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Global existence and asymptotic behavior of the solutions to the 3D bipolar non-isentropic Euler–Poisson equation*

Yeping Li

Department of Mathematics
East China University of Science and Technology
Shanghai 200237, China
ypleemei@gmail.com

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Abstract. In this paper, the global existence of smooth solutions for the three-dimensional (3D) non-isentropic bipolar hydrodynamic model is showed when the initial data are close to a constant state. This system takes the form of non-isentropic Euler–Poisson with electric field and frictional damping added to the momentum equations. Moreover, the L^2-decay rate of the solutions is also obtained. Our approach is based on detailed analysis of the Green function of the linearized system and elaborate energy estimates. To our knowledge, it is the first result about the existence and L^2-decay rate of global smooth solutions to the multi-dimensional non-isentropic bipolar hydrodynamic model.

Keywords: non-isentropic bipolar Euler–Poisson system, asymptotic behavior, Green function, smooth solution, energy estimates.

1 Introduction

In this paper, we consider the following three-dimensional non-isentropic bipolar hydrodynamic model:

$$\begin{aligned}
&\partial_t\rho_1 + \nabla\cdot m_1 = 0,\\
&\partial_t m_1 + \nabla\cdot\left(\frac{m_1\otimes m_1}{\rho_1}\right) + \nabla(\rho_1 T_1) = \rho_1\nabla\phi - \frac{m_1}{\tau_1},\\
&\partial_t T_1 + \frac{m_1}{\rho_1}\nabla T_1 + \frac{2}{3}T_1\nabla\cdot\left(\frac{m_1}{\rho_1}\right) - \frac{2}{3\rho_1}\nabla\cdot(\kappa\nabla T_1)\\
&\quad = \frac{2\tau_2-\tau_1}{3\tau_1\tau_2}\left|\frac{m_1}{\rho_1}\right|^2 - \frac{T_1 - T_{L_1}}{\tau_2},
\end{aligned} \tag{1_1}$$

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\begin{aligned}
&\partial_t \rho_2 + \nabla \cdot m_2 = 0, \\
&\partial_t m_2 + \nabla \cdot \left(\frac{m_2 \otimes m_2}{\rho_2} \right) + \nabla(\rho_2 T_2) = -\rho_2 \nabla \phi - \frac{m_2}{\tau_1}, \\
&\partial_t T_2 + \frac{m_2}{\rho_2} \nabla T_2 + \frac{2}{3} T_2 \nabla \cdot \left(\frac{m_2}{\rho_2} \right) - \frac{2}{3\rho_2} \nabla \cdot (\kappa \nabla T_2) \\
&\quad = \frac{2\tau_2 - \tau_1}{3\tau_1 \tau_2} \left| \frac{m_2}{\rho_2} \right|^2 - \frac{T_2 - T_{L_2}}{\tau_2}, \\
&\lambda^2 \Delta \phi = \rho_1 - \rho_2.
\end{aligned}
\tag{1_2}
$$

Here $(t, x) \in \mathbb{R}_+ \times \mathbb{R}^3$, and the unknown variables ρ_i, m_i, T_i $(i = 1, 2)$, and ϕ are the charge densities, current densities, temperatures, and electrostatic potential. The coefficients τ_1, τ_2, κ and λ are the momentum relaxation time, the energy relaxation limit, the heat conduction and the Debye length, respectively. The constants T_{L_1} and T_{L_2} stand for the lattice temperature. The non-isentropic bipolar hydrodynamic model plays an important role in simulating the behavior of charge carries in submicron semiconductor devices, see the pioneering work by Blotekjaer in [4], and also see [2,3]. The model takes the nonisentropic Euler–Poisson form, and consists of a set of nonlinear conservation laws for particle number, momentum, and energy, plus Poisson's equation for the electric potential. Moreover, it is worth mentioning that there are a lot of simplified models in the fields of applied and computational mathematics, i.e., we can refer to [11, 16, 20], etc.

Recently, many efforts were made for the isentropic bipolar hydrodynamic equations from semiconductors or plasmas. Zhou and Li [25] and Tsuge [22] discussed the unique existence of the stationary solutions for the one-dimensional bipolar hydrodynamic model with proper boundary conditions. Natalini [18] and Hsiao and Zhang [6] established the global entropy weak solutions in the framework of compensated compactness on the whole real line and bounded domain respectively. Hattori and Zhu [26] proved the stability of steady-state solutions for a recombined one-dimensional bipolar hydrodynamical model. Gasser, Hsiao and Li [5] investigated the large time behavior of smooth "small" solutions for the one-dimensional bipolar hydrodynamic model, and they found that the frictional damping is the key to the nonlinear diffusive phenomena of hyperbolic waves. Huang and Li [7] also studied the large-time behavior and quasi-neutral limit of L^∞ solution for large initial data with vacuum. Huang, Mei and Wang [8] discussed the large time behavior of solution to n-dimensional bipolar hydrodynamic model for semiconductors in switch-on case. Ali and Jüngel [1] and Li and Zhang [15] studied the global smooth solutions of the Cauchy problem for multidimensional bipolar hydrodynamic models in the Sobolev space $H^l(\mathbb{R}^d)$ $(l > 1 + d/2)$ and in the Besov space, respectively. Ju [10] discussed the global existence of smooth solutions to the IBVP for the 3D bipolar Euler–Poisson system (1). Li and Yang [14] discussed the global existence and L^2-decay rates of smooth solutions for the three-dimensional isentropic bipolar hydrodynamic model. To our knowledge, there are very few results about the non-isentropic bipolar hydrodynamic model (1). Li [13] investigated the global existence and nonlinear diffusive waves of smooth solutions for the initial value problem of the one-dimensional non-isentropic bipolar hydrodynamic model. Jiang, etc. [9] discussed the quasi-neutral limit of the full bipolar

Euler–Poisson system and obtained the local existence of smooth solutions for the initial value problem. In this paper, we will discuss the global existence and asymptotic behavior of smooth solutions of the initial value problem for the three-dimensional hydrodynamic model (1) here. For the sake of simplicity, we assume $\tau_2 = \kappa = 1$, $\tau_1 = 2\tau_2$ and $T_{L_1} = T_{L_2} = T_L$, then we can rewrite (1) as

$$\begin{aligned}
&\partial_t \rho_1 + \nabla\cdot m_1 = 0,\\
&\partial_t m_1 + \nabla\cdot\left(\frac{m_1\otimes m_1}{\rho_1}\right) + \nabla(\rho_1 T_1) = \rho_1\nabla\phi - m_1,\\
&\partial_t T_1 + \frac{m_1}{\rho_1}\nabla T_1 + \frac{2}{3}T_1\nabla\cdot\left(\frac{m_1}{\rho_1}\right) - \frac{2}{3\rho_1}\Delta T_1 + T_1 - T_L = 0,\\
&\partial_t \rho_2 + \nabla\cdot m_2 = 0,\\
&\partial_t m_2 + \nabla\cdot\left(\frac{m_2\otimes m_2}{\rho_2}\right) + \nabla(\rho_2 T_2) = -\rho_2\nabla\phi - m_2,\\
&\partial_t T_2 + \frac{m_2}{\rho_2}\nabla T_2 + \frac{2}{3}T_2\nabla\cdot\left(\frac{m_2}{\rho_2}\right) - \frac{2}{3\rho_2}\Delta T_2 + T_2 - T_L = 0,\\
&\Delta\phi = \rho_1 - \rho_2.
\end{aligned}\tag{2}$$

We also prescribe the initial data as

$$\begin{aligned}
&\rho_1(t=0,x) = \rho_{10}(x) > 0, \quad \rho_2(t=0,x) = \rho_{20}(x) > 0, \quad x\in\mathbb{R}^3,\\
&(m_1,T_1,m_2,T_2)(t=0,x) = (m_{10},T_{10},m_{20},T_{20})(x), \qquad \phi\to 0 \quad \text{as } |x|\to\infty.
\end{aligned}\tag{3}$$

The main result in this paper is stated in the following theorem.

Theorem 1. *Let $(\bar\rho,\bar m,T_L)$ be constant state with $\bar\rho > 0$ and $T_L > 0$. Assume that $\Theta_0 =: \|(\rho_{10}-\bar\rho,\ m_{10}-\bar m,\ T_{10}-T_L,\ \rho_{20}-\bar\rho,\ m_{20}-\bar m,\ T_{20}-T_L)\|_{H^4\cap L^1}$ is small enough. Then, there is a unique global classical solution $(\rho_1,m_1,T_1,\rho_2,m_2,T_2,\phi)$ of the IVP* (2)–(3) *satisfying*

$$\begin{gathered}
\rho_1-\bar\rho,\ \rho_2-\bar\rho \in C^0\big(\mathbb{R}_+, H^4(\mathbb{R}^3)\big)\cap C^1\big(\mathbb{R}_+, H^3(\mathbb{R}^3)\big),\\
m_1-\bar m,\ m_2-\bar m \in C^0\big(\mathbb{R}_+, H^4(\mathbb{R}^3)\big)\cap C^1\big(\mathbb{R}_+, H^3(\mathbb{R}^3)\big),\\
T_1-T_L,\ T_2-T_L \in C^0\big(\mathbb{R}_+, H^4(\mathbb{R}^3)\big)\cap C^1\big(\mathbb{R}_+, H^2(\mathbb{R}^3)\big),\\
\phi\in C^0\big(\mathbb{R}_+, L^6(\mathbb{R}^3)\big), \qquad \nabla\phi\in C^0\big(\mathbb{R}_+, H^5(\mathbb{R}^3)\big),
\end{gathered}$$

and there is some positive constant $C > 0$ such that, for $i = 1,2$,

$$\big\|\partial_x^\alpha(\rho_i-\bar\rho)(t)\big\| \leqslant C\Theta_0(1+t)^{-3/4-|\alpha|/2}, \quad |\alpha|\leqslant 2, \tag{4}$$

$$\big\|\partial_x^\alpha(m_i-\bar m)(t)\big\| \leqslant C\Theta_0(1+t)^{-1/4-|\alpha|/2}, \quad |\alpha|\leqslant 2, \tag{5}$$

$$\big\|\partial_x^\alpha(T_i-T_L)(t)\big\| \leqslant C\Theta_0(1+t)^{-3/4-|\alpha|/2}, \quad |\alpha|\leqslant 2, \tag{6}$$

$$\big\|\partial_x^\beta\nabla\phi(t)\big\| \leqslant C\Theta_0(1+t)^{-1/4-|\beta|/2}, \quad |\beta|\leqslant 3.$$

