Modeling and prescribed H_∞ tracking control for strict feedback nonlinear systems*

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Abstract. By utilizing backstepping technique, an H_∞ robust controller with improved prescribed performance and dynamic surface control is designed for a class of strict feedback nonlinear systems. The transient and steady state performance for the tracking errors of nonlinear system can be guaranteed by using improved prescribed performance constraint. The dynamic surface control is used to overcome the differential explosion problem in the backstepping procedure. The impacts of uncertainties in the system are attenuated by H_∞ control. The performance and stability analysis proves that the controller design procedure is simple with low complexity and robustness. Finally, the simulation results verify the effectiveness of the controller. By comparing with the existing method, the proposed method has a faster convergence speed and better steady state performance, and also the controller design process is simpler.

Keywords: H_∞ control, backstepping, prescribed performance, dynamic surface control, nonlinear system.

1 Introduction

In recent years, prescribed performance control (PPC) is one of hot research topics in current control area, its main idea is guaranteeing the transient and steady state behavior on the premise of ensuring the stabilization of the system. The relative remarkable PPC method was developed by Rovithakis et al. In [10], an adaptive dynamic output feedback neural network controller is designed for a class of MIMO affine in the control uncertain nonlinear systems with prescribed performance. In [1], due to the force/position tracking problem, the prescribed performance is used to achieve prescribed performance

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bounds on the tracking errors and guarantee contact maintenance. In [3], the prescribed performance tracking problem is solved for SISO, unknown, non-affine systems in the presence of exogenous disturbances. In these papers, the prescribed error transformation function $S(\varepsilon(t)) = e(t)/\rho(t)$ is adopted, and it can regulate the tracking errors of the transient and steady state performance. The inverse of the transformation function $\varepsilon(t) = S^{-1}(e(t)/\rho(t))$ is provided for the controller design to guarantee the prescribed performance and stability of the closed-loop system. In [8], Han and Lee pointed out that the constraint method proposed by Rovithakis et al. is complex due to the use of the transformation function, and its inverse function has a singularity problem in the inverse transformation function with certain constraint conditions. Therefore, Han and Lee proposed an improved prescribed performance constraint (IPPC) method by using a new transformation function, which successfully avoids the singularity problem and makes the design process simpler.

Backstepping is a systematic control design method for nonlinear systems, which is a combination of the selection of Lyapunov function and controller design. It starts from the lowest order differential equation system, and the concept of virtual control is introduced. The requirements of virtual control are satisfied based on step by step design. Finally, the real control law can be designed [5, 6, 11, 25, 28–30, 32]. Unfortunately, the conventional backstepping control method has a large number of complex terms with high order system due to differentiations of virtual control functions.

Dynamic surface control (DSC) method introduces a low-pass filter during the recursive process to avoid the differentiations of desired virtual control functions, which can overcome the differential explosion problem in the backstepping control, and simplify the design of the control law, therefore, it has a large numbers of applications. In [13, 26], the observer-based output feedback control schemes are designed by considering all state vectors measurable. In [31, 33], DSC is applied to the non-affine systems in pure feedback form. In [4], an adaptive DSC is proposed for reinforcing robustness.

By considering the unknown and time-varing uncertainties, some approximation methods are adopted for the controller design to attenuate the impacts of uncertainties, such as fuzzy logic systems [8, 12, 14, 15, 17, 18, 24, 27] and neural networks [2, 9, 19, 22]. In [8], an adaptive fuzzy system is used to obtain required approximation performances. By exploiting the neural network density property [2], the unknown nonlinearities are substituted, without loss of generality, by neural networks linear in weights plus a modeling error term. In [9], the fuzzy echo state network method is proposed to improve the approximation performance in conventional neural network algorithms. In [22], the uncertainties are eliminated by compensation signals that are constructed by a low-pass filter. Most studies with above-mentioned methods are combined with PPC or funnel control. H_∞ robust control is another effective method to deal with the uncertainties, the main objective of H_∞ robust control is to minimize the effect of model uncertainties and external disturbances on the tracking performance. In [23], H_∞ robust control is applied to a single machine infinite bus system with var compensator, damping coefficient uncertainty, and external disturbances. In [20], H_∞ robust control is applied to a class of strict feedback nonlinear systems with mismatching nonlinear uncertainties that may not be linearly parameterized based on neural networks. In [16], H_∞ control is applied to

irregular buildings with AMD via LMI approach. Currently, the researches of PPC or IPPC with H_∞ robust control are few.

Based on the above observations, the prescribed tracking control for strict feedback system is considered. In the controller design, the performance function in [8] is selected to guarantee the transient and steady state performance. Due to the differential explosion problem in the controller design procedure of existing method, the dynamic surface control is adopted. Comparing with the existing results, the proposed control scheme can avoid the repeated differentiations of virtual controls. Therefore, the proposed control scheme is simpler. Then H_∞ control is used to deal with the uncertainties in the system. Finally, the simulation results verify the effectiveness of the controller. The proposed method has a better transient and steady state performance than that of the existing methods. The contributions of this work are including: (i) In order to improve the system performance and robustness, it is the first attempt to combine IPPC method with H_∞ robust control for nonlinear systems. (ii) By comparing with the controller design procedures with PPC methods proposed by Rovithakis et al., IPPC method with surface control has a low computational complexity, IPPC method can avoid the repeated differentiations of inverse of the transformation function $S^{-1}(e(t)/\rho(t))$ in the recursive steps, and surface control can avoid the repeated differentiations of virtual controls in backstepping design. The combination of these two methods simplifies the controller design.

The remainder of this paper is organized as follows: Section 2 describes the system formulation, performance function and error transformation. An H_∞ robust controller is designed and the stability of the system is analyzed in Section 3. Section 4 verifies the effectiveness of the controller through the simulation, and Section 5 is the summary of the proposed work and further remarks.

2 Problem formulation and preliminaries

2.1 System formulation

Consider a strict feedback nonlinear system

$$
\begin{aligned}
\dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \Delta_i, \\
\dot{x}_n &= f_n(\bar{x}_n) + g_n(\bar{x}_n)u + \Delta_n, \\
y &= x_1,
\end{aligned}
\tag{1}
$$

where $x_i \in \Re$, $i = 1, \ldots, n$, is the system state, $\bar{x}_i = [x_1, \ldots, x_i]^{\mathrm{T}} \in \Re^i$; $u \in \Re$ and $y \in \Re$ represent the control input and output, respectively; $f_i(\cdot)$ and $g_i(\cdot)$ are known, continuous, and smooth functions. Δ_i, $i = 1, \ldots, n$, are the unknown bounded disturbances.

Assumption 1. (See [8].) The signs of $g_i(\cdot)$ are known, and there exist constants $0 < g_{\min} < g_{\max}$ such that $g_{\min} \leqslant |g_i(\cdot)| \leqslant g_{\max}$, $i = 1, \ldots, n$. Without losing generality, assume that $g_{\min} \leqslant g_i(\cdot) \leqslant g_{\max}$.

