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Impulsive control of nonlinear systems with impulse time window and bounded gain error*

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Abstract. In this paper, we establish a new sufficient condition for the stability of impulsive systems with impulse time window and bounded gain error. The proposed result is more general and more applicable than some existing results. Finally, a numerical example is given to show the effectiveness of our result.

Keywords: impulse time window, bounded gain error, impulsive control.

1 Introduction

In this paper, we mainly adopt the notation and terminology in [8]. Within the last three decades, impulsive control theory had been intensively studied because impulsive control can be applied in many fields, such as chaotic systems [9, 15, 24, 30, 34], HIV prevention modles [5], complex dynamical systems [2–4, 11, 13, 14, 16, 18, 22, 23, 25, 31, 35–38]. The nonlinear impulsive control systems with impulses at fixed times is given by

$$\begin{aligned} &\dot{\boldsymbol{x}} = A\boldsymbol{x} + \phi(\boldsymbol{x}), \quad t \neq \tau_k, \\ &\Delta\boldsymbol{x} = U(k, \boldsymbol{x}), \quad t = \tau_k,\ k = 1, 2, \dots, \\ &\boldsymbol{x}(t_0) = \boldsymbol{x}_0. \end{aligned} \tag{1}$$

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where $\boldsymbol{x} \in \mathbb{R}^n$ is the state variable, $t_0 < \tau_1 < \tau_2 < \cdots, \lim_{k\to\infty} \tau_k = \infty$ denote the moments when impulsive control occurs, A is an $n \times n$ constant matrix, $\phi : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous nonlinear function satisfying $\phi(t, 0) = 0$ and $\|\phi(\boldsymbol{x})\|^2 \leqslant \boldsymbol{x}^{\mathrm{T}} L \boldsymbol{x}$, where L is a diagonal matrix, and $U(k, \boldsymbol{x})$ is the impulsive control law. Many researchers have studied impulsive control system (1) [1, 17, 32, 33]. But we cannot guarantee the impulses without any error due to the limit of equipment and technology.

Recently, Feng, Li, and Huang [8] discussed the following nonlinear impulsive control systems with impulse time window:

$$\begin{aligned} &\dot{\boldsymbol{x}}(t) = A\boldsymbol{x} + \phi(\boldsymbol{x}), \quad mT \leqslant t < mT + \tau_m, \\ &\boldsymbol{x}(t) = J_m \boldsymbol{x}(t^-), \quad t = mT + \tau_m, \\ &\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + \phi(\boldsymbol{x}), \quad mT + \tau_m < t < (m+1)T, \end{aligned} \tag{2}$$

where $T > 0$ denotes the control period, τ_m is unknown within impulse time window $(mT,\ (m+1)T)$, and $J_m \in \mathbb{R}^{n\times n}$ is impulsive control gain. The impulsive effects can be stochastically occurred in a impulse time window in system (2), which is more general than ones impulses occurred at fixed times. Some results related to impulse time window can be found in [6, 7, 12, 26–29, 39].

In many practical applications, the impulsive control gain J_m may also contain errors, so we should take into account the influence of impulsive control gain errors on the systems. In this paper, we consider a class of impulsive control systems with impulse time window and bounded gain error as follows:

$$\begin{aligned} &\dot{\boldsymbol{x}}(t) = A\boldsymbol{x} + \phi(\boldsymbol{x}), \quad mT \leqslant t < mT + \tau_m, \\ &\boldsymbol{x}(t) = (J_m + \varDelta J_m)\boldsymbol{x}(t^-), \quad t = mT + \tau_m, \\ &\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + \phi(\boldsymbol{x}), \quad mT + \tau_m < t < (m+1)T, \end{aligned} \tag{3}$$

where $\varDelta J_m$ is gain error, which is often time-varying and bounded. As pointed out in [17], we can assume that $\varDelta J_m = mF(t)J_m$, $m > 0$, and $F^{\mathrm{T}}(t)F(t) \leqslant I$.