Remark 1. Furthermore, if we assume that $\Theta_1 =: \|(\rho_{10} - \bar\rho, m_{10} - \bar m, \rho_{20} - \bar\rho, m_{20} - \bar m)\|_{H^{s+l}\cap L^1}$ ($s = 2$, $l \geqslant 2$) is small, then there is a unique global classical solution $(\rho_1, m_1, T_1, \rho_1, m_2, T_2, \phi)$ of the IVP (2)–(3) satisfying

$$\begin{gathered}
\rho_1 - \bar\rho,\, \rho_2 - \bar\rho \in C^0\big(\mathbb{R}_+, H^{s+l}\big(\mathbb{R}^3\big)\big) \cap C^1\big(\mathbb{R}_+, H^{s+l-1}\big(\mathbb{R}^3\big)\big),\\
m_1 - \bar m,\, m_2 - \bar m \in C^0\big(\mathbb{R}_+, H^{s+l}\big(\mathbb{R}^3\big)\big) \cap C^1\big(\mathbb{R}_+, H^{s+l-1}\big(\mathbb{R}^3\big)\big),\\
T_1 - T_L,\, T_2 - T_L \in C^0\big(\mathbb{R}_+, H^{s+l}\big(\mathbb{R}^3\big)\big) \cap C^1\big(\mathbb{R}_+, H^{s+l-2}\big(\mathbb{R}^3\big)\big),\\
\phi \in C^0\big(\mathbb{R}_+, L^6\big(\mathbb{R}^3\big)\big), \qquad \nabla\phi \in C^0\big(\mathbb{R}_+, H^{s+l+1}\big(\mathbb{R}^3\big)\big),
\end{gathered}$$

and there is some positive constant $C > 0$ such that, for $i = 1, 2$ and $|\alpha| \leqslant l$, $|\beta| \leqslant l + 1$,

$$\begin{aligned}
\big\|\partial_x^\alpha(\rho_i - \bar\rho)(t)\big\| &\leqslant C\Theta_1(1+t)^{-3/4-|\alpha|/2},\\
\big\|\partial_x^\alpha(m_i - \bar m)(t)\big\| &\leqslant C\Theta_1(1+t)^{-1/4-|\alpha|/2},\\
\big\|\partial_x^\alpha(T_i - T_L)(t)\big\| &\leqslant C\Theta_1(1+t)^{-3/4-|\alpha|/2},\\
\big\|\partial_x^\beta\nabla\phi(t)\big\| &\leqslant C\Theta_1(1+t)^{-1/4-|\beta|/2}.
\end{aligned}$$

Remark 2. Compared with the Euler equations with damping in [23], we find that the interaction of the two particles and the additional electric field reduce the decay rate of the momentums, which are seen in the isentropic bipolar case in [14]. Moreover it is interesting studying the existence and stability of the planar diffusion waves for the multi-dimensional full bipolar Euler–Poisson system in switch-on case as in [8], which is left for the forthcoming future.

The idea of the proof is outlined as follows. First, we present local-in-time existence of the initial value problem (7)–(8) by the standard argument of contracting map theorem as in [12]. Next, combining the local existence and global a-priori estimates, we apply the continuity argument to establish global existence of smooth solutions for the nonlinear problem. The key point is to derive the a-priori estimates, in which we show the estimates of the lower order derivatives of solutions by the spectral analysis of the corresponding linearized equations, and obtain the estimates of the higher order derivatives of solutions by elaborate energy estimates.

The rest of this paper is outlined as follows. In Section 2, we reformulate the original problem in terms of the perturbed variable, and present the L^2 decay rate of the linearized equations. The global existence and L^2-convergence rates of smooth solutions will be shown in Section 3.

Notations. Throughout this paper, $C > 0$ denotes a generic positive constant independent of time. $L^p(\mathbb{R}^3)$ ($1 \leqslant p < \infty$) denotes the space of measurable functions whose p-powers are integrable on $\mathbb{R}^3$, with the norm $\|\cdot\|_{L^p} = (\int_{\mathbb{R}^3} |\cdot|^p\,\mathrm{d}x)^{1/p}$, and L^∞ is the space of bounded measurable functions on $\mathbb{R}^3$, with the norm $\|\cdot\|_{L^\infty} = \operatorname{esssup}_x|\cdot|$, and also simply denote $\|\cdot\|_{L^2}$ by $\|\cdot\|$. $H^k (k \geqslant 0)$ stands for usual Sobolev space with the norm $\|\cdot\|_s$. Moreover, we denote $\|\cdot\|_s + \|\cdot\|_{L^1}$ by $\|\cdot\|_{H^s\cap L^1}$. Finally, for $f \in L^2$, the Fourier transform of f is $\hat f(\xi) = \mathcal{F}[f](\xi) = (2\pi)^{-3/2}\int_{\mathbb{R}^3} f(x)\mathrm{e}^{-\mathrm{i}x\cdot\xi}\,\mathrm{d}x$.

2 Solutions of the linearized equation

Firstly, we reformulate the nonlinear system (2) for $(\rho_1, m_1, T_1, \rho_2, m_2, T_2)$ around the equilibrium state $(\bar\rho, \bar m, T_L, \bar\rho, \bar m, T_L)$. Without loss of generality, we can take $(\bar\rho, \bar m, T_L, \bar\rho, \bar m, T_L) = (1,0,1,1,0,1)$. Denote

$$n_i = \rho_i - 1, \quad m_i = m_i, \quad \theta_i = T_i - 1, \quad i = 1,2, \qquad \Phi = \phi,$$

then the IVP problem for $(n_1, m_1, \theta_1, n_2, m_2, \theta_2, \Phi)$ is given by

$$\begin{aligned}
&\partial_t n_i + \nabla\cdot m_i = 0, \quad i = 1,2,\\
&\partial_t m_i + \nabla n_i + \nabla\theta_i - (-1)^i\nabla\Phi + m_i = f_i, \quad i = 1,2,\\
&\partial_t\theta_i + \frac{2}{3}\nabla\cdot m_i - \frac{2}{3}\Delta\theta_i + \theta_i = g_i, \quad i = 1,2,\\
&\Delta\Phi = n_1 - n_2, \qquad \lim_{|x|\to\infty}\Phi(x,t) = 0,
\end{aligned} \tag{7}$$

with the initial data

$$\begin{aligned}
&(n_1, m_1, \theta_1, n_2, m_2, \theta_2)(x,0)\\
&\quad = (n_{10}, m_{10}, \theta_{10}, n_{20}, m_{20}, \theta_{20})(x)\\
&\quad = (\rho_{10} - 1,\ m_{10},\ T_{10} - 1,\ \rho_{20} - 1,\ m_{20},\ T_{20} - 1)(x).
\end{aligned} \tag{8}$$

Here the nonlinear terms f_i, g_i $(i = 1,2)$ are defined by

$$f_i = (-1)^{i-1} n_i\nabla\Phi - \nabla\cdot\left(\frac{m_i\otimes m_i}{n_i+1}\right) - \nabla(n_i\theta_i),$$
$$g_i = -\frac{m_i}{n_i+1}\nabla\theta_i - \frac{2}{3}\theta_i\nabla\cdot\frac{m_i}{n_i+1} - \frac{2}{3}\nabla\cdot\left(\frac{m_i}{n_i+1} - m_i\right) + \frac{2}{3}\left(\frac{1}{n_i+1} - 1\right)\Delta\theta_i.$$

For simplicity, we replace $\nabla\Phi$ with the following formulation:

$$\nabla\Phi = \nabla\Delta^{-1}(n_1 - n_2). \tag{9}$$

Inserting (9) into (7) and neglecting the nonlinear terms, we have the following linearized nonisentropic bipolar Euler–Poisson system:

$$\begin{aligned}
&\partial_t\bar n_i + \nabla\cdot\bar m_i = 0, \quad i = 1,2,\\
&\partial_t\bar m_i + \nabla\bar n_i + \nabla\bar\theta_i - (-1)^i\nabla\Delta^{-1}(\bar n_1 - \bar n_2) + \bar m_i = 0, \quad i = 1,2,\\
&\partial_t\bar\theta_i + \frac{2}{3}\nabla\cdot\bar m_i - \frac{2}{3}\Delta\bar\theta_i + \bar\theta_i = 0, \quad i = 1,2,
\end{aligned} \tag{10}$$

with

$$(\bar n_1, \bar n_2, \bar\theta_1, \bar m_1, \bar m_2, \bar\theta_2)(x,0) = (n_{10}, m_{10}, \theta_{10}, n_{20}, m_{20}, \theta_{20})(x). \tag{11}$$

By setting $\bar U = (\bar n_1, \bar m_1, \bar\theta_1, \bar n_2, \bar m_2, \bar\theta_2)^t$, IVP (10)–(11) can be expressed as

$$\bar U_t = \mathcal{B}\bar U, \quad \bar U(0) = U_0, \quad t \geqslant 0. \tag{12}$$

In particular, there is a solution of the following IVP: $U_t = \mathcal{B}U$, $U(0) = \delta(x)I_{10*10}$, which was always called Green function and denoted as $G(x,t)$. And the solution of (12) can be expressed as

$$\bar{U}(x,t) = G(\cdot,t) * U_0(\cdot), \tag{13}$$

where $*$ is the convolution in x. In the following, we focus on analyzing the properties of $G(\cdot,t) * U_0(\cdot)$. For this aim, from (10), we have

$$\begin{aligned}
&\partial_t(\bar{n}_1 + \bar{n}_2) + \nabla\cdot(\bar{m}_1 + \bar{m}_2) = 0,\\
&\partial_t(\bar{m}_1 + \bar{m}_2) + \nabla(\bar{n}_1 + \bar{n}_2) + \nabla(\bar{\theta}_1 + \bar{\theta}_2) + \bar{m}_1 + \bar{m}_2 = 0,\\
&\partial_t(\bar{\theta}_1 + \bar{\theta}_2) + \frac{2}{3}\nabla\cdot(\bar{m}_1 + \bar{m}_2) - \frac{2}{3}\Delta(\bar{\theta}_1 + \bar{\theta}_2) + \bar{\theta}_1 + \bar{\theta}_2 = 0
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
&\partial_t(\bar{n}_1 - \bar{n}_2) + \nabla\cdot(\bar{m}_1 - \bar{m}_2) = 0,\\
&\partial_t(\bar{m}_1 - \bar{m}_2) + \nabla(\bar{n}_1 - \bar{n}_2) + \nabla(\bar{\theta}_1 - \bar{\theta}_2)\\
&\quad + \bar{m}_1 - \bar{m}_2 - 2\nabla\Delta^{-1}(\bar{n}_1 - \bar{n}_2) = 0,\\
&\partial_t(\bar{\theta}_1 - \bar{\theta}_2) + \frac{2}{3}\nabla\cdot(\bar{m}_1 - \bar{m}_2) - \frac{2}{3}\Delta(\bar{\theta}_1 - \bar{\theta}_2) + \bar{\theta}_1 - \bar{\theta}_2 = 0,
\end{aligned} \tag{15}$$

By setting $\bar{U}_1 = (\bar{n}_1+\bar{n}_2, \bar{m}_1+\bar{m}_2, \bar{\theta}_1+\bar{\theta}_2)^t$, and $\bar{U}_2 = (\bar{n}_1-\bar{n}_2, \bar{m}_1-\bar{m}_2, \bar{\theta}_1-\bar{\theta}_2)^t$, IVP (14) and (15) can be expressed as

$$\bar{U}_{1t} = \mathcal{B}_1\bar{U}_1, \quad \bar{U}_1(0) = U_{10} = (\bar{n}_{10} + \bar{n}_{20}, \bar{m}_{10} + \bar{m}_{20}, \bar{\theta}_{10} + \bar{\theta}_{20})^t, \quad t \geqslant 0, \tag{16}$$

and

$$\bar{U}_{2t} = \mathcal{B}_2\bar{U}_2, \quad \bar{U}_2(0) = U_{20} = (\bar{n}_{10} - \bar{n}_{20}, \bar{m}_{10} - \bar{m}_{20}, \bar{\theta}_{10} - \bar{\theta}_{20})^t, \quad t \geqslant 0, \tag{17}$$

Let us denote the solution of the following IVP:

$$U_{it} = \mathcal{B}_i U_i, \quad U_i(0) = \delta(x)I_{5*5}$$

by $G^i(x,t)$. Then the solution of (14) and (15) can be expressed as

$$U_i(x,t) = G^i(\cdot,t) * (n_{10} \pm n_{20},\, m_{10} \pm m_{20},\, \theta_{10} \pm \theta_{20})(\cdot), \tag{18}$$

where $*$ is the convolution in x.

