were investigated for all values of parameters γ_1 and γ_2 in [143]. For this problem:

1) all eigenvalues are simple and real;
2) if $\gamma<2$, then $\lambda\in(0,4/h^2]$;
3) if $\gamma\nearrow 2/h$, then $\lambda_1\to-\infty$;
4) if $\gamma=2/h$, then boundary conditions (51)–(52) are not linearly independent;
5) if $\gamma\searrow 2/h$, then $\lambda_1\to+\infty$;
6) if $\gamma>2/h$, then all the eigenvalues λ are positive.

Chanane [30] used the regularized sampling method introduced recently to compute the eigenvalues of Sturm–Liouville problems

$$-u'' + q(x)u = \lambda u$$

with NBCs

$$\int_0^1 u(x)\,\mathrm{d}\psi_0(x) + u'(x)\,\mathrm{d}\psi_0(x) = 0, \qquad \int_0^1 u(x)\,\mathrm{d}\varphi_0(x) + u'(x)\,\mathrm{d}\varphi_0(x) = 0.$$

In [108], the second-order singular Sturm–Liouville integral boundary value problem was concerned and the existence of at least one positive solution was proved by Jiang, Liu, Wub. Ma and An [123] considered the nonlinear eigenvalue problems

$$-u'' = \lambda h(t) f(u), \quad t \in (0,1), \qquad u(0) = 0, \qquad u(1) = \int_0^1 u(s)\,\mathrm{d}A(s).$$

They investigated the global structure of positive solutions by using global bifurcation techniques. Hao, Liu, Wub, Sun [70] similar n-order problem considered under some conditions concerning the first eigenvalue corresponding to the relevant linear operator. The existence of positive solutions is obtained by means of the fixed point index theory in cones. Hao, Liu, Wub, Xu in [71] considered a class of singular n-order nonlocal BVPs in Banach spaces. The existence of multiple positive solutions for the problem was obtained by using the fixed point index theory of strict set contraction operators. Karulina [111] considered the Sturm–Liouville problem with symmetric boundary conditions and an integral condition and estimated the first eigenvalue of this problem for different values of the parameters. Nizhnik solved the inverse spectral problem for a class of Sturm–Liouville operators with singular nonlocal potentials and NBCs on a star graph [142]. By employing known Guo–Krasnoselskii fixed point theorem, Wang, Liu and Zhang investigated the eigenvalue interval for the existence and non existence of at least one positive solution of nonlinear fractional differential equation with integral boundary conditions [225]. Yan determined the principal eigenvalue of the linear and nonlinear fourth-order eigenvalue problems with integral type NBCs [235].

5 Some other recent differential and discrete problems with NBCs

Note that many of such articles are connected with various applications when NBCs have clear physical or mechanical meaning [27–29, 237]. In this section, we mention some articles on BVP with NCs.

Hyperbolic problems with integral type NBCs were investigated in [147] by Panejah and Panejah. Various hyperbolic equations with NBCs were investigated by a lot of authors: Duzheva considered BVP with integral BC of the first order [48]; Khaleel and Khtan in [112] used numerical method for solving the initial value problem that consists of the multi-dimensional hyperbolic equation with nonlinear integral NBCs. This method depends on Crank–Nikolson FDS and Taylor's expansion; Martín-Vaquero and Wade [134] derived a new family of high-order stable three-level algorithms to solve the wave equation. Pulkina analysed BVPs for a hyperbolic equation with NBCs of the first and the second kind [160, 161]. The unique existence of classical solution of initial-boundary value problem for wave equation with a special integral NBC is proved in the work of Korzyuk [114]. Ashyralyev and Prenov studied FDS for initial-boundary value problem for a hyperbolic system with NBCs [8], Ashyralyev and Ozdemir analysed FDS for hyperbolic-parabolic equations with multi-point NBCs [7]. In [33] Cheniguel presented new technique for solving wave equation with NBCs.

Pao and Wang considered a class of fourth-order nonlinear elliptic equation with multi-point NBCs in [148]. They considered a second-order elliptic equation with NBC and the usual multi-point boundary problem in ODEs, too. The aim of the paper was to show the existence of maximal and minimal solutions, the uniqueness of a positive solution, and the method of construction for these solutions. The monotone iterative schemes can be developed into computational algorithms for numerical solutions of the problem by either the FDM or the FEM. The paper of D. Gordeziani, E. Gordeziani, Davitashvili and Meladze [53] deals with the formulation and analysis of a generalized nonlocal problem for the elliptic equations with variable coefficients. Berikelashvili and Khomeriki in [15,16] considered a nonlocal BVP for the Poisson equation in a rectangular domain. Dirichlet and Neumann conditions are posed on a pair of adjacent sides of a rectangle, and integral constraints are given instead of BCs on the other pair. The corresponding difference scheme is constructed and investigated.

Rassias and Karimov investigated some BVPs with nonlocal initial condition for model and degenerate parabolic equations with parameter [162]. Karatay, Bayramoglu, Yildiz and Kokluce study matrix stability of the first-order and the second-order FDS for parabolic BVPs with NBCs [110]. Pseudo-parabolic equation and FDS for this equation with NBCs was investigated by Mamedov [131] (Bitsadze–Samarskii and Samarskii–Ionkin type NBCs), Beshtokov [18].

In paper [211], the existence was proved for a solution of the nonlocal problem with integral conditions for linear PDE of the third order by Sopuev and Arkabaev. Moldojarov used the method of integral equations and the contraction mapping was used for proof unique solvability of nonlocal problem with integral conditions for a nonlinear PDE of the third order [140]. Smirnov investigated the third-order nonlinear BVP with two-point NBC [209, 210]. An estimation of the number of solutions to BVP and their nodal structure are established.

Mokin considered the eigenvalue problem for a nonselfadjoint difference operator with nonconstant coefficient [137]. Multiplicity of eigenvalues is discussed and a region, where all eigenvalues reside is defined. Stability and convergence of FDS approximating a two-parameter nonlocal BVP for time-fractional diffusion equation were studied by Alikhanov [2]. El-Shahed and Shammakh investigated the nonlinear fractional nonlocal BVP [50]. They obtain the results on the existence of one and two positive solution by utilizing the results of Webb and Lan [231] involving comparison with the principal characteristic value of a related linear problem to the fractional case. We then use the theory worked out by Webb and Infante in [228, 229] to study the general NBCs.

Henderson and Luca [76] investigated the existence of positive solutions for systems of singular nonlinear higher-order differential equations subject to multi-point BCs. Bolojan-Nica, Infante and Pietramala [21] studied the existence of solutions for nonlinear first-order impulsive systems with NBCs. Bolojan-Nica, Infante and Precup in [22] were developed an existence theory for first-order differential systems with coupled NCs given by Stieltjes integrals. The approach was based on the fixed point theorems of Perov, Schauder and Schaefer and on a vector method for treating systems, which uses matrices having spectral radius less than one.

Green's functionals and reproducing kernel method was used for BVP with NBCs in the articles of Niu [141], Özen and Oruçoğlu [145,146]. Sergejeva investigated Fučìk type problem with Bitsadze–Samarskii type NBC [201] and with a damping term and integral type NBC [203]. Sajavičius in his doctoral dissertation [170] analysed numerical solution of PDEs with nonlocal conditions (stability of FDS, radial basis function method).

Numerical analysis for boundary value problems with nonlocal conditions

This Special Issue contains a variety of contributions within this area of research. The following articles deal with BVPs with NBCs for ordinary, discrete, impulsive, elliptic, parabolic, pseudo-parabolic, hyperbolic and nonlinear equations.

Ashyralyev and Agirseven investigate the inverse problem of a delay parabolic equation with NBCs [5]. The stability estimates are established.

Ashyralyev and Ashyralyyev study the BVP of determining the parameter of an elliptic equation in an arbitrary Banach space [6].

Berikelashvili and Khomeriki consider the Poisson equation in a rectangular domain with an integral constraints [17]. The corresponding FDS scheme is constructed.

Čiegis, Subоč and Bugajev investigate three-dimensional parabolic and pseudo-parabolic equations with classical, periodic and NBCs [37]. Equations are approximated by the full approximation backward Euler method, locally one dimensional and Douglas ADI splitting schemes.

Henderson is concerned with differentiating solutions of BVPs with respect to boundary data for the nth-order ordinary differential equation, satisfying the Dirichlet and multi-strip integral boundary conditions [73].