The purpose of this paper is to find some conditions on control gain J_m and impulse time window T such that the origin of impulsive control system (3) is asymptotically stable. We establish a new sufficient condition for the stability of system (3). Compared with the results shown in [8, 17, 19], our result is more general and more applicable. Finally, a numerical example is given to show the effectiveness of our result.

2 Main results

In this section, we will give the main results. To do this, we need the following lemmas.

Lemma 1. (*See* [21].) *Let* $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$ *and* $\varepsilon > 0$. *Then*

$$2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} \leqslant \varepsilon \boldsymbol{x}^{\mathrm{T}}\boldsymbol{x} + \frac{1}{\varepsilon}\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y}.$$

Lemma 2. *Let $A, P, B \in \mathbb{R}^{n\times n}$ such that P is a symmetric and positive definite matrix and $\mu > 0$. Then*

$$A^{\mathrm{T}}PB + B^{\mathrm{T}}PA \leqslant \mu A^{\mathrm{T}}PA + \frac{1}{\mu}B^{\mathrm{T}}PB.$$

Proof. Note that for any $X \in \mathbb{R}^{n\times n}$, the matrix X^TX is positive semidefinite. It follows that

$$\left(\sqrt{\mu}P^{1/2}A - \frac{1}{\sqrt{\mu}}P^{1/2}B\right)^{\mathrm{T}}\left(\sqrt{\mu}P^{1/2}A - \frac{1}{\sqrt{\mu}}P^{1/2}B\right) \geqslant 0.$$

Small calculations show that the result holds. This completes the proof. □

Lemma 3. (*See* [10].) *Let H be a real symmetrical matrix, and $\lambda_{\max}(H) \geqslant \lambda_{\min}(H)$ be the largest and the smallest eigenvalues of H, respectively. Then for any $\boldsymbol{x} \in \mathbb{R}^n$, we have*

$$\lambda_{\min}(H)\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x} \leqslant \boldsymbol{x}^{\mathrm{T}}H\boldsymbol{x} \leqslant \lambda_{\max}(H)\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}.$$

Theorem 1. *Let $P \in \mathbb{R}^{n\times n}$ be a symmetric and positive definite matrix. If there exit $g, \mu, \varepsilon > 0$ such that the following hold*:

(i) $PA + A^{\mathrm{T}}P + \varepsilon P^2 + \varepsilon^{-1}L - gP \leqslant 0$,
(ii) $gT + \ln\lambda < 0$,

where $\lambda = \max\{\lambda_m(1+\mu+m^2(1+1/\mu))\}$, $\lambda_m = \lambda_{\max}(P) \times \lambda_{\max}(P^{-1}J_m^{\mathrm{T}}J_m)$, then the origin of system (3) *is asymptotically stable.*

Proof. Let us construct the following Lyapunov function:

$$V\bigl(\boldsymbol{x}(t)\bigr) = \boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t).$$

If $mT \leqslant t < mT + \tau_m$, then by Lemma 1 and condition (i) we have

$$\begin{aligned}
D^+\bigl(V\bigl(\boldsymbol{x}(t)\bigr)\bigr) &= 2\boldsymbol{x}^{\mathrm{T}}(t)P\bigl(A\boldsymbol{x}(t) + \phi\bigl(\boldsymbol{x}(t)\bigr)\bigr) = 2\boldsymbol{x}^{\mathrm{T}}(t)PA\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)P\phi(\boldsymbol{x}) \\
&= \boldsymbol{x}^{\mathrm{T}}(t)\bigl(PA + A^TP\bigr)\boldsymbol{x}(t) + 2\boldsymbol{x}^TP\phi(\boldsymbol{x}) \\
&\leqslant \boldsymbol{x}^{\mathrm{T}}(t)\bigl(PA + A^TP\bigr)\boldsymbol{x}(t) + \varepsilon\boldsymbol{x}^TP^2\boldsymbol{x} + \frac{1}{\varepsilon}\phi\bigl(\boldsymbol{x}(t)\bigr)^{\mathrm{T}}\phi\bigl(\boldsymbol{x}(t)\bigr) \\
&\leqslant \boldsymbol{x}^{\mathrm{T}}(t)\bigl(PA + A^TP\bigr)\boldsymbol{x}(t) + \varepsilon\boldsymbol{x}^TP^2\boldsymbol{x}(t) + \frac{1}{\varepsilon}\boldsymbol{x}^{\mathrm{T}}(t)L\boldsymbol{x}(t) \\
&= \boldsymbol{x}^{\mathrm{T}}(t)\left(PA + A^TP + \varepsilon P^2 + \frac{1}{\varepsilon}L - gP\right)\boldsymbol{x}(t) + g\boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t) \\
&\leqslant gV\bigl(\boldsymbol{x}(t)\bigr),
\end{aligned}$$

