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An advanced delay-dependent approach of impulsive genetic regulatory networks besides the distributed delays, parameter uncertainties and time-varying delays*

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Abstract. In this typescript, we concerned the problem of delay-dependent approach of impulsive genetic regulatory networks besides the distributed delays, parameter uncertainties and time-varying delays. An advanced Lyapunov–Krasovskii functional are defined, which is in triple integral form. Combining the Lyapunov–Krasovskii functional with convex combination method and free-weighting matrix approach the stability conditions are derived with the help of linear matrix inequalities (LMIs). Some available software collections are used to solve the conditions. Lastly, two numerical examples and their simulations are conferred to indicate the feasibility of the theoretical concepts.

Keywords: genetic regulatory networks (GRNs), time-varying delays, distributed delays, parameter uncertainty, convex combination method, impulses, linear matrix inequalities (LMIs).

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1 Introduction

In genetic regulatory networks, DNA, RNA, and collection of molecules are interact with each other and result in the process of the expression of genes. Earlier, the research was popularized by Macdonald in 1989. In recent years, the study of genetic regulatory networks has fascinated noticeable attention in the biological and biomedical sciences. Generally, genetic regulatory networks (GRNs) act as a main role in great number of ordinary life processes as well as cell discrimination, the cell cycle, signal transduction and metabolism; hence, indicative exertions have been formed to establish mathematical approaches for their resolution. Moreover, GRNs include different types of models, i.e., discrete model or Boolean model, continuous model or differential equation model, Petri net model and Bayesian network model have been considered and employed in [1–4, 28]. Basically, Boolean model and differential equation model are mainly used in genetic regulatory networks.

During the construction of genetic network models, the extrinsic noise and the intrinsic noise may bring parameter uncertainties. At the same time, data errors, parameter fluctuations and uncertainties such as external perturbations are unavoidable. That is, one has to analyze the uncertain systems in the way of robust stability [5, 15, 29, 30, 35–37]. In the gene regulation process, time-delays are inevitable because the process of transcription and translation. Also, time-delays leads to poor performances and instability of genetic regulatory networks, see [11, 14, 17, 31, 32, 39, 41]. In GRNs, the activity of proteins and the observed oscillatory expression are driven by using the transcriptional delays. In the dynamical systems, delays have a great effect. Therefore, the stability problem of GRNs with time-varying delays are analyzed.

The study of impulsive differential equations are found in many domains of applied science, as reported in [16, 18, 23, 32, 38]. It is known that impulses can make unstable systems stable or exponentially stable, or otherwise, stable systems can become unstable after impulse effects, see [19, 22, 26]. In GRNs, Wang et al. [32] analyzed the nonlinear disturbance and time-varying delays using delay-dependent approach. In [24], the authors discussed the impulsive perturbations in genetic regulatory networks using delay-dependent method. In [33], Wang et al. investigated the uncertain genetic regulatory networks with time-varying delays in the sense of robust stability analysis. In GRNs, Liang et al. [21] discussed the uncertain mode transition rates and state estimation for Markov type with delays. In [7], the authors presented the combinational measurements in event-triggered systems with distributed delays. In [10], Hu et al. investigated the state estimation for nonlinear systems with discrete and distributed delays. In [9], the authors investigated the stability analysis of genetic regulatory networks with distributed delay. In [37], some robust stability criteria are given to the uncertain genetic regulatory networks with time-varying delays. In [8], Feng et al. derived the stability analysis problem by using convex combination method in GRNs. In GRNs, Koo et al. [13] investigated the delay-dependent approach and time-varying delays in the way of robust stability criterion.

Induced by the beyond deliberation, in this work, we design an advanced delay-dependent GRNs with distributed delays and impulses. A new triple-integral Lyapunov–Krasovskii functional are constructed, which helps us to reduce the conservatism effected by the distributed delays and time-varying delays. By taking the time-varying delays into account, the stability criteria are granted by using the delay-dependent approach, convex combination and free-weighting matrix method combined with Jensen's inequality. Finally, numerical simulations are worn to show the less conservativeness of the attained results. The significant of the manuscript is given as follows:

(i) An advanced delay-dependent genetic regulatory networks with parameter uncertainties, which includes distributed delays and impulsive effects are investigated using delay-dependent approach.
(ii) Based on the contemporary Lyapunov–Krasovskii functional and integral inequality techniques, some sufficient conditions for asymptotical stability of delay-dependent genetic regulatory networks are derived in the form of LMIs. In addition, compared to the existing results, the derived outcomes are different and advanced.
(iii) In this chapter, the feasibility of the obtained LMIs for asymptotic stability can be easily solved by the aid of MATLAB LMI control toolbox.
(iv) By handled the time-varying delay and distributed time-varying delay terms in our concerned genetic regulatory networks, the allowable upper bounds of time-delays are maximum in comparison with some existing literatures, see Table 1 in Example 2. This can be expressed that the approach developed in this chapter is more effective and less conserved.