Applying the Fourier transform to system (16) with respect to x, we have

$$\partial_t\hat{U}_1(\xi,t) = A_1(\xi)\hat{U}_1(\xi,t) \quad \bigl(\xi = (\xi_1,\xi_2,\xi_3)\bigr), \qquad \hat{U}_1(0) = I_{5*5},$$

where

$$A_1(\xi) = \begin{pmatrix} 0 & -\mathrm{i}\xi & 0\\ -\mathrm{i}\xi^{\mathrm{T}} & -I_3 & -\mathrm{i}\xi^{\mathrm{T}}\\ 0 & -2\mathrm{i}\xi/3 & -(2|\xi|^2/3 + 1)\end{pmatrix}$$

The eigenvalues of the matrix $A_1(\xi)$ can be computed from the determinant

$$\begin{aligned}&\det\bigl(\lambda I_5 - A_1(\xi)\bigr)\\&\quad = (\lambda+1)^2\left(\lambda^3 + \left(\frac{2}{3}|\xi|^2+2\right)\lambda^2 + \left(\frac{7}{3}|\xi|^2+1\right)\lambda + |\xi|^2\left(\frac{2}{3}|\xi|^2+1\right)\right) = 0,\end{aligned}$$

which implies $\lambda_1 = -1$ (double), and there exist positive constants r_1 and r_2 satisfying $r_1 < r_2$ such that when $|\xi| > r_2$,

$$\lambda_2 = \frac{-b-\sqrt[3]{Y_1}-\sqrt[3]{Y_2}}{3a}, \qquad \lambda_{3,4} = \frac{1}{6a}(-2b+\sqrt[3]{Y_1}+\sqrt[3]{Y_2}) \pm \frac{\sqrt{-3}}{6}(\sqrt[3]{Y_1}-\sqrt[3]{Y_2}),$$

and when $|\xi| < r_1$,

$$\lambda_2 = \frac{-b-2\sqrt{A}\cos\frac{\vartheta}{3}}{3a}, \qquad \lambda_{3,4} = \frac{1}{3a}\left(-b+\sqrt{A}\left(\cos\frac{\vartheta}{3} \pm \sqrt{3}\sin\frac{\vartheta}{3}\right)\right).$$

Here

$$\begin{gathered}a=1, \qquad b=\frac{2}{3}|\xi|^2+2, \qquad c=\frac{7}{3}|\xi|^2+1, \qquad d=|\xi|^2\left(\frac{2}{3}|\xi|^2+1\right),\\ A=b^2-3ac, \qquad B=bc-9ad, \qquad C=c^2-3bd,\\ Y_{1,2}=Ab+\frac{3a}{2}(-B\pm\sqrt{B^2-4AC}), \qquad \vartheta=\arccos\frac{2Ab-3aB}{2\sqrt{A^3}}.\end{gathered}$$

The semigroup $S_1(t) = \mathrm{e}^{tA_1}$ is expressed as

$$\mathrm{e}^{tA_1(\xi)} = \sum_{i=1}^{4} \mathrm{e}^{\lambda_i t} P_i(\xi) = \begin{pmatrix} \hat G^1_{11} & \hat G^1_{12} & \hat G^1_{13}\\ \hat G^1_{21} & \hat G^1_{22} & \hat G^1_{23}\\ \hat G^1_{31} & \hat G^1_{32} & \hat G^1_{33}\end{pmatrix}, \tag{19}$$

where the project operators $P_i(\xi)$ $(i = 1,2,3,4)$ can be computed as

$$P_i(\xi) = \prod_{j\neq i} \frac{A_1(\xi)-\lambda_j I}{\lambda_i-\lambda_j}.$$

Similarly, for

$$A_2(\xi) = \begin{pmatrix} 0 & -\mathrm{i}\xi & 0\\ -\mathrm{i}\xi^{\mathrm{T}}(1+2|\xi|^{-2}) & -I_3 & -\mathrm{i}\xi^{\mathrm{T}}\\ 0 & -2\mathrm{i}\xi/3 & -(2|\xi|^2/3+1)\end{pmatrix},$$

we have

$$\mathrm{e}^{tA_2(\xi)} = \sum_{i=1}^{4} \mathrm{e}^{\bar\lambda_i t} \bar P_i(\xi) = \begin{pmatrix} \hat G^2_{11} & \hat G^2_{12} & \hat G^2_{13}\\ \hat G^2_{21} & \hat G^2_{22} & \hat G^2_{23}\\ \hat G^2_{31} & \hat G^2_{32} & \hat G^2_{33}\end{pmatrix}, \tag{20}$$

where $\bar\lambda_1 = 2$ and $\bar\lambda_{2,3,4}$ satisfy

$$\bar\lambda^3 + \left(\frac{2}{3}|\xi|^2 + 2\right)\bar\lambda^2 + \left(\frac{7}{3}|\xi|^2 + 1\right)\bar\lambda + \left(|\xi|^2 + 1\right)\left(\frac{2}{3}|\xi|^2 + 1\right) = 0,$$

and the project operators $\bar P_i(\xi)$ $(i = 1, 2, 3, 4)$ can be computed as

$$\bar P_i(\xi) = \prod_{j\neq i} \frac{A_2(\xi) - \bar\lambda_j I}{\bar\lambda_i - \bar\lambda_j}.$$

Noting in (19) and (20), G^i_{12} $(i = 1, 2)$ are 1×3-matrix, G^i_{22} $(i = 1, 2)$ are 3×3-matrix, and G^i_{21}, G^i_{23} $(i = 1, 2)$ are 3×1-matrix, the other are 1×1-matrix.

Then, from (19) and (20), we have

$$G(\xi, t) = \begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} \\ G_{21} & G_{22} & G_{23} & G_{24} & G_{25} & G_{26} \\ G_{31} & G_{32} & G_{33} & G_{34} & G_{35} & G_{36} \\ G_{41} & G_{42} & G_{43} & G_{44} & G_{45} & G_{46} \\ G_{51} & G_{52} & G_{53} & G_{54} & G_{55} & G_{56} \\ G_{61} & G_{62} & G_{63} & G_{64} & G_{65} & G_{66} \end{pmatrix}, \tag{21}$$

where

$$\begin{aligned} G_{ij} &= \frac{1}{2}\big(G^1_{ij} + G^2_{ij}\big), \quad i = 1, 2, 3,\ j = 1, 2, 3, \\ G_{ij} &= \frac{1}{2}\big(G^1_{il} + G^2_{il}\big), \quad i = 1, 2, 3,\ j = 4, 5, 6,\ l = j - 3, \\ G_{ij} &= \frac{1}{2}\big(G^1_{lj} - G^2_{lj}\big), \quad i = 4, 5, 6,\ j = 1, 2, 3,\ l = i - 3, \\ G_{ij} &= \frac{1}{2}\big(G^1_{kl} - G^2_{kl}\big), \quad i = 4, 5, 6,\ j = 4, 5, 6,\ k = i - 3,\ l = j - 3. \end{aligned}$$

Using the idea and argument of [14, 17, 19, 24], we have L^2-estimate of solution for the IVP (10)–(11) as follows.

Lemma 1. *If $U_0 \in L^1(\mathbb{R}^3) \cap H^4(\mathbb{R}^3)$, then for $i = 1, 3, 4, 6$, $j = 2, 5$, we have*

$$\begin{aligned}
&\big\|\partial^\alpha_x (G_{i1} * U_0, G_{i4} * U_0)\big\| \leqslant C(1 + t)^{-3/4-|\alpha|/2}\big(\|U_0\|_{L^1} + \big\|\partial^\alpha_x U_0\big\|\big),\\
&\big\|\partial^\alpha_x (G_{i2} * U_0, G_{i5} * U_0)\big\| \leqslant C(1 + t)^{-(3/2)(1/m-1/2)-1/2-|\alpha|/2}\big(\|U_0\|_{L^m} + \big\|\partial^\alpha_x U_0\big\|\big),\\
&\big\|\partial^\alpha_x (G_{i3} * U_0, G_{i6} * U_0)\big\| \leqslant C(1 + t)^{-(3/2)(1/m-1/2)-1/2-|\alpha|/2}\big(\|U_0\|_{L^m} + \big\|\partial^\alpha_x U_0\big\|\big),\\
&\big\|\partial^\alpha_x (G_{i3} * U_0, G_{i6} * U_0)\big\| \leqslant C(1 + t)^{-1/2-|\alpha|/2}\big(\|U_0\| + \big\|\partial^\beta_x U_0\big\|\big), \quad |\alpha| \geqslant 1,\\
&\big\|\partial^\alpha_x (G_{j1} * U_0, G_{j4} * U_0)\big\| \leqslant C(1 + t)^{-1/4-|\alpha|/2}\big(\|U_0\|_{L^1} + \big\|\partial^\alpha_x U_0\big\|\big),\\
&\big\|\partial^\alpha_x (G_{j2} * U_0, G_{j5} * U_0)\big\| \leqslant C(1 + t)^{-(3/2)(1/m-1/2)-|\alpha|/2}\big(\|U_0\|_{L^m} + \big\|\partial^\alpha_x U_0\big\|\big),\\
&\big\|\partial^\alpha_x (G_{j3} * U_0, G_{j6} * U_0)\big\| \leqslant C(1 + t)^{-3/2(1/m-1/2)-1/2-|\alpha|/2}\big(\|U_0\|_{L^m} + \big\|\partial^\alpha_x U_0\big\|\big),\\
&\big\|\partial^\alpha_x (G_{j3} * U_0, G_{j6} * U_0)\big\| \leqslant C(1 + t)^{-1/2-|\alpha|/2}\big(\|U_0\| + \big\|\partial^\beta_x U_0\big\|\big), \quad |\alpha| \geqslant 1,
\end{aligned}$$

where $|\alpha| \leqslant 4$, $|\beta| = |\alpha| - 1$, and $m = 1, 2$.

Indeed, with the express of $\hat{G}(\xi,t)$, and applying Hausdorff–Young inequality, we can show Lemma 1, and the details can be found in [14, 17, 19, 24].

Moreover, from (9) and (21), the Fourier transform for the electric field is

$$\begin{aligned}\hat{\hat{E}} &= -\frac{\mathrm{i}\xi}{|\xi|^2}(\hat{\hat{n}}_1 - \hat{\hat{n}}_2)\\ &= -\frac{\mathrm{i}\xi}{|\xi|^2}(G_{11}, G_{12}, G_{13}, G_{14}, G_{15}, G_{16})\hat{U}_0 + \frac{\mathrm{i}\xi}{|\xi|^2}(G_{41}, G_{42}, G_{43}, G_{44}, G_{45}, G_{46})\hat{U}_0.\end{aligned}$$

From the above equality, we can define

$$\hat{\hat{E}} = \hat{L}\hat{U}_0 = (\hat{\mathcal{L}} + \hat{\mathfrak{L}})\hat{U}_0,$$

where

$$\begin{aligned}\hat{\mathcal{L}} &= -\frac{i\xi}{|\xi|^2}(G_{11} - G_{41},\ 0,\ 0,\ G_{14} - G_{44},\ 0,\ 0),\\ \hat{\mathfrak{L}} &= -\frac{i\xi}{|\xi|^2}(0,\ G_{12} - G_{42},\ G_{13} - G_{43},\ 0,\ G_{15} - G_{45},\ G_{16} - G_{46}).\end{aligned}$$

Here $\hat{L}$, $\hat{\mathcal{L}}$, $\hat{\mathfrak{L}}$ are the Fourier transform of function L, $\mathcal{L}$, $\mathfrak{L}$, respectively. From Lemma 1, the estimates of L,$\mathcal{L}$, $\mathfrak{L}$ is given as following.