which implies that

$$V\bigl(\boldsymbol{x}(t)\bigr) \leqslant V\bigl(\boldsymbol{x}(mT)\bigr)\mathrm{e}^{g(t-mT)}. \tag{4}$$

Similarly, if $mT + \tau_m < t < (m+1)T$, we also have

$$D^+\bigl(V\bigl(\boldsymbol{x}(t)\bigr)\bigr) \leqslant gV\bigl(\boldsymbol{x}(t)\bigr),$$

which implies that

$$V\big(\boldsymbol{x}(t)\big) \leqslant V\big(\boldsymbol{x}(mT+\tau_m)\big)\mathrm{e}^{g(t-mT-\tau_m)}. \tag{5}$$

If $t = mT + \tau_m$, then by Lemmas 2 and 3 we have

$$\begin{aligned}
V\big(\boldsymbol{x}(t)\big) &= \big((J_m + \Delta J_m)\boldsymbol{x}(t^-)\big)^{\mathrm{T}} P (J_m + \Delta J_m)\boldsymbol{x}(t^-) \\
&= \boldsymbol{x}(t^-)^{\mathrm{T}}\big(J_m^{\mathrm{T}} P + \Delta J_m^{\mathrm{T}} P\big)(J_m + \Delta J_m)\boldsymbol{x}(t^-) \\
&= \boldsymbol{x}(t^-)^{\mathrm{T}}\big(J_m^{\mathrm{T}} P J_m + J_m^{\mathrm{T}} P \Delta J_m + \Delta J_m^{\mathrm{T}} P J_m + \Delta J_m^{\mathrm{T}} P \Delta J_m\big)\boldsymbol{x}(t^-) \\
&\leqslant \boldsymbol{x}(t^-)^{\mathrm{T}}\left((1+\mu)J_m^{\mathrm{T}} P J_m + \left(1+\frac{1}{\mu}\right)\Delta J_m^{\mathrm{T}} P \Delta J_m\right)\boldsymbol{x}(t^-) \\
&\leqslant \lambda_{\max}(P)\boldsymbol{x}(t^-)^{\mathrm{T}}\left((1+\mu)J_m^{\mathrm{T}} J_m + \left(1+\frac{1}{\mu}\right)\Delta J_m^{\mathrm{T}} \Delta J_m\right)\boldsymbol{x}(t^-) \\
&\leqslant \lambda_{\max}(P)\boldsymbol{x}(t^-)^{\mathrm{T}}\left((1+\mu)J_m^{\mathrm{T}} J_m + m^2\left(1+\frac{1}{\mu}\right)J_m^{\mathrm{T}} F_t^{\mathrm{T}} F_t J_m\right)\boldsymbol{x}(t^-) \\
&\leqslant \lambda_{\max}(P)\boldsymbol{x}(t^-)^{\mathrm{T}}\left((1+\mu)J_m^{\mathrm{T}} J_m + m^2\left(1+\frac{1}{\mu}\right)J_m^{\mathrm{T}} J_m\right)\boldsymbol{x}(t^-) \\
&= \lambda_m\left(1+\mu+m^2\left(1+\frac{1}{\mu}\right)\right)V\big(\boldsymbol{x}(t^-)\big) \leqslant \lambda V\big(\boldsymbol{x}(t^-)\big).
\end{aligned} \tag{6}$$

It follows from (5) and (6) that

$$V\big(\boldsymbol{x}(t)\big) \leqslant \lambda V\big(\boldsymbol{x}\big((mT+\tau_m)^-\big)\big)\mathrm{e}^{g(t-mT-\tau_m)}, \tag{7}$$

where $mT + \tau_m \leqslant t < (m+1)T$.