Assumption 2. (See [8].) The desired trajectory $y_r(t)$ is a known and bounded function with time, and its derivatives are also known and bounded.

Remark 1. Actually, many real control systems are able to fall into the class given in (1), such as single force mechanical arm [7], inverted pendulum system [21], and so on. From the mathematical models of these applications, Assumption 1 can be satisfied. For Assumption 2, the function of desired trajectory can be easily defined in the controller design for real systems.

2.2 Performance function and error transformation

A continuous and smooth function $\rho(t) : \Re_+ \to \Re_+$ with $\lim_{t \to \infty} \rho(t) = \rho_\infty$ can be defined as follows:

$$\rho(t) = (\rho_0 - \rho_\infty)\mathrm{e}^{-lt} + \rho_\infty, \tag{2}$$

where ρ_0, ρ_∞, and l are appropriately selected positive constants. The transient and steady state performance can be guaranteed by the following prescribed constraint conditions:

$$-\delta\rho(t) < e(t) < \rho(t) \quad \text{if } e(0) \geqslant 0$$

or

$$-\rho(t) < e(t) < \delta\rho(t) \quad \text{if } e(0) < 0,$$

where $e(t)$ is the output tracking error, and $0 < \delta < 1$ is the designed parameter. The prescribed performance function is shown in Fig. 1.

According to [8], the transformed error ε can be defined as follows:

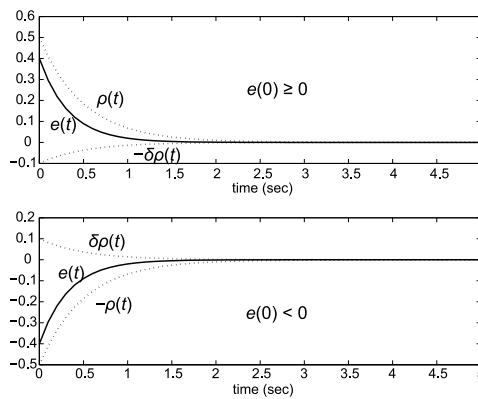$$\varepsilon(t) = \frac{e(t)}{\zeta(t)}, \qquad \zeta = a\zeta_H + (1-a)\zeta_L,$$

Figure 1. Behavior of prescribed performance function.

where $a = 1$ if $e(t) \geqslant 0$, and $a = 0$ if $e(t) < 0$. ζ_H and ζ_L are defined as follows:

$$\zeta_H = \begin{cases} \rho(t) & \text{if } e(0) \geqslant 0, \\ \delta\rho(t) & \text{if } e(0) < 0, \end{cases} \qquad \zeta_L = \begin{cases} -\delta\rho(t) & \text{if } e(0) \geqslant 0, \\ -\rho(t) & \text{if } e(0) < 0. \end{cases}$$

In [8], it has proved that the transformed error ε satisfies the inequality

$$0 < \varepsilon(t) < 1 \quad \forall t > 0. \tag{3}$$

Remark 2. In equation (2), the constant ρ_∞ confines the maximum allowable steady state error, l regulates the convergence speed of tracking error, and $\delta\rho_0$ is the upper bound of maximum overshoot. Therefore, the steady state error of system is able to converge into a prescribed area by selecting appropriate ρ_0, ρ_∞, and l. Thus the maximum overshoot and convergence speed can be guaranteed to satisfy the requirements of prescribed performance.

3 H_∞ robust controller design

A group of dynamic surface variables are defined as follows:

$$S_1 = \frac{\varepsilon_1}{1 - \varepsilon_1}, \qquad S_i = x_i - x_{i,\text{out}}, \tag{4}$$

where $\varepsilon_1 = e_1/\zeta_1$ and $x_{i,\text{out}}$, $i = 2, \ldots, n$, are virtual filtering control functions. The time derivatives of surface variables are given below:

$$\dot{S}_1 = \frac{\dot{\varepsilon}_1}{(1 - \varepsilon_1)^2} = \frac{\dot{e}_1 \zeta_1 - e_1 \dot{\zeta}_1}{(1 - \varepsilon_1)^2 \zeta_1^2} = \frac{1}{(1 - \varepsilon_1)^2 \zeta_1}(\dot{x}_1 - \dot{y}_r - \varepsilon_1 \dot{\zeta}_1),$$

$$\dot{S}_i = \dot{x}_i - \dot{x}_{i,\text{out}}.$$

Substituting above derivatives into system (1), a transformation system is provided by

$$\begin{aligned} \dot{S}_1 &= \Xi\big(f_1(\bar{x}_1) + g_1(\bar{x}_1)x_2 + \Delta_1 - \dot{y}_r - \varepsilon_1 \dot{\zeta}_1\big), \\ \dot{S}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + \Delta_i - \dot{x}_{i,\text{out}}, \\ \dot{S}_n &= f_n(\bar{x}_n) + g_n(\bar{x}_n)u + \Delta_n - \dot{x}_{n,\text{out}}, \\ z &= S_1, \end{aligned} \tag{5}$$

where $\Xi = 1/((1 - \varepsilon_1)^2 \zeta_1)$ and z is control output. The derivatives of virtual filtering controls are obtained by passing designed virtual controls $x_{i,\text{in}}$, $i = 2, \ldots, n$, through first-order filters with time constants τ_i, $i = 2, \ldots, n$, such that

$$\tau_{i-1}\dot{x}_{i,\text{out}} + x_{i,\text{out}} = x_{i,\text{in}}, x_{i,\text{out}}(0) = x_{i,\text{in}}(0). \tag{6}$$

The approximation error of above filters are determined by

$$\sigma_i = x_{i,\text{out}} - x_{i,\text{in}}, \quad i = 2, \ldots, n. \tag{7}$$

The H_∞ control problem is defined as follows.

Definition 1. If there exist a control law $u = \alpha(x)$, an appropriate Lyapunov function candidate V_N, and a positive constant γ so that the following three objectives (O1)–(O3) can be achieved, then the H_∞ control problem is solvable.

(O1) All signals are bounded in the closed loop system.

(O2) The output tracking error $e(t) = y(t) - y_r(t)$ satisfies the prescribed performance during both transient process and steady state with the desired trajectory $y_r(t)$.

(O3) The L_2 gain from the external disturbances and modeling errors to the output is less than or equal to γ, that is,

$$V_N - V_N(0) \leqslant \int_0^{\mathrm{T}} \left(\gamma^2 \|\bar{\Delta}\|^2 - \|z\|^2 \right) \mathrm{d}t \tag{8}$$

for any final time $T > 0$ with $\bar{\Delta} = [\Delta_1, \ldots, \Delta_n, \sigma_2, \ldots, \sigma_n]^{\mathrm{T}}$.