The remaining things of this work is classified well as follows: In Section 2, GRNs with distributed delays and impulses are described, and we introduced some assumptions and lemmas for proving our required criteria. In Section 3, we define an advanced Lyapunov–Krasovskii functional, which is in triple integral form, and derived sufficient conditions, which can be expressed in the form of LMIs. Additionally, two mathematical examples are shown in Section 4 to demonstrate the advantages of our stability conditions. Lastly, conclusions are shown in Section 5.

Notations. The superscript "T" act as the transpose of matrix. $\mathbb{R}^n$ indicates the Euclidean space with n dimensions, and $\mathbb{R}^{n\times m}$ denotes the set of all $n\times m$ real matrices. I means the identity matrix of appropriate dimensions. $\mathrm{diag}\{\cdot\}$ is the diagonal matrix. The symbol "$*$" denotes the symmetric term. In this paper, the matrices are assumed to be with appropriate dimensions.

2 Model description and preliminaries

Now, we consider the continuous-time genetic regulatory networks with time-varying delays described by the following equations:

$$\begin{aligned}\dot{m}_i(\mathcal{T}) &= -g_{1i}m_i(\mathcal{T})\\ &\quad + h_{1i}\big(p_1\big(\mathcal{T}-\xi(\mathcal{T})\big), p_2\big(\mathcal{T}-\xi(\mathcal{T})\big), \ldots, p_n\big(\mathcal{T}-\xi(\mathcal{T})\big)\big),\\ \dot{p}_i(\mathcal{T}) &= -g_{2i}p_i(\mathcal{T}) + h_{2i}m_i\big(\mathcal{T}-\eta(\mathcal{T})\big), \quad i=1,2,\ldots,n.\end{aligned} \tag{1}$$

Here $m_i(\mathcal{T})$ and $p_i(\mathcal{T})$ are the concentrations of mRNAs and proteins, respectively. g_{1i} and g_{2i} are the degradation rates of mRNAs and proteins, respectively. h_{2i} defines the translation rate, $\xi(\mathcal{T})$ and $\eta(\mathcal{T})$ are the transcriptional and translational delay, respectively. The regulatory function is defined as h_{1i}, which is nonlinear, and the sum logic is $h_{1i}(p_1(\mathcal{T}), p_2(\mathcal{T}), \dots, p_n(\mathcal{T})) = \sum_{j=1}^{n} h_{ij}(p_j(\mathcal{T}))$, which is in [12, 40]. In [6], a monotone function of the Hill form $h_{ij}(p_j(\mathcal{T}))$ is defined as

$$h_{ij}(p_j(\mathcal{T}) = \begin{cases} \beta_{ij}\frac{(p_j(\mathcal{T})/\gamma_j)^{H_{fj}}}{1+(p_j(\mathcal{T})/\gamma_j)^{H_{fj}}} & \text{if } j \text{ is an activator of gene } i, \\ \beta_{ij}\frac{1}{1+(p_j(\mathcal{T})/\gamma_j)^{H_{fj}}} & \text{if } j \text{ is a repressor of gene } i, \end{cases}$$

where j is the transcription factor, β_{ij} is a bounded constant, γ_j is a positive scalar, H_{fj} is the Hill coefficient. Therefore, Eq. (1) can be changed accordingly as

$$\begin{aligned} \dot{m}_i(\mathcal{T}) &= -g_{1i}m_i(\mathcal{T}) + \sum_{i=1}^{n} H_{ij}f_j\big(p_j\big(\mathcal{T} - \xi(\mathcal{T})\big)\big) + w_i, \\ \dot{p}_i(\mathcal{T}) &= -g_{2i}p_i(\mathcal{T}) + h_{2i}m_i\big(\mathcal{T} - \eta(\mathcal{T})\big), \quad i = 1, 2, \dots, n, \end{aligned} \tag{2}$$