By using inequalities (4) and (7) we can derive the following results.

When $m = 0$, if $t \in [0, \tau_0)$, then

$$V\big(\boldsymbol{x}(t)\big) \leqslant V(\boldsymbol{x}_0)\mathrm{e}^{gt}$$

and so

$$V\big(\boldsymbol{x}(\tau_0^-)\big) \leqslant V\big(\boldsymbol{x}_0\big)\mathrm{e}^{g\tau_0}.$$

If $t \in [\tau_0, T)$, then

$$V\big(\boldsymbol{x}(t)\big) \leqslant \lambda V\big(\boldsymbol{x}(\tau_0^-)\big)\mathrm{e}^{g(t-\tau_0)} \leqslant \lambda V(\boldsymbol{x}_0)\mathrm{e}^{gt}$$

and so

$$V\big(\boldsymbol{x}(T)\big) \leqslant \lambda V(\boldsymbol{x}_0)\mathrm{e}^{gT}$$

When $m = 1$, if $t \in [T,\, T + \tau_1)$, then

$$V\big(\boldsymbol{x}(t)\big) \leqslant V\big(\boldsymbol{x}(T)\big)\mathrm{e}^{g(t-T)} \leqslant \lambda V(\boldsymbol{x}_0)\mathrm{e}^{gt}$$

and so

$$V\big(\boldsymbol{x}\big((T+\tau_1)^-\big)\big) \leqslant \lambda V(\boldsymbol{x}_0)\mathrm{e}^{g(T+\tau_1)}.$$

If $t \in [T+\tau_1,\, 2T)$, then

$$V\big(\boldsymbol{x}(t)\big) \leqslant \lambda V\big(\boldsymbol{x}\big((T+\tau_1)^-\big)\big)\mathrm{e}^{g(t-T-\tau_1)} \leqslant \lambda^2 V(\boldsymbol{x}_0)\mathrm{e}^{gt}$$

and so

$$V\big(\boldsymbol{x}(2T)\big) \leqslant \lambda^2 V(\boldsymbol{x}_0)\mathrm{e}^{2gT}.$$

In general, when $m = k$, $k = 0, 1, \dots$, if $t \in [kT,\, kT+\tau_k)$, then we have

$$\begin{aligned} V\big(\boldsymbol{x}(t)\big) &\leqslant \lambda^k V(\boldsymbol{x}_0)\mathrm{e}^{gt} \leqslant \lambda^k V(\boldsymbol{x}_0)\mathrm{e}^{g(k+1)T} \\ &= V(\boldsymbol{x}_0)\mathrm{e}^{gT+kgT+k\ln\lambda} = V(\boldsymbol{x}_0)\mathrm{e}^{gT+k(gT+\ln\lambda)}. \end{aligned} \tag{8}$$

If $t \in [kT+\tau_k,\, (k+1)T)$, we obtain

$$\begin{aligned} V\big(\boldsymbol{x}(t)\big) &\leqslant \lambda^{k+1} V(\boldsymbol{x}_0)\mathrm{e}^{gt} \leqslant \lambda^{k+1} V(\boldsymbol{x}_0)\mathrm{e}^{g(k+1)T} \\ &= V(\boldsymbol{x}_0)\mathrm{e}^{(k+1)gT+(k+1)\ln\lambda} = V(\boldsymbol{x}_0)\mathrm{e}^{(k+1)(gT+\ln\lambda)}. \end{aligned} \tag{9}$$

It follows from (8), (9), and condition (ii) that

$$\lim_{t\to\infty} V\big(\boldsymbol{x}(t)\big) = 0.$$

This completes the proof. □

If we choose $P = I$ in Theorem 1, then the condition of Theorem 1 can be simplified as follows.