Remark 3. $\bar{\Delta}_i$ is the system uncertainty composed by external disturbances and approximation errors of surface control. A better robust performance can be achieved by selecting a smaller γ.

The H_∞ controller with improved prescribed performance is designed by using backstepping control method, and the design procedure is divided into n steps.

Step 1. Consider the first subsystem in (5) and choose the following Lyapunov function candidate:

$$V_1(S_1) = \frac{1}{2} S_1^2.$$

Define the following function:

$$H_1 = \frac{1}{2} z^2 - \frac{\gamma^2}{2} \Delta_1^2 + \dot{V}_1(S_1). \tag{9}$$

Substituting the derivative of $V_1(S_1)$, $z = S_1$, $x_2 = S_2 + x_{2,\mathrm{out}}$, and $x_{2,\mathrm{out}} = \sigma_2 + x_{2,\mathrm{in}}$ into (9), we obtain

$$
\begin{aligned}
H_1 &= \frac{1}{2} S_1^2 - \frac{\gamma^2}{2} \Delta_1^2 + S_1 \Xi \big(f_1(\bar{x}_1) + g_1(\bar{x}_1) x_2 - \dot{y}_r - \varepsilon_1 \dot{\zeta}_1 + \Delta_1 \big) \\
&= -\frac{1}{4} \gamma^2 \Delta_1^2 - \left(\frac{\gamma}{2} \Delta_1 - \frac{S_1 \Xi}{\gamma} \right)^2 \\
&\quad + S_1 \Xi \big[\Gamma_1 S_1 + f_1(\bar{x}_1) + g_1(\bar{x}_1)(S_2 + \sigma_2 + x_{2,\mathrm{in}}) \big] + S_1 \Xi (-\dot{y}_r - \varepsilon_1 \dot{\zeta}_1) \\
&\leqslant -\frac{1}{4} \gamma^2 \Delta_1^2 - \left(\frac{\gamma}{2} \Delta_1 - \frac{S_1 \Xi}{\gamma} \right)^2 + S_1 \Xi g_1(\bar{x}_1) S_2 + S_1^2 \frac{\Xi^2 g_1^2(\bar{x}_1)}{2\gamma^2} + \frac{\gamma^2}{2} \sigma_2^2 \\
&\quad + S_1 \Xi \big(\Gamma_1 S_1 + f_1(\bar{x}_1) + g_1(\bar{x}_1) x_{2,\mathrm{in}} - \dot{y}_r - \varepsilon_1 \dot{\zeta}_1 \big),
\end{aligned} \tag{10}
$$

where $\Gamma_1 = \Xi/\gamma^2 + 1/(2\Xi)$, and the inequality above is obtained by using the following relation:

$$S_1 \Xi g_1(\bar{x}_1)\sigma_2 \leqslant \frac{S_1^2 \Xi^2 g_1^2(\bar{x}_1)}{2\gamma^2} + \frac{\gamma^2}{2}\sigma_2^2.$$

Introduce the virtual control $x_{2,\text{in}}$ as follows:

$$x_{2,\text{in}} = -\frac{1}{g_1(\bar{x}_1)}\left(k_1 \zeta_1 S_1 + \Gamma_1 S_1 + f_1(\bar{x}_1) - \dot{y}_r - \varepsilon_1 \dot{\zeta}_1 + \frac{S_1 \Xi g_1^2(\bar{x}_1)}{2\gamma^2}\right), \quad (11)$$

where k_1 is a positive design parameter. It follows from substituting $x_{2,\text{in}}$ into (10) that

$$H_1 \leqslant -\frac{1}{4}\gamma^2 \Delta_1^2 - \left(\frac{\gamma}{2}\Delta_1 - \frac{S_1 \Xi}{\gamma}\right)^2 - \frac{k_1}{(1-\varepsilon_1)^2}S_1^2$$
$$+ S_1 \Xi g_1(\bar{x}_1)S_2 + \frac{\gamma^2}{2}\sigma_2^2. \quad (12)$$

Step 2. Consider the second subsystem in (5) and choose the following Lyapunov function candidate:

$$V_2(S_1, S_2) = V_1(S_1) + \frac{1}{2}S_2^2. \quad (13)$$

By differentiating both sides of (13), the following relation can be obtained:

$$\dot{V}_2(S_1, S_2) = \dot{V}_1(S_1) + S_2 \dot{S}_2.$$

Substituting (9) into the equation above gives

$$\dot{V}_2(S_1, S_2) = H_1 - \frac{1}{2}z^2 + \frac{\gamma^2}{2}\Delta_1^2 + S_2 \dot{S}_2. \quad (14)$$

Define the following function:

$$H_2 = \frac{1}{2}z^2 - \frac{\gamma^2}{2}\sum_{j=1}^{2}\left(\Delta_j^2 + \sigma_j^2\right) + \dot{V}_2(S_1, S_2). \quad (15)$$

Then, by substituting (14) into (15) and taking (12) into consideration, it follows that

$$H_2 = -\frac{\gamma^2}{2}\Delta_2^2 - \frac{\gamma^2}{2}\sigma_2^2 + H_1 + S_2 \dot{S}_2$$
$$\leqslant \Psi_1 - \frac{\gamma^2}{4}\Delta_2^2 - \left(\frac{\gamma}{2}\Delta_2 - \frac{S_2}{\gamma}\right)^2$$
$$+ S_2\left(\frac{S_2}{\gamma^2} + f_2(\bar{x}_2) + g_2(\bar{x}_2)x_3 + S_1 \Xi g_1(\bar{x}_1) - \dot{x}_{2,\text{out}}\right), \quad (16)$$

where $\Psi_1 = -(\gamma^2/4)\Delta_1^2 - ((\gamma/2)\Delta_1 - (\Xi/\gamma)S_1)^2 - (k_1/(1-\varepsilon_1)^2)S_1^2$. Substituting $x_3 = S_3 + x_{3,\text{out}}$ and $x_{3,\text{out}} = \sigma_3 + x_{3,\text{in}}$ into (16) produces

$$
\begin{aligned}
H_2 \leqslant\ & \Psi_1 - \frac{\gamma^2}{4}\Delta_2^2 - \left(\frac{\gamma}{2}\Delta_2 - \frac{S_2}{\gamma}\right)^2 \\
& + S_2\left[\frac{S_2}{\gamma^2} + f_2(\bar{x}_2) + g_2(\bar{x}_2)(S_3 + \sigma_3 + x_{3,\text{in}})\right] + S_2\big(S_1\Xi g_1(\bar{x}_1) - \dot{x}_{2,\text{out}}\big) \\
\leqslant\ & \Psi_1 - \frac{\gamma^2}{4}\Delta_2^2 - \left(\frac{\gamma}{2}\Delta_2 - \frac{S_2}{\gamma}\right)^2 + S_2 g_2(\bar{x}_2)S_3 + \frac{\gamma^2}{2}\sigma_3^2 \\
& + S_2\left(\frac{S_2}{\gamma^2} + f_2(\bar{x}_2) + g_2(\bar{x}_2)x_{3,\text{in}} + S_1\Xi g_1(\bar{x}_1)\right) \\
& + S_2\left(\frac{S_2 g_2^2(\bar{x}_2)}{2\gamma^2} - \dot{x}_{2,\text{out}}\right),
\end{aligned}
\tag{17}
$$

where the inequality is obtained by using the following relation:

$$
S_2 g_2(\bar{x}_2)\sigma_3 \leqslant \frac{S_2^2 g_2^2(\bar{x}_2)}{2\gamma^2} + \frac{\gamma^2}{2}\sigma_3^2.
$$

