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Gaussian solitary waves to Boussinesq equation with dual dispersion and logarithmic nonlinearity

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Abstract. This paper discusses shallow water waves that is modeled with Boussinesq equation that comes with dual dispersion and logarithmic nonlinearity. The extended trial function scheme retrieves exact Gaussian solitary wave solutions to the model.

Keywords: Gaussons, Boussinesq equation, logarithmic nonlinearity.

1 Introduction

There are several models that address the dynamics of shallow water waves along lake shores and sea beaches. A few of these models that carry a lot of visibility and frequented upon are the Korteweg–de Vries (KdV) equation, Kadomtsev–Petviashvili (KP) equation, Kawahara equation, Boussinesq equation (BE), Benjamin–Bona–Mahoney equation, and others. All of these models have been studied with algebraic forms of nonlinearity. The current practice is to study these models with logarithmic nonlinearity; a trend that was first introduced by Wazwaz during 2014 [20]. Thus, KdV equation, KP equation, and Boussinesq equations have all been studied for logarithmic nonlinearities using a variety of rich mathematical schemes such as soliton perturbation theory, semi-inverse variational method, traveling wave hypothesis, and several others. In fact, there exists a plethora of mathematical techniques that are applied to several nonlinear evolution equations in fluid dynamics, nonlinear optics, nuclear physics, and other areas to address them [1–23]. This paper studies the dynamics of Gaussian solitary waves due to BE with logarithmic nonlinear form by the aid of another powerful mathematical principle, namely, the extended trial equation algorithm. The next couple of sections details the scheme that yields Gaussian solitary waves to the BE.

2 Governing equation

BE with logarithmic nonlinearity and dual dispersion reads as follows [2, 21]:

$$q_{tt} - k^2 q_{xx} + a(q \ln q)_{xx} + b_1 q_{xxxx} + b_2 q_{xxtt}. \tag{1}$$

This dynamical model was introduced by Wazwaz [21]. In Eq. (1), $q(x,t)$ represents the wave profile, where the independent variables x and t represent spatial and temporal coordinates, respectively. The first two terms in Eq. (1) constitute the wave operator. The coefficient of a is the logarithmic nonlinear term. The coefficients of b_1 and b_2 are dispersion terms, where, in particular, the coefficient of b_2 gives the spatio–temporal dispersion.

2.1 Mathematical analysis

To secure Gaussons or solitary wave solutions to Eq. (1), the starting hypothesis is

$$q(x,t) = g(x - vt) = g(s), \tag{2}$$

where

$$s = x - vt.$$

In Eq. (2), v represents the speed of the wave, and the functional form of g will give the solitary wave solution. Substituting hypothesis (2) into (1) and integrating twice yields

$$\left(v^2 - k^2\right)g + ag \ln g + \left(b_1 + b_2 v^2\right)g'' = 0, \tag{3}$$

where $g'' = \mathrm{d}^2 g/\mathrm{d}s^2$. The integration constant is taken to be zero, both times, since the search is for a localized solitary wave solution.

To obtain a closed form analytic solution, we employ a transformation formula

$$\psi^{-1}(x,t) = \ln g(x,t),$$

equivalent to

$$g(x,t) = \exp \frac{1}{\psi(x,t)}.$$

This formula carries (3) into

$$a\psi^3 + \left(v^2 - k^2\right)\psi^4 + \left(b_1 + b_2 v^2\right)\left\{(\psi')^2 + 2\psi(\psi')^2 - \psi^2\psi''\right\} = 0. \tag{4}$$

Equation (4) will now be studied by the aid of extended trial equation scheme (ETES) [4–6, 13, 14, 18, 20] in the next section.

3 Extended trial equation scheme

To start with the ETES, the following initial assumption for the solution structure of (4) is considered:

$$\psi=\sum_{i=0}^{\varsigma}\tau_i\Theta^i, \tag{5}$$

where

$$(\Theta')^2=\Delta(\Theta)=\frac{\Phi(\Theta)}{\Upsilon(\Theta)}=\frac{\mu_\sigma\Theta^\sigma+\cdots+\mu_1\Theta+\mu_0}{\chi_\rho\Theta^\rho+\cdots+\chi_1\Theta+\chi_0}. \tag{6}$$