Infante and Pietramala study the existence of nonnegative solutions for a system of impulsive differential equations subject to nonlinear, NBCs [84]. The system has a coupling in the differential equations and in the boundary conditions.

Makarov, Sytnyk and Vasylyk study a nonlocal-in-time evolutional problem for the first-order differential equation in Banach space [130]. It results in the necessary and sufficient conditions for the existence of a generalized solution to the given nonlocal problem.

Martín-Vaquero numerically study polynomial-based mean weighted residuals methods for the solution of elliptic problems with NBCs in rectangular domains [133].

Novickij and Štikonas consider the stability of a weighted FDS for a linear hyperbolic equation with integral NBCs [143].

Rutkauskas studies Dirichlet type problem in a bounded domain for the system of linear elliptic equations of the second order, which degenerate into the first-order system at a line crossing the domain [169]. The existence and uniqueness of a solution are proved without any additional condition at line of degeneracy.

Serbina considers the nonlocal initial and BVP for Lavrentiev–Bitsadze equation [200]. This problem models the nonstationary one-dimensional motion of a groundwater with horizontal stopping.

Sergejeva considers the Fučík spectrum for the second-order BVP with NBC [202]. The explicit formulas for the spectrum of this problem are given.

Štikonienė, Sapagovas and Čiupaila consider convergence of iterative processes for elliptic differential problem with nonlocal conditions [219].

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References

1. A.M. Abu-Saman, Solution of linearly-dependent equations by generalized inverse of a matrix, *International Journal of Science and Emerging Technologies*, **4**(2):138–142, 2012.

2. A.A. Alikhanov, Stability and convergence of difference schemes approximating a two-parameter nonlocal boundary value problem for time-fractional diffusion equation, 2013, arXiv:1312.4864.

3. D.R. Anderson, Green's function for a third-order generalized right focal problem, *J. Math. Anal. Appl.*, **288**(1):1–14, 2003, http://dx.doi.org/10.1016/S0022-247X(03)00132-X.

4. D.R. Anderson, T.O. Anderson, M.M. Kleber, Green's function and existence of solutions for a functional focal differential equation, *Electron. J. Differ. Equ.*, **2006**(12):1–14, 2006.

5. A. Ashyralyev, D. Agirseven, On source identification problem for a delay parabolic equation, *Nonlinear Anal. Model. Control*, **19**(3):335–349, 2014, http://www.mii.lt/na/issues/NA_1403/NA19302.pdf.

6. A. Ashyralyev, C. Ashyralyyev, On the problem of determining the parameter of an elliptic equation in a Banach space, *Nonlinear Anal. Model. Control*, **19**(3):350–366, 2014, http://www.mii.lt/na/issues/NA_1403/NA19303.pdf.

7. A. Ashyralyev, Y. Ozdemir, On numerical solutions for hyperbolic-parabolic equations with the multipoint nonlocal boundary condition, *J. Franklin Inst.*, **351**(2):602–630, 2014, `http://dx.doi.org/10.1016/j.jfranklin.2012.08.007`.

8. A. Ashyralyev, R. Prenov, Finite-difference method for the hyperbolic system of equations with nonlocal boundary conditions, *Adv. Difference Equ.*, **2014**(26), 2014.

9. N.S. Bahvalov, N.P. Zhidkov, G.M. Kobelkov, *Chislennye Metody*, Binom, Moscow, 2011 (in Russian).

10. Z. Bai, Positive solutions of some nonlocal fourth-order boundary value problem, *Appl. Math. Comput.*, **215**:4191–4197, 2010, `http://dx.doi.org/10.1016/j.amc.2009.12.040`.

11. B. Bandyrskii, I. Lazurchak, V. Makarov, M. Sapagovas, Eigenvalue problem for the second order differential equation with nonlocal conditions, *Nonlinear Anal. Model. Control*, **11**(1): 115–122, 2006, `http://www.mii.lt/na/issues/NA_1101/NA11102.pdf`.

12. R.W. Beals, Nonlocal elliptic boundary value problems, *Bull. Am. Math. Soc.*, **70**:693–696, 1964.

13. R.W. Beals, *Nonlocal Elliptic Boundary Value Problems*, PhD thesis, Yale University, 1964.

14. A. Ben-Israel, T.N.E. Greville, *Generalized Inverses: Theory and Applications*, 2nd edition, Springer Verlag, New York, 2003.

15. G. Berikelashvili, N. Khomeriki, On the convergence of difference schemes for one nonlocal boundary-value problem, *Lith. Math. J.*, **52**(4):353–362, 2012, `http://dx.doi.org/10.1007/s10986-012-9178-0`.

16. G. Berikelashvili, N. Khomeriki, On a numerical solution of one nonlocal boundary-value problem with mixed Dirichlet–Neumann conditions, *Lith. Math. J.*, **53**(4):367–380, 2013, `http://dx.doi.org/10.1007/s10986-013-9214-8`.

17. G. Berikelashvili, N. Khomeriki, Difference schemes for one fully nonlocal boundary-value problem, *Nonlinear Anal. Model. Control*, **19**(3):367–381, 2014, `http://www.mii.lt/na/issues/NA_1403/NA19304.pdf`.

18. M.X. Beshtokov, Apriornye ocenki resheniya nelokal'nykh kraevykh zadach dlya psevdoparabolicheskogo uravneniya, *Vladikavkaz. Mat. Zh.*, **15**(3):19–36, 2013 (in Russian).

19. A.V. Bitsadze, A.A. Samarskii, Some elementary generalizations of linear elliptic boundary value problems, *Dokl. Akad. Nauk SSSR*, **185**:739–740, 1969 (in Russian).

20. A. Boichuk, J. Diblík, D. Khusainov, M. Ružičková, Boundary value problems for delay differential systems, *Adv. Difference Equ.*, **2010**, Article ID 593834, 20 pp., 2010, `http://dx.doi.org/10.1155/2010/593834`.

21. O. Bolojan-Nica, G. Infante, P. Pietramala, Existence results for impulsive systems with initial nonlocal conditions, *Math. Model. Anal.*, **18**(5):599–611, 2013, `http://dx.doi.org/10.3846/13926292.2013.865678`.

22. O. Bolojan-Nica, G. Infante, R. Precup, Existence results for systems with coupled nonlocal initial conditions, *Nonlinear Anal., Theory Methods Appl.*, **94**:231–242, 2014, `http://dx.doi.org/10.1016/j.na.2013.08.019`.

23. R.C. Brown, Duality theory for N-th order differential operators under Stieltjes boundary conditions. II: Nonsmooth coefficients and nonsingular measures, *Ann. Mat. Pura Appl.* **105**(1):141–170, 1975.

24. R.C. Brown, Generalized Green's functions and generalized inverses for linear differential systems with Stieltjes boundary conditions, *J. Differ. Equ.*, **16**:335–341, 1974.

25. B. Cahlon, D.M. Kulkarni, P. Shi, Stepwise stability for the heat equation with a nonlocal constrain, *SIAM J. Numer. Anal.*, **32**(2):571–593, 1995, `http://dx.doi.org/10.1137/0732025`.

26. J.R. Cannon, The solution of the heat equation subject to specification of energy, *Q. Appl. Math.*, **21**(2):155–160, 1963.

27. J.R. Cannon, J. Van der Hoek, An implicit finite scheme for the diffusion equation in a portion of domain, in: J. Noye (Ed.), *Numerical Solutions of Partial Differential Equations*, North-Holland Publishing, Amsterdam, 1982, pp. 527–539.

28. J.R. Cannon, Y. Lin, A Galerkin procedure for diffusion equations with boundary integral conditions, *Int. J. Eng. Sci.*, **28**(7):579–587, 1990.

29. J.R. Cannon, A. Matheson, A numerical procedure for diffusion subject to the specification of mass, *Int. J. Eng. Sci.*, **131**:347–355, 1993.

30. B. Chanane, Computing the eigenvalues of a class of nonlocal Sturm–Liouville problems, *Math. Comput. Modelling*, **50**:225–232, 2009, `http://dx.doi.org/10.1016/j.mcm.2008.10.021`.

31. R. Chegis, The numerical solution of the heat equation with nonclassical condition, *Lith. Math. J.*, **24**(4):206–215, 1984.

32. R. Chegis, Numerical solution of a problem with small parameter for the highest derivative and nonlocal condition, *Lith. Math. J.*, **28**(1):90–96, 1988, `http://dx.doi.org/10.1007/BF00972255`.