Define the virtual control $x_{3,\text{in}}$ as follows:

$$
x_{3,\text{in}} = -\frac{1}{g_2(\bar{x}_2)}\left(k_2 S_2 + \frac{S_2}{\gamma^2} + f_2(\bar{x}_2) + S_1\Xi g_1(\bar{x}_1) + \frac{S_2 g_2^2(\bar{x}_2)}{2\gamma^2} - \dot{x}_{2,\text{out}}\right),
$$

where k_1 is a positive design parameter. It follows from substituting $x_{3,\text{in}}$ into (17) that

$$
H_2 \leqslant \Psi_1 - \frac{\gamma^2}{4}\Delta_2^2 - \left(\frac{\gamma}{2}\Delta_2 - \frac{S_2}{\gamma}\right)^2 - k_2 S_2^2 + S_2 g_2(\bar{x}_2)S_3 + \frac{\gamma^2}{2}\sigma_3^2.
$$

Step i $(i = 3, \ldots, n-1)$. Suppose that at Step $i-1$, the function

$$
H_{i-1} = \frac{1}{2}z^2 - \frac{\gamma^2}{2}\sum_{j=1}^{i-1}\big(\Delta_j^2 + \sigma_j^2\big) + \dot{V}_{i-1}(S_1, \ldots, S_{i-1})
\tag{18}
$$

satisfies the inequality

$$
\begin{aligned}
H_{i-1} \leqslant\ & \Psi_1 - \sum_{j=2}^{i-1}\left[\frac{\gamma^2}{4}\Delta_j^2 + \left(\frac{\gamma}{2}\Delta_j - \frac{S_j}{\gamma}\right)^2\right] \\
& - \sum_{j=2}^{i-1} k_j S_j^2 + S_{i-1} g_{i-1}(\bar{x}_{i-1})S_i + \frac{\gamma^2}{2}\sigma_i^2,
\end{aligned}
\tag{19}
$$

where

$$
V_{i-1}(S_1, \ldots, S_{i-1}) = V_{i-2}(S_1, \ldots, S_{i-2}) + \frac{1}{2}S_{i-1}^2.
$$

Consider the ith subsystem in (5) and choose the following Lyapunov function candidate:

$$V_i(S_1, \ldots, S_i) = V_{i-1}(S_1, \ldots, S_{i-1}) + \frac{1}{2}S_i^2. \tag{20}$$

By differentiating both sides of (20), the following can be obtained:

$$\dot{V}_i(S_1, \ldots, S_i) = \dot{V}_{i-1}(S_1, \ldots, S_{i-1}) + S_i\dot{S}_i.$$

Substituting (5) and (18) into the equation above yields

$$\dot{V}_i(S_1, \ldots, S_i) = H_{i-1} - \frac{1}{2}z^2 + \frac{\gamma^2}{2}\sum_{j=1}^{i-1}\left(\Delta_j^2 + \sigma_j^2\right) + S_i\dot{S}_i. \tag{21}$$

Define the function

$$H_i = \frac{1}{2}z^2 - \frac{\gamma^2}{2}\sum_{j=1}^{i}\left(\Delta_j^2 + \sigma_j^2\right) + \dot{V}_i(S_1, \ldots, S_i).$$

Then, by using (21) and (19), it can be easily verified that

$$
\begin{aligned}
H_i &= \frac{1}{2}z^2 - \frac{\gamma^2}{2}\sum_{j=1}^{i}\left(\Delta_j^2 + \sigma_j^2\right) + H_{i-1} - \frac{1}{2}z^2 + \frac{\gamma^2}{2}\sum_{j=1}^{i-1}\left(\Delta_j^2 + \sigma_j^2\right) + S_i\dot{S}_i \\
&\leqslant \Psi_1 - \sum_{j=2}^{i}\left[\frac{\gamma^2}{4}\Delta_j^2 + \left(\frac{\gamma}{2}\Delta_j - \frac{S_j}{\gamma}\right)^2\right] - \sum_{j=2}^{i-1}k_jS_j^2 \\
&\quad + S_i\left(\frac{S_i}{\gamma^2} + f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + S_{i-1}g_{i-1}(\bar{x}_{i-1}) - \dot{x}_{i,\text{out}}\right).
\end{aligned} \tag{22}
$$

Substituting $x_{i+1} = S_{i+1} + x_{i+1,\text{out}}$ and $x_{i+1,\text{out}} = \sigma_{i+1} + x_{i+1,\text{in}}$ into (22) gives

$$
\begin{aligned}
H_i &\leqslant \Psi_1 - \sum_{j=2}^{i}\left[\frac{\gamma^2}{4}\Delta_j^2 + \left(\frac{\gamma}{2}\Delta_j - \frac{S_j}{\gamma}\right)^2\right] - \sum_{j=2}^{i-1}k_jS_j^2 \\
&\quad + S_ig_i(\bar{x}_i)S_{i+1} + S_ig_i(\bar{x}_i)\sigma_{i+1} \\
&\quad + S_i\left(\frac{S_i}{\gamma^2} + f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1,\text{in}} + S_{i-1}g_{i-1}(\bar{x}_{i-1}) - \dot{x}_{i,\text{out}}\right) \\
&\leqslant \Psi_1 - \sum_{j=2}^{i}\left[\frac{\gamma^2}{4}\Delta_j^2 + \left(\frac{\gamma}{2}\Delta_j - \frac{S_j}{\gamma}\right)^2\right] - \sum_{j=2}^{i-1}k_jS_j^2 + S_ig_i(\bar{x}_i)S_{i+1} + \frac{\gamma^2}{2}\sigma_{i+1}^2 \\
&\quad + S_i\left(\frac{S_i}{\gamma^2} + f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1,\text{in}} + S_{i-1}g_{i-1}(\bar{x}_{i-1})\right) \\
&\quad + S_i\left(\frac{S_ig_i^2(\bar{x}_i)}{2\gamma^2} - \dot{x}_{i,\text{out}}\right),
\end{aligned} \tag{23}
$$

where the inequality is obtained by

$$S_i g_i(\bar{x}_i)\sigma_{i+1} \leqslant \frac{S_i^2 g_i^2(\bar{x}_i)}{2\gamma^2} + \frac{\gamma^2}{2}\sigma_{i+1}^2.$$