Corollary 1. *If there exit $g, \mu, \varepsilon > 0$ such that the following hold*:

(i) $A + A^{\mathrm{T}} + (\varepsilon - g)I + \varepsilon^{-1}L \leqslant 0$,
(ii) $gT + \ln\lambda < 0$,

where $\lambda = \max\{\lambda_m(1+\mu+m^2(1+1/\mu))\}$, $\lambda_m = \lambda_{\max}(J_m^{\mathrm{T}}J_m)$, then the origin of system (3) *is asymptotically stable.*

The condition of Corollary 1 is similar to Theorem 1 shown in [19]. Since impulsive effects can be stochastically occurred in a impulse time window in system (3), Corollary 1 is more general than Theorem 1 shown in [19].

Sometimes, for the sake of convenience, the impulsive control gain J_m is always selected as a constant matrix J, then we have the following.

Corollary 2. *Let $P \in \mathbb{R}^{n\times n}$ be a symmetric and positive definite matrix. If there exit $g, \mu, \varepsilon > 0$ such that the following hold*:

(i) $PA + A^{\mathrm{T}}P + \varepsilon P^2 + \varepsilon^{-1}L - gP \leqslant 0$,
(ii) $gT + \ln\lambda < 0$,

where $\lambda = \lambda_{\max}(P)\lambda_{\max}(P^{-1}J^{\mathrm{T}}J)(1+\mu+m^2(1+1/\mu))$, then the origin of system (3) *is asymptotically stable.*

The condition of Corollary 2 is similar to Theorem 1 shown in [8]. Since we take into account the influence of impulsive control gain errors on the systems, Corollary 2 is more practical than Theorem 1 shown in [8].

In many practical applications, the parameters of impulsive control of nonlinear systems contain errors. In what follows, we will consider system (3) with parameter uncertainty. The corresponding system can be described as

$$\begin{aligned}
&\dot{\boldsymbol{x}}(t) = (A + \Delta A)\boldsymbol{x} + \phi(\boldsymbol{x}), \quad mT \leqslant t < mT + \tau_m, \\
&\boldsymbol{x}(t) = (J_m + \Delta J_m)\boldsymbol{x}(t^-), \quad t = mT + \tau_m, \\
&\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + \phi(\boldsymbol{x}), \quad mT + \tau_m < t < (m+1)T,
\end{aligned} \tag{10}$$

where ΔA is the parametric uncertainty and has the following form: $\Delta A = GF(t)H$, where $F^{\mathrm{T}}(t)F(t) \leqslant I$, while G and H are appropriate known matrices.

Theorem 2. *Let $P \in \mathbb{R}^{n\times n}$ be a symmetric and positive definite matrix. If there exit $g, \mu, \varepsilon > 0$ such that the following hold*:

(i) $PA + A^{\mathrm{T}}P + (1+\varepsilon)P^2 + \varepsilon^{-1}L + \varpi I - gP \leqslant 0$,
(ii) $gT + \ln\lambda < 0$,

where $\varpi = \lambda_{\max}(G^{\mathrm{T}}G)\lambda_{\max}(H^{\mathrm{T}}H)$, $\lambda = \max\{\lambda_m(1 + \mu + m^2(1 + 1/\mu))\}$, $\lambda_m = \lambda_{\max}(P)\lambda_{\max}(P^{-1}J_m^{\mathrm{T}}J_m)$, then the origin of system (10) *is asymptotically stable.*

Proof. Let us construct the following Lyapunov function:

$$V\big(\boldsymbol{x}(t)\big) = \boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t).$$