33. A. Cheniguel, Numerical method for solving wave equation with non local boundary conditions, in: *Proceedings of the International MultiConference of Engineers and Computer Scientists (Hong Kong, March 13–15, 2013)*, Vol. II, `http://www.iaeng.org/publication/IMECS2013/IMECS2013_pp1190-1193.pdf`.

34. R. Čiegis, Finite-difference schemes for nonlinear parabolic problem with nonlocal boundary conditions, in: M. Ahues, C. Constanda, A. Largillier (Eds.), *Integral Methods in Science and Engineering: Analytic and Numerical Techniques*, Birkhauser, Boston, 2004, pp. 47–52.

35. R. Čiegis, A. Štikonas, O. Štikonienė, O. Subač, Stationary problems with nonlocal boundary conditions, *Math. Model. Anal.*, **6**(2):178–191, 2001, `http://dx.doi.org/10.1080/13926292.2001.9637157`.

36. R. Čiegis, A. Štikonas, O. Štikonienė, O. Subač, A monotonic finite-difference scheme for a parabolic problem with nonlocal conditions, *Differ. Equ.*, **38**(7):1027–1037, 2002, `http://dx.doi.org/10.1023/A:1021167932414`.

37. R. Čiegis, O. Subač, A. Bugajev, Parallel algorithms for three-dimensional parabolic and pseudoparabolic problems with different boundary conditions, *Nonlinear Anal. Model. Control*, **19**(3):382–395, 2014, `http://www.mii.lt/na/issues/NA_1403/NA19305.pdf`.

38. R. Čiegis, N. Tumanova, Numerical solution of parabolic problems with nonlocal boundary conditions, *Numer. Funct. Anal. Optim.*, **31**(12):1318–1329, 2010, http://dx.doi.org/10.1080/01630563.2010.526734.

39. R. Čiegis and N. Tumanova. On construction and analysis of finite difference schemes for pseudoparabolic problems with nonlocal boundary conditions, *Math. Model. Anal.*, **19**(2):281–297, 2014, http://dx.doi.org/10.3846/13926292.2014.910562.

40. R. Čiupaila, M. Sapagovas, O. Štikonienė, Numerical solution of nonlinear elliptic equation with nonlocal condition, *Nonlinear Anal. Model. Control*, **18**(4):412–426, 2013, http://www.mii.lt/na/issues/NA_1804/NA18402.pdf.

41. R. Čiupaila, Ž. Jesevičiūtė, M. Sapagovas, On the eigenvalue problem for one-dimensional differential operator with nonlocal integral condition, *Nonlinear Anal. Model. Control*, **9**(2): 109–116, 2004, http://www.mii.lt/na/issues/NA_0902/NA09201.pdf.

42. E.A. Coddington, N. Levinson, *Theory of Ordinary Differential Equations*, McGraw Hill Book Co., New York, Toronto, London, 1955.

43. W. Day, Extensions of a property of the heat equation to linear thermo-elasticity and order theories, *Q. Appl. Math.*, **40**:319–330, 1982.

44. W. Day, A decreasing property of solutions of the parabolic equations with applications to thermoelasticity, *Q. Appl. Math.*, **41**:468–475, 1983.

45. M. Dehghan, A computational study of the one-dimensional parabolic equation subject to nonclassical boundary specifications, *Numer. Methods Partial Differ. Equations*, **2**(1):220–257, 2006.

46. X. Deng, Z. Tang, Moving surface spline interpolation based on Green's function, *Math. Geosci.*, **43**(6):663, 2011, http://dx.doi.org/10.1007/s11004-011-9346-5.

47. D.G. Duffy, *Green's Functions with Applications*, Chapman & Hall/CRC, 2001.

48. A.V. Duzheva, On certain nonlocal problem for hyperbolic equation with integral conditions of the first kind, *Vestn. Samar. Gos. Univ., Estestvennonauchn. Ser.*, **5**(86):29–36, 2011 (in Russian).

49. G. Ekolin, Finite difference methods for a nonlocal boundary value problem for the heat equation, *BIT*, **31**:245–261, 1991.

50. M. El-Shahed, W.M. Shammakh, Multiple positive solutions for nonlinear fractional eigenvalue problem with nonlocal conditions, *J. Fract. Calc. Appl.*, **3**(10):1–13, 2012.

51. G. Fairweather, J.C. Lopez-Marcos, Galerkin methods for a semilinear parabolic problem with nonlocal conditions, *Adv. Comput. Math.*, **6**:243–262, 1996.

52. D. Franco, G. Infante, F.M. Minhós, Nonlocal boundary value problems, *Bound. Value Probl.*, **2012**, Article ID 23, 2012, http://dx.doi.org/10.1186/1687-2770-2012-23.

53. D. Gordeziani, E. Gordeziani, T. Davitashvili, G. Meladze, O reshenii nekotorykh nelokal'nykh kraevykh i nachal'no-kraevykh zadach, *Computer Science and Telecommunications*, **3**(26):161–169, 2010 (in Russian).

54. A.V. Gulin, Stability of nonlocal difference schemes in a subspace, *Differ. Equ.*, **48**(7):956–965, 2012, http://dx.doi.org/10.1134/S0012266112070051.

55. A.V. Gulin, On the spectral stability in subspaces for difference schemes with nonlocal boundary conditions, *Differ. Equ.*, **49**(7):844–852, 2013, http://dx.doi.org/10.1134/S0012266112070051.

56. A.V. Gulin, N.I. Ionkin, V.A. Morozova, Difference schemes with nonlocal boundary conditions, *Comput. Methods Appl. Math.*, **1**(1):62–71, 2001.

57. A.V. Gulin, N.I. Ionkin, V.A. Morozova, Study of the norm in stability problems for nonlocal difference schemes, *Differ. Equ.*, **42**(7):914–923, 2006, http://dx.doi.org/10.1134/S0012266106070068.

58. A.V. Gulin, A.Yu. Mokin, Stability of a family of weighted finite-difference schemes, *Comput. Math. Model.*, **20**(2):152–172, 2009, http://dx.doi.org/10.1007/s10598-009-9026-1.

59. A.V. Gulin, V.A. Morozova, A nonlocal finite-difference boundary-value problem, *Comput. Math. Model.*, **14**(4):410–416, 2003, http://dx.doi.org/dx.doi.org/10.1023/A:1026153824288.

60. A.V. Gulin, V.A. Morozova, On the stability of a nonlocal finite-difference boundary value problem, *Differ. Equ.*, **39**(7):962–967, 2003, http://dx.doi.org/10.1023/B:DIEQ.0000009192.30909.13.

61. A.V. Gulin, V.A. Morozova, Family of self-adjoint nonlocal finite-difference schemes, *Differ. Equ.*, **44**(9):1247–1254, 2008, http://dx.doi.org/10.1134/S0012266108090103,

62. A.V. Gulin, V.A. Morozova, On family of nonlocal difference schemes, *Differ. Equ.*, **45**(7):1001–1013, 2009, http://dx.doi.org/10.1134/S001226610907009X.

63. A.V. Gulin, V.A. Morozova, Stability of the two-parameter set of nonlocal difference schemes, *Comput. Methods Appl. Math.*, **9**(1):79–99, 2009, http://dx.doi.org/10.2478/cmam-2009-0005.

64. A.V. Gulin, V.A. Morozova, N.S. Udovichenko, Difference scheme for the Samarskii–Ionkin problem with a parameter, *Differ. Equ.*, **44**(7):991–998, 2010, http://dx.doi.org/10.1134/S0012266108070112.

65. A.V. Gulin, V.A. Morozova, N.S. Udovichenko, Stability criterion for a family of nonlocal difference schemes, *Differ. Equ.*, **46**(7):973–990, 2010, http://dx.doi.org/10.1134/S0012266110070050.

66. A.V. Gulin, V.A. Morozova, N.S. Udovichenko, Stability of a nonlocal difference problem with a complex parameter, *Differ. Equ.*, **47**(8):1116–1129, 2011, http://dx.doi.org/10.1134/S0012266111080064.

67. L.-J. Guo, J.-P. Sun, Y.-H. Zhao, Multiple positive solutions for nonlinear third-order three-point boundary-value problems, *Electron. J. Differ. Equ.*, **2007**:1–7, 2007.

68. Y. Guo, F. Yang, Positive solutions for third-order boundary-value problems with the integral boundary conditions and dependence on the first-order derivatives, *J. Appl. Math.*, **2013**, Article ID 721909, 6 pp., 2013, http://dx.doi.org/10.1155/2013/721909.