Here $\tau_0,\dots,\tau_\varsigma;\mu_0,\dots,\mu_\sigma$ and $\chi_0,\dots,\chi_\rho$ are constants to be determined later. From (5) and (6), terms $(\psi')^2$ and ψ'' can be derived as

$$(\psi')^2=\frac{\Phi(\Theta)}{\Upsilon(\Theta)}\left(\sum_{i=0}^{\varsigma}i\tau_i\Theta^{i-1}\right)^2,$$

and

$$\psi''=\frac{\Phi'(\Theta)\Upsilon(\Theta)-\Phi(\Theta)\Upsilon'(\Theta)}{2\Upsilon^2(\Theta)}\sum_{i=0}^{\varsigma}i\tau_i\Theta^{i-1}+\frac{\Phi(\Theta)}{\Upsilon(\Theta)}\sum_{i=0}^{\varsigma}i(i-1)\tau_i\Theta^{i-2},$$

where $\Phi(\Theta)$ and $\Upsilon(\Theta)$ are polynomials of Θ. Equation (6) can be reduced the following integral form:

$$\pm(s-s_0)=\int\frac{\mathrm{d}\Theta}{\sqrt{\Delta(\Theta)}}=\int\sqrt{\frac{\Upsilon(\Theta)}{\Phi(\Theta)}}\,\mathrm{d}\Theta. \tag{7}$$

Balancing the order of $\psi^2\psi''$ and ψ^4 in Eq. (4) yields

$$\sigma=\rho+\varsigma+2. \tag{8}$$

Case 1. Let us choose $\sigma=3$, $\rho=0$, and $\varsigma=1$ in Eq. (8). Then, Eq. (4) have the solution in the form

$$\psi=\tau_0+\tau_1\Theta, \tag{9}$$

where τ_0 and τ_1 are constants to be determined later such that $\tau_1\neq0$, and Θ satisfies Eq. (6). Substituting (9) into (4) and solving the resulting system, the sets are derived as

$$\mu_2=\mu_2,\qquad \chi_0=\chi_0,\qquad \tau_1=\tau_1,$$

$$\mu_0=\frac{\mu_2^3(2b_1+b_2(a+2k^2))^2}{108a^2\tau_1^2\chi_0^2},\qquad \mu_1=-\frac{\mu_2^2(2b_1+b_2(a+2k^2))}{6a\tau_1\chi_0},$$

$$\mu_3=-\frac{2a\tau_1\chi_0}{2b_1+b_2(a+2k^2)},\qquad \tau_0=-\frac{\mu_2(2b_1+b_2(a+2k^2))}{6a\chi_0},\qquad v=\sqrt{\frac{a+2k^2}{2}}.$$

Substituting these results into (6) and (7) leads to

$$\pm(s - s_0) = \sqrt{\Omega} \int \frac{\mathrm{d}\Theta}{\sqrt{\Delta(\Theta)}},$$

where

$$\Omega = \frac{\chi_0}{\mu_3}, \qquad \Delta(\Theta) = \Theta^3 + \frac{\mu_2}{\mu_3}\Theta^2 + \frac{\mu_1}{\mu_3}\Theta + \frac{\mu_0}{\mu_3}.$$

In view of these results, traveling wave solutions to BE with logarithmic nonlinearity are derived in the following forms:

For $\Delta(\Theta) = (\Theta - \gamma_1)^3$,

$$q(x,t) = \exp\left[-\frac{\mu_2(2b_1 + b_2(a+2k^2)) - 6a\chi_0\tau_1\gamma_1}{6a\chi_0} + \frac{4\tau_1\Omega}{(x - \sqrt{\frac{a+2k^2}{2}}t - s_0)^2}\right]^{-1}. \tag{10}$$

If $\Delta(\Theta) = (\Theta - \gamma_1)^2(\Theta - \gamma_2)$ and $\gamma_2 > \gamma_1$,

$$\begin{aligned} q(x,t) = \exp\Bigg[&-\frac{\mu_2(2b_1 + b_2(a+2k^2)) - 6a\chi_0\tau_1\gamma_2}{6a\chi_0} \\ &+ \tau_1(\gamma_1 - \gamma_2)\tanh^2\left(\frac{1}{2}\sqrt{\frac{\gamma_1 - \gamma_2}{\Omega}}\left[x - \sqrt{\frac{a+2k^2}{2}}t - s_0\right]\right)\Bigg]^{-1}. \end{aligned} \tag{11}$$