Lemma 2. *If $U_0 \in L^1(\mathbb{R}^3) \cap H^4(\mathbb{R}^3)$, then for $m = 1, 2$, we have*

$$\begin{aligned}\left\|\partial_x^\alpha(L * U_0)\right\| &\leqslant C(1+t)^{-1/4-|\alpha|/2}\left(\|U_0\|_{L^1} + \left\|\partial_x^\alpha U_0\right\|\right),\\ \left\|\partial_x^\alpha(\mathfrak{L} * U_0)\right\| &\leqslant C(1+t)^{-3/4-|\alpha|/2}\left(\|U_0\|_{L^1} + \left\|\partial_x^\alpha U_0\right\|\right).\end{aligned}$$

3 Global existence and L^2-decay rate

In this section, we are going to establish the global existence and show the L^2-decay rate of the solution of nonlinear problem (7)–(8).

First of all, we give the local existence theory, which can be established in the framework as in [12]. The key point is the electric field $\nabla\Phi$ can be expressed by the Riesz potential as a nonlocal term

$$\nabla\Phi = \nabla\Phi(t=0,x) + \nabla(-\Delta)^{-1}\nabla\cdot\int_0^t (m_1 - m_2)\,\mathrm{d}s,$$

which together with the L^p estimates of Riesz potential leads to

$$\left\|\nabla(-\Delta)^{-1}\nabla\cdot\int_0^t (m_1 - m_2)\,\mathrm{d}s\right\|_k \leqslant C\left\|\int_0^t (m_1 - m_2)\,\mathrm{d}s\right\|_k.$$

Then, we can prove the following local-in-time existence of the initial value problem (7)–(8) by the standard argument of contracting map theorem as in [12]. The details are omitted.

Theorem 2. *Assume that* $(n_{10}, m_{10}, \theta_{10}, n_{20}, m_{20}, \theta_{20})(x) \in H^4(\mathbb{R}^3)$. *Then, there is a time* $T > 0$ *such that the IVP* (7)–(8) *has a unique global smooth solution* $(n_1, m_1, \theta_1, n_2, m_2, \theta_2, \Phi)$:

$$\begin{gathered} n_1, m_1, n_2, m_2 \in C^0\big([0,T], H^4(\mathbb{R}^3)\big) \cap C^1\big([0,T], H^3(\mathbb{R}^3)\big), \\ \theta_1, \theta_2 \in C^0\big([0,T], H^4(\mathbb{R}^3)\big) \cap C^1\big([0,T], H^2(\mathbb{R}^3)\big), \\ \Phi \in C^0\big([0,T], L^6(\mathbb{R}^3)\big), \qquad \nabla\Phi \in C^0\big([0,T], H^5(\mathbb{R}^3)\big), \end{gathered}$$

satisfying $\inf_{(t,x)\in[0,T]\times\mathbb{R}^3} n_i(t,x) > 0$, *and*

$$\big\|(n_1, m_1, \theta_1, n_2, m_2, \theta_2)(\cdot,t)\big\|_4 + \big\|\nabla\Phi(\cdot,t)\big\|_5 + \big\|\Phi(\cdot,t)\big\|_{L^6} \leqslant C.$$

To extend the local existence of solution to be a global solution in time, we need to establish some uniform a priori estimates. For this aim, we will look for the solution in the following space

$$\mathcal{S} = \big\{(n_1, m_1, \theta_1, n_2, m_2, \theta_2, \nabla\Phi) \in \big(H^4\big)^6 \times H^5 \mid \Lambda(t) < +\infty\big\},$$

where

$$\begin{aligned} \Lambda(t) = \sup_{0\leqslant s\leqslant t} \bigg\{ & (1+s)^{1/4}\big\|\nabla\Phi(s)\big\| + \sum_{|\alpha|\leqslant 2} \big[(1+s)^{3/4+|\alpha|/2}\big\|D^\alpha(n_1, \theta_1, n_2, \theta_2)(s)\big\| \\ & + (1+s)^{1/4+|\alpha|/2}\big\|D^{|\alpha|}(m_1, m_2)(s)\big\|\big] + (1+s)^{3/4}\big\|D^3(n_1, m_1, \theta_1, n_2, m_2, \theta_2)\big\| \\ & + \big\|D^4(n_1, m_1, \theta_1, n_2, m_2, \theta_2)\big\| \bigg\}. \end{aligned} \tag{22}$$

Due to the property of Riesz potential (see [21]), we have

$$\big\|D^k\nabla\Phi\big\| \leqslant C\big\|D^{k-1}(n_1 - n_2)\big\|, \quad k \geqslant 1. \tag{23}$$

It seems that the estimates of the high order derivatives of $\nabla\Phi$ come from the bounds of n_1 and n_2. That is, if $(n_1, m_1, n_2, m_2, \nabla\Phi) \in \mathcal{S}$, it is obviously that for all $0 \leqslant s \leqslant t$,

$$\begin{aligned} &\big\|D^k\nabla\Phi(s)\big\| \leqslant (1+s)^{-1/4-k/2}\Lambda(t) \quad (k = 1, 2, 3), \\ &\big\|D^4\nabla\Phi(s)\big\| \leqslant (1+s)^{-3/4}\Lambda(t). \end{aligned} \tag{24}$$

So we should obtain the estimate of $\nabla\Phi$ itself.

Lemma 3 (The a priori estimate of lower order derivatives of solution). *Under the assumption of Theorem* 1, *suppose that* $(n_1, m_1, \theta_1, n_2, m_2, \theta_2, \nabla\Phi) \in \mathcal{S}$ *is the solution of IVP* (7)–(8) *on* $[0,T]$ *for any* $T > 0$, *which satisfies the following assumption*:

$$(n_1, m_1, \theta_1, n_2, m_2, \theta_2, \nabla\Phi) \in \mathcal{S}, \qquad \Lambda(t) \leqslant \delta_0 \quad \textit{for } \delta_0 \ll 1. \tag{25}$$

Then, for any $t \in [0,T]$, there exists some constant C such that

$$\begin{aligned}
\big\|\partial_x^k(n_1,n_2,\theta_1,\theta_2)(t)\big\| &\leqslant C\Theta_0(1+t)^{-3/4-k/2} + C\big(\Lambda(t)\big)^2(1+t)^{-3/4-k/2},\\
\big\|\partial_x^3(n_1,n_2,\theta_1,\theta_2)(t)\big\| &\leqslant C\Theta_0(1+t)^{-3/4} + C\big(\Lambda(t)\big)^2(1+t)^{-3/4},\\
\big\|\partial_x^k(m_1,m_2)(t)\big\| &\leqslant C\Theta_0(1+t)^{-1/4-k/2} + C\big(\Lambda(t)\big)^2(1+t)^{-1/4-k/2},\\
\|\partial_x^3(m_1,m_2)(t)\| &\leqslant C\Theta_0(1+t)^{-3/4} + C\big(\Lambda(t)\big)^2(1+t)^{-3/4},\\
\big\|\nabla\Phi(t)\big\| &\leqslant C\Theta_0(1+t)^{-1/4} + C\big(\Lambda(t)\big)^2(1+t)^{-1/4}.
\end{aligned}$$

Here $k = 0,1,2$.

Proof. By Duhamel principle, it is easy to verify that the solution $U = (n_1,m_1,\theta_1,n_2,m_2,\theta_2,\nabla\Phi)$ of the IVP problem (7)–(8) can be expressed as

$$\begin{aligned}
n_1 &= (G_{11},G_{12},G_{13},G_{14},G_{15},G_{16}) * U_0\\
&\quad + \int_0^t (G_{12}*f_1 + G_{13}*g_1 + G_{15}*f_2 + G_{16}*g_2)(s)\,\mathrm{d}s, \qquad (26)\\
m_1 &= (G_{21},G_{22},G_{23},G_{24},G_{25},G_{26}) * U_0\\
&\quad + \int_0^t (G_{22}*f_1 + G_{23}*g_1 + G_{25}*f_2 + G_{26}*g_2)(s)\,\mathrm{d}s, \qquad (27)\\
\theta_1 &= (G_{31},G_{32},G_{33},G_{34},G_{35},G_{36}) * U_0\\
&\quad + \int_0^t (G_{32}*f_1 + G_{33}*g_1 + G_{35}*f_2 + G_{36}*g_2)(s)\,\mathrm{d}s, \qquad (28)\\
n_2 &= (G_{41},G_{42},G_{43},G_{44},G_{45},G_{46}) * U_0\\
&\quad + \int_0^t (G_{42}*f_1 + G_{43}*g_1 + G_{45}*f_2 + G_{46}*g_2)(s)\,\mathrm{d}s, \qquad (29)\\
m_2 &= (G_{51},G_{52},G_{53},G_{54},G_{55},G_{56}) * U_0\\
&\quad + \int_0^t (G_{52}*f_1 + G_{53}*g_1 + G_{55}*f_2 + G_{56}*g_2)(s)\,\mathrm{d}s, \qquad (30)\\
\theta_2 &= (G_{61},G_{62},G_{63},G_{64},G_{65},G_{66}) * U_0\\
&\quad + \int_0^t (G_{62}*f_1 + G_{63}*g_1 + G_{65}*f_2 + G_{66}*g_2)(s)\,\mathrm{d}s, \qquad (31)
\end{aligned}$$

and

$$\nabla\Phi = L * U_0 + \int_0^t \mathfrak{L}(t-s) * (0,f_1,g_1,0,f_2,g_2)(s)\,\mathrm{d}s. \qquad (32)$$

First, from (26) and Lemma 1, we have

$$\left\|D_x^k(G_{11},G_{12},G_{13},G_{14},G_{15},G_{16})*U_0\right\| \leqslant C(1+t)^{-3/4-|k|/2}\left(\|U_0\|_{L^1}+\|U_0\|_k\right).$$