69. X. Hao, L. Liu, Y. Wu, On positive solutions of an m-point nonhomogeneous singular boundary value problem, *Nonlinear Anal., Theory Methods Appl.*, **73**:2532–2540, 2010, http://dx.doi.org/10.1016/j.na.2010.06.028.

70. X. Hao, L. Liu, Y. Wu, Q. Sun, Positive solutions for nonlinear nth-order singular eigenvalue problem with nonlocal conditions, *Nonlinear Anal., Theory Methods Appl.*, **73**(6):1653–1662, 2010, http://dx.doi.org/10.1016/j.na.2010.04.074.

71. X. Haoa, L. Liu, Y. Wu, N. Xu, Multiple positive solutions for singular nth-order nonlocal boundary value problems in Banach spaces, *Comput. Math. Appl.*, **61**:1880–1890, 2011, http://dx.doi.org/10.1016/j.camwa.2011.02.017.

72. J. Henderson, Existence theorems for boundary value problems for nth-order nonlinear difference equations, *SIAM J. Math. Anal.*, **20**(2):468–478, 1989, http://dx.doi.org/10.1137/0520032.

73. J. Henderson, Smoothness of solutions with respect to multi-strip integral boundary conditions for nth order ordinary differential equations, *Nonlinear Anal. Model. Control*, **19**(3): 396–412, 2014, http://www.mii.lt/na/issues/NA_1403/NA19306.pdf.

74. J. Henderson, R. Luca, Existence and multiplicity for positive solutions of a multi-point boundary value problem, *Appl. Math. Comput.*, **218**:10572–10585, 2012, http://dx.doi.org/10.1016/j.amc.2012.04.019.

75. J. Henderson, R. Luca, Positive solutions for a system of second-order multi-point boundary value problems, *Appl. Math. Comput.*, **218**:6083–6094, 2012, http://dx.doi.org/10.1016/j.amc.2011.11.092.

76. J. Henderson, R. Luca, Positive solutions for singular systems of higher-order multi-point boundary value problems, *Math. Model. Anal.*, **18**(3):309–324, 2013, http://dx.doi.org/10.3846/13926292.2013.804009.

77. J. Henderson, S.K. Ntouyas, Positive solutions for systems of nth order three-point nonlocal boundary value problems, *Electron. J. Qual. Theory Differ. Equ.*, **2007**(18):1–12, 2007.

78. V.A. Il'in, Necessary and sufficient properties of being a basis of a subsystem of eigenfunctions and adjoint functions for Keldysh bundle for ordinary differential operators, *Dokl. Akad. Nauk SSSR*, **227**(4):796–799, 1976 (in Russian).

79. V.A. Il'in, On a connection between the form of the boundary conditions and the basis property of equiconvergence with a trigonometric series of expansions in root functions of a nonself-adjoint differential operator, *Differ. Uravn.*, **30**(9):1402–1413, 1994 (in Russian).

80. V.A. Il'in, E.I. Moiseev, Nonlocal boundary value problem of the first kind for a Sturm–Liouville operator in its differential and finite difference aspects, *Differ. Equ.*, **23**(7):803–810, 1987.

81. V.A. Il'in, E.I. Moiseev, Nonlocal boundary value problem of the second kind for a Sturm–Liouville operator, *Differ. Equ.*, **23**(8):979–987, 1987.

82. G. Infante, Eigenvalues of some non-local boundary-value problems, *Proc. Edinb. Math. Soc., II Ser.*, **46**:75–86, 2003, http://dx.doi.org/10.1017/S0013091501001079.

83. G. Infante, Eigenvalues and positive solutions of ODEs involving integral boundary conditions, *Discrete Contin. Dyn. Syst.*, **2005**:436–442, 2005.

84. G. Infante, P. Pietramala, Nonnegative solutions for a system of impulsive BVPs with nonlinear nonlocal BCs, *Nonlinear Anal. Model. Control*, **19**(3):413–431, 2014, http://www.mii.lt/na/issues/NA_1403/NA19307.pdf.

85. G. Infante, J.R.L. Webb, Positive solutions of some nonlocal boundary value problems, *Abstr. Appl. Anal.*, **18**:1047–1060, 2003, http://dx.doi.org/10.1155/S1085337503301034.

86. G. Infante, J.R.L. Webb, Three-point boundary value problems with solutions that change sign, *J. Integral Equations Appl.*, **15**(1):37–57, 2003, http://dx.doi.org/10.1216/jiea/1181074944.

87. N.I. Ionkin, Difference schemes for one nonclassical problem, *Mosc. Univ. Comput. Math. Cybern.*, **2**:20–32, 1977 (in Russian).

88. N.I. Ionkin, Solution of a certain boundary value problem of the theory of heat conduction with a nonclassical boundary condition, *Differ. Uravn.*, **13**(2):204–304, 1977 (in Russian).

89. N.I. Ionkin, A problem for the equation of heat conduction with a nonclassical (nonlocal) boundary condition, Technical report No. 14, Numerikus Modzerek, Budapest, 1979.

90. N.I. Ionkin, E.I. Moiseev, Problem for the heat-transfer equation with two point boundary conditions, *Differ. Uravn.*, **15**(7):1284–1295, 1979 (in Russian).

91. N.I. Ionkin, V.A. Morozova, Two dimensional heat equation with nonlocal boundary conditions, *Differ. Uravn.*, **36**(7):884–888, 2000 (in Russian).

92. N.I. Ionkin, E.A. Valikova, On eigenvalues and eigenfunctions of a non-classical boundary value problem, *Mat. Model.*, **8**(1):53–63, 1996 (in Russian).

93. F. Ivanauskas, T. Meškauskas, M. Sapagovas, Stability of difference schemes for two-dimensional parabolic equations with non-local boundary conditions, *Appl. Math. Comput.*, **215**(7):2716–2732, 2009, http://dx.doi.org/10.1016/j.amc.2009.09.012.

94. F.F. Ivanauskas, Yu.A. Novitski, M.P. Sapagovas, On the stability of an explicit difference scheme for hyperbolic equations with nonlocal boundary conditions, *Differ. Equ.*, **49**(7):849–856, 2013, http://dx.doi.org/10.1134/S0012266113070070.

95. J. Jachimavičienė, The finite-difference method for a third-order pseudoparabolic equation with integral conditions, in: V. Kleiza, S. Rutkauskas, A. Štikonas (Eds.), *Proceedings of the International Conference on Differential Equations and Their Applications (Panevėžys, Lithuania, September 10–12, 2009)*, Technologija, Kaunas, 2009, pp. 49–58.

96. J. Jachimavičienė, *Solution of a pseudoparabolic equation with nonlocal integral conditions by the finite difference method*, PhD thesis, Vilnius University, 2013, http://www.mii.lt/files/mii_dis_2013_jachimaviciene.pdf.

97. J. Jachimavičienė, M. Sapagovas, Locally one-dimensional difference scheme for a pseudoparabolic equation with nonlocal conditions, *Lith. Math. J.*, **52**(1):53–61, 2012, http://dx.doi.org/10.1007/s10986-012-9155-7.

98. J. Jachimavičienė, M. Sapagovas, A. Štikonas, O. Štikonienė, On the stability of explicit finite difference schemes for a pseudoparabolic equation with nonlocal conditions, *Nonlinear Anal. Model. Control*, **19**(2):225–240, 2014. http://www.mii.lt/na/issues/NA_1902/NA19206.pdf.

99. J. Jachimavičienė, Ž. Jesevičiūtė, M. Sapagovas, The stability of finite-difference schemes for a pseudoparabolic equation with nonlocal conditions, *Numer. Funct. Anal. Optim.*, **30**(9):988–1001, 2009, http://dx.doi.org/10.1080/01630560903405412.

100. K. Jakubėlienė, *Solution of a Two-Dimensional Parabolic Equation with an Integral Condition by the Finite-Difference Method*, PhD thesis, Vilnius University, 2013, http://www.mii.lt/files/mii_dis_2013_jakubeliene.pdf.

101. K. Jakubėlienė, R. Čiupaila, M. Sapagovas, Solution of a two-dimensional elliptic equation with a nonlocal condition, in: V. Kleiza, S. Rutkauskas, A. Štikonas (Eds.), *Proceedings of the International Conference on Differential Equations and Their Applications (Panevėžys, Lithuania, September 10–12, 2009)*, Technologija, Kaunas, 2009, pp. 49–58.

