

Modeling the effect of economic efforts to control population pressure and conserve forestry resources

Kusum Lata^{1,2}, Arvind Kumar Misra

Department of Mathematics, Institute of Science,
Banaras Hindu University,
Varanasi-221005, India
kusum41@gmail.com; akmisra_knp@yahoo.com

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Abstract. The increase in human population poses a great menace for the forestry resources. Therefore, protection and conservation of forestry resources is a challenging subject across the globe. Keeping this point in view, we propose and analyze a nonlinear mathematical model to assess the effect of economic efforts applied to control population pressure as well as to increase forestry resources by plantation on the conservation of forestry resources. The condition when one should spend more money on plantation/reducing population pressure has been obtained. Numerical simulation is also performed to support analytical findings and identify the important parameters useful for the conservation of forestry resources.

Keywords: mathematical model, forestry resources, economic efforts, population pressure, stability.

1 Introduction

Sustainable forest management is crucial for the survival of species on the planet Earth because forests sequester carbon dioxide during photosynthesis and release oxygen and thus maintaining the air quality in the ecosystem. Apart from this, forests provide food, wood from trees, medicinal plants, habitats to diverse animal species, prevent soil erosion, help in maintaining the water cycle, etc. Hence, rural communities depend upon the forest resources for their livelihood [5, 24, 33]. The increase in human population needs various kinds of infrastructure, like railways, roads, housing complexes, agricultural lands, industrialization, etc., and the required land for the development of this infrastructure mainly comes from clearing forests [31]. 11% of the globe's arable land, near about the size of both India and China has been destroyed due to the human related activities. As a consequence, every year farmers afford the extra burden of forage for 77 million people

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²Corresponding author.

with 27 billion fewer tons of topsoil every year [35]. In the past few decades, forests have been considerably decreased and are continuously declining with an alarming rate due to the rapid increase in human population [23, 25]. To maintain forestry resources, Forest and Wildlife Department, Government of Uttar Pradesh, India, have introduced plantation programmes in rainy season with the help of students, farmers, women and localities to increase the green belt, forest area and eco-tourism [32].

Some studies have been conducted using mathematical models to see the effect of population [9–11, 15, 17–19, 26, 28], pollution [8, 10, 22, 26] and industrialization [3, 10, 16, 27, 28] on the forestry resources. In these studies, it is concluded that as these factors increase, the forestry resources decrease. Further, mathematical models have been proposed to see the effect of population and population pressure augmented industrialization on forestry resources [11, 19]. In these studies, the main assumption was that as population increases, the demand of population (population pressure) increases. To fulfil this demand of population, industrialization increases, which ultimately leads to deforestation [11]. In this regard, Misra et al. [19] have suggested that economic efforts (i.e., providing incentives to the people in the form of subsidies on the alternate products of forestry resources, like tin, fibre, fuel efficient stoves, biogas, etc.) are the useful tools to fulfil the demand of population and thus population pressure can be reduced. As population pressure reduces, the industrialization decreases and as a result forestry resources increase. According to Adebayo [2], population pressure and related poverty, both of them are the prime reasons behind the deforestation because, due to these, people and government are bound to mowing the forest resources and convert the forest land into the agricultural land to accomplish the increasing demand of food. Moreover, as time passes, soil loses its fertility, which leads to the declination in agriculture productivity [12]. Therefore, this forces the farmers to clear the other forest land, which are virgin and fertile, and so, for the protection of forestry resources, alternate incentives can be implemented in different ways, like user rights, collaborative or joint management, complete or partial privatization of some parts of the forests and polluter pay principles [21], motivation and empowerment of stakeholders' to invest in other business, which is the alternate of forestry resources [20]. Recently, Misra et al. [18] have presented a mathematical model to study the depletion of forestry resources due to population and population pressure. Further, they have extended their model for the conservation of forestry resources by using economic efforts, which provide subsidies to the peoples on fuel efficient stoves, biogas, tin, fibre, etc., so population switch over to the alternate of forestry resources. Therefore, the application of economic efforts reduces the population pressure and thus found to be helpful in protecting the forestry resources as well as forests. Misra and Lata [17] have also extended this study for the sustainable management of forestry resources by applying technological (i.e., genetically engineered plants) and economic efforts. They have shown that technological efforts directly increase the forestry resources and economic efforts reduce the population pressure, which indirectly helps to conserve forestry resources. Hassan et al. [13] have suggested that increase in the reforestation efforts and optimal exploitation of forests for the needs of society along with the other opportunities like substitute of fuel, are necessary to cope up the problem of deforestation. Moreover, Costa Rica is one of the example where to tackle the problem of deforestation and

biodiversity loss, some forest based policies are proposed by providing financial incentives to generate the income of local communities, good health and habitat for species with the conservation and sustainable use of forest [1]. Further, by clear, selective and shelter wood cutting, starting reforestation and afforestation programmes [24], use of chemical spray and antibodies [34], use of alternative of forest resources [4, 7, 18, 19], planting fast growing trees [30], providing fuel efficient stoves, biogas [6] and spreading the knowledge regarding the importance of forestry resources among the indigenous people [29], all are also important for the protection of forestry resources.