where $f_j(x) = (x/\gamma_j)^{H_{fj}}/(1 + (x/\gamma_j)^{H_{fj}})$, $w_i = \sum_{j\in U_i} \beta_{ij}$, and U_i is the basel rate, which is defined as $U_i = \sum_{j\in w_i} \beta_{ij}$. The matrix $h_1 = (H_{ij}) \in \mathbb{R}^{n\times n}$ of GRNs is defined as

$$H_{ij} = \begin{cases} \gamma_{ij} & \text{if } j \text{ is an activator of gene } i, \\ 0 & \text{if no link from } j \text{ to } i, \\ -\gamma_{ij} & \text{if } j \text{ is a repressor of gene } i. \end{cases}$$

Equation (2) changed into the compact matrix form, we have

$$\begin{aligned} \dot{m}(\mathcal{T}) &= -G_1m(\mathcal{T}) + H_1f\big(p\big(\mathcal{T} - \xi(\mathcal{T})\big)\big) + w, \\ \dot{p}(\mathcal{T}) &= -G_2p(\mathcal{T}) + H_2m\big(\mathcal{T} - \eta(\mathcal{T})\big), \end{aligned} \tag{3}$$

where $G_1 = \mathrm{diag}\{g_{11}, g_{12}, \dots, g_{1n}\}$, $w = \mathrm{diag}\{w_1, w_2, \dots, w_n\}$, $G_2 = \mathrm{diag}\{g_{21}, g_{22}, \dots, g_{2n}\}$, $H_2 = \mathrm{diag}\{h_{21}, h_{22}, \dots, h_{2n}\}$, $m(\mathcal{T}) = (m_1(\mathcal{T}), \dots, m_n(\mathcal{T}))^{\mathrm{T}}$, $p(\mathcal{T}) = (p_1(\mathcal{T}), \dots, p_n(\mathcal{T}))^{\mathrm{T}}$, $f(p(\mathcal{T})) = (f_1(p_1(\mathcal{T}), \dots, f_n(p_n(\mathcal{T}))))^{\mathrm{T}}$. Here monotonically increasing function $f_j(x) = (x/\gamma_j)^{H_{fj}}/(1 + (x/\gamma_j)^{H_{fj}})$ is bounded with $H_{fj} \geqslant 1$ and have the continuous derivatives for $x \geqslant 0$. Completely the direct algebraic directions, we have

$$r_j = \max_{x\geqslant 0} f_j(x) = \frac{(H_{fj} - 1)^{(H_{fj}-1)/H_{fj}}(H_{fj} + 1)^{(H_{fj}+1)/H_{fj}}}{4\gamma_j H_{fj}} > 0.$$

Let (m^*, p^*) is an equilibrium point of the GRN (3). Then we have

$$\begin{aligned} &-G_1m^* + H_1f(p^*) + w = 0, \\ &-G_2p^* + H_2m^* = 0. \end{aligned} \tag{4}$$

Shift equilibrium point (m^*, p^*) to the origin and let $x(\mathcal{T}) = m(\mathcal{T}) - m^*$, $y(\mathcal{T}) = p(\mathcal{T}) - p^*$. Therefore, Eqs. (3) will be rewritten as

$$\begin{aligned}
&\dot{x}(\mathcal{T}) = -G_1 x(\mathcal{T}) + H_1 g\bigl(y\bigl(\mathcal{T} - \xi(\mathcal{T})\bigr)\bigr),\\
&\dot{y}(\mathcal{T}) = -G_2 y(\mathcal{T}) + H_2 x\bigl(\mathcal{T} - \eta(\mathcal{T})\bigr),\\
&x_0 = x(\theta) = \psi(\theta), \quad y_0 = y(\theta) = \pi(\theta) \quad \forall \theta \in [-\varpi, 0],
\end{aligned}$$

where $x(\mathcal{T}) = (x_1(\mathcal{T}), x_2(\mathcal{T}), \dots, x_n(\mathcal{T}))^{\mathrm{T}}$, $y(\mathcal{T}) = (y_1(\mathcal{T}), y_2(\mathcal{T}), \dots, y_n(\mathcal{T}))^{\mathrm{T}}$, $g_j(y_j(\mathcal{T})) = f_j(y_j(\mathcal{T}) + p_j^*) - f_j(p^*)$, $\varpi = \max[\eta_2, \xi_2]$, the initial functions $\psi(\cdot)$ and $\pi(\cdot)$ are continuously differentiable on $[\varpi, 0]$.