102. K. Jakubėlienė, M. Sapagovas, On the stability of a difference scheme for a two-dimensional parabolic equation with an integral condition, *Lith. Math. J.*, **53**(3):311–323, 2013, http://dx.doi.org/10.1007/s10986-013-9210-z.

103. T. Jankowski, Existence of positive solutions to third order differential equations with advanced arguments and nonlocal boundary conditions, *Nonlinear Anal., Theory Methods Appl.*, **75**(2):913–923, 2012, http://dx.doi.org/10.1016/j.na.2011.09.025.

104. T. Jankowski, Nonnegative solutions to nonlocal boundary value problems for systems of second-order differential equations dependent on the first-order derivatives, *Nonlinear Anal., Theory Methods Appl.*, **87**:83–101, 2013, http://dx.doi.org/10.1016/j.na.2013.04.004.

105. Ž. Jesevičiūtė, On one eigenvalue problem for a differential operator with integral conditions, in: V. Kleiza, S. Rutkauskas, A. Štikonas (Eds.), *Proceedings of the International Conference on Differential Equations and Their Applications (Panevėžys, Lithuania, September 10–12, 2009)*, Technologija, Kaunas, 2009, pp. 49–58.

106. Ž. Jesevičiūtė, *The Eigenvalue Problem for Differential Operator with Nonlocal Integral Conditions*, PhD thesis, Vilnius University, 2010, http://www.mii.lt/files/mii_dis_2010_jeseviciute.pdf.

107. Ž. Jesevičiūtė, M. Sapagovas, On the stability of the finite-difference schemes for parabolic equations subject to integral conditions with applications for thermoelasticity, *Comp. Meth. Appl. Math.*, **8**(4):360–373, 2008, http://dx.doi.org/10.2478/cmam-2008-0026.

108. J. Jiang, L. Liu, Y. Wu, Second-order nonlinear singular Sturm–Liouville problems with integral boundary conditions, *Appl. Math. Comput.*, **215**:1573–1582, 2009, http://dx.doi.org/10.1016/j.amc.2009.07.024.

109. L.I. Kamynin, A boundary value problem in the theory of the heat conduction with nonclassical boundary condition, *Zh. Vychisl. Mat. Mat. Fiz.*, **4**(6):1006–1024, 1964 (in Russian).

110. I. Karatay, S.R. Bayramoglu, B. Yildiz, B. Kokluce, Matrix stability of the difference schemes for nonlocal boundary value problems for parabolic differential equations, *Int. J. Phys. Sci.*, **6**(4):819–827, 2011.

111. E. Karulina, Some estimates for the minimal eigenvalue of the Sturm–Liouville problem with third-type boundary conditions, *Math. Bohem.*, **136**(4):377–384, 2011.

112. A.J. Khaleel, J.A. Khtan, A numerical method for solving the multi-dimensional hyperbolic equations with nonlocal non-linear conditions, *J. Kerbala University*, **10**(4):53–61, 2012.

113. L. Kong, Q. Kong, Higher order boundary value problems with nonhomogeneous boundary conditions, *Nonlinear Anal., Theory Methods Appl.*, **72**:240–261, 2010, http://dx.doi.org/10.1016/j.na.2009.06.050.

114. V. Korzyuk, V. Erofeenko, J. Sheika, Classical solution for initial-boundary value problem for wave equation with integral boundary condition, *Math. Model. Anal.*, **17**(3):309–329, 2012, http://dx.doi.org/10.3846/13926292.2012.685955.

115. Y. Li, Z.-L. Zhang, Digraph Laplacian and the degree of asymmetry, *Internet Math.*, **8**(4):321–481, 2012.

116. H. Liu, Z. Ouyang, Existence of solutions for second-order three-point integral boundary value problems at resonance, *Boundary Value Probl.*, **2013**(197), 2013, http://dx.doi.org/10.1186/1687-2770-2013-197.

117. Y. Liu, D. O'Regan, Multiplicity results using bifurcation techniques for a class of fourth-order m-point boundary value problems, *Boundary Value Probl.*, **2009**:1–20, 2009, http://dx.doi.org/10.1155/2009/970135.

118. Y. Liu, Z. Weiguo, S. Chunfang, Monotone and convex positive solutions for fourth-order multi-point boundary value problems, *Boundary Value Probl.*, **2011**(21), 2011, http://dx.doi.org/10.1186/1687-2770-2011-21.

119. J. Locker, On constructing least squares solutions to two-point boundary value problems, *Trans. Am. Math. Soc.*, **203**:175–183, 1975.

120. J. Locker, The generalized Green's function for an n-th order linear differential operator, *Trans. Am. Math. Soc.*, **228**:243–268, 1977.

121. Q. Ma, Existence of positive solutions for the symmetry three-point boundary-value problem, *Electron. J. Differ. Equ.*, **2007**(154):1–8, 2007.

122. R. Ma, Nonlinear periodic boundary value problems with sign-changing Green's function, *Nonlinear Anal., Theory Methods Appl.*, **74**(5):1714–1720, 2011, http://dx.doi.org/10.1016/j.na.2010.10.043.

123. R. Ma, Y. An, Global structure of positive solutions for nonlocal boundary value problems involving integral conditions, *Nonlinear Anal., Theory Methods Appl.*, **71**:4364–4376, 2009, http://dx.doi.org/10.1016/j.na.2009.02.113.

124. R.Y. Ma, Existence of positive solutions for superlinear semipositone m-point boundary value problems, *Proc. Edinb. Math. Soc., II Ser.*, **46**:279–292, 2003, http://dx.doi.org/10.1017/S0013091502000391.

125. R.Y. Ma, A survey on nonlocal boundary value problems, *Appl. Math. E-Notes*, **7**:257–279, 2007.

126. R.Y. Ma, B. Thompson, Global behavior of positive solutions of nonlinear three-point boundary value problems, *Nonlinear Anal., Theory Methods Appl.*, **60**(4):685–701, 2005, http://dx.doi.org/10.1016/j.na.2004.09.044.

127. V.L. Makarov, D.T. Kulyev, The line method for quasilinear parabolic equation with nonlocal boundary condition, *Ukr. Math. J.*, **37**(1):42–48, 1985.

128. V.L. Makarov, D.T. Kulyev, The solution of one boundary value problem for quasilinear parabolic equation with nonclassical boundary condition, *Differ. Equ.*, **21**(2):296–305, 1985.

129. V.L. Makarov, D.T. Kulyev, T. Arazmydarov, The solution of one boundary value problem for quasilinear ordinary differential equation with nonclassical boundary conditions, *Comput. Appl. Math.*, (19):31–42, 1984 (in Russian).

130. V.L. Makarov, D. Sytnyk, V. Vasylyk, Existence of the solution to a nonlocal-in-time evolutional problem, *Nonlinear Anal. Model. Control*, **19**(3):432–447, 2014, `http://www.mii.lt/na/issues/NA_1403/NA19308.pdf`.

131. I.G. Mamedov, Non-local combined problem, 2013, arXiv:1302.3151 (in Russian).

132. D. Maroncelli, J. Rodríguez, A least squares solution to linear boundary value problems with impulses, *Differ. Equ. Appl.*, **5**(4):519–525, 2013, `http://dx.doi.org/10.7153/dea-05-30`.

133. J. Martín-Vaquero, Polynomial-based mean weighted residuals methods for elliptic problems with nonlocal boundary conditions in the rectangle, *Nonlinear Anal. Model. Control*, **19**(3): 448–459, 2014, `http://www.mii.lt/na/issues/NA_1403/NA19309.pdf`.

134. J. Martín-Vaquero, B.A. Wade, On efficient numerical methods for an initial-boundary value problem with nonlocal boundary conditions, *Appl. Math. Model.*, **36**(8):3411–3418, 2012, `http://dx.doi.org/10.1016/j.apm.2011.10.021`.

135. X. Mary, On generalized inverses and Greens relations, *Linear Algebra Appl.*, **434**(8):1836–1844, 2011, `http://dx.doi.org/10.1016/j.laa.2010.11.045`.

136. R. Mennicken, M. Möller, *Non-Self-Adjoint Boundary Eigenvalue Problems*, Elsevier, 2003.

137. A.Ju. Mokin, On spectral properties of a nonselfadjoint difference operator, *Computer Research and Modeling*, **2**(2):143–150, 2010 (in Russian).

138. A.Yu. Mokin, Application of nonclassical separation of variables to a nonlocal heat problem, *Differ. Equ.*, **49**(1):60–67, 2011, `http://dx.doi.org/10.1134/S0012266113010060`.