For the estimates of the nonlinear terms, we first have for $i=1,2$,

$$\left\|(n_i,\theta_i)\right\|_\infty \leqslant C(1+t)^{-3/2}\Lambda(t), \qquad \left\|(m_i,\nabla\Phi)\right\|_\infty \leqslant C(1+t)^{-1}\Lambda(t), \tag{33}$$

$$\left\|(Dn_i,D\theta_i)\right\|_\infty \leqslant C(1+t)^{-1}\Lambda(t), \qquad \left\|D(m_i,\nabla\Phi)\right\|_\infty \leqslant C(1+t)^{-3/4}\Lambda(t) \tag{34}$$

with the aid of

$$\|u\|_{L^\infty} \leqslant C\|Du\|_{L^2}^{1/2}\left\|D^2u\right\|^{1/2} \quad \text{for } u\in H^2\left(\mathbb{R}^3\right),$$

which appeared in [21]. Further, from (25) and (33)–(34), we can get that, for $i=1,2$,

$$\begin{aligned}\left\|f_i(U)\right\| &\leqslant C\big(\|n_i\|\|\nabla\Phi\|_{L^\infty}+\|Dm_i\|\|m_i\|_{L^\infty}+\|Dn_i\|\|m_i\|_{L^\infty}^2\\&\quad+\|Dn_i\|\|n_i\|_{L^\infty}+\|Dn_i\|\|\theta_i\|_{L^\infty}+\|n_i\|_{L^\infty}\|D\theta_i\|\big)\\&\leqslant C(1+t)^{-7/4}\big(\Lambda(t)\big)^2, &(35)\\ \left\|f_i(U)\right\|_{L^1} &\leqslant C\big(\|n_i\|\|\nabla\Phi\|+\|Dm_i\|\|m_i\|+\|Dn_i\|\|m_i\|\|m_i\|_{L^\infty}\\&\quad+\|Dn_i\|\|n_i\|+\|Dn_i\|\|\theta_i\|+\|n_i\|\|D\theta_i\|\big)\\&\leqslant C(1+t)^{-1}\big(\Lambda(t)\big)^2, &(36)\\ \left\|g_i(U)\right\| &\leqslant C\big(\|m_i\|_{L^\infty}\|\nabla\theta_i\|+\|Dm_i\|\big(\|\theta_i\|_{L^\infty}+\|n_i\|_{L^\infty}\big)\\&\quad+\|Dn_i\|\|m_i\|_{L^\infty}\|\theta_i\|_{L^\infty}+\|\Delta\theta_i\|\|n_i\|_{L^\infty}+\|Dn_i\|\|m_i\|_{L^\infty}\\&\quad+\|Dn_i\|\|m_i\|_{L^\infty}\|n_i\|_{L^\infty}\big)\\&\leqslant C\big(\Lambda(t)\big)^2(1+t)^{-9/4}, &(37)\end{aligned}$$

and

$$\begin{aligned}\left\|g_i(U)\right\|_{L^1} &\leqslant C\big(\|m_i\|\|\nabla\theta_i\|+\|Dm_i\|\|\theta_i\|+\|Dn_i\|\|m_i\|_{L^\infty}\|\theta_i\|+\|\Delta\theta_i\|\|n_i\|\\&\quad+\|Dn_i\|\|m_i\|+\|n_i\|\|Dm_i\|+\|Dn_i\|\|m_i\|\|n_i\|_{L^\infty}\big)\\&\leqslant C\big(\Lambda(t)\big)^2(1+t)^{-3/2}. &(38)\end{aligned}$$

Thus, one have

$$\begin{aligned}\|n_1\| &\leqslant \left\|(G_{11},G_{12},G_{13},G_{14},G_{15},G_{16})*U_0\right\|\\&\quad+\int_0^t\big(\|G_{12}*f_1\|+\|G_{13}*g_1\|+\|G_{15}*f_2\|+\|G_{16}*g_2\|\big)\,\mathrm{d}\tau\\&\leqslant C\|U_0\|_{L^1\cap L^2}(1+t)^{-3/4}\\&\quad+C\int_0^t(1+t-\tau)^{-5/4}\sum_{i=1}^2\big(\left\|f_i(U)(\tau)\right\|_{L^1\cap L^2}+\left\|g_i(U)(\tau)\right\|_{L^1\cap L^2}\big)\,\mathrm{d}\tau\end{aligned}$$

$$\leqslant C\Theta_0(1+t)^{-3/4} + C\bigl(\Lambda(t)\bigr)^2 \times \int_0^t (1+t-\tau)^{-5/4}\bigl[(1+\tau)^{-7/4} + (1+\tau)^{-1} + (1+\tau)^{-9/4} + (1+\tau)^{-3/2}\bigr]\,\mathrm{d}\tau$$
$$\leqslant C\Theta_0(1+t)^{-3/4} + C\bigl(\Lambda(t)\bigr)^2(1+t)^{-1}.$$

Next, we have the following estimate of the nonlinear terms $f_i(U)$ $(i = 1,2)$ and $g_i(U)$ $(i = 1,2)$:

$$\bigl\|Df_i(U)\bigr\| \leqslant C\bigl(\Lambda(t)\bigr)^2(1+t)^{-3/2}, \tag{39}$$
$$\bigl\|Dg_i(U)\bigr\| \leqslant C\bigl(\Lambda(t)\bigr)^2(1+t)^{-2}, \tag{40}$$
$$\bigl\|D^2f_i(U)\bigr\| \leqslant C\bigl(\Lambda(t)\bigr)^2(1+t)^{-7/4}, \tag{41}$$

and

$$\bigl\|D^2g_i(U)\bigr\| \leqslant C\bigl(\Lambda(t)\bigr)^2(1+t)^{-1}, \tag{42}$$

which together with (26) yield

$$\begin{aligned}
\bigl\|Dn_1(t)\bigr\| &\leqslant \bigl\|D(G_{11},G_{12},G_{13},G_{14},G_{15},G_{16}) * U_0\bigr\| \\
&\quad + \int_0^t \bigl\|D^k(G_{12}*f_1 + G_{13}*g_1 + G_{15}*f_2 + G_{16}*g_2)(\tau)\bigr\|\,\mathrm{d}\tau \\
&\leqslant C(1+t)^{-5/4}\|U_0\|_{L^1\cap H^1} \\
&\quad + C\int_0^{t/2}(1+t-\tau)^{-7/4}\sum_{i=1}^2\bigl(\bigl\|f_i(U)\bigr\|_{L^1} + \bigl\|Df_i(U)(\tau)\bigr\|\bigr)\,\mathrm{d}\tau \\
&\quad + C\int_{t/2}^t(1+t-\tau)^{-1}\sum_{i=1}^2\bigl(\bigl\|f_i(U)(\tau)\bigr\| + \bigl\|Df_i(U)(\tau)\bigr\|\bigr)\,\mathrm{d}\tau \\
&\quad + C\int_0^{t/2}(1+t-\tau)^{-7/4}\sum_{i=1}^2\bigl(\bigl\|g_i(U)\bigr\|_{L^1} + \bigl\|Dg_i(U)(\tau)\bigr\|\bigr)\,\mathrm{d}\tau \\
&\quad + C\int_{t/2}^t(1+t-\tau)^{-1}\sum_{i=1}^2\bigl(\bigl\|g_i(U)\bigr\| + \bigl\|Dg_i(U)(\tau)\bigr\|\bigr)\,\mathrm{d}\tau \\
&\leqslant C(1+t)^{-5/4}\|U_0\|_{L^1\cap H^1} \\
&\quad + C\bigl(\Lambda(t)\bigr)^2\int_0^{t/2}(1+t-\tau)^{-7/4}\bigl[(1+\tau)^{-1} + (1+\tau)^{-3/2}\bigr]\,\mathrm{d}\tau
\end{aligned}$$

$$
\begin{aligned}
&+ C\big(\Lambda(t)\big)^2 \int\limits_{t/2}^{t} (1+t-\tau)^{-1}\big[(1+\tau)^{-7/4} + (1+\tau)^{-3/2}\big]\,\mathrm{d}\tau \\
&+ C\big(\Lambda(t)\big)^2 \int\limits_{0}^{t/2} (1+t-\tau)^{-7/4}\big[(1+\tau)^{-2} + (1+\tau)^{-3/2}\big]\,\mathrm{d}\tau \\
&+ C\big(\Lambda(t)\big)^2 \int\limits_{t/2}^{t} (1+t-\tau)^{-1}\big[(1+\tau)^{-9/4} + (1+\tau)^{-2}\big]\,\mathrm{d}\tau \\
&\leqslant C\Theta_0(1+t)^{-5/4} + C\big(\Lambda(t)\big)^2(1+t)^{-3/2}
\end{aligned}
$$

and

$$
\begin{aligned}
\big\|D^2 n_1(t)\big\| &\leqslant \big\|D^2(G_{11}, G_{12}, G_{13}, G_{14}, G_{15}, G_{16}) * U_0\big\| \\
&\quad + \int\limits_0^t \big\|D^2(G_{12} * f_1 + G_{13} * g_1 + G_{15} * f_2 + G_{16} * g_2)(\tau)\big\|\,\mathrm{d}\tau \\
&\leqslant C(1+t)^{-7/4}\|U_0\|_{L^1\cap H^2} \\
&\quad + C\int\limits_0^{t/2} (1+t-\tau)^{-9/4} \sum_{i=1}^{2} \big(\big\|f_i(U)\big\|_{L^1} + \big\|D^2 f_i(U)(\tau)\big\|\big)\,\mathrm{d}\tau \\
&\quad + C\int\limits_{\frac{t}{2}}^{t} (1+t-\tau)^{-3/2} \sum_{i=1}^{2} \big(\big\|f_i(U)(\tau)\big\| + \big\|D^2 f_i(U)(\tau)\big\|\big)\,\mathrm{d}\tau \\
&\quad + C\int\limits_0^{t/2} (1+t-\tau)^{-9/4} \sum_{i=1}^{2} \big(\big\|g_i(U)\big\|_{L^1} + \big\|D^2 g_i(U)\big\|\big)\,\mathrm{d}\tau \\
&\quad + C\int\limits_{t/2}^{t} (1+t-\tau)^{-3/2} \sum_{i=1}^{2} \big(\big\|g_i(U)\big\| + \big\|D g_i(U)\big\|\big)\,\mathrm{d}\tau \\
&\leqslant C(1+t)^{-7/4}\|U_0\|_{L^1\cap H^2} \\
&\quad + C\big(\Lambda(t)\big)^2 \int\limits_0^{t/2} (1+t-\tau)^{-9/4}\big[(1+\tau)^{-1} + (1+\tau)^{-7/4}\big]\,\mathrm{d}\tau \\
&\quad + C\big(\Lambda(t)\big)^2 \int\limits_{t/2}^{t} (1+t-\tau)^{-3/2}\big[(1+\tau)^{-7/4} + (1+\tau)^{-7/4}\big]\,\mathrm{d}\tau
\end{aligned}
$$

$$
\begin{aligned}
&+ C\big(\Lambda(t)\big)^2 \int_0^{t/2} (1+t-\tau)^{-9/4}\big[(1+\tau)^{-1} + (1+\tau)^{-3/2}\big]\,\mathrm{d}\tau \\
&+ C\big(\Lambda(t)\big)^2 \int_{t/2}^{t} (1+t-\tau)^{-3/2}\big[(1+\tau)^{-7/4} + (1+\tau)^{-9/4}\big]\,\mathrm{d}\tau \\
&\leqslant C\Theta_0(1+t)^{-7/4} + C\big(\Lambda(t)\big)^2(1+t)^{-7/4}
\end{aligned}
$$

with the help of (35)–(38).