Design the virtual control $x_{i+1,\mathrm{in}}$ as follows:

$$\begin{aligned}
x_{i+1,\mathrm{in}} = &-\frac{1}{g_i(\bar{x}_i)}\left(k_i S_i + \frac{S_i}{\gamma^2} + f_i(\bar{x}_i) + S_{i-1}g_{i-1}(\bar{x}_{i-1})\right) \\
&-\frac{1}{g_i(\bar{x}_i)}\left(\frac{S_i g_i^2(\bar{x}_i)}{2\gamma^2} - \dot{x}_{i,\mathrm{out}}\right),
\end{aligned}$$

where k_i is a positive design parameter. It follows from substituting $x_{i+1,\mathrm{in}}$ into (23) that

$$\begin{aligned}
H_i \leqslant \Psi_1 &- \sum_{j=2}^{i}\left[\frac{\gamma^2}{4}\Delta_j^2 + \left(\frac{\gamma}{2}\Delta_j - \frac{S_j}{\gamma}\right)^2\right] \\
&- \sum_{j=2}^{i} k_j S_j^2 + S_i g_i(\bar{x}_i)S_{i+1} + \frac{\gamma^2}{2}\sigma_{i+1}^2.
\end{aligned} \tag{24}$$

Step n. Consider the nth subsystem in (5) and choose the following Lyapunov function candidate:

$$V_n(S_1,\dots,S_n) = V_{n-1}(S_1,\dots,S_{n-1}) + \frac{1}{2}S_n^2. \tag{25}$$

The following relation can be obtained by differentiating both sides of (25) and using (24) with $i = n - 1$:

$$\begin{aligned}
\dot{V}_n(S_1,\dots,S_n) &= \dot{V}_{n-1}(S_1,\dots,S_{n-1}) + S_i\dot{S}_i \\
&= H_{n-1} - \frac{1}{2}z^2 + \frac{\gamma^2}{2}\sum_{j=1}^{n-1}\left(\Delta_j^2 + \sigma_j^2\right) + S_n\dot{S}_n.
\end{aligned} \tag{26}$$

Define the following function:

$$H_n = \frac{1}{2}z^2 - \frac{\gamma^2}{2}\sum_{j=1}^{n}\left(\Delta_j^2 + \sigma_j^2\right) + \dot{V}_n(S_1,\dots,S_n). \tag{27}$$

Then, by using (26) and (24) with $i = n - 1$, it can be proved that

$$\begin{aligned}
H_n = &\frac{1}{2}z^2 - \frac{\gamma^2}{2}\sum_{j=1}^{n}\left(\Delta_j^2 + \sigma_j^2\right) + H_{n-1}\frac{1}{2}z^2 + \frac{\gamma^2}{2}\sum_{j=1}^{n-1}\left(\Delta_j^2 + \sigma_j^2\right) + S_n\dot{S}_n \\
\leqslant &-\frac{\gamma^2}{2}\Delta_n^2 - \frac{\gamma^2}{2}\sigma_n^2 + \Psi_1 - \sum_{j=2}^{n-1}\left[\frac{\gamma^2}{4}\Delta_j^2 + \left(\frac{\gamma}{2}\Delta_j - \frac{S_j}{\gamma}\right)^2\right] - \sum_{j=2}^{n-1}k_j S_j^2 \\
&+ S_{n-1}g_{n-1}(\bar{x}_{n-1})S_n + \frac{\gamma^2}{2}\sigma_n^2 + S_n\left(f_n(\bar{x}_n) + g_n(\bar{x}_n)u + \Delta_n - \dot{x}_{n,\mathrm{out}}\right)
\end{aligned}$$

$$\leqslant \Psi_1 - \sum_{j=2}^n \left[\frac{\gamma^2}{4} \Delta_j^2 + \left(\frac{\gamma}{2} \Delta_j - \frac{S_j}{\gamma} \right)^2 \right] - \sum_{j=2}^{n-1} k_j S_j^2$$
$$+ S_n \left(\frac{S_n}{\gamma^2} + f_n(\bar{x}_n) + g_n(\bar{x}_n)u + g_{n-1}(\bar{x}_{n-1})S_{n-1} - \dot{x}_{i,\text{out}} \right). \tag{28}$$

Choose the control input u as follows:

$$u = -\frac{1}{g_n(\bar{x}_n)} \left(k_n S_n + \frac{S_n}{\gamma^2} + f_n(\bar{x}_n) + g_{n-1}(\bar{x}_{n-1})S_{n-1} - \dot{x}_{n,\text{out}} \right),$$

where k_n is a positive design parameter. It follows from substituting u into (28) that

$$H_n \leqslant \Psi_1 - \sum_{j=2}^n \left[\frac{\gamma^2}{4} \Delta_j^2 + \left(\frac{\gamma}{2} \Delta_j - \frac{S_j}{\gamma} \right)^2 \right] - \sum_{j=2}^n k_j S_j^2 \leqslant 0. \tag{29}$$

Select $V_N(S_1, \ldots, S_n) = 2V_n(S_1, \ldots, S_n)$. Then it follows from (27) that the derivative of V_N satisfies

$$\dot{V}_N(S_1, \ldots, S_n) = 2H_n - \left(\|z\|^2 - \gamma^2 \|\Delta\|^2 - \gamma^2 \|\sigma\|^2 \right).$$

Because of $H_n \leqslant 0$, the following inequality is obtained:

$$\dot{V}_N(S_1, \ldots, S_n) \leqslant \left(\gamma^2 \|\bar{\Delta}\|^2 - \|z\|^2 \right). \tag{30}$$

By integrating both sides of inequality (30), inequality (8) in Definition 1 can be obtained with the initial condition $V_N(0) = 2V_n(0)$, which indicates that the L_2 gain from uncertainties $\bar{\Delta}_i$ to output z is smaller than or equal to a positive constant γ.

Now it is ready to make the following conclusion.