139. A.Yu. Mokin, Stability of a family of difference schemes for the Samarskii–Ionkin problem with variable coefficient, *Differ. Equ.*, **50**(2):254–263, 2014, `http://dx.doi.org/10.1134/S001226611402013X`.

140. U.D. Moldojarov, Nelokal'naya zadacha s integral'nymi usloviyami dlya nelineinogo uravneniya v chastnykh proizvodnykh tret'ego poryadka, *Izvestiya Tomskogo Politekhnicheskogo Universiteta*, **321**(2):14–17, 2012 (in Russian).

141. J. Niu, Y.Z. Lin, C.P. Zhang, Approximate solution of nonlinear multi-point boundary value problem on the half-line, *Math. Model. Anal.*, **17**(2):190–202, 2012, `http://dx.doi.org/10.3846/13926292.2012.660889`.

142. L.P. Nizhnik, Inverse eigenvalue problems for nonlocal Sturm–Liouville operators on a star graph, *Methods Funct. Anal. Topol.*, **18**(1):68–78, 2012.

143. J. Novickij, A. Štikonas, On the stability of a weighted finite difference scheme for wave equation with nonlocal boundary conditions, *Nonlinear Anal. Model. Control*, **19**(3):460–475, 2014, http://www.mii.lt/na/issues/NA_1903/NA19310.pdf.

144. S.K. Ntouyas, Nonlocal initial and boundary value problems: A survey, in: A. Cañada, P. Drábek, A. Fonda (Eds.), *Handbook of Differential Equations: Ordinary Differential Equations*, Vol. 2, Elsevier, 2005, pp. 461–558, http://dx.doi.org/10.1016/S1874-5725(05)80008-2.

145. K. Özen, K. Oruçoğlu, A representative solution to m-order linear ordinary differential equation with nonlocal conditions by Green's functional concept, *Math. Model. Anal.*, **17**(4): 571–588, 2012, http://dx.doi.org/10.3846/13926292.2012.709471.

146. K. Özen, K. Oruçoğlu, Approximate solution to a multi-point boundary value problem involving nonlocal integral conditions by reproducing kernel method, *Math. Model. Anal.*, **18**(4):529–536, 2013, http://dx.doi.org/10.3846/13926292.2013.840867.

147. B. Panejah, P. Panejah, Nelokal'nye zadachi v teorii giperbolicheskikh differencialnykh uravnenii, *Tr. Mosk. Mat. O.-va* , **70**:183–234, 2009 (in Russian).

148. C.V. Pao, Y.-M. Wang, Nonlinear fourth-order elliptic equations with nonlocal boundary conditions, *J. Math. Anal. Appl.*, **372**:351–365, 2010, http://dx.doi.org/10.1016/j.jmaa.2010.07.027.

149. G. Paukštaitė, A. Štikonas, Generalized Green's functions for the second order discrete problems with nonlocal conditions, *Lith. Math. J.*, **54**(2):203–219, 2014, http://dx.doi.org/10.1007/s10986-014-9238-8.

150. G. Paukštaitė, A. Štikonas, Generalized Green's functions for second-order discrete boundary-value problems with nonlocal boundary conditions, *Liet. matem. rink. Proc. LMS, Ser. A*, **53**:96–101, 2012, ftp://ftp.science.mii.lt/pub/Publications/LMR/53(2012)/Series_A/MATHEMATICS/Paukstaite_Stikonas.pdf.

151. G. Paukštaitė, A. Štikonas, Investigation of matrix nullity for the second order discrete nonlocal boundary value problem, *Liet. matem. rink. Proc. LMS, Ser. A*, **54**:49–54, 2013, ftp://ftp.science.mii.lt/pub/Publications/LMR/54(2013)/Series_A/MATHEMATICS/Paukstaite_Stikonas.pdf.

152. S. Pečiulytė, *Investigation and Numerical Analysis of Boundary Poblems with Various Types Nonlocal Conditions*, PhD thesis, Institute of Mathematics and Informatics, Vilnius, 2007, http:/www.mii.lt/files/mii_dis_2007_peciulyte.pdf.

153. S. Pečiulytė, A. Štikonas, Sturm–Liouville problem for stationary differential operator with nonlocal two-point boundary conditions, *Nonlinear Anal. Model. Control*, **11**(1):47–78, 2006, http://www.mii.lt/na/issues/NA_1101/NA11104.pdf.

154. S. Pečiulytė, A. Štikonas, On positive eigenfunctions of Sturm–Liouville problem with nonlocal two-point boundary condition, *Math. Model. Anal.*, **12**(2):215–226, 2007, http://dx.doi.org/10.3846/1392-6292.2007.12.215-226.

155. S. Pečiulytė, A. Štikonas, On the spectrum of the Sturm–Liouville problem with one nonlocal boundary condition, in: V. Kleiza, S. Rutkauskas, A. Štikonas (Eds.), *Proceedings of the International Conference on Differential Equations and Their Applications (Panevėžys, Lithuania, September 10–12, 2009)*, Technologija, Kaunas, 2009, pp. 49–58.

156. S. Pečiulytė, O. Štikonienė, A. Štikonas, Sturm–Liouville problem for stationary differential operator with nonlocal integral boundary condition, *Math. Model. Anal.*, **10**(4):377–392, 2005, http://dx.doi.org/10.1080/13926292.2005.9637295.

157. S. Pečiulytė, O. Štikonienė, A. Štikonas, Sturm–Liouville problem for stationary differential operator with nonlocal integral boundary condition, in: R. Čiegis (Ed.), *Proceedings of the 10th International Conference MMA2005&CMAM2 (Trakai, Lithuania, June 1–5, 2005)*, Technika, Vilnius, 2005, pp. 199–204, http://inga.vgtu.lt/ art/proc/file/PeciSi.pdf.

158. S. Pečiulytė, O. Štikonienė, A. Štikonas, Investigation of negative critical points of the characteristic function for problems with nonlocal boundary conditions, *Nonlinear Anal. Model. Control*, **13**(4):467–490, 2008, http://www.mii.lt/na/issues/NA_1304/NA13405.pdf.

159. M. Picone, Su un problema al contorno nelle equazioni differenziali lineari ordinarie del secondo ordine, *Ann. Sc. Norm. Super. Pisa, Cl. Sci.*, **10**:1–95, 1908.

160. L.S. Pul'kina, Boundary-value problems for a hyperbolic equation with nonlocal conditions of the I and II kind, *Russian Math.*, **56**(4):62–69, 2012, http://dx.doi.org/10.3103/S1066369X12040081.

161. L.S. Pul'kina, A nonlocal problem for a hyperbolic equation with integral conditions of the 1st kind with time-dependent kernels, *Russian Math.*, **56**(10):26–37, 2012, http://dx.doi.org/10.3103/S1066369X12100039.

162. J.M. Rassias, E.T. Karimov, Boundary-value problems with non-local initial condition for parabolic equations with parameter, *European J. Pure Appl. Math.*, **3**(6):948–957, 2010.

163. S. Roman, *Green's Functions for Boundary-Value Problems with Nonlocal Boundary Conditions*, PhD thesis, Vilnius University, 2011, http:/www.mii.lt/files/s_roman_mii_disertacija.pdf.

164. S. Roman, Linear differential equation with additional conditions and formulae for Green's function, *Math. Model. Anal.*, **16**(3):401–417, 2011, http://dx.doi.org/10.3846/13926292.2011.602125.

165. S. Roman, A. Štikonas, Green's functions for stationary problems with four-point nonlocal boundary conditions, in: V. Kleiza, S. Rutkauskas, A. Štikonas (Eds.), *Proceedings of the International Conference on Differential Equations and Their Applications (Panevėžys, Lithuania, September 10–12, 2009)*, Technologija, Kaunas, 2009, pp. 49–58.

166. S. Roman, A. Štikonas, Green's functions for stationary problems with nonlocal boundary conditions, *Lith. Math. J.*, **49**(2):190–202, 2009, http://dx.doi.org/10.1007/s10986-009-9041-0.

167. S. Roman, A. Štikonas, Third-order linear differential equation with three additional conditions and formula for Green's function, *Lith. Math. J.*, **50**(4):426–446, 2010, http://dx.doi.org/10.1007/s10986-010-9097-x.

168. S. Roman, A. Štikonas, Green's function for discrete second-order problems with nonlocal boundary conditions, *Boundary Value Probl.*, **2011**, Article ID 767024, 23 pp., 2011, http://dx.doi.org/10.1155/2011/767024.