Hence, the situation of deforestation mainly occurs due to the human related activities. Human population use the forestry resources and forest land for various purposes. Here, it is noteworthy fact that if people can use the alternate of forestry resources, then the extra burden upon the forestry resources can be reduced. As a consequence, the forestry resources may increase, and we get a healthy environment. Therefore, to cope up the problem of deforestation, our aim in the present study is to assess the impact of economic efforts on the conservation of forestry resources. The applied economic efforts in the form of money are used to reduce the population pressure, and some part of the money is also used in plantation and to encourage the private landowners to use their land for plantation. Moreover, the population pressure can be reduced by using money to aware the population about ill effects of deforestation, to give up the slash and burn techniques, using the products, which are not made of woods, like plastic doors and furniture and providing the incentives to the population to purchase fuel efficient stoves, biogas, etc.

2 Mathematical model

In this section, we present a nonlinear mathematical model to show how the use of economic efforts are beneficial to reduce the population pressure and increase the forestry resources. Our aim behind this model is to focus on the problem of the depletion and conservation of forestry resources along with fulfilling the need of population. Consider a habitat where forestry resources are continuously depleting due to rapid increase in population as well as its pressure (commonly known as population pressure). The forestry resources and population increase logistically, whereas population pressure increases with the increase in population. It is well known fact that as population size increases, their need for food, products and space increases. Therefore, to reduce this population pressure, new housing complexes, roads and railway lines are constructed, and the agricultural land is made available to population by clearing forests. Hence, to reduce the population pressure and increase the forestry resources, the use of economic efforts may be a fruitful step. To reduce the population pressure and maintaining the forestry resources, the economic efforts in the form of money are employed in two ways. Firstly, in reducing the population pressure by using money to aware the population regarding the ill effects of deforestation, to give up the slash and burn techniques, using the products, which are not made of woods, like plastic doors and furniture and providing the incentives to the population to purchase fuel efficient stoves, biogas, etc., to minimize the extra burden on the forestry resources. Moreover, secondly, by using some money to run the plantation programs and

also encourage the private land owners to use their land for plantation to manage the demand of population, etc., which increases the forestry resources in the region under consideration.

In the modeling process, $B(t)$ and $N(t)$ denote the densities of forestry resources and population, respectively at any time t in the region under consideration. It is assumed that forestry resources follow logistic growth with intrinsic growth rate s and carrying capacity L , whereas population also follow the logistic model with intrinsic growth rate r and carrying capacity K . Consider that $P(t)$ denotes the intensity of population pressure at time t in the region under consideration, which directly increases proportional to human population at a constant rate λ . Moreover, land is cleared from forests for housing complexes, space for cattle population, new road and railway lines, agricultural purposes can never be used for reforestation, and this leads to reduction in the carrying capacity of forestry resources in the region under consideration, which is the main reason behind the depletion of forestry resources. So, it is considered that the reduction in the carrying capacity of forestry resources is proportional to the population pressure at a constant rate λ_2 . Further, let $E(t)$ be the measure of economic efforts at time t in the region under consideration. It is considered that these economic efforts are implemented according to the depleted level of forestry resources at a constant rate ϕ . Further, it is considered that some part of economic efforts get diminish due to its use to reduce population pressure and other to increase forestry resources. Keeping these aspects in view, the system dynamics may be governed by the following nonlinear ordinary differential equations:

$$\begin{aligned}\frac{dB}{dt} &= sB\left(1 - \frac{B}{L}\right) - \alpha_1BN - \lambda_2B^2P + \pi_1\phi_0E, \\ \frac{dN}{dt} &= rN\left(1 - \frac{N}{K}\right) + \pi\alpha_1BN, \\ \frac{dP}{dt} &= \lambda N - \lambda_0P - \pi_2\gamma_1PE, \\ \frac{dE}{dt} &= \phi(L - B) - \phi_0E - \gamma_1PE,\end{aligned}\tag{1}$$

where $B(0) \geq 0$, $N(0) \geq 0$, $P(0) \geq 0$, $E(0) \geq 0$.

Here, we assume that the intrinsic growth rate and carrying of forestry resources get affected due to their excessive use for fuel, food, medicinal plants, wood, fodder, etc. by population at a constant rate α_1 , and it is also considered that population increases proportional to forestry resources with proportionality constant π (assume $0 < \pi < 1$). We assume that some population pressure will naturally diminished with time at a constant rate λ_0 , and some population pressure also reduces due to the use of economic efforts with proportionality constant π_2 (assume $0 < \pi_2 < 1$). Moreover, the constant γ_1 represents the depletion rate coefficient of economic efforts due to population pressure. Further, the constant ϕ_0 represents the depletion rate coefficient in the economic efforts due to the use of economic efforts to increase the forestry resources, while π_1 (assume $0 < \pi_1 < 1$) is the proportionality constant, which represents the increase in forestry resources due to use of economic efforts. All the above constants are assumed to be positive.