However, when $\Delta(\Theta) = (\Theta - \gamma_1)(\Theta - \gamma_2)^2$ and $\gamma_1 > \gamma_2$,

$$\begin{aligned} q(x,t) = \exp\Bigg[&-\frac{\mu_2(2b_1 + b_2(a+2k^2)) - 6a\chi_0\tau_1\gamma_1}{6a\chi_0} \\ &+ \tau_1(\gamma_1 - \gamma_2)\,\mathrm{cosech}^2\left(\frac{1}{2}\sqrt{\frac{\gamma_1 - \gamma_2}{\Omega}}\left[x - \sqrt{\frac{a+2k^2}{2}}t\right]\right)\Bigg]^{-1}. \end{aligned} \tag{12}$$

Whenever $\Delta(\Theta) = (\Theta - \gamma_1)(\Theta - \gamma_2)(\Theta - \gamma_3)$ and $\gamma_1 > \gamma_2 > \gamma_3$,

$$\begin{aligned} q(x,t) = \exp\Bigg[&-\frac{\mu_2(2b_1 + b_2(a+2k^2)) - 6a\chi_0\tau_1\gamma_3}{6a\chi_0} \\ &+ \tau_1(\gamma_2 - \gamma_3)\,\mathrm{sn}^2\left(\mp\frac{1}{2}\sqrt{\frac{\gamma_1 - \gamma_3}{\Omega}}\left[x - \sqrt{\frac{a+2k^2}{2}}t - s_0\right], l\right)\Bigg]^{-1}, \end{aligned} \tag{13}$$

where

$$l^2 = \frac{\gamma_2 - \gamma_3}{\gamma_1 - \gamma_3}.$$

It is important to emphasize that γ_i for $i = 1, 2, 3$ are the roots of the equation

$$\Delta(\Theta) = 0.$$

For

$$s_0 = 0, \qquad \mu_2\bigl(2b_1 + b_2\bigl(a + 2k^2\bigr)\bigr) - 6a\chi_0\tau_1\gamma_1 = 0,$$

solutions (10)–(12) can be reduced to the following exact solutions, respectively:

$$q(x,t) = \exp\left[\frac{1}{4\tau_1\Omega}\left(x - \sqrt{\frac{a+2k^2}{2}}t\right)^2\right],$$

$$q(x,t) = \exp\left[\frac{1}{\tau_1(\gamma_2-\gamma_1)}\cosh^2\left(\frac{1}{2}\sqrt{\frac{\gamma_1-\gamma_2}{\Omega}}\left(x - \sqrt{\frac{a+2k^2}{2}}t\right)\right)\right],$$

and

$$q(x,t) = \exp\left[\frac{1}{\tau_1(\gamma_1-\gamma_2)}\sinh^2\left(\frac{1}{2}\sqrt{\frac{\gamma_1-\gamma_2}{\Omega}}\left(x - \sqrt{\frac{a+2k^2}{2}}t\right)\right)\right].$$

Moreover, when

$$s_0 = 0, \qquad \mu_2\bigl(2b_1 + b_2\bigl(a + 2k^2\bigr)\bigr) - 6a\chi_0\tau_1\gamma_3 = 0,$$

the exact solutions given by (13) can be reduced to

$$q(x,t) = \exp\left[\frac{1}{\tau_1(\gamma_2-\gamma_3)}\,\mathrm{ns}^2\left(\mp\frac{1}{2}\sqrt{\frac{\gamma_1-\gamma_3}{\Omega}}\left[x - \sqrt{\frac{a+2k^2}{2}}t\right], \frac{\gamma_2-\gamma_3}{\gamma_1-\gamma_3}\right)\right].$$

Remark 1. When the modulus $l \to 1$, exact solutions fall out

$$q(x,t) = \exp\left[\frac{1}{\tau_1(\gamma_2-\gamma_3)}\coth^2\left(\mp\frac{1}{2}\sqrt{\frac{\gamma_1-\gamma_3}{\Omega}}\left[x - \sqrt{\frac{a+2k^2}{2}}t\right]\right)\right]$$

for $\gamma_1 = \gamma_2$.