169. S. Rutkauskas, Dirichlet type problem for the system of elliptic equations, which order degenerate at a line, *Nonlinear Anal. Model. Control*, **19**(3):476–487, 2014, http://www.mii.lt/na/issues/NA_1403/NA19311.pdf.

170. S. Sajavičius, *Numerical Analysis of Differential Problems with Nonlocal Initial and Boundary Conditions*, PhD thesis, Vilnius University, 2013.

171. S. Sajavičius, M. Sapagovas, Numerical analysis of the eigenvalue problem for one-dimensional differential operator with nonlocal integral conditions, *Nonlinear Anal. Model. Control*, **14**(1):115–122, 2009, http://www.mii.lt/na/issues/NA_1401/NA14109.pdf.

172. A.A. Samarskii, *The Theory of Difference Schemes*, Marcel Dekker, New York, Basel, 2001.

173. A.A. Samarskii, E.S. Nikolaev, *Metody Resheniya Setochnykh Uravnenii*, Nauka, Moscow, 1978 (in Russian).

174. M.P. Sapagovas, The alternately-triangular method for the solving of the problems with nonlinear or nonselfadjoined operator, *Differ. Uravn. Primen.*, **21**:61–89, 1978 (in Russian).

175. M.P. Sapagovas, The determination of free surface of the drop by finite elements method, in: G.I. Marchuk (Ed.), *Variacionno-Raznostnye Metody v Matematicheskoi Fizike*, Novosibirsk, 1978, pp. 117–128 (in Russian).

176. M.P. Sapagovas, The difference method for the solution of the problem on the equilibrium of a drop of liquid, *Differ. Uravn. Primen.*, **31**:63–72, 1978 (in Russian).

177. M.P. Sapagovas, Difference scheme for two-dimensional elliptic problem with an integral condition, *Lith. Math. J.*, **23**(3):317–320, 1983, http://dx.doi.org/10.1007/BF00966473.

178. M.P. Sapagovas, Investigation of nonclassical drop equation, *Differ. Equ.*, **19**(7):1271–1276, 1983.

179. M.P. Sapagovas, Numerical methods for the solution of the equation of surface with prescribed mean curvature, *Lith. Math. J.*, **23**(3):160–166, 1983.

180. M.P. Sapagovas, The numerical method for the solution of the problem on the equilibrium of a drop of liquid, *Banach Cent. Publ.*, Computational Mathematics, **13**:45–59, 1984 (in Russian).

181. M.P. Sapagovas, Numerical methods for two-dimensional problem with nonlocal conditions, *Differ. Equ.*, **20**(7):1258–1266, 1984.

182. M.P. Sapagovas, The solution of nonlinear ordinary differential equation with integral condition, *Lith. Math. J.*, **24**(1):60–68, 1984, http://dx.doi.org/10.1007/BF00970337.

183. M.P. Sapagovas, The solution of difference equations, arising from differential problem with integral condition, in: G.I. Marchuk (Ed.), *Comput. Process and Systems*, No. 6, Nauka, Moscow, 1988, pp. 245–253 (in Russian).

184. M.P. Sapagovas, The boundary value problem with a nonlocal condition for a system of ordinary differential equations, *Differ. Uravn.*, **36**(7):971–978, 2000 (in Russian).

185. M.P. Sapagovas, The eigenvalues of some problem with a nonlocal condition, *Differ. Equ.*, **38**(7):1020–1026, 2002, http://dx.doi.org/10.1023/A:1021115915575.

186. M. Sapagovas, *Diferencialinių lygčių kraštiniai uždaviniai su nelokaliosiomis sąlygomis*, Mokslo aidai, Vilnius, 2007 (in Lithuanian).

187. M. Sapagovas, On the stability of a finite-difference scheme for nonlocal parabolic boundary-value problems, *Lith. Math. J.*, **48**(3):339–356, 2008, http://dx.doi.org/10.1007/s10986-008-9017-5.

188. M.P. Sapagovas, Difference method of increased order of accuracy for the Poisson equation with nonlocal conditions, *Differ. Equ.*, **44**(7):1018–1028, 2008, http://dx.doi.org/10.1134/S0012266108070148.

189. M. Sapagovas, On the spectral properties of three-layer difference schemes for parabolic equations with nonlocal conditions, *Differ. Equ.*, **48**(7):1018–1027, 2012, http://dx.doi.org/10.1134/S0012266112070129.

190. M.P. Sapagovas, R. Čiegis, The numerical solution of some nonlocal problems, *Lith. Math. J.*, **27**(2):348–356, 1987.

191. M.P. Sapagovas, R. Čiegis, On some boundary problems with nonlocal conditions, *Differ. Equ.*, **23**(7):1268–1274, 1987.

192. M. Sapagovas, R. Čiupaila, Ž. Jokšienė, T. Meškauskas, Computational experiment for stability analysis of difference schemes with nonlocal conditions, *Informatica*, **24**(2):275–290, 2013.

193. M. Sapagovas, K. Jakubėlienė, Alternating direction method for two-dimensional parabolic equation with nonlocal integral condition, *Nonlinear Anal. Model. Control*, **17**(1):91–98, 2012, http://www.mii.lt/na/issues/NA_1701/NA17107.pdf.

194. M. Sapagovas, G. Kairytė, O. Štikonienė, A. Štikonas, Alternating direction method for a two-dimensional parabolic equation with a nonlocal boundary condition, *Math. Model. Anal.*, **12**(1):131–142, 2007, http://dx.doi.org/10.3846/1392-6292.2007.12.131-142.

195. M. Sapagovas, T. Meškauskas, F. Ivanauskas, Numerical spectral analysis of a difference operator with non-local boundary conditions, *Appl. Math. Comput.*, **218**(14):7515–7527, 2012, http://dx.doi.org/10.1016/j.amc.2012.01.017.

196. M. Sapagovas, A. Štikonas, O. Štikonienė, Alternating direction method for the Poisson equation with variable weight coefficients in an integral condition, *Differ. Equ.*, **47**(8):1163–1174, 2011, http://dx.doi.org/10.1134/S0012266111080118.

197. M.P. Sapagovas, A.D. Štikonas, On the structure of the spectrum of a differential operator with a nonlocal condition, *Differ. Equ.*, **41**(7):1010–1018, 2005, http://dx.doi.org/10.1007/s10625-005-0242-y.

198. M. Sapagovas, O. Štikonienė, A fourth-order alternating-direction method for difference schemes with nonlocal condition, *Lith. Math. J.*, **49**(3):309–317, 2009, http://dx.doi.org/10.1007/s10986-009-9057-5.

199. M. Sapagovas, O. Štikonienė, Alternating-direction method for a mildly nonlinear elliptic equation with nonlocal integral conditions, *Nonlinear Anal. Model. Control*, **16**(2):220–230, 2011, http://www.mii.lt/na/issues/NA_1602/NA16208.pdf.

200. L. Serbina, The solution of an initial-boundary value problem of the filtration theory with nonlocal boundary condition, *Nonlinear Anal. Model. Control*, **19**(3):488–502, 2014, http://www.mii.lt/na/issues/NA_1403/NA19312.pdf.

201. N. Sergejeva, The regions of solvability for some three point problem, *Math. Model. Anal.*, **18**(2):191–203, 2013, http://dx.doi.org/10.3846/13926292.2013.780189.

202. N. Sergejeva, The Fučík spectrum for nonlocal BVP with Sturm–Liouville boundary condition, *Nonlinear Anal. Model. Control*, **19**(3):503–516, 2014, http://www.mii.lt/na/issues/NA_1403/NA19313.pdf.

203. N. Sergejeva, On solvability of the damped Fučìk type problem with integral condition, *Math. Model. Anal.*, **19**(3):417–429, 2014, http://dx.doi.org/10.3846/13926292.2014.927014.

204. A.A. Shkalikov, Bases formet by eigenfunctions of ordinary differential operators with integral boundary conditions, *Vestn. Mosk. Univ., Ser. I*, **6**:12–21, 1982 (in Russian).

205. A. Skučaitė, K. Skučaitė-Bingelė, S. Pečiulytė, A. Štikonas, Investigation of the spectrum for the Sturm–Liouville problem with one integral boundary condition, *Nonlinear Anal. Model. Control*, **15**(4):501–512, 2010, http://www.mii.lt/na/issues/NA_1504/NA15412.pdf.