Now, we discuss the following impulsive genetic regulatory networks with distributed delays and time-varying delays:

$$\begin{aligned}
&\dot{x}(\mathcal{T}) = -G_1 x(\mathcal{T}) + H_1 g\bigl(y\bigl(\mathcal{T} - \xi(\mathcal{T})\bigr)\bigr) + E_1 \int\limits_{\mathcal{T}-r(\mathcal{T})}^{\mathcal{T}} J\bigl(y(s)\bigr)\,\mathrm{d}s,\\
&\dot{y}(\mathcal{T}) = -G_2 y(\mathcal{T}) + H_2 x\bigl(\mathcal{T} - \eta(\mathcal{T})\bigr) + E_2 \int\limits_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}} x(s)\,\mathrm{d}s,\\
&x(\mathcal{T}_k) = \mathcal{D}_1 x(\mathcal{T}_k)^-, \quad y(\mathcal{T}_k) = \mathcal{D}_2 y(\mathcal{T}_k)^-, \quad k \in \mathbb{Z}^+,\\
&x_0 = x(\theta) = \psi(\theta), \quad y_0 = y(\theta) = \pi(\theta) \quad \forall \theta \in [-\varpi, 0].
\end{aligned} \tag{5}$$

$E_1 = \mathrm{diag}\{e_{11}, e_{12}, \dots, e_{1n}\}$ and $E_2 = \mathrm{diag}\{e_{21}, e_{22}, \dots, e_{2n}\}$ are weight matrices. The bounded function $r(\mathcal{T})$ and $l(\mathcal{T})$ represents the distributed delay of systems with $0 \leqslant r(\mathcal{T}) \leqslant \bar{r}$ and $0 \leqslant l(\mathcal{T}) \leqslant \bar{l}$. Here $\bar{r}$ and $\bar{l}$ are constants. $\hat{J}_1(y(\mathcal{T})) = (J_{11}(y_1(\mathcal{T})), \dots, J_{1n}(y_n(\mathcal{T})))^{\mathrm{T}}$ denotes the activation function, $\mathcal{T}_k$ denotes the sequence of time, which satisfies $0 < \mathcal{T}_0 < \mathcal{T}_1 < \cdots < \mathcal{T}_k < \mathcal{T}_{k-1} < \cdots$ and $\lim_{k\to\infty} \mathcal{T}_k = \infty$. The impulses are denoted by $x(\sqcup_k)$ and $y(\sqcup_k)$. $D_1, D_2 \in \mathbb{R}^n$ are the sudden change effects of the state of the above system.

Assumption 1. A monotonically increasing function $\hat{f}_i(\cdot)$, $i \in \{1, 2, \dots, n\}$, with saturation satisfies

$$0 \leqslant \frac{f_i(l_1) - f_i(l_2)}{l_1 - l_2} \leqslant q_i, \quad \hat{f}_i(0) = 0$$

for all $l_1, l_2 \in \mathbb{R}$ with $l_1 \neq l_2$, where q_i are known constants.

Assumption 2. $\eta(\mathcal{T})$ and $\xi(\mathcal{T})$ are the time-varying delays, which satisfy $0 \leqslant \eta_1 \leqslant \eta(\mathcal{T}) \leqslant \eta_2$, $0 \leqslant \xi_1 \leqslant \xi(\mathcal{T}) \leqslant \xi_2$, $\dot{\eta}(\mathcal{T}) \leqslant \lambda < \infty$, $\dot{\xi}(\mathcal{T}) \leqslant \delta < \infty$, where $0 \leqslant \eta_1 \leqslant \eta_2$, $0 \leqslant \xi_1 \leqslant \xi_2$, $\lambda > 0$ and $\delta > 0$.