Case 2. When $\sigma = 4$, $\rho = 0$, and $\varsigma = 2$ in Eq. (8), we have

$$\psi = \tau_0 + \tau_1\Theta + \tau_2\Theta^2, \tag{14}$$

and then

$$(\psi')^2 = \frac{(\tau_1 + 2\tau_2\Theta)^2(\mu_4\Theta^4 + \mu_3\Theta^3 + \mu_2\Theta^2 + \mu_1\Theta + \mu_0)}{\chi_0}, \tag{15}$$

$$\psi'' = \frac{(\tau_1 + 2\tau_2\Theta)(4\mu_4\Theta^3 + 3\mu_3\Theta^2 + 2\mu_2\Theta + \mu_1)}{2\chi_0} + \frac{2\tau_2(\mu_4\Theta^4 + \mu_3\Theta^3 + \mu_2\Theta^2 + \mu_1\Theta + \mu_0)}{\chi_0}, \tag{16}$$

where $\mu_4 \neq 0$ and $\chi_0 \neq 0$. Substituting Eqs. (14)–(16) into Eq. (4) and solving the resulting system, the sets are obtained as

$$\chi_0 = \chi_0, \qquad \tau_1 = \tau_1, \qquad \tau_2 = \tau_2,$$

$$\mu_0 = -\frac{a\tau_1^4\chi_0}{32\tau_2^3(2b_1 + b_2(a+2k^2))}, \qquad \mu_1 = -\frac{a\tau_1^3\chi_0}{4\tau_2^2(2b_1 + b_2(a+2k^2))},$$
$$\mu_2 = -\frac{3a\tau_1^2\chi_0}{4\tau_2(2b_1 + b_2(a+2k^2))}, \qquad \mu_3 = -\frac{a\tau_1\chi_0}{2b_1 + b_2(a+2k^2)},$$
$$\mu_4 = -\frac{a\tau_2\chi_0}{4b_1 + 2b_2(a+2k^2)}, \qquad \tau_0 = \frac{\tau_1^2}{4\tau_2}, \qquad v = \sqrt{\frac{a+2k^2}{2}}.$$

Substituting these results into (6) and (7) leads to

$$\pm(s - s_0) = \Omega_1 \int \frac{\mathrm{d}\Theta}{\sqrt{\Delta(\Theta)}}, \tag{17}$$

where

$$\Omega_1 = \sqrt{\frac{\chi_0}{\mu_4}}, \qquad \Delta(\Theta) = \Theta^4 + \frac{\mu_3}{\mu_4}\Theta^3 + \frac{\mu_2}{\mu_4}\Theta^2 + \frac{\mu_1}{\mu_4}\Theta + \frac{\mu_0}{\mu_4}.$$

Integrating Eq. (17) and taking $s_0 = 0$, exact solutions to BE with logarithmic nonlinearity are secured as the following:

For $\Delta(\Theta) = (\Theta - \gamma_1)^4$,

$$q(x,t) = \exp\left[\sum_{i=0}^{2} \tau_i \left(\gamma_1 \pm \frac{\Omega_1}{x - \sqrt{\frac{a+2k^2}{2}}t}\right)^i\right]^{-1}.$$

If $\Delta(\Theta) = (\Theta - \gamma_1)^3(\Theta - \gamma_2)$ and $\gamma_2 > \gamma_1$,

$$q(x,t) = \exp\left[\sum_{i=0}^{2} \tau_i \left(\gamma_1 + \frac{4\Omega_1^2(\gamma_2 - \gamma_1)}{4\Omega_1^2 - [(\gamma_1 - \gamma_2)(x - \sqrt{\frac{a+2k^2}{2}}t)]^2}\right)^i\right]^{-1}.$$

However, when $\Delta(\Theta) = (\Theta - \gamma_1)^2(\Theta - \gamma_2)^2$,

$$q(x,t) = \exp\left[\sum_{i=0}^{2} \tau_i \left(\gamma_2 + \frac{\gamma_2 - \gamma_1}{\exp[\frac{\gamma_1 - \gamma_2}{\Omega_1}(x - \sqrt{\frac{a+2k^2}{2}}t)] - 1}\right)^i\right]^{-1},$$

and

$$q(x,t) = \exp\left[\sum_{i=0}^{2} \tau_i \left(\gamma_1 + \frac{\gamma_1 - \gamma_2}{\exp[\frac{\gamma_1 - \gamma_2}{\Omega_1}(x - \sqrt{\frac{a+2k^2}{2}}t)] - 1}\right)^i\right]^{-1}.$$