Finally, we can show

$$
\big\|D^3 f_i(U)\big\| \leqslant C\big(\Lambda(t)\big)^2(1+t)^{-1},
$$

which together with (35)–(38), and (42) leads to

$$
\begin{aligned}
\big\|D^3 n_1(t)\big\| &\leqslant \big\|D^3(G_{11}, G_{12}, G_{13}, G_{14}, G_{15}, G_{16}) * U_0\big\| \\
&\quad + \int_0^t \big\|D^k(G_{12} * f_1 + G_{13} * g_1 + G_{15} * f_2 + G_{16} * g_2)(\tau)\big\|\,\mathrm{d}\tau \\
&\leqslant C(1+t)^{-11/4}\|U_0\|_{L^1\cap H^3} \\
&\quad + C\int_0^{t/2} (1+t-\tau)^{-11/4} \sum_{i=1}^{2} \big(\big\|f_i(U)\big\|_{L^1} + \big\|D^3 f_i(U)(\tau)\big\|\big)\,\mathrm{d}\tau \\
&\quad + C\int_{t/2}^{t} (1+t-\tau)^{-3/2} \sum_{i=1}^{2} \big(\big\|f_i(U)(\tau)\big\| + \big\|D^3 f_i(U)(\tau)\big\|\big)\,\mathrm{d}\tau \\
&\quad + C\int_0^{t} (1+t-\tau)^{-2} \sum_{i=1}^{2} \big(\big\|g_i(U)(\tau)\big\| + \big\|D^2 g_i(U)(\tau)\big\|\big)\,\mathrm{d}\tau \\
&\leqslant C(1+t)^{-11/4}\|U_0\|_{L^1\cap H^3} \\
&\quad + C\big(\Lambda(t)\big)^2 \int_0^{t/2} (1+t-\tau)^{-11/4}\big[(1+\tau)^{-1} + (1+\tau)^{-1}\big]\,\mathrm{d}\tau \\
&\quad + C\big(\Lambda(t)\big)^2 \int_{t/2}^{t} (1+t-\tau)^{-2}\big[(1+\tau)^{-7/4} + (1+\tau)^{-7/4} + (1+\tau)^{-1}\big]\,\mathrm{d}\tau \\
&\quad + C\big(\Lambda(t)\big)^2 \int_0^{t} (1+t-\tau)^{-2}\big[(1+\tau)^{-7/4} + (1+\tau)^{-7/4} + (1+\tau)^{-1}\big]\,\mathrm{d}\tau \\
&\leqslant C\Theta_0(1+t)^{-3/4} + C\big(\Lambda(t)\big)^2(1+t)^{-3/4}.
\end{aligned}
$$

Moreover, in the completely same way, from (27), we can obtain

$$\begin{aligned}\|m_1(t)\| &\leqslant C\Theta_0(1+t)^{-1/4}+C\big(\Lambda_1(t)\big)^2(1+t)^{-1},\\ \|D^\alpha m_1(t)\| &\leqslant C\Theta_0(1+t)^{-1/4-k/2}+C\big(\Lambda_1(t)\big)^2(1+t)^{-1/4-k/2}, \quad |\alpha|=k=1,2,\\ \|D^3 m_1(t)\|_{L^2} &\leqslant C\Theta_0(1+t)^{-7/4}+C\big(\Lambda_1(t)\big)^2(1+t)^{-1},\end{aligned}$$

and from (28), we can show

$$\begin{aligned}\|\theta_1(t)\| &\leqslant C\Theta_0(1+t)^{-3/4}+C\big(\Lambda_1(t)\big)^2(1+t)^{-1},\\ \|D^\alpha \theta_1(t)\| &\leqslant C\Theta_0(1+t)^{-3/4-k/2}+C\big(\Lambda_1(t)\big)^2(1+t)^{-3/4-k/2}, \quad |\alpha|=k=1,2,\\ \|D^3 \theta_1(t)\|_{L^2} &\leqslant C\Theta_0(1+t)^{-9/4}+C\big(\Lambda_1(t)\big)^2(1+t)^{-1},\end{aligned}$$

The estimates of n_2, m_2 and θ_2 can be obtained by the completely similar way. We omit the details here. For the time decay rate for $\nabla\Phi$, from Lemma 2 and (35)–(38), it is easy to get

$$\begin{aligned}\|\nabla\Phi(t)\| &\leqslant \|L*U_0\| + \int_0^t \big\|\mathfrak{L}(t-\tau)*(0,f_1,g_1,0,f_2,g_2)(\tau)\big\|\,\mathrm{d}\tau\\ &\leqslant C(1+t)^{-1/4}\|U_0\|_{L^1\cap L^2}\\ &\quad + C\int_0^t (1+t-\tau)^{-3/4}\sum_{i=1}^2\big(\|f_i(\tau)\|_{L^1\cap L^2}+\|g_i(\tau)\|_{L^1\cap L^2}\big)\,\mathrm{d}\tau\\ &\leqslant C\Theta_0(1+t)^{-1/4}+C\big(\Lambda(t)\big)^2(1+t)^{-1/4}.\end{aligned}$$

This completes the proofs. □

Next, we are going to derive the estimates of higher order derivatives of $(n_1, m_1, \theta_1, n_2, m_2, \theta_2)$. For simplicity, we denote $u_i = m_i/(n_i+1)$, $i=1,2$. From (7)–(8), we derive the system for $(n_1, u_1, n_2, u_2, \Phi)$ as

$$\partial_t n_i + \nabla\cdot u_i = -\nabla\cdot(n_i u_i), \quad i=1,2, \tag{43}$$

$$\partial_t u_i + (u_i\cdot\nabla)u_i + \frac{\nabla((n_i+1)(\theta_i+1))}{n_i+1} - (-1)^i\nabla\Phi = -u_i, \quad i=1,2, \tag{44}$$

$$\partial_t\theta_i + u_i\nabla\theta_i + \frac{2}{3}(\theta_i+1)\nabla\cdot u_i - \frac{2}{3(n_i+1)}\Delta\theta_i = \frac{1}{3}|u_i|^2-\theta_i, \quad i=1,2, \tag{45}$$

$$\Delta\Phi = n_1 - n_2, \qquad \Phi\to 0, \quad |x|\to\infty. \tag{46}$$

Further, we have

$$n_{itt} - \Delta n_i + n_{it} + (-1)^{i-1}(n_1-n_2) - \Delta\theta_i = R_i, \tag{47}$$

$$u_{it} + u_i + \nabla n_i + \nabla\theta_i - (-1)^i\nabla\Phi = Q_i, \tag{48}$$

$$\partial_t\theta_i + \frac{2}{3}\nabla\cdot u_i - \frac{2}{3}\Delta\theta_i + \theta_i = S_i, \tag{49}$$

for $i = 1, 2$, here

$$\begin{aligned} R_i &= \nabla \cdot \left(\frac{\theta_i + 1}{n_i + 1} \nabla n_i - \nabla n_i \right) - \nabla \cdot \big(n_i u_i + (n_i u_i)_t \big) \\ &\quad + \sum_{l,k} u_{ix_l}^k u_{ix_k}^l + \sum_k u_i^k (\nabla \cdot u_i)_{x_k}, \\ Q_i &= \left(1 - \frac{\theta_i + 1}{n_i + 1} \right) \nabla n_i - (u_i \cdot \nabla) u_i, \\ S_i &= -u_i \nabla \theta_i + \left(\frac{2}{3(n_i + 1)} - \frac{2}{3} \right) \Delta \theta_i - \frac{2}{3} \theta_i \nabla \cdot u_i. \end{aligned}$$

In the following, we define

$$\begin{aligned} \mathcal{E}(t) &= \big\| (n_{1t}, n_{2t}) \big\|_3^2 + \big\| (n_1, u_1, \theta_1, n_2, u_2, \theta_2) \big\|_4^2, \\ \mathcal{D}(t) &= \big\| (n_{1t}, n_{2t}, \nabla n_1, \nabla n_2, \theta_{1t}, \theta_{2t}) \big\|_3^2 + \big\| (u_1, u_2) \big\|_4^2 + \big\| (\theta_1, \theta_2) \big\|_5^2. \end{aligned}$$

Then, we have the following estimate of the solution by basic energy estimate.

Lemma 4 (The a priori estimate of the high order derivatives of solution). *Under the assumption of Theorem* 1*, suppose* $(n_1, m_1, n_2, m_2, \nabla \Phi) \in \mathcal{S}$ *is the solution of IVP* (7)–(8) *on* $[0, T]$ *for any* $T > 0$*, which satisfies* (25)*. Then, for any* $t \in [0, T]$*, there exists* C *such that*

$$\frac{\mathrm{d}}{\mathrm{d}t} \big(\mathcal{E}(t) + \big\| \nabla \Phi(t) \big\|_4^2 \big) + \mathcal{D}(t) \leqslant C \Lambda(t) \mathcal{D}(t) + C(1 + t)^{-3/2} \big\| (u_1, u_2, \nabla \Phi) \big\|_{L^2}^2. \quad (50)$$

Proof. From (22)–(25) and Sobolev inequality, we know that

$$\sum_{|\alpha| \leqslant 2} \big\| \partial_x^\alpha (n_1, u_1, \theta_1, n_2, u_2, \theta_2) \big\|_{L^\infty} \leqslant C \Lambda(t). \quad (51)$$

From equalities (43)–(45) and assumption (25), we also have

$$\sum_{|\alpha| \leqslant 1} \big\| \partial_x^\alpha (n_{1t}, u_{1t}, \theta_{1t}, n_{2t}, u_{2t}, \theta_{2t}) \big\|_{L^\infty} \leqslant C \Lambda(t). \quad (52)$$

Moreover, it is easy to see that

$$\left| \frac{\theta_i + 1}{1 + n_i} - 1 \right| \leqslant C \big(|n_i| + |\theta_i| \big) \leqslant C \Lambda(t), \quad i = 1, 2. \quad (53)$$

In the following, we will obtain five elementary estimates, denoted by estimates A, B, C, D and E. Then the estimates of the higher derivatives will be considered.

Estimate A. Multiplying (47) with $i = 1$ by $n_{1t} + \lambda n_1$ ($0 < \lambda \ll 1$) and integrating it by parts over $\mathbb{R}^3$ yields

$$\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\big(n_{1t}^2 + |\nabla n_1|^2 + \lambda n_1^2\big)\,\mathrm{d}x + \int_{\mathbb{R}^3}\big(n_{1t}^2 + \lambda|\nabla n_1|^2\big)\,\mathrm{d}x \\
&\quad + \int_{\mathbb{R}^3}(n_1 - n_2)(n_{1t} + \lambda n_1)\,\mathrm{d}x - \int_{\mathbb{R}^3}\Delta\theta_1(n_{1t} + \lambda n_1)\,\mathrm{d}x \\
&= \int_{\mathbb{R}^3}(n_{1t} + \lambda n_1)R_1\,\mathrm{d}x \\
&= -\int_{\mathbb{R}^3}(n_{1t} + \lambda n_1)\nabla\cdot\left(\left(1 - \frac{\theta_1 + 1}{n_1 + 1}\right)\nabla n_1\right)\mathrm{d}x \\
&\quad + \int_{\mathbb{R}^3}(n_{1t} + \lambda n_1)\left(\sum_{l,k} u_{1x_l}^k u_{1x_k}^l + \sum_k u_1^k(\nabla\cdot u_1)_{x_k}\right)\mathrm{d}x \\
&\quad - \int_{\mathbb{R}^3}(n_{1t} + \lambda n_1)\nabla\cdot\big(n_1 u_1 + (n_1 u_1)_t\big)\,\mathrm{d}x \\
&=: I_1 + I_2 + I_3.
\end{aligned} \tag{54}$$

First, we have

$$-\int_{\mathbb{R}^3}\Delta\theta_1(n_{1t} + \lambda n_1)\,\mathrm{d}x \leqslant \epsilon\int_{\mathbb{R}^3}\big(n_{1t}^2 + (\nabla n_1)^2\big)\,\mathrm{d}x + C(\epsilon)\int_{\mathbb{R}^3}\big(\big(D^2\theta_1\big)^2 + (D\theta_1)^2\big)\,\mathrm{d}x.$$

By integrating by parts, we can obtain

$$\begin{aligned}
I_1 &= \int_{\mathbb{R}^3}(n_{1t} + \lambda n_1)\nabla\cdot\left(\left(1 - \frac{\theta_1+1}{1+n_1}\right)\nabla n_1\right)\mathrm{d}x \\
&= \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\left(1 - \frac{\theta_1+1}{1+n_1}\right)|\nabla n_1|^2\,\mathrm{d}x + \int_{\mathbb{R}^3}\left[\lambda\left(1 - \frac{\theta_1+1}{1+n_1}\right) - \frac{1}{2}\left(\frac{\theta_1+1}{1+n_1}\right)_t\right]|\nabla n_1|^2\,\mathrm{d}x.
\end{aligned}$$

So, (51)–(53) give

$$I_1 \leqslant \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\left(1 - \frac{\theta_1 + 1}{1 + n_1}\right)|\nabla n_1|^2\,\mathrm{d}x + C\Lambda(t)\big\|\nabla n_1(t)\big\|^2.$$