Theorem 1. *Suppose that Assumptions 1, 2 are satisfied. If the initial condition $e(0)$ satisfies (3), then the H_∞ control problem is solvable. All signals are bounded in the closed loop system, and the output tracking error $e(t)$ satisfies the prescribed performance with the desired trajectory $y_r(t)$.*

Proof. (i) By substituting $\Psi_1 = -(\gamma^2/4)\Delta_1^2 - (\gamma/2)\Delta_1 - (\Xi/\gamma)S_1^2 - (k_1/(1-\varepsilon_1)^2)S_1^2$ into (29), the following relation can be derived:

$$H_n \leqslant -\frac{\gamma^2}{4} \Delta_1^2 - \left(\frac{\gamma}{2} \Delta_1 - \frac{\Xi}{\gamma} S_1 \right)^2 - \frac{k_1}{(1-\varepsilon_1)^2} S_1^2$$
$$- \sum_{j=2}^n \left[\frac{\gamma^2}{4} \Delta_j^2 + \left(\frac{\gamma}{2} \Delta_j - \frac{S_j}{\gamma} \right)^2 \right] - \sum_{j=2}^n k_j S_j^2$$
$$\leqslant -\frac{k_1}{(1-\varepsilon_1)^2} S_1^2 - \sum_{j=2}^n k_j S_j^2.$$

Replacing H_n by (27) gives

$$\frac{1}{2}z^2 - \frac{\gamma^2}{2}\sum_{j=1}^{n}\left(\Delta_j^2 + \sigma_j^2\right) + \dot{V}_n(S_1,\dots,S_n) \leqslant -\frac{k_1}{(1-\varepsilon_1)^2}S_1^2 - \sum_{j=2}^{n}k_j S_j^2.$$

It is easy to derive the following:

$$\dot{V}_n(S_1,\dots,S_n) \leqslant -\frac{k_1}{(1-\varepsilon_1)^2}S_1^2 - \sum_{j=2}^{n}k_j S_j^2 - \frac{1}{2}z^2 + \frac{\gamma^2}{2}\sum_{j=1}^{n}\left(\Delta_j^2 + \sigma_j^2\right)$$

$$\leqslant -\frac{k_1}{(1-\varepsilon_1)^2}S_1^2 - \sum_{j=2}^{n}k_j S_j^2 + \frac{\gamma^2}{2}\sum_{j=1}^{n}\left(\Delta_j^2 + \sigma_j^2\right). \qquad (31)$$

Select the control gains as follows:

$$k_1 = (1-\varepsilon_1)^2\Lambda_1, \qquad k_j = \Lambda_j, \quad j=2,\dots,n. \qquad (32)$$

By considering (32), (31) becomes

$$\dot{V}_n(S_1,\dots,S_n) \leqslant -2\Lambda V_n(S_1,\dots,S_n) + \vartheta, \qquad (33)$$

where $\Lambda = \min[\Lambda_1,\dots,\Lambda_n]$ and $\vartheta = (\gamma^2/2)\sum_{j=1}^{n}(\Delta_j^2 + \sigma_j^2)$.

Solving inequality (33), we get

$$V_n(t) \leqslant \left(V_n(0) - \frac{\vartheta}{2\Lambda}\right)\mathrm{e}^{-2\Lambda t} + \frac{\vartheta}{2\Lambda} \leqslant V_n(0)\mathrm{e}^{-2\Lambda t} + \frac{\vartheta}{2\Lambda} \quad \forall t > 0. \qquad (34)$$

From (34), it shows that all signals in the closed-loop system are semiglobally, uniformly and ultimately bounded, which can be explained as follows.

It is easy to see that $V_n(t) \geqslant 0$ and $V_n(t)$ is bounded by $\vartheta/(2\Lambda)$, which implies that $\vartheta/(2\Lambda)$ can be made arbitrarily small by selecting appropriate design parameters.

From the first equation in (4), $\varepsilon_1 = S_1/(1+S_1) = e_1/\zeta_1$ can be derived. By using the conclusion in (3), S_1 cannot be equal to -1, hence ε_1 is bounded by the boundness of S_1, which implies that e_1 is bounded because ζ_1 is bounded in the definition of performance function. From $e_1 = x_1 - y_r$ and Assumption 2, it can be proven that x_1 is bounded and $f_1(\bar{x}_1)$ is also bounded.

From (11), it is easy to derive that $x_{2,\mathrm{in}}$ is bounded by the boundness of ζ_1, S_1, $f_1(\bar{x}_1)$, $g_1(\bar{x}_1)$, ε_1, $\dot{y}_r$, and $\dot{\zeta}_1$. According to (6), $x_{2,\mathrm{out}}$ is bounded by the boundness of $x_{2,\mathrm{in}}$, furthermore, x_2 is bounded from (4), and the modeling error σ_2 is also bounded from (7). Similarly, $x_{i,\mathrm{in}}$, $x_{i,\mathrm{out}}$, σ_i, x_i, $i=3,\dots,n$, and u are all bounded. The objective (O1) is achieved.

(ii) According to (34), the following equality is satisfied:

$$V_1 = \frac{1}{2}S_1^2 = \frac{1}{2}\frac{\varepsilon_1^2}{(1-\varepsilon_1)^2} \leqslant V_n(0)\mathrm{e}^{-2\Lambda t} + \frac{\vartheta}{2\Lambda}. \qquad (35)$$

Then (35) can be written as

$$\varepsilon_1^2 \leqslant 2(1 - \varepsilon_1)^2 \left[V_n(0)\mathrm{e}^{-2\Lambda t} + \frac{\vartheta}{2\Lambda} \right].$$

Substituting $\varepsilon_1(t) = e_1(t)/\zeta_1(t)$ into the above inequality, we get

$$\left(\frac{e_1(t)}{\zeta_1(t)} \right)^2 \leqslant 2(1 - \varepsilon_1)^2 \left[V_n(0)\mathrm{e}^{-2\Lambda t} + \frac{\vartheta}{2\Lambda} \right],$$

$$\frac{|e_1(t)|}{|\zeta_1(t)|} \leqslant \sqrt{2}(1 - \varepsilon_1)\sqrt{V_n(0)\mathrm{e}^{-2\Lambda t} + \frac{\vartheta}{2\Lambda}},$$

$$|e_1(t)| \leqslant |\zeta_1(t)|\sqrt{2}(1 - \varepsilon_1)\sqrt{V_n(0)\mathrm{e}^{-2\Lambda t} + \frac{\vartheta}{2\Lambda}}. \tag{36}$$

For $t \to \infty$, $V_n(0)\mathrm{e}^{-2\Lambda t} = 0$, then it follows from (36) that

$$|e_1(t)| \leqslant |\zeta_1(t)|\sqrt{2}(1 - \varepsilon_1)\sqrt{\frac{\vartheta}{2\Lambda}}. \tag{37}$$

According to the conclusion $0 < \varepsilon_1 < 1$, (37) becomes

$$|e_1(t)| \leqslant |\zeta_1(t)|\sqrt{\frac{\vartheta}{\Lambda}}. \tag{38}$$

If the selected design parameters satisfies $\Lambda \geqslant \vartheta$, (38) yields

$$|e_1(t)| \leqslant |\zeta_1(t)|.$$

Therefore, the output tracking errors are smaller than the prescribed bounds, and the errors can be arbitrarily small by selecting appropriate design parameters. The objective (O2) is achieved.