If $mT \leqslant t < mT + \tau_m$, then by Lemma 1 and condition (i) we have

$$\begin{aligned}
D^+\big(V\big(\boldsymbol{x}(t)\big)\big) &= 2\boldsymbol{x}^{\mathrm{T}}(t)P\big((A+\Delta A)\boldsymbol{x}(t) + \phi\big(\boldsymbol{x}(t)\big)\big) \\
&= 2\boldsymbol{x}^{\mathrm{T}}(t)PA\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)P\Delta A\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)P\phi(\boldsymbol{x}) \\
&= \boldsymbol{x}^{\mathrm{T}}(t)\big(PA + A^{\mathrm{T}}P\big)\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)P\Delta A\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)P\phi(\boldsymbol{x}) \\
&\leqslant \boldsymbol{x}^{\mathrm{T}}(t)\big(PA + A^{\mathrm{T}}P\big)\boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}P^2\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}\Delta A^{\mathrm{T}}\Delta A\boldsymbol{x} \\
&\quad + \varepsilon\boldsymbol{x}^{\mathrm{T}}P^2\boldsymbol{x} + \frac{1}{\varepsilon}\phi\big(\boldsymbol{x}(t)\big)^{\mathrm{T}}\phi\big(\boldsymbol{x}(t)\big) \\
&= \boldsymbol{x}^{\mathrm{T}}(t)\big(PA + A^{\mathrm{T}}P\big)\boldsymbol{x}(t) + (1+\varepsilon)\boldsymbol{x}^{\mathrm{T}}P^2\boldsymbol{x} + \frac{1}{\varepsilon}\phi\big(\boldsymbol{x}(t)\big)^{\mathrm{T}}\phi\big(\boldsymbol{x}(t)\big) \\
&\quad + \boldsymbol{x}^{\mathrm{T}}H^{\mathrm{T}}F^{\mathrm{T}}(t)G^{\mathrm{T}}GF(t)H\boldsymbol{x} \\
&\leqslant \boldsymbol{x}^{\mathrm{T}}(t)\big(PA + A^{\mathrm{T}}P\big)\boldsymbol{x}(t) + (1+\varepsilon)\boldsymbol{x}^{\mathrm{T}}P^2\boldsymbol{x}(t) + \frac{1}{\varepsilon}\boldsymbol{x}^{\mathrm{T}}(t)L\boldsymbol{x}(t) \\
&\quad + \lambda_{\max}\big(G^{\mathrm{T}}G\big)\boldsymbol{x}^{\mathrm{T}}H^{\mathrm{T}}F^{\mathrm{T}}(t)F(t)H\boldsymbol{x} \\
&\leqslant \boldsymbol{x}^{\mathrm{T}}(t)\big(PA + A^{\mathrm{T}}P\big)\boldsymbol{x}(t) + (1+\varepsilon)\boldsymbol{x}^{\mathrm{T}}P^2\boldsymbol{x}(t) + \frac{1}{\varepsilon}\boldsymbol{x}^{\mathrm{T}}(t)L\boldsymbol{x}(t) \\
&\quad + \lambda_{\max}\big(G^{\mathrm{T}}G\big)\boldsymbol{x}^{\mathrm{T}}H^{T}H\boldsymbol{x}
\end{aligned}$$

$$\begin{aligned}
&\leqslant \boldsymbol{x}^{\mathrm{T}}(t)\big(PA+A^{\mathrm{T}}P\big)\boldsymbol{x}(t)+(1+\varepsilon)\boldsymbol{x}^{\mathrm{T}}P^2\boldsymbol{x}(t)+\frac{1}{\varepsilon}\boldsymbol{x}^{\mathrm{T}}(t)L\boldsymbol{x}(t)\\
&\quad+\lambda_{\max}\big(G^{\mathrm{T}}G\big)\lambda_{\max}\big(H^{\mathrm{T}}H\big)\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}\\
&=\boldsymbol{x}^{\mathrm{T}}(t)\bigg(PA+A^{\mathrm{T}}P+(1+\varepsilon)P^2+\frac{1}{\varepsilon}L+\varpi I-gP\bigg)\boldsymbol{x}(t)+gV\big(\boldsymbol{x}(t)\big)\\
&\leqslant gV\big(\boldsymbol{x}(t)\big),
\end{aligned}$$

which implies that

$$V\big(\boldsymbol{x}(t)\big)\leqslant V\big(\boldsymbol{x}(mT)\big)\mathrm{e}^{g(t-mT)}.$$

The rest of proof is same as that of Theorem 1, so we omit it here for simplicity. This completes the proof. □

3 A numerical example

In this section, we will illustrate the effectiveness of our result by showing simulation results employing the Chua's system. Throughout this section, we assume that $\boldsymbol{x}=[x,y,z]^{\mathrm{T}}$.