For the estimate of I_2, we will use the following equality

$$\nabla\cdot u_1 = \frac{-1}{1 + n_1}(n_{1t} + u_1\cdot\nabla n_1), \tag{55}$$

which comes from (43). And it implies that

$$\begin{aligned}
&\int\limits_{\mathbb{R}^3} n_{1t} \sum_k u_1^k (\nabla \cdot u_1)_{x_k}\,\mathrm{d}x \\
&\quad= \sum_k \int\limits_{\mathbb{R}^3} \left(n_{1t}^2 \left[\frac{u_1^k}{2(1+n_1)} \right]_{x_k} + \frac{n_{1x_k} n_{1t}^2 u_1^k}{(1+n_1)^2} \right) \mathrm{d}x \\
&\qquad+ \sum_k \int\limits_{\mathbb{R}^3} u_1^k \left(\frac{n_{1t} n_{1x_k} u_1 \cdot \nabla n_1}{(1+n_1)^2} - \frac{n_{1t} n_{1x_k} \nabla \cdot u_1 + n_{1t} u_1 \cdot \nabla n_{1x_k}}{1+n_1} \right) \mathrm{d}x.
\end{aligned}$$

Since the last term above is

$$\begin{aligned}
&\sum_k \int\limits_{\mathbb{R}^3} \frac{u_1^k n_{1t} u_1}{1+n_1} \cdot \nabla n_{1x_k}\,\mathrm{d}x \\
&\quad= \sum_{k,l} \int\limits_{\mathbb{R}^3} \frac{u_1^k u_1^l n_{1t} n_{1x_l x_k}}{1+n_1}\,\mathrm{d}x \\
&\quad= -\sum_{k,l} \int\limits_{\mathbb{R}^3} \frac{u_1^k u_1^l n_{1x_k t} n_{1x_l}}{1+n_1}\,\mathrm{d}x - \sum_{k,l} \int\limits_{\mathbb{R}^3} \left(\frac{u_1^k u_1^l}{1+n_1} \right)_{x_k} n_{1t} n_{1x_l}\,\mathrm{d}x,
\end{aligned}$$

and

$$\sum_{k,l} u_1^k u_1^l n_{1x_k t} n_{1x_l} = \frac{1}{2} \sum_{k=l} \left(u_1^k\right)^2 \left(n_{1x_k}^2\right)_t + \sum_{k>l} u_1^k u_1^l (n_{1x_k} n_{1x_l})_t.$$

Then we have

$$\begin{aligned}
I_2 \leqslant{}& \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \sum_{k=l} \int\limits_{\mathbb{R}^3} \frac{(u_1^k n_{1x_k})^2}{1+n_1}\,\mathrm{d}x + \frac{\mathrm{d}}{\mathrm{d}t} \sum_{k>l} \int\limits_{\mathbb{R}^3} \frac{u_1^k u_1^l n_{1x_k} n_{1x_l}}{1+n_1}\,\mathrm{d}x \\
&+ C\Lambda(t)\bigl(\|\nabla n_1\|^2 + \|n_{1t}\|^2 + \|\nabla u_1\|^2\bigr).
\end{aligned}$$

Now, we rewrite I_3 as follows,

$$\begin{aligned}
I_3 &= \int\limits_{\mathbb{R}^3} (n_{1t} + \lambda n_1) \nabla \cdot \bigl(n_1 u_1 + (n_1 u_1)_t\bigr)\,\mathrm{d}x \\
&= \int\limits_{\mathbb{R}^3} n_{1t} \bigl(\nabla n_{1t} \cdot u_1 + n_{1t} \nabla \cdot u_1 + \nabla \cdot u_{1t} n_1 + \nabla n_1 \cdot u_{1t} + \nabla \cdot (n_1 u_1)\bigr)\,\mathrm{d}x \\
&\quad- \int\limits_{\mathbb{R}^3} \lambda \nabla n_1 \bigl(n_1 u_1 + (n_1 u_1)_t\bigr)\,\mathrm{d}x \\
&= \sum_{j=1}^{7} I_{3,j},
\end{aligned}$$

where $I_{3,j}$ represents every term in the above equality respectively. First,

$$|I_{3,1} + I_{3,2}| = \left| \frac{1}{2} \int_{\mathbb{R}^3} n_{1t}^2 \nabla \cdot u_1 \, \mathrm{d}x \right| \leqslant C\Lambda(t) \|n_{1t}\|^2.$$

By using (55) for $\nabla \cdot u_{1t}$, we have

$$I_{3,3} \leqslant \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} \frac{n_{1t} n_{1t}^2}{1 + n_1} \, \mathrm{d}x + C\Lambda(t) \bigl(\|\nabla n_1\|^2 + \|n_{1t}\|^2 \bigr).$$

The estimation of other terms, $\{I_{3,j}\}_{j \geqslant 4}$ is similar, so we omit the details. Combining above inequalities gives

$$I_3 \leqslant \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} \frac{n_{1t} n_{1t}^2}{1 + n_1} \, \mathrm{d}x + C\Lambda(t) \bigl(\|\nabla n_1\|^2 + \|n_{1t}\|^2 + \|\nabla u_1\|^2 + \|u_1\|^2 \bigr).$$

Thus, if $\Lambda(T) \leqslant \delta_0$ is sufficiently small, we obtain for all $t \in [0, T]$

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} \bigl(n_{1t}^2 + |\nabla n_1|^2 + \lambda n_1^2 \bigr) \, \mathrm{d}x + \int_{\mathbb{R}^3} \bigl(n_{1t}^2 + \lambda |\nabla n_1|^2 \bigr) \, \mathrm{d}x + \int_{\mathbb{R}^3} \bigl(n_1 - n_2 \bigr)\bigl(n_{1t} + \lambda n_1 \bigr) \, \mathrm{d}x \\ &\quad \leqslant C\Lambda(t) \|u_1\|_1^2 + C(\epsilon) \bigl\| \bigl(D^2\theta_1, \nabla\theta_1 \bigr) \bigr\|^2. \end{aligned} \tag{56}$$

Similarly, from (47) with $i = 2$, we can show

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} \bigl(n_{2t}^2 + |\nabla n_2|^2 + \lambda n_2^2 \bigr) \, \mathrm{d}x + \int_{\mathbb{R}^3} \bigl(n_{2t}^2 + \lambda |\nabla n_2|^2 \bigr) \, \mathrm{d}x - \int_{\mathbb{R}^3} (n_1 - n_2)(n_{2t} + \lambda n_2) \, \mathrm{d}x \\ &\quad \leqslant C\Lambda(t) \|u_2\|_1^2 + C(\epsilon) \bigl\| \bigl(D^2\theta_2, \nabla\theta_2 \bigr) \bigr\|^2. \end{aligned} \tag{57}$$

Moreover, noting that

$$\int_{\mathbb{R}^3} (n_1 - n_2)(n_{1t} + \lambda n_1 - n_{2t} - \lambda n_2) \, \mathrm{d}x = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} (n_1 - n_2)^2 \, \mathrm{d}x + \lambda \int_{\mathbb{R}^3} (n_1 - n_2)^2 \, \mathrm{d}x,$$

which together with (56)–(57) yields

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} \biggl[\sum_{i=1,2} \bigl(n_{it}^2 + |\nabla n_i|^2 + \lambda n_i^2 \bigr) + \frac{(n_1 - n_2)^2}{2} \biggr] \, \mathrm{d}x \\ &\qquad + \int_{\mathbb{R}^3} \biggl[\sum_{i=1,2} \bigl(n_{it}^2 + \lambda |\nabla n_i|^2 \bigr) + \lambda (n_1 - n_2)^2 \biggr] \, \mathrm{d}x \\ &\quad \leqslant C\Lambda(t) \bigl(\|u_1\|_1^2 + \|u_2\|_1^2 \bigr) + C(\epsilon) \bigl\| \bigl(D^2\theta_1, \nabla\theta_1, D^2\theta_2, \nabla\theta_2 \bigr) \bigr\|^2. \end{aligned} \tag{58}$$

Estimate B. Next, multiplying (48) by u_i ($i = 1, 2$) and integrating it over $\mathbb{R}^3$ gives

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3} u_i^2\,\mathrm{d}x + \int_{\mathbb{R}^3} u_i^2\,\mathrm{d}x - \int_{\mathbb{R}^3}(n_i+\theta_i)\nabla\cdot u_i\,\mathrm{d}x + (-1)^i\int_{\mathbb{R}^3}\nabla\Phi u_i\,\mathrm{d}x = \int_{\mathbb{R}^3} u_i\cdot Q_i\,\mathrm{d}x.$$

By (43), (46), we have

$$\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\left(u_i^2+n_i^2\right)\mathrm{d}x + \int_{\mathbb{R}^3} u_i^2\,\mathrm{d}x - \int_{\mathbb{R}^3}\theta_i\nabla\cdot u_i\,\mathrm{d}x + (-1)^i\int_{\mathbb{R}^3}\nabla\Phi u_i\,\mathrm{d}x\\
&= \int_{\mathbb{R}^3} u_i\left(1-\frac{1+\theta_i}{1+n_i}\right)\nabla n_i\,\mathrm{d}x - \int_{\mathbb{R}^3} u_i\cdot(u_i\cdot\nabla u_i)\,\mathrm{d}x - \int_{\mathbb{R}^3}\nabla n_i\cdot(n_i u_i)\,\mathrm{d}x\\
&=: H_1 + H_2 + H_3.
\end{aligned}$$

From (53), we have

$$H_1 \leqslant C\Lambda(t)\left(\|\nabla n_i\|^2 + \|u_i\|^2\right).$$

It is easy to see that

$$H_2 + H_3 \leqslant C\Lambda(t)\left(\|\nabla n_i\|^2 + \|u_i\|^2\right).$$

By combining the above estimates and the assumption (25), we get for $i = 1, 2$

$$\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\left(u_i^2+n_i^2\right)\mathrm{d}x + \int_{\mathbb{R}^3} u_i^2\,\mathrm{d}x - \int_{\mathbb{R}^3}\theta_i\nabla\cdot u_i\,\mathrm{d}x + (-1)^i\int_{\mathbb{R}^3}\nabla\Phi u_i\,\mathrm{d}x\\
&\leqslant C\Lambda(t)\left(\|\nabla n_i\|^2 + \|u_i\|^2\right).
\end{aligned}\tag{59}$$

Moreover, we can deal with the coupled term as follows:

$$\begin{aligned}
&-\int_{\mathbb{R}^3}\nabla\Phi(u_1-u_2)\\
&= \int_{\mathbb{R}^3}\Phi(\nabla\cdot u_1 - \nabla\cdot u_2) = -\int_{\mathbb{R}^3}\Phi\Delta\Phi_t + \int_{\mathbb{R}^3}\nabla\Phi(u_1n_1-u_2n_2)\\
&\geqslant \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}|\nabla\Phi|^2\,\mathrm{d}x - \max\left\{\|n_1\|_{L^\infty},\|n_2\|_{L^\infty}\right\}\left[\|\nabla\Phi\|^2 + \|u_1\|_{L^2}^2 + \|u_2\|^2\right]\\
&\geqslant \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}|\nabla\Phi|^2\,\mathrm{d}x - C\Lambda(t)(1+t)^{-3/2}\left\|(u_1,u_2,\nabla\Phi)\right\|^2,
\end{aligned}$$

which together with (59) gives

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\big(u_1^2+n_1^2+u_2^2+n_2^2+|\nabla\Phi|^2\big)\,\mathrm{d}x+\int_{\mathbb{R}^3}\big(u_1^2+u_2^2\big)\,\mathrm{d}x \\ -\int_{\mathbb{R}^3}\big(\theta_1\nabla\cdot u_1+\theta_2\nabla\cdot u_2\big)\,\mathrm{d}x \\ \leqslant C\Lambda(t)\big\|(\nabla n_1,\nabla n_2,u_1,u_2)\big\|^2+C(1+t)^{-3/2}\big\|(u_1,u_2,\nabla\Phi)\big\|^2. \tag{60}$$