3 Boundedness of the system

Boundedness of the model system ensures its validity. Hence, to establish the uniform boundedness of the model system (1), we have the following theorem.

Theorem 1. *All the solutions of model system (1), which initiate in \mathbb{R}_+^4 , are uniformly bounded. That is, there exists a constant $M > 0$ such that $B(t) \leq M$, $N(t) \leq M$, $P(t) \leq M$ and $E(t) \leq M$ for each solution $(B(t), N(t), P(t), E(t))$ of model system (1) with all t large enough.*

For proof of this theorem see Appendix A.

4 Equilibrium analysis

The above model system (1) has two non-negative equilibria, which are listed as follows:

- (i) $F_0(L, 0, 0, 0)$ exists without any condition.
- (ii) $F^*(B^*, N^*, P^*, E^*)$ exists without any condition.

The existence of equilibria F_0 is obvious hence omitted. Now in the following, we show the existence of equilibrium F^* . In equilibrium $F^*(B^*, N^*, P^*, E^*)$, B^* , N^* , P^* and E^* may be obtained by solving following algebraic equations:

$$sB \left(1 - \frac{B}{L}\right) - \alpha_1 BN - \lambda_2 B^2 P + \pi_1 \phi_0 E = 0, \tag{2}$$

$$r \left(1 - \frac{N}{K}\right) + \pi \alpha_1 B = 0, \tag{3}$$

$$\lambda N - \lambda_0 P - \pi_2 \gamma_1 P E = 0, \tag{4}$$

$$\phi(L - B) - \phi_0 E - \gamma_1 P E = 0. \tag{5}$$

From equation (3) we get

$$N = \frac{K}{r} [r + \pi \alpha_1 B]. \tag{6}$$

Putting this value in equation (4), we have

$$P = \frac{\lambda K}{r} \frac{(r + \pi \alpha_1 B)}{(\lambda_0 + \pi_2 \gamma_1 E)}. \tag{7}$$

Now using the above value of P in equation (5) and solving for E , we get

$$E = \frac{1}{2\pi_2 \phi_0 \gamma_1} \left[-Z_1 + \sqrt{Z_1^2 + 4\pi_2 \phi \phi_0 \lambda_0 \gamma_1 (L - B)} \right] = h(B) \quad (\text{say}), \tag{8}$$

where $Z_1 = \{\phi_0 \lambda_0 + (\lambda \gamma_1 K / r)(r + \pi \alpha_1 B) - \pi_2 \phi \gamma_1 (L - B)\}$

Using the values of N , P and E in equation (2), we get the following equation in B :

$$F(B) = sB \left(1 - \frac{B}{L}\right) - \frac{\alpha_1 K}{r} (r + \pi\alpha_1 B) B - \frac{\lambda_2 \lambda K (r + \pi\alpha_1 B)}{r(\lambda_0 + \pi_2 \gamma_1 h(B))} B^2 + \pi_1 \phi_0 h(B) = 0. \quad (9)$$

From equation (9) the following facts may be easily noted:

- (i) $F(0) > 0$,
- (ii) $F(L) < 0$.

The above points (i) and (ii) implies that equation (9) has at least one positive root in open interval $(0, L)$. In addition to this, for the existence of exactly one positive root $B = B^*$ (say) in $(0, L)$, we can easily obtain that

- (iii) $F'(B) < 0$.

Hence, after knowing the value of $B = B^*$, we get the positive values of $N = N^*$, $P = P^*$, and $E = E^*$ from equations (6), (7), and (8), respectively.

Remark 1. Here, it is interesting to note that $dB^*/d\phi_0 > 0$ if $B^* < \sqrt{\pi_1 \lambda_0 / (\pi_2 \lambda_2)}$ and $dB^*/d\gamma_1 > 0$ if $B^* > \sqrt{\pi_1 \lambda_0 / (\pi_2 \lambda_2)}$. From the above it is clear that if the use of forestry resources as well as the clearance of forest land is small due to increase in the demand of population (i.e., λ_2), then the use of economic efforts to increase the forestry resources through plantation is beneficial. Moreover, if the use of forestry resources and the clearance of forest land, due to the population pressures, is large, then the use of economic efforts to reduce the population pressure is beneficial for the conservation of forestry resources. Therefore, the above inequalities clearly indicate that to increase the forestry resources when one should spend more money on the plantation and when the increment should be made in reducing the population pressure.

5 Stability analysis

5.1 Local stability analysis

Now, we analyze the local stability of equilibria F_0 and F^* . The local stability of each equilibrium is determined by the nature of eigenvalues of the Jacobian matrix corresponding to each equilibrium point. The Jacobian matrix J for the model system (1) is given as follows:

$$J = \begin{pmatrix} a_{11} & -\alpha_1 B & -\lambda_2 B^2 & \pi_1 \phi_0 \\ \pi\alpha_1 N & a_{22} & 0 & 0 \\ 0 & \lambda & -(\lambda_0 + \pi_2 \gamma_1 E) & -\pi_2 \gamma_1 P \\ -\phi & 0 & -\gamma_1 E & -(\phi_0 + \gamma_1 P) \end{pmatrix},$$

where

$$a_{11} = \left[s \left(1 - \frac{2B}{L}\right) - \alpha_1 N - 2\lambda_2 B P \right], \quad a_{22} = \left[r \left(1 - \frac{2N}{K}\right) + \pi\alpha_1 B \right].$$

Here, it is easy to note that one eigenvalue of the above Jacobian matrix J (evaluated at F_0) is $(r + \pi\alpha_1L)$, which is clearly positive, so equilibrium F_0 is always unstable in N -direction.