(iii) The objective (O3) was proved before the theorem. $\qquad\square$

4 Simulation

Consider a rigid robot manipulator system, its mathmatical model can be described as follows [7]:

$$\begin{aligned}
\dot{x}_1 &= x_2 + \Delta_1, \\
\dot{x}_2 &= -\frac{m_r g_v l_r}{J}\cos x_1 + \frac{u}{J} + \Delta_2, \\
y &= x_1,
\end{aligned} \tag{39}$$

where x_1 is the angular position of manipulator, x_2 is the relative angular velocity, m_r is the load mass, g_v is the gravity, l_r is the length of manipulator, and $J = 4m_r l_r^2/3$ is the inertia coeffecient. Δ_1 and Δ_2 are the external disturbances.
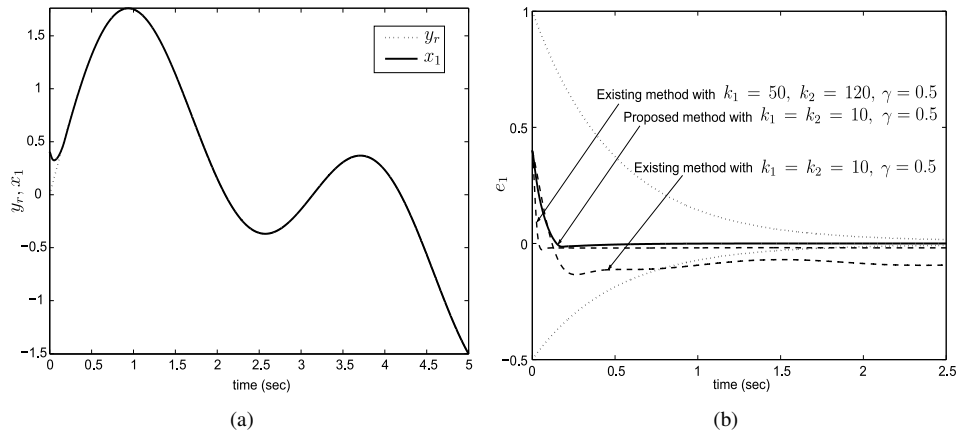
Figure 2. Tracking performance of system (39) and comparisons with existing method.

According to Theorem 1, the robust controller for system (39) is designed. In order to verify the effectiveness and feasibility of the proposed method, the simulation results of the proposed method are compared with the existing backstepping method with H_∞ control.

The selected parameters are as follows: the initial conditions are

$$x_1(0) = 0.4 \quad \text{and} \quad x_2(0) = 0.$$

The desired trajectory is

$$y_r(t) = \sin t + \sin(2t).$$

$\Delta_1 = 0$ and $\Delta_2 = 0.01 \cos t$. The prescribed performances of tracking errors are set as follows: the initial value of performance function $\rho_0 = 1$, the steady state error is no more than $\rho_\infty = 0.01$, the minimum convergence speed is $l = 2$, and the overshoot is $\delta = 0.5$, therefore, the selected performance function is

$$\rho(t) = (1 - 0.01)\mathrm{e}^{-2t} + 0.01.$$

The time constant of filter is $\tau = 0.01$. The control gains are selected as $k_1 = k_2 = 10$. The best disturbance attenuation constant is $\gamma = 0.5$.

The tracking performance with IPPC and the comparisons of tracking errors between the proposed method and existing backstepping method with H_∞ control are shown in Fig. 2. It is found that (i) if the selected control gains are $k_1 = 10$ and $k_2 = 10$, the proposed method has a faster convergence speed and better steady state performance than the existing backstepping method, (ii) the steady state performance of the existing backstepping method can be improved by setting the control gains with $k_1 = 50$ and $k_2 = 120$. However, the steady state of the system is still out of the range of performance functions. The comparisons of the proposed method with different value of γ and control gains are shown in Fig. 3. It shows that the smaller value of γ has a faster the convergence speed, and the smallest value of γ is 0.5. It also indicates that the tracking errors can
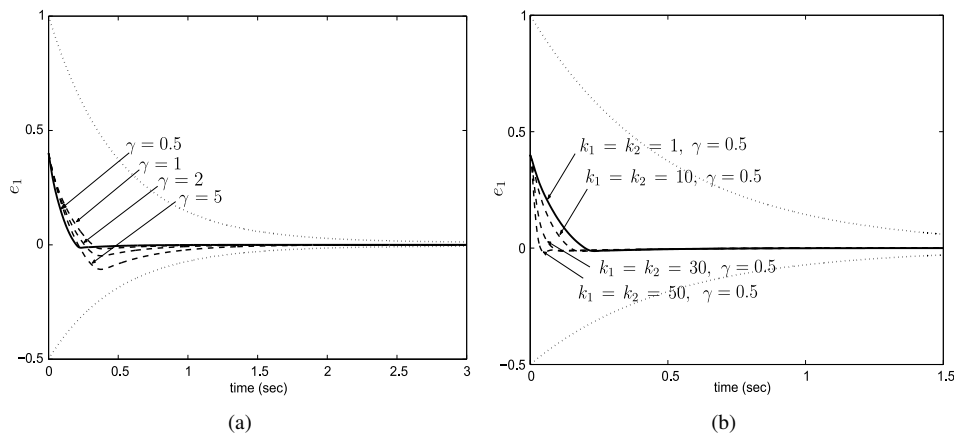
Figure 3. Comparisons of the proposed method with different γ and k.

converge to a very small value, and the larger the control gains have the faster convergence rate. Therefore, it is easy to see that the proposed method can achieve a better performance with small control gains, and the proposed control design scheme is feasible and effective.

5 Conclusion

In this paper, a backstepping control scheme has been designed for the H_∞ control problem of strict feedback nonlinear systems. An improved prescribed performance constraint method has been adopted to achieve prescribed performance bounds on the tracking errors. Surface control has been used to avoid the differentiation of virtual control in each recursive step of backstepping design, H_∞ robust control has been introduced to attenuate the impacts of the unknown disturbances and modeling errors. By selecting appropriate parameters, a better tracking performance has been obtained by simulation. The simulation results have shown that the proposed design scheme is effective and feasible. For the further work, a new prescribed performance function can be designed based on IPPC method to improve the transient and steady state performance of the system, and also adaptive neural or fuzzy control method can be used to approximate the uncertain terms in system (1).

References

1. C.P. Bechlioulis, G.A. Rovithakis,   Guaranteeing prescribed performance and contact maintenance via an approximation free robot force/position controller, *Automatica*, **48**(2):360–365, 2012.

2. C.P. Bechlioulis, G.A. Rovithakis, A priori guaranteed evolution within the neural network approximation set and robustness expansion via prescribed performance control, *IEEE Trans. Neural Netw. Learn. Syst.*, **23**(4):669–675, 2012.

3. C.P. Bechlioulis, G.A. Rovithakis, Robust adaptive fuzzy control of nonaffine systems guaranteeing transient and steady state error bounds, *Int. J. Adapt. Control Signal Process.*, **26**(7):576–591, 2012.