Whenever $\varDelta(\Theta) = (\Theta - \gamma_1)^2(\Theta - \gamma_2)(\Theta - \gamma_3)$ and $\gamma_1 > \gamma_2 > \gamma_3$,

$$q(x,t) = \exp\left[\sum_{i=0}^{2} \tau_i \left(\gamma_1 - \frac{2(\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)}{\bar{\gamma}_1 + (\gamma_3 - \gamma_2)\cosh[\mathcal{P}_1(x - \sqrt{\frac{a+2k^2}{2}}t)]}\right)^i\right]^{-1},$$

where

$$\bar{\gamma}_1 = 2\gamma_1 - \gamma_2 - \gamma_3, \qquad \mathcal{P}_1 = \frac{\sqrt{(\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)}}{\varOmega_1}.$$

Finally, if $\varDelta(\Theta) = (\Theta - \gamma_1)(\Theta - \gamma_2)(\Theta - \gamma_3)(\Theta - \gamma_4)$ and $\gamma_1 > \gamma_2 > \gamma_3 > \gamma_4$,

$$q(x,t) = \exp\left[\sum_{i=0}^{2} \tau_i \left(\gamma_2 + \frac{\bar{\gamma}_2(\gamma_1 - \gamma_2)}{\bar{\gamma}_2 + (\gamma_1 - \gamma_4)\,\mathrm{sn}^2[\mathcal{P}_j(x - \sqrt{\frac{a+2k^2}{2}}t), l]}\right)^i\right]^{-1},$$

where

$$\bar{\gamma}_2 = \gamma_4 - \gamma_2, \qquad l^2 = \frac{(\gamma_2 - \gamma_3)(\gamma_1 - \gamma_4)}{(\gamma_1 - \gamma_3)(\gamma_2 - \gamma_4)},$$

$$\mathcal{P}_j = \frac{(-1)^j\sqrt{(\gamma_1 - \gamma_3)(\gamma_2 - \gamma_4)}}{2\varOmega_1} \quad \text{for } j = 1, 2.$$

It is important to note that γ_i for $i = 1, \dots, 4$ are the roots of the equation

$$\varDelta(\Theta) = 0.$$

Remark 2. When the modulus $l \to 1$, the following solutions fall out:

$$q(x,t) = \exp\left[\sum_{i=0}^{2} \tau_i \left(\gamma_2 + \frac{\bar{\gamma}_2(\gamma_1 - \gamma_2)}{\bar{\gamma}_2 + (\gamma_1 - \gamma_4)\tanh^2[\mathcal{P}_j(x - \sqrt{\frac{a+2k^2}{2}}t)]}\right)^i\right]^{-1}$$

for $\gamma_3 = \gamma_4$.

Remark 3. However, if $l \to 0$, the solutions are obtained as

$$q(x,t) = \exp\left[\sum_{i=0}^{2} \tau_i \left(\gamma_2 + \frac{\bar{\gamma}_2(\gamma_1 - \gamma_2)}{\bar{\gamma}_2 + (\gamma_1 - \gamma_4)\sin^2[\mathcal{P}_j(x - \sqrt{\frac{a+2k^2}{2}}t)]}\right)^i\right]^{-1}$$

for $\gamma_2 = \gamma_3$.

4 Conclusions

This paper secured Gaussian solitary wave solutions to BE that is considered with logarithmic nonlinearity and dual dispersion. The powerful extended trial function method is the integration scheme that has been implemented to retrieve the solitary wave solutions that are also referred to as Gaussons in this context. These solutions appear with constraint conditions that guarantee their existence.

The results of this manuscript are new and are being reported for the first time in this paper. In regards to the physical meaning of the model, this represents shallow water wave dynamics along lake shores and beaches. This is generalized model to the regular Boussinesq equation that is known. Upon carrying out the Taylor series expansion of the logarithmic function about $q = 1$ and retaining till the first term, the regular Boussinesq equation, with drifting term, falls out. Thus the model of study incorporates all of the previously established results.

The results of this paper paves way to carry out further research in this avenue. Later, perturbation terms will be included in this model and thus the perturbed BE will be addressed using this integration scheme as well as various other integration algorithms. These will be Lie symmetry analysis, Kudryashov's method, modified simple equation method, and several others. Additionally, this model will be studied with time-dependent coefficients along with stochastic perturbation terms. The results of those research will be reported in the future.

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