Further, it may be noted that to determine the nature of the eigenvalues of the Jacobian matrix J (evaluated at F^*) is difficult task. Therefore, we have used the Lyapunov's stability theory to check the local stability behavior of equilibrium F^* . The following theorem gives sufficient condition for local stability of equilibrium F^* .

Theorem 2. *The interior equilibrium F^* is locally asymptotically stable, provided the following inequality holds:*

$$\max\left\{\frac{\lambda_2^2 B^{*2}}{\frac{s}{L} + \lambda_2 P^* + \frac{\pi_1 \phi_0 E^*}{B^{*2}}}, \frac{2\pi_1 \phi_0 \gamma_1^2 E^{*2}}{\phi B^* (\phi_0 + \gamma_1 P^*)}\right\} < (\lambda_0 + \pi_2 \gamma_1 E^*)^2 \min\left\{\frac{r}{\pi K \lambda^2}, A_1\right\}, \tag{10}$$

where A_1 is defined in the proof.

5.2 Global stability analysis

Now, in the following, we prove the global stability behavior of interior equilibrium F^* . For this, we need the following lemma, which is stated without proof [14].

Lemma 1. *The set*

$$\Omega = \left\{ (B, N, P, E) : 0 \leq B \leq L, 0 \leq N \leq N_m, 0 \leq P \leq P_m, 0 \leq E \leq \frac{\phi L}{\phi_0} \right\},$$

where $N_m = (K/r)(r + \pi\alpha_1L)$ and $P_m = (\lambda/\lambda_0)N_m$, is the region of attraction for model (1) and attracts all solutions initiating in the interior of the positive orthant.

Theorem 3. *The interior equilibrium F^* is globally asymptotically stable inside the region of attraction Ω , provided the following inequality holds:*

$$\max\left\{\frac{\lambda_2^2 L^2}{\frac{s}{L} + \lambda_2 P^*}, \frac{2\pi_1 \gamma_1^2 E^{*2}}{\phi B^*}\right\} < \lambda_0^2 \min\left\{\frac{r}{\pi \lambda^2 K}, \frac{\pi_1 \phi_0^2}{2\phi \pi_2^2 \gamma_1^2 B^* P^{*2}}\right\}. \tag{11}$$

For proof of Theorems 2 and 3, see Appendices B and C, respectively.

Remark 2. Here, it may be noted that stability conditions (10) and (11) will be easily satisfied for small values of λ_2 and γ_1 . This implies that depletion in carrying capacity of forestry resources and economic efforts due to population pressure, respectively, have destabilizing effect on the system.

6 Numerical simulation

In this section, we illustrate the feasibility of our analytical findings such as local as well as global stability of interior equilibrium F^* by choosing a set of parameter values, which is given as follows:

$$\begin{aligned}
 s &= 0.8, & L &= 50, & \pi &= 0.004, & \pi_1 &= 0.03, & \pi_2 &= 0.09, \\
 r &= 0.5, & K &= 100, & \lambda &= 0.007, & \lambda_0 &= 0.4, & \lambda_2 &= 0.0007, \\
 \phi &= 0.006, & \phi_0 &= 0.4, & \alpha_1 &= 0.0031, & \gamma_1 &= 0.0002.
 \end{aligned}
 \tag{12}$$

For the above set of parameter values, it may be checked that the condition for global stability is satisfied. The equilibrium values corresponding to F^* are obtained as

$$B^* = 28.4408, \quad N^* = 100.0705, \quad P^* = 1.7512, \quad E^* = 0.3231.$$

Further, the eigenvalues of Jacobian matrix corresponding to equilibrium F^* of the model system (1) are obtained as

$$\begin{aligned}
 &-0.4945 + 0.00522i, & &-0.4945 - 0.00522i, \\
 &-0.4009 + 0.0008i, & &-0.4009 - 0.0008i.
 \end{aligned}$$

Here, it may be noted that all eigenvalues of Jacobian matrix J_{F^*} are with negative real part. Hence, the interior equilibrium F^* is locally asymptotically stable for the set of parameter values given in (12). Moreover, to show the global stability behavior of equilibrium F^* , we have plotted the solution trajectories with different initial starts in BPE - and BNE -space in Fig. 1. From this figure it is manifested that all solution trajectories initiating inside the region of attraction are approaching towards the equilibrium values.