4. C.P. Bechlioulis, G.A. Rovithakis, Reinforcing robustness of adaptive dynamic surface control, *Int. J. Adapt. Control Signal Process.*, **27**(4):323–339, 2013.

5. H. Chen, C.L. Wang, B.W. Zhang, D.K. Zhang, Saturated tracking control for nonholonomic mobile robots with dynamic feedback, *Trans. Inst. Meas. Control*, **35**(2):105–116, 2013.

6. W.S. Chen, Adaptive backstepping dynamic surface control for systems with periodic disturbances using neural networks, *IET Control Theory Appl.*, **3**(10):1383–1394, 2009.

7. V. Feliu, K.S. Rattan, H.B. Brown, Control of flexible arms with friction in the joints, *IEEE Trans. Robotic Autom.*, **9**(4):467–475, 1993.

8. S.I. Han, J.M. Lee, Improved prescribed performance constraint control for a strict feedback non-linear dynamic system, *IET Control Theory Appl.*, **7**(14):1818–1827, 2013.

9. S.I. Han, J.M. Lee, Fuzzy echo state neural networks and funnel dynamic surface control for prescribed performance of a nonlinear dynamic system, *IEEE Trans. Ind. Electron.*, **61**(2):1099–1112, 2014.

10. A.K. Kostarigka, G.A. Rovithakis, Adaptive dynamic output feedback neural network control of uncertain MIMO nonlinear systems with prescribed performance, *IEEE Trans. Neural Netw. Learn. Syst.*, **23**(1):138–149, 2012.

11. K. Kristic, I. Kanellakopoulos, P.V. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.

12. Y.M. Li, S.C. Tong, Adaptive fuzzy output-feedback stabilization control for a class of switched non-strict-feedback nonlinear systems, *IEEE Trans. Cybern.*, 2016, https://doi.org/10.1109/TCYB.2016.2536628.

13. Y.M. Li, S.C. Tong, T.S. Li, Adaptive fuzzy output feedback dynamic surface control of interconnected nonlinear pure-feedback systems, *IEEE Trans. Cybern.*, **45**(1):138–149, 2015.

14. Y.M. Li, S.C. Tong, T.S. Li, Composite adaptive fuzzy output feedback control design for uncertain nonlinear strict-feedback systems with input saturation, *IEEE Trans. Cybern.*, **45**(10):2299–2308, 2015.

15. Y.M. Li, S.C. Tong, T.S. Li, Hybrid fuzzy adaptive output feedback control design for uncertain MIMO nonlinear systems with time-varying delays and input saturation, *IEEE Trans. Fuzzy Syst.*, **24**(4):841–853, 2016.

16. Z.J. Li, S.L. Wang, Robust optimal H_∞ control for irregular buildings with AMD via LMI approach, *Nonlinear Anal. Model. Control*, **19**(2):256–271, 2014.

17. Y.J. Liu, S.C. Tong, Adaptive fuzzy control for a class of nonlinear discrete-time systems with backlash, *IEEE Trans. Fuzzy Syst.*, **22**(5):1359–1365, 2014.

18. Y.J. Liu, S.C. Tong, Adaptive fuzzy identification and control for a class of nonlinear pure-feedback MIMO systems with unknown dead-zones, *IEEE Trans. Fuzzy Syst.*, **23**(5):1387–1398, 2015.

19. H.F. Min, N. Dua, Adaptive nn output-feedback control for stochastic time-delay nonlinear systems with unknown control coefficients and perturbations, *Nonlinear Anal. Model. Control*, **21**(4):515–530, 2016.

20. Y.G. Niu, L. James, X.Y. Wang, W.C.H. Daniel, Adaptive H_∞ control using backstepping design and neural networks, *J. Dyn. Syst. Meas. Control*, **127**(3):478–485, 2005.

21. P.A. Phan, T. Gale, Two-mode adaptive fuzzy control with approximation error estimator, *IEEE Trans. Fuzzy Syst.*, **15**(5):943–955, 2007.

22. H.T. Song, T. Zhang, G. L. Zhang, C. Lu, Robust dynamic surface control of nonlinear systems with prescribed performance, *Nonlinear Dyn.*, **76**(1):599–608, 2014.

23. L.Y. Sun, S.C. Tong, Y. Liu, Adaptive backstepping sliding mode H_∞ control of static var compensator, *IEEE Trans. Control Syst. Technol.*, **19**(5):1178–1185, 2011.

24. S.C. Tong, B.Y. Huo, Y.M. Li, Observer-based adaptive decentralized fuzzy fault-tolerant control of nonlinear large-scale systems with actuator failures, *IEEE Trans. Fuzzy Syst.*, **22**(1):1–15, 2014.

25. S.C. Tong, Y. Li, Y.M. Li, Y.J. Liu, Observer-based adaptive fuzzy backstepping control for a class of stochastic nonlinear strict-feedback systems, *IEEE Trans. Syst. Man Cybern., Part B*, **41**(6):1693–1704, 2011.

26. S.C. Tong, Y.M. Li, G. Feng, T.S. Li, Observer-based adaptive fuzzy backstepping dynamic surface control for a class of mimo nonlinear systems, *IEEE Trans. Syst. Man Cybern., Part B*, **41**(4):1124–1135, 2011.

27. S.C. Tong, S. Sui, Y.M. Li, Fuzzy adaptive output feedback control of MIMO nonlinear systems with partial tracking errors constrained, *IEEE Trans. Fuzzy Syst.*, **23**(4):729–742, 2015.

28. H.Q. Wang, X.P. Liu, K.F. Liu, Adaptive fuzzy tracking control for a class of pure-feedback stochastic nonlinear systems with non-lower triangular structure, *Fuzzy Sets Syst.*, **302**:101–120, 2016.

29. H.Q. Wang, X.P. Liu, K.F. Liu, Robust adaptive neural tracking control for a class of stochastic nonlinear interconnection systems, *IEEE Trans. Neural Netw. Learn. Syst.*, **27**(3):510–523, 2016.

30. H.Q. Wang, P. Shi, H. Li, Q. Zhou, Adaptive neural tracking control for a class of nonlinear systems with unmodeled dynamics, *IEEE Trans. Cybern.*, 2016, `https://doi.org/10.1109/TCYB.2016.2607166`.

31. M. Wang, X.P. Liu, P. Shi, Adaptive neural control of pure-feedback nonlinear time-delay systems via dynamic surface technique, *IEEE Trans. Syst. Man Cybern., Part B*, **41**(6):1681–1692, 2011.

32. S. Yin, P. Shi, H.Y. Yang, Adaptive fuzzy control of strict-feedback nonlinear time-delay systems with unmodeled dynamics, *IEEE Trans. Cybern.*, **46**(8):1926–1938, 2015.

33. Q.C. Zhao, Y. Lin, Adaptive dynamic surface control for pure-feedback systems, *Int. J. Robust Nonlinear Control*, **22**(14):1647–1660, 2012.