206. A. Skučaitė, A. Štikonas, Investigation of the spectrum of the Sturm–Liouville problem with a nonlocal integral condition, *Liet. matem. rink. Proc. LMS,* Ser. A, **54**:67–72, 2013, ftp://ftp.science.mii.lt/pub/Publications/LMR/54(2013)/Series_A/MATHEMATICS/Skucaite_Stikonas.pdf.

207. K. Skučaitė-Bingelė, A. Štikonas, Investigation of complex eigenvalues for a stationary problem with two-point nonlocal boundary condition, *Liet. matem. rink. LMD darbai*, **52**:303–308, 2011, ftp://ftp.science.mii.lt/pub/Publications/52_TOMAS(2011)/SKAICIAVIMO_MATEMATIKA/sm_SkucK_Stik.pdf.

208. K. Skučaitė-Bingelė, A. Štikonas, Inverstigation of the spectrum for Sturm–Liouville problems with a nonlocal boundary condition, *Liet. matem. rink. Proc. LMS,* Ser. A, **54**:73–78, 2013, ftp://ftp.science.mii.lt/pub/Publications/LMR/54(2013)/Series_A/MATHEMATICS/Skucaite-Bingele_Stikonas.pdf.

209. S. Smirnov, On some spectral properties of third order nonlinear boundary value problems, *Math. Model. Anal.*, **17**(1):78–89, 2012, http://dx.doi.org/10.3846/13926292.2012.645076.

210. S. Smirnov, Nonlocal third order boundary value problems with solutions that change sign, *Math. Model. Anal.*, **19**(2):145–154, 2014, http://dx.doi.org/10.3846/13926292.2014.908792.

211. A. Sopuev, N.K. Arkabaev, Nelokal'naya zadacha s integral'nym usloviem dlya lineinogo uravneniya v chastnykh proizvodnykh tret'ego poryadka, *Vestnik KRSU*, **10**(9):150–153, 2010 (in Russian).

212. I. Stakgold, *Green's Functions and Boundary Value Problems*, Wiley-Interscience, New York, 1979.

213. I. Stakgold, M. Holst, *Green's Functions and Boundary Value Problems*, Wiley-Interscience, Hoboken, NJ, 2010.

214. A. Štikonas, The Sturm–Liouville problem with a nonlocal boundary condition, *Lith. Math. J.*, **47**(3):336–351, 2007, http://dx.doi.org/10.1007/s10986-007-0023-9.

215. A. Štikonas, Investigation of characteristic curve for Sturm–Liouville problem with nonlocal boundary conditions on torus, *Math. Model. Anal.*, **16**(1):1–22, 2011, http://dx.doi.org/10.3846/13926292.2011.552260.

216. A. Štikonas, S. Roman, Stationary problems with two additional conditions and formulae for Green's functions, *Numer. Funct. Anal. Optim.*, **30**(9):1125–1144, 2009, http://dx.doi.org/10.1080/01630560903420932.

217. A. Štikonas, S. Roman, Green's functions for discrete mth-order problems, *Lith. Math. J.*, **52**(3):334–351, 2012, http://dx.doi.org/10.1007/s10986-012-9177-1.

218. A. Štikonas, O. Štikonienė, Characteristic functions for Sturm–Liouville problems with nonlocal boundary conditions, *Math. Model. Anal.*, **14**(2):229–246, 2009, http://dx.doi.org/10.3846/1392-6292.2009.14.229-246.

219. O. Štikonienė, M. Sapagovas, R. Čiupaila, On iterative methods for some elliptic equations with nonlocal conditions, *Nonlinear Anal. Model. Control*, **19**(3):517–535, 2014, http://www.mii.lt/na/issues/NA_1903/NA19314.pdf.

220. O. Subož, *Some Problems with Non-Local Boundary Conditions*, PhD thesis, Vilnius Gediminas Technical University, 2002.

221. Y. Sun, Eigenvalues and symmetric positive solutions for a three-point boundary-value problem, *Electron. J. Differ. Equ.*, **2005**(127):1–7, 2005.

222. L.X. Truong, L.T.P. Ngoc, N.T. Long, Positive solutions for an m-point boundary-value problem, *Electron. J. Differ. Equ.*, **2008**(111):1–11, 2008.

223. N. Tumanova, *The Numerical Analysis of Nonlinear Mathematical Models on Graphs*, PhD thesis, Vilnius Gediminas Technical University, 2012.

224. F. Wang, Y. An, On positive solutions for nonhomogeneous m-point boundary value problems with two parameters, *Boundary Value Probl.*, **2012**(87), 2012, http://dx.doi.org/10.1186/1687-2770-2012-87.

225. G. Wang, S. Liu, L. Zhang, Eigenvalue problem for nonlinear fractional differential equations with integral boundary conditions, *Abstr. Appl. Anal.*, **2014**, Article ID 916260, 6 pp., 2014, http://dx.doi.org/10.1155/2014/916260.

226. J.R.L. Webb, Remarks on positive solutions of some three point boundary value problems, in: W. Feng, S. Hu, X. Lu (Eds.), *Proceedings of the Fourth International Conference on Dynamical Systems and Differential Equations (Wilmington, NC, USA, May 24–27, 2002)*, AIMS, 2003, pp. 905–915.

227. J.R.L. Webb, Positive solutions of some higher order nonlocal boundary value problems, *Electron. J. Qual. Theory Differ. Equ.*, **29**:1–15, 2009.

228. J.R.L. Webb, G. Infante, Positive solutions of nonlocal boundary value problems involving integral conditions, *NoDEA, Nonlinear Differ. Equ. Appl.*, **15**:45–67, 2008, http://dx.doi.org/10.1007/s00030-007-4067-7.

229. J.R.L. Webb, G. Infante, Nonlocal boundary value problems of arbitrary order, *J. London Math. Soc.*, **79**:238–258, 2009, http://dx.doi.org/10.1112/jlms/jdn066.

230. J.R.L. Webb, G. Infante, D. Franco, Positive solutions of nonlinear fourth order boundary value problems with local and nonlocal boundary conditions, *Proc. R. Soc. Edinb., Sect. A, Math.*, **138**:427–446, 2008, http://dx.doi.org/10.1017/S0308210506001041.

231. J.R.L. Webb, K. Lan, Eigenvalue criteria for existence of multiple positive solutions of nonlinear boundary value problems of local and nonlocal type, *Topol. Methods Nonlinear Anal.*, **27**:91–116, 2006.

232. W.M. Whyburn, Differential equations with general boundary conditions, *Bull. Am. Math. Soc.*, **48**:692–704, 1942.

233. D. Xie, Y. Liu, C. Bai, Green's function and positive solutions of a singular nth-order three-point boundary value problem on time scales, *Electron. J. Differ. Equ.*, **2009**(38):1–14, 2009.

234. H. Xu, S.-T. Yau, Discrete Green's functions and random walks on graphs, *J. Comb. Theory, Ser. A*, **120**(2):483–499, 2013, http://dx.doi.org/10.1016/j.jcta.2012.10.002.

235. D. Yan, On linear and nonlinear fourth-order eigenvalue problems with nonlocal boundary condition, *J. Funct. Spaces Appl.*, **2013**, 2013, Article ID 932145, 10 pp., http://dx.doi.org/10.1155/2013/932145.

236. B. Yang, Positive solutions of a third-order three-point boundary-value problem, *Electron. J. Differ. Equ.*, **2008**(99):1–10, 2008.

237. N.I. Yurchuk, Mixed problem with an integral condition for certain parabolic equations, *Differ. Uravn.*, **22**(12):2117–2126, 1986 (in Russian).

238. X. Zhang, W. Ge, Symmetric positive solutions of boundary value problems with integral boundary conditions, *Appl. Math. Comput.*, **219**(8):3553–3564, 2012, http://dx.doi.org/10.1016/j.amc.2012.09.037.

239. J. Zhao, F. Geng, J. Zhao, W. Ge, Positive solutions to a new kind Sturm–Liouville-like four-point boundary value problem, *Appl. Math. Comput.*, **217**:811–819, 2010, http://dx.doi.org/ 10.1016/j.amc.2010.06.020.

240. Z. Zhao, Positive solutions for singular three-point boundary-value problems, *Electron. J. Differ. Equ.*, **2007**(156):1–8, 2007.

241. Z. Zhao, Exact solutions of a class of second-order nonlocal boundary value problems and applications, *Appl. Math. Comput.*, **215**:1926–1936, 2009, http://dx.doi.org/10.1016/j.amc.2009.07.043