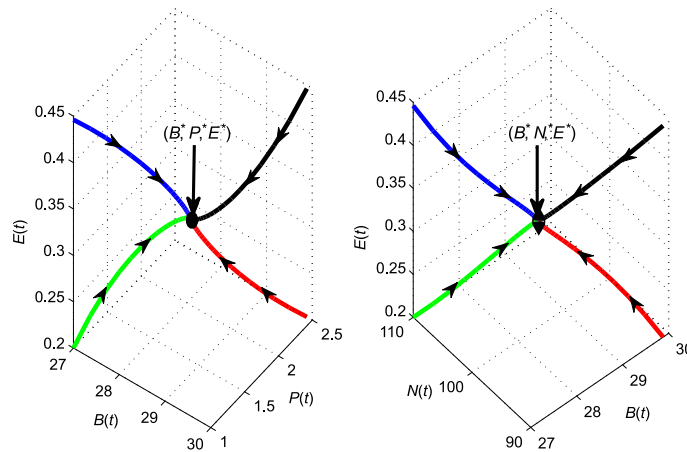


Figure 1. Global stability of interior equilibrium F^* in BPE - and BNE -spaces. All parameters are same as given in (12).

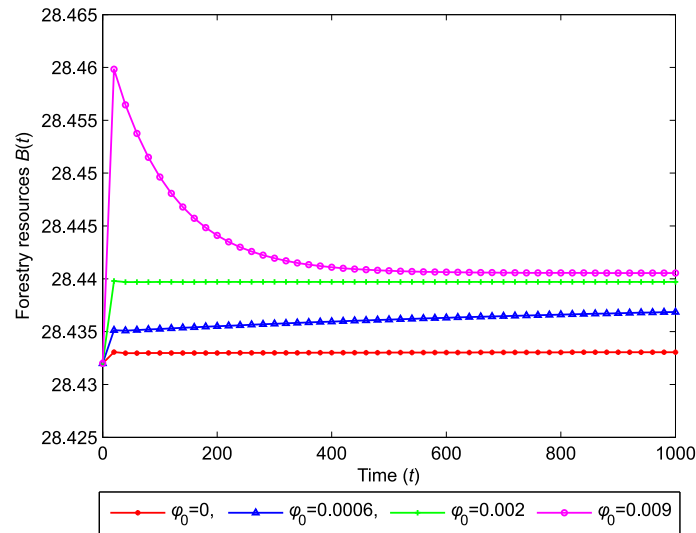


Figure 2. Variation of forestry resources and population pressure with time for different values of ϕ_0 by taking $\pi_2 = 0.001$. The remaining parameters are same as given in (12).

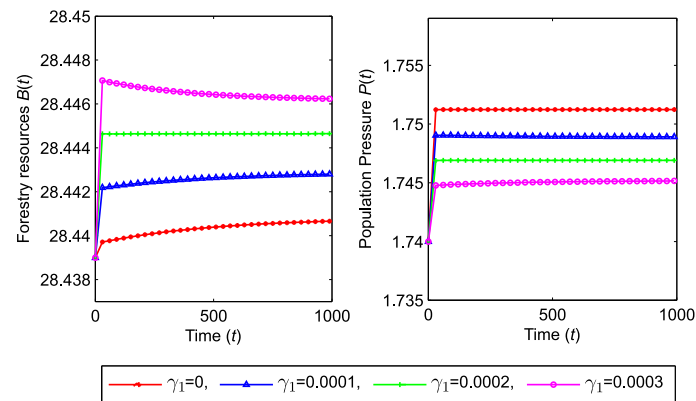


Figure 3. Variation of forestry resources and population pressure with time for different values of γ_1 by taking $\pi_2 = 0.09$, $\phi_0 = 0.002$. The remaining parameters are same as given in (12).

Further, to see the effect of various parameters, like ϕ_0 , γ_1 , ϕ and λ , on the system dynamics, we have plotted Figs. 2, 3, 4 and 5, respectively. The variation of forestry resources B with respect to time t for different values of ϕ_0 is given in Fig. 2 by taking the value of $\pi_2 = 0.001$, and remaining parameters are same as given in (12). From this figure it is apparent that on increasing the value of ϕ_0 , the forestry resources increase. Further, from Fig. 3 we have shown the variation of forestry resources and population pressure with respect to time t for the different values of γ_1 and also taking $\pi_2 = 0.09$, $\phi_0 = 0.002$, and remaining parameters are same as given in (12). From this figure it

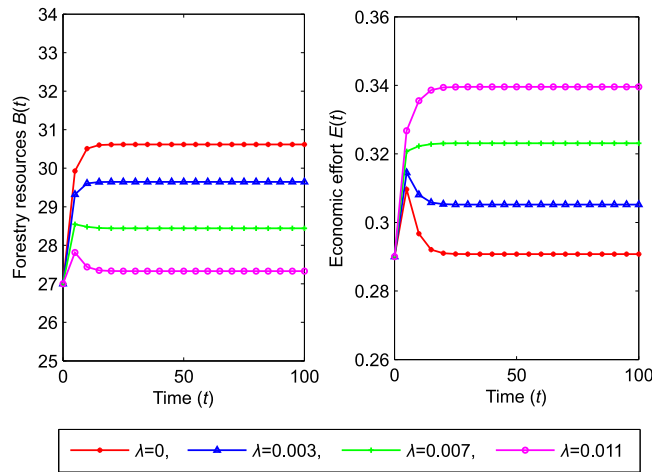


Figure 4. Variation of forestry resources and economic efforts with time for different values of λ . The remaining parameters are same as given in (12).