The original and dimensionless form of a Chua's oscillator [20] is given by

$$\dot{x}=\alpha\big(y-x-g(x)\big),\qquad \dot{y}=x-y+z,\qquad \dot{z}=-\beta y, \tag{11}$$

where α and β are parameters, and $g(x)$ is the piecewise linear characteristics of the Chua's diode, which is defined by

$$g(x)=bx+0.5(a-b)\big(|x+1|-|x-1|\big),$$

where $a<b<0$ are two constants.

By decomposing the linear and nonlinear parts of the system in (11), we rewrite it as

$$\dot{\boldsymbol{x}}(t)=A\boldsymbol{x}+\phi(\boldsymbol{x}),$$

where

$$A=\begin{bmatrix}-\alpha-\alpha b & \alpha & 0\\ 1 & -1 & 1\\ 0 & -\beta & 0\end{bmatrix},\qquad \phi(\boldsymbol{x})=\begin{bmatrix}-0.5\alpha(a-b)(|x+1|-|x-1|)\\ 0\\ 0\end{bmatrix}.$$

Simple calculations show that

$$\begin{aligned}
\big\|\phi(\boldsymbol{x})\big\|^2&=0.25\alpha^2(a-b)^2\big[(x+1)^2+(x-1)^2-2\big|(x+1)(x-1)\big|\big]\\
&=0.5\alpha^2(a-b)^2\big(x^2+1-|x^2-1|\big)\\
&=\begin{cases}\alpha^2(a-b)^2, & x^2>1,\\ \alpha^2(a-b)^2x^2, & x^2\leqslant 1\end{cases}\\
&\leqslant \alpha^2(a-b)^2x^2.
\end{aligned}$$

Thus, we can choose $L=\operatorname{diag}(\alpha^2(a-b)^2,0,0)$.

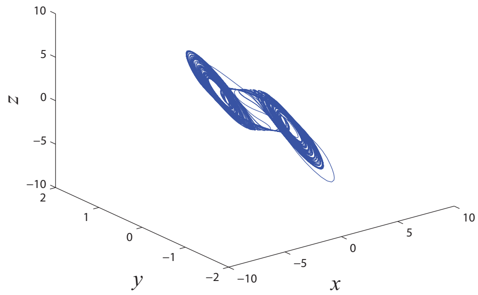

Figure 1. The chaotic phenomenon of Chua's oscillator with the initial condition $x(0) = (0.5, 0.3, -0.5)^{\mathrm{T}}$.

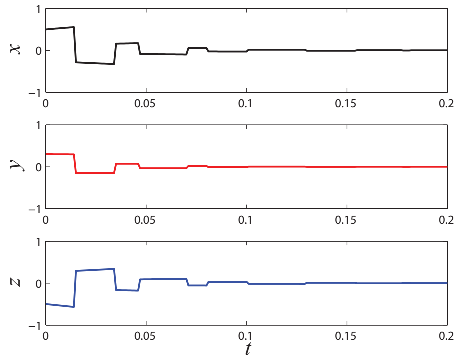

Figure 2. Time response curves of controlled Chua's system with the initial condition $x(0) = (0.5, 0.3, -0.5)^{\mathrm{T}}$.

In this example, we set the system parameters as

$$\alpha = 9.2156, \qquad \beta = 15.9946, \qquad a = -1.24905, \qquad b = -0.75735,$$

which make Chua's circuit (11) chaotic. Figure 1 shows the chaotic phenomenon of Chua's oscillator with the initial condition $x(0) = (0.5, 0.3, -0.5)^{\mathrm{T}}$.

Meanwhile, we choose $P = I$, $\mu = \varepsilon = 1$, and

$$J_m = J = \operatorname{diag}(-0.5, -0.5, -0.5).$$

For the sake of simplicity, the impulsive control gain error $\varDelta J_m$ is specified as

$$\varDelta J_m = \varDelta J = 0.05 \sin tJ,$$

and so $\lambda = 0.5012$. In order to satisfy condition (i) of Theorem 1, we can choose $g = 31$. From the following inequality

$$gT + \ln \lambda < 0$$

we have $T < 0.0233$. Thus, by Theorem 1 we know that the origin of system (3) is asymptotically stable. The simulation results with $T = 0.0200$ are shown in Fig. 2.

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