Estimate C. By differentiating (48) with respect to x_l and integrating its product with u_{ix_l} $(i=1,2)$ over $\mathbb{R}^3$, respectively, we have

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\big(|u_{ix_l}|^2+|n_{ix_l}|^2\big)\,\mathrm{d}x+\int_{\mathbb{R}^3}|u_{ix_l}|^2\,\mathrm{d}x-\int_{\mathbb{R}^3}\theta_{ix_l}\nabla\cdot u_{ix_l}\,\mathrm{d}x \\ +\int_{\mathbb{R}^3}\nabla n_{ix_l}u_{ix_l}\,\mathrm{d}x+(-1)^i\int_{\mathbb{R}^3}\nabla\Phi_{x_l}u_{ix_l}\,\mathrm{d}x \\ =\int_{\mathbb{R}^3}u_{ix_l}\cdot Q_{ix_l}\,\mathrm{d}x.$$

Similar to the proof of (59), by (43), we have

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\big(|\nabla u_i|^2+|\nabla n_i|^2\big)\,\mathrm{d}x+\int_{\mathbb{R}^3}|\nabla u_i|^2\,\mathrm{d}x+(-1)^i\int_{\mathbb{R}^3}\nabla\Phi_{x_l}u_{1x_l}\,\mathrm{d}x-\int_{\mathbb{R}^3}\theta_{ix_l}\nabla\cdot u_{ix_l}\,\mathrm{d}x \\ \leqslant C\sum_l\bigg[\bigg|\int_{\mathbb{R}^3}u_{ix_l}\cdot Q_{ix_l}\,\mathrm{d}x\bigg|+\bigg|\int_{\mathbb{R}^3}\frac{1}{(1+n_i)^2}n_{ix_l}^2n_{it}\,\mathrm{d}x\bigg| \\ +\bigg|\int_{\mathbb{R}^3}n_{ix_l}\bigg(\frac{1}{n_i+1}(u_i\cdot\nabla n_i)_{x_l}\bigg)\,\mathrm{d}x\bigg|\bigg].$$

By symmetry, such as

$$\int_{\mathbb{R}^3}\frac{u_i}{n_i+1}\cdot(\nabla n_{ix_l})n_{ix_l}\,\mathrm{d}x=-\frac{1}{2}\int_{\mathbb{R}^3}\nabla\cdot\bigg(\frac{u_i}{n_i+1}\bigg)(n_{ix_l})^2\,\mathrm{d}x,$$

we have after some tedious but straightforward calculation that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\big(|\nabla u_i|^2+|\nabla n_i|^2\big)\,\mathrm{d}x+\int_{\mathbb{R}^3}|\nabla u_i|^2\,\mathrm{d}x-\int_{\mathbb{R}^3}\theta_{ix_l}\nabla\cdot u_{ix_l}\,\mathrm{d}x+(-1)^i\int_{\mathbb{R}^3}\nabla\Phi_{x_l}u_{1x_l}\,\mathrm{d}x \\ \leqslant C\Lambda(t)\|\nabla n_i\|^2.$$

From (23) and (24), we also have the following estimate:

$$
\begin{aligned}
&-\int\limits_{\mathbb{R}^3} \nabla\Phi_{x_l}(u_{1x_l} - u_{2x_l}) \\
&\quad = \int\limits_{\mathbb{R}^3} \Phi_{x_l}\big(\nabla\cdot u_{1x_l} - \nabla\cdot u_{2x_l}\big) = -\int\limits_{\mathbb{R}^3} \Phi_{x_l}\Delta\Phi_{x_l t} + \int\limits_{\mathbb{R}^3} \nabla\Phi_{x_l}(u_1 n_1 - u_2 n_2)_{x_l} \\
&\quad \geqslant \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{\mathbb{R}^3} |\nabla\Phi_{x_l}|^2\,\mathrm{d}x - C\Lambda(t)\big\|(\nabla n_1, \nabla n_2, u_1, u_2, \nabla u_1, \nabla u_2)\big\|_{L^2}^2.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{\mathbb{R}^3}\bigg(\sum_{i=1,2}\big(|\nabla u_i|^2 + |\nabla n_i|^2\big) + |\nabla^2\Phi|^2\bigg)\,\mathrm{d}x + \sum_{i=1,2}\int\limits_{\mathbb{R}^3} |\nabla u_i|^2\,\mathrm{d}x \\
&\qquad - \sum_{i=1,2}\int\limits_{\mathbb{R}^3} \theta_{ix_l}\nabla\cdot u_{ix_l}\,\mathrm{d}x \\
&\qquad \leqslant C\Lambda(t)\big(\|\nabla n_1\|^2 + \|\nabla n_2\|^2 + \|u_1\|_1^2 + \|u_2\|_1^2\big).
\end{aligned} \tag{61}
$$

Estimate D. Multiplying (49) by $(3/2)\theta_i$ $(i = 1,2)$, and taking the derivatives of (49) with respect to x_l and multiplying the resultant equation by $(3/2)\theta_{ix_l}$ $(i = 1,2)$, then integrating it over $\mathbb{R}^3$, after a length and straightforward computation, we have

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{\mathbb{R}^3} \frac{3}{4}\theta_i^2\,\mathrm{d}x + \int\limits_{\mathbb{R}^3}\bigg(\frac{3}{2}\theta_i^2 + (\nabla\theta_i)^2\bigg)\,\mathrm{d}x + \int\limits_{\mathbb{R}^3} \theta_i\nabla\cdot u_i\,\mathrm{d}x \\
&\quad = \frac{3}{2}\int\limits_{\mathbb{R}^3} S_i\theta_i\,\mathrm{d}x \leqslant C\Lambda(t)\int\limits_{\mathbb{R}^3}\big(\theta_i^2 + (\nabla\theta_i)^2 + (\nabla n_i)^2 + (\nabla u_i)^2 + u_i^2\big)\,\mathrm{d}x
\end{aligned} \tag{62}
$$

and

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{\mathbb{R}^3} \frac{3}{4}(\theta_{ix_l})^2\,\mathrm{d}x + \int\limits_{\mathbb{R}^3}\bigg(\frac{3}{2}(\theta_{ix_l})^2 + (\nabla\theta_{ix_l})^2\bigg)\,\mathrm{d}x + \int\limits_{\mathbb{R}^3} \theta_{ix_l}\nabla\cdot u_{ix_l}\,\mathrm{d}x \\
&\quad = \frac{3}{2}\int\limits_{\mathbb{R}^3} S_{ix_l}\theta_{ix_l}\,\mathrm{d}x \leqslant C\Lambda(t)\int\limits_{\mathbb{R}^3}\big((\theta_{ix_l})^2 + (\nabla\theta_{ix_l})^2 + (\nabla n_i)^2 + (\nabla u_i)^2\big)\,\mathrm{d}x.
\end{aligned} \tag{63}
$$

Combining (58), (60) and (61)–(63), we obtain

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\sum_{i=1,2}\big[\|n_{it}\|^2 + \big\|(n_i, u_i, \theta_i)\big\|_1^2 + \|\nabla\Phi\|^2\big] + \sum_{i=1,2}\big[\big\|(n_{it}, \nabla n_i,)\big\|^2 + \|u_i\|_1^2 + \|\theta_i\|_2^2\big] \\
&\quad \leqslant C\Lambda(t)\sum_{i=1,2}\big[\big\|(n_{it}, \nabla n_i)\big\|^2 + \|u_i\|_1^2 + \|\theta_i\|_2^2\big] \\
&\qquad + C\Lambda(t)(1+t)^{-3/2}\big\|(u_1, u_2, \nabla\Phi)\big\|^2.
\end{aligned} \tag{64}
$$

Estimate E. Similarly, taking the $|\alpha|$th derivatives of (47), the $(|\alpha|+1)$th derivatives of (48), and the $(|\alpha|+1)$th derivatives of (49) for $1 \leqslant |\alpha| \leqslant 3$, with respect to the space variable, respectively, furthermore, multiplying the resultant equations by $D^{|\alpha|}n_{it} + \lambda D^{|\alpha|}n_i$, $D^{|\alpha|+1}u_i$ and $D^{|\alpha|+1}\theta_i$ with $i = 1,2$, respectively, then integrating them over $\mathbb{R}^3$, we have

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\biggl[\sum_{i=1,2}\bigl(\bigl\|D^{|\alpha|}n_{it}\bigr\|^2 + \bigl\|D^{|\alpha|}(n_i,\theta_i,u_i)\bigr\|_1^2\bigr) + \bigl\|D^{|\alpha|}\nabla\Phi\bigr\|_1^2\biggr] \\
&\quad + \sum_{i=1,2}\bigl[\bigl\|D^{|\alpha|}n_{it}\bigr\|^2 + \bigl\|D^{|\alpha|}(n_i,n_i)\bigr\|_1^2 + \bigl\|D^{|\alpha|}\theta_i\bigr\|_2^2\bigr] \\
&\leqslant C\Lambda(t)\sum_{i=1,2}\bigl[\bigl\|(\nabla n_i, n_{it})\bigr\|_3^2 + \|u_i\|_4^2 + \|\theta_i\|_5^2\bigr]. \qquad (65)
\end{aligned}$$

Summing up together for all $0 \leqslant |\alpha| \leqslant 3$, estimate (50) follows immediately. □

Proof of Theorem 1. Suppose that $(n_1, m_1, \theta_1, n_2, m_2, \theta_2, \nabla\Phi) \in (H^4)^6 \times H^5$ correspond respectively to the smooth solutions of the bipolar non-isentropic Euler–Poisson system (7) for $t \in [0,T]$, subject to initial data (8). First, if $\Lambda(t)$ is rather small, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\bigl(\mathcal{E}(t) + \bigl|\nabla\Phi(t)\bigr|_4^2\bigr) + C\mathcal{D}(t) \leqslant C(1+t)^{-3/2}\Lambda(t)\bigl|\nabla\Phi(t)\bigr|^2. \qquad (66)$$

By the Gronwall's inequality, we get

$$\mathcal{E}(t) + \bigl|\nabla\Phi(t)\bigr|_4^2 + C\int_0^t \mathcal{D}(\tau)\,\mathrm{d}\tau \leqslant \mathcal{E}(0) + \bigl|\nabla\Phi(0)\bigr|_4^2 + C\Lambda^3(t), \qquad (67)$$

where $\nabla\Phi$ is explained as (30) as $t = 0$ and $\mathcal{E}(0) + |\nabla\Phi(0)|_4 \leqslant C\Theta_0$. Hence, from (67) and Lemma 3, we have

$$\Lambda(t) \leqslant C\Theta_0 + C\bigl(\Lambda(t)\bigr)^2, \quad t \in [0,T]. \qquad (68)$$

By standard continuous argument, we know that there exist constant C such that

$$\Lambda(t) \leqslant C\Theta_0, \quad t \in [0,T] \qquad (69)$$

when the initial data $\Theta_0 > 0$ is sufficiently small. And estimate (68) implies that assumption (25) is valid for all the time $t \in [0,T]$. Furthermore, it is easy to get estimates (4)–(6). The proof of Theorem 1 is completed. □

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