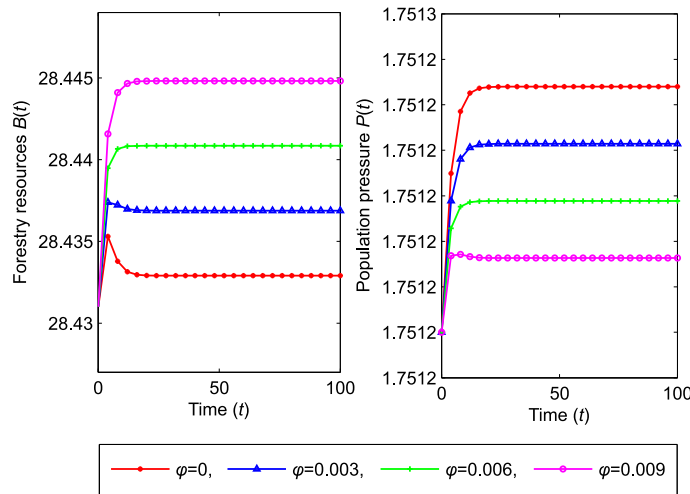


Figure 5. Variation of forestry resources and population pressure with time for different values of ϕ . The remaining parameters are same as given in (12).

is evident that as γ_1 increases, the forestry resources increase, and population pressure decreases. It is because as economic efforts are used to reduce the population pressure, the population pressure will reduce. Therefore, dependency of population on forestry resources also reduces, so the forestry resources indirectly increase. Also, from Fig. 4 it is clear that if we increase the value of λ , the forestry resources decrease, and economic efforts increase. In the last figure (Fig. 5), it is apparent that if we implement the economic efforts at sufficiently large scale, then we may control the problem of the depletion of

forestry resources along with fulfilling the demand of population. Hence, from above discussion it may be inferred that the population pressure is the major driver behind the depletion of forestry resources. Therefore, it is necessary to take steps to control the population pressure along with protection of forestry resources by using economic efforts.

7 Conclusion

Forests provide habitat to millions of species, and some of them are wholly/partially depend on forestry resources for their survival. But at present time, the depletion of forestry resources as well as forest land, due to rapid increase in population along its pressure, have become a hazardous problem in front of the several developing countries across the world. Due to population pressure, people are compelled to clear the forests and moving the forest land at large scale for agriculture and infrastructure. Therefore, in this paper, we have studied the influence of economic efforts on the control of population pressure and conservation of forestry resources. In the modelling process, it is considered that economic efforts are implemented in accordance to the depleted level of forestry resources. These economic efforts are applied in two ways, first for reducing population pressure and second for increasing the forestry resources. Basically, we have considered that economic efforts in the form of money are spent to aware the population about ill effects of deforestation to give up the slash and burn techniques using the products, which are not made of woods, like plastic doors and furniture and provide incentives to the population to purchase fuel efficient stoves, biogas, etc. Moreover, on the other hand, some money is used to run the plantation programs and encourage the private land owners to use their land for plantation. As a result, population pressure will be reduced, and forestry resources increase. The proposed model is analyzed using stability theory of differential equations. By setting the growth rate of all dynamical variables equal to zero, it is found that the proposed model exhibits two non-negative equilibria. The interior equilibrium is locally as well as globally asymptotically stable under certain condition. It is found that as population pressure increases, the forestry resources decrease sufficiently at a large scale in absence of economic efforts. Therefore, to control the population pressure, along with the protection of forestry resources, economic efforts (like money) are necessary. The condition regarding the expenditure of money for the conservation of forestry resources is established, which clearly states that when one should spent more money on plantation to increase the forestry resources and when the money should be spent to reduce the population pressure. The model analysis suggests that if economic efforts are applied with enough potential to reduce the population pressure along with conservation of forestry resources, the population pressure can be controlled significantly and forestry resources can be conserved. Moreover, by using suitable economic efforts, we can also achieve the sustainable forest management. Hence, it is recommended to take steps in applying economic efforts to save our one of the important natural resources (forestry resources) as well as health of our environment. As a result, we may be able to protect the forestry resources and indirectly save the life of various species, which are completely/partially dependent on it.

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Appendix A: Proof of Theorem 1

Consider $(B(t), N(t), P(t), E(t))$ be any solution of model system (1) with positive initial conditions and

$$U = B + N + P + E.$$

Therefore,

$$\begin{aligned} \frac{dU}{dt} &= \frac{dB}{dt} + \frac{dN}{dt} + \frac{dP}{dt} + \frac{dE}{dt} \\ &= (s - \phi)B - \frac{sB^2}{L} - \alpha_1(1 - \pi)BN - \lambda_2 B^2 P - \phi_0(1 - \pi_1)E \\ &\quad + (r + \lambda)N - \frac{rN^2}{K} - \lambda_0 P - \gamma_1(1 + \pi_2)PE + \phi L. \end{aligned}$$

Therefore,

$$\frac{dU}{dt} \leq (s - \phi)B - \frac{sB^2}{L} + (r + \lambda)N - \frac{rN^2}{K} - \lambda_0 P - \phi_0(1 - \pi_1)E + \phi L,$$

i.e.,

$$\frac{dU}{dt} \leq -\theta(B + N + P + E) + 2(s - \phi)B - \frac{s}{L}B^2 + 2(r + \lambda)N - \frac{r}{K}N^2 + \phi L,$$

where $\theta = \min\{(s - \phi), (r + \lambda), \lambda_0, \phi_0(1 - \pi_1)\} > 0$, provided $s > \phi$. Therefore,

$$\frac{dU}{dt} + \theta U \leq M_0,$$

where $M_0 = (L/s)(s - \phi)^2 + (K/r)(r + \lambda)^2 + \phi L > 0$.

Applying the theory of differential inequality, we obtain

$$U(t) \leq \frac{M_0}{\theta}(1 - e^{-\theta t}) + U(0)e^{-\theta t}.$$

For $t \rightarrow \infty$, we get $U(t) \leq M_0/\theta$.

So $U(t)$ is ultimately bounded. Therefore, there exists a positive constant $M = M_0/\theta$ such that each solution of model system (1), $B(t) \leq M$, $N(t) \leq M$, $P(t) \leq M$ and $E(t) \leq M$ when t is sufficiently large.

Appendix B: Proof of Theorem 2

To prove this theorem, we use the Lyapunov's stability theorem. Now linearizing the model system (1) about F^* by using the following transformations:

$$B = B^* + x_1, \quad N = N^* + x_2, \quad P = P^* + x_3, \quad E = E^* + x_4,$$

where x_1, x_2, x_3 and x_4 are small perturbations around the equilibrium F^* .

Further, we construct the following Lyapunov function, which is zero at equilibrium F^* and also positive definite:

$$V = \int_0^{x_1} \frac{a_1}{B^*} da_1 + k_1 \int_0^{x_2} \frac{a_2}{N^*} da_2 + k_2 \int_0^{x_3} a_3 da_3 + k_3 \int_0^{x_4} a_4 da_4$$

(k_1, k_2 and k_3 are some positive constants to be chosen appropriately).

Differentiating above equation with respect to t along the solutions of linearized system of (1), we get

$$\begin{aligned} \frac{dV}{dt} = & - \left(\frac{s}{L} + \lambda_2 P^* + \frac{\pi_1 \phi_0 E^*}{B^{*2}} \right) x_1^2 - k_1 \frac{r}{K} x_2^2 - k_2 (\lambda_0 + \pi_2 \gamma_1 E^*) x_3^2 \\ & - k_3 (\phi_0 + \gamma_1 P^*) x_4^2 - \alpha_1 (1 - k_1 \pi) x_1 x_2 - \lambda_2 B^* x_1 x_3 - \left(k_3 \phi - \frac{\pi_1 \phi_0}{B^*} \right) x_1 x_4 \\ & + k_2 \lambda x_2 x_3 - k_2 \pi_2 \gamma_1 P^* x_3 x_4 - k_3 \gamma_1 E^* x_3 x_4. \end{aligned}$$

Choosing $k_1 = 1/\pi$ and $k_3 = \pi_1 \phi_0 / \phi B^*$, dV/dt simplified as

$$\begin{aligned} \frac{dV}{dt} = & - \left(\frac{s}{L} + \lambda_2 P^* + \frac{\pi_1 \phi_0 E^*}{B^{*2}} \right) x_1^2 - \frac{r}{\pi K} x_2^2 - k_2 (\lambda_0 + \pi_2 \gamma_1^*) x_3^2 \\ & - \frac{\pi_1 \phi_0}{\phi B^*} (\phi_0 + \gamma_1 P^*) x_4^2 - \lambda_2 B^* x_1 x_3 + k_2 \lambda x_2 x_3 - k_2 \pi_2 \gamma_1 P^* x_3 x_4 \\ & - \frac{\pi_1 \phi_0}{\phi B^*} \gamma_1 E^* x_3 x_4. \end{aligned}$$

Now dV/dt will be negative definite, provided the following inequalities are satisfied:

$$\lambda_2^2 B^{*2} < k_2 (\lambda_0 + \pi_2 \gamma_1 E^*) \left(\frac{s}{L} + \lambda_2 P^* + \frac{\pi_1 \phi_0 E^*}{B^{*2}} \right), \tag{B.1}$$

$$k_2 \lambda^2 < \frac{r}{\pi K} (\lambda_0 + \pi_2 \gamma_1 E^*), \tag{B.2}$$

$$k_2 \pi_2^2 \gamma_1^2 P^{*2} < \frac{\pi_1 \phi_0}{2 \phi B^*} (\lambda_0 + \pi_2 \gamma_1 E^*) (\phi_0 + \gamma_1 P^*), \tag{B.3}$$

$$\frac{\pi_1 \phi_0 \gamma_1^2 E^{*2}}{\phi B^*} < \frac{k_2}{2} (\lambda_0 + \pi_2 \gamma_1 E^*) (\phi_0 + \gamma_1 P^*). \tag{B.4}$$

From inequalities (B.1)–(B.4) we may easily choose the positive value of k_2 if

$$\begin{aligned} & \max \left\{ \frac{\lambda_2^2 B^{*2}}{\left(\frac{s}{L} + \lambda_2 P^* + \frac{\pi_1 \phi_0 E^*}{B^{*2}} \right)}, \frac{2 \pi_1 \phi_0 \gamma_1^2 E^{*2}}{\phi B^* (\phi_0 + \gamma_1 P^*)} \right\} \\ & < (\lambda_0 + \pi_2 \gamma_1 E^*)^2 \min \left\{ \frac{r}{\pi K \lambda^2}, A_1 \right\}, \end{aligned} \tag{B.5}$$

where $A_1 = \pi_1 \phi_0 (\phi_0 + \gamma_1 P^*) / (2 \phi \pi_2^2 \gamma_1^2 B^* P^{*2})$. From inequality (B.5) we assert that dV/dt is negative definite under condition (10), which proves the theorem.

Appendix C: Proof of Theorem 3

To prove the global stability of equilibrium F^* , let us consider the following Lyapunov function corresponding to the model system (1) about equilibrium $F^*(B^*, N^*, P^*, E^*)$:

$$W(B, N, P, E) = \int_{B^*}^B \frac{b_1 - B^*}{b_1} db_1 + m_1 \int_{N^*}^N \frac{b_2 - N^*}{b_2} db_2 + m_2 \int_{P^*}^P (b_3 - P^*) db_3 \\ + m_3 \int_{E^*}^E (b_4 - E^*) db_4,$$

where m_1 , m_2 and m_3 are positive constants to be chosen appropriately. It can be easily checked that the Lyapunov function W is zero at the equilibrium $F^*(B^*, N^*, P^*, E^*)$ and is positive for all other positive values of B , N , P and E .

Now differentiating W with respect to t along the trajectories of model system (1) and choosing $m_1 = 1/\pi$ and $m_3 = \pi_1\phi_0/\phi B^*$, after a simple manipulation, we get

$$\frac{dW}{dt} = -\frac{\pi_1\phi_0}{BB^*}E(B - B^*)^2 - \left(\frac{s}{L} + \lambda_2 P^*\right)(B - B^*)^2 - \frac{r}{\pi K}(N - N^*)^2 \\ - m_2(\lambda_0 + \pi_2\gamma_1 E)(P - P^*)^2 - \frac{\pi_1\phi_0}{\phi B^*}(\phi_0 + \gamma_1 P)(E - E^*)^2 \\ - \lambda_2 B(B - B^*)(P - P^*) + m_2\lambda(P - P^*)(N - N^*) \\ - m_2\pi_2\gamma_1 P^*(P - P^*)(E - E^*) - \frac{\pi_1\phi_0}{\phi B^*}\gamma_1 E^*(P - P^*)(E - E^*).$$

Now dW/dt will be negative definite inside the region of attraction Ω , provided the following inequalities are satisfied:

$$\lambda_2^2 L^2 < m_2 \left(\frac{s}{L} + \lambda_2 P^*\right)(\lambda_0 + \pi_2\gamma_1 E), \quad (C.1)$$

$$m_2\lambda^2 < \frac{r}{\pi K}(\lambda_0 + \pi_2\gamma_1 E), \quad (C.2)$$

$$\frac{\pi_1\phi_0}{\phi B^*}\gamma_1^2 E^{*2} < \frac{1}{2}m_2(\lambda_0 + \pi_2\gamma_1 E)(\phi_0 + \gamma_1 P), \quad (C.3)$$

$$m_2\pi_2^2\gamma_1^2 P^{*2} < \frac{1}{2}\frac{\pi_1\phi_0}{\phi B^*}(\lambda_0 + \pi_2\gamma_1 E)(\phi_0 + \gamma_1 P). \quad (C.4)$$

From inequalities (C.1)–(C.4) we may choose positive m_2 if the following inequality holds:

$$\max\left\{\frac{\lambda_2^2 L^2}{\frac{s}{L} + \lambda_2 P^*}, \frac{2\pi_1\gamma_1^2 E^{*2}}{\phi B^*}\right\} < \lambda_0^2 \min\left\{\frac{r}{\pi\lambda^2 K}, \frac{\pi_1\phi_0^2}{2\phi\pi_2^2\gamma_1^2 B^* P^{*2}}\right\}. \quad (C.5)$$

Now, from inequality (C.5) we assert that dW/dt is negative definite under condition (11). Hence the proof.

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