Definition 1. If, for any $\epsilon > 0$, there is $\delta(\epsilon) > 0$, then system (5) is *stable* such that

$$\mathbf{E}\bigl\|\bigl(x^{\mathrm{T}}(\mathcal{T}), y^{\mathrm{T}}(\mathcal{T})\bigr)^{\mathrm{T}}\bigr\|^2 < \epsilon \quad \text{when} \quad \sup_{-\tau \leqslant \delta \leqslant 0} \mathbf{E}\bigl\|\varphi(s)\bigr\|^2 < \delta,$$

where $\varphi \in L^2([-\tau,0];\mathbb{R}^{2n})$. System (5) is *asymptotically stable* if

$$\lim_{t\to 0}\mathbf{E}\big\|(x^{\mathrm T}(\mathcal T),y^{\mathrm T})(\mathcal T)^{\mathrm T}\big\|^2=0.$$

Lemma 1 [Schur complement]. (*See* [25].) *Let Ξ_1, Ξ_2, Ξ_3 by constant matrices, where $\Xi_1=\Xi_1^{\mathrm T}$ and $0<\Xi_2=\Xi_2^{\mathrm T}$, then $\Xi_1+\Xi_3^{\mathrm T}\Xi_2^{-1}\Xi_3<0$ iff*

$$\begin{bmatrix}\hat\Xi_1 & \hat\Xi_3^{\mathrm T}\\ \hat\Xi_3 & -\hat\Xi_2\end{bmatrix}<0 \quad or \quad \begin{bmatrix}-\hat\Xi_2 & \hat\Xi_3\\ \hat\Xi_3^{\mathrm T} & \hat\Xi_1\end{bmatrix}<0.$$

Lemma 2 [Jensen's inequality]. (*See* [27].) *For any real matrix $W\in\mathbb{R}^{n\times n}$, $W=W^{\mathrm T}>0$, there exist a scalar $q>0$ and a function $\psi:[0,q]\to\mathbb{R}^n$ such that*

$$q\int\limits_q^0\psi^{\mathrm T}(s)W\psi(s)\,\mathrm ds\geqslant\Bigg(\int\limits_q^0\psi(s)\,\mathrm ds\Bigg)^{\mathrm T}W\Bigg(\int\limits_q^0\psi(s)\,\mathrm ds\Bigg).$$

3 Asymptotic stability criterion

In this portion, we discuss the asymptotic stability criterion for impulsive GRNs with distributed delays and time-varying delays by using matrix analysis techniques and Lyapunov stability theory.

Theorem 1. *With the help of Assumptions* 1 *and* 2*, for given positive scalars $\eta_2>\eta_1$, $\xi_2>\xi_1$, λ and δ, system* (5) *becomes globally asymptotically stable if there exists positive-definite matrices $R=[R_{ij}]_{6\times 6}$, P_i ($i=1,2,\dots,5$), Q_i ($i=1,2\dots,6$), S_i ($i=1,2,\dots,8$) and U_i ($i=1,\dots,4$), matrices Q_7, S_i ($i=9,10,11,12$), K_1, K_2, K_3, L_1, L_2, L_3, M_i ($i=1,\dots,4$) and positive definite diagonal matrices $\Omega=\mathrm{diag}\{z_{1i},z_{2i},\dots,z_{ni}\}$ ($i=1,2$) such that the following LMIs hold*:

$$F_{ik}^{\mathrm T}B_iF_{ik}-B_i<0,$$

$$\begin{bmatrix}Q_1 & Q_7\\ * & Q_2\end{bmatrix}\geqslant 0,\qquad \begin{bmatrix}S_1 & S_9\\ * & S_3\end{bmatrix}\geqslant 0,\qquad \begin{bmatrix}S_2 & S_{10}\\ * & S_4\end{bmatrix}\geqslant 0,\qquad \begin{bmatrix}S_5 & S_{11}\\ * & S_7\end{bmatrix}\geqslant 0,$$

$$\begin{bmatrix}S_6 & S_{12}\\ * & S_8\end{bmatrix}\geqslant 0,\qquad \Psi_i=\begin{bmatrix}\Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14}^{(i)}\\ * & \Psi_{22} & 0 & 0\\ * & * & \Psi_{33} & 0\\ * & * & * & \Psi_{44}\end{bmatrix}<0\quad (i=1,2,3,4)$$

with

$$\Psi_{11}=[\Omega_{ij}]_{21\times 21},\quad \Psi_{12}=[\Lambda_1N_1\ \ \Lambda_2N_2],\quad \Psi_{13}=\bigg[\frac{\eta_1^2}{2}M_1\ \ \eta_\sigma M_2\ \ \frac{\xi_1^2}{2}M_3\ \ \xi_\sigma M_4\bigg],$$

$$\Psi_{22}=\mathrm{diag}\{-N_1,\,-N_2\},\quad \Psi_{33}=\mathrm{diag}\bigg\{-\frac{\eta_1^2}{2}U_1,\,-\eta_\sigma U_2,\,-\frac{\xi_1^2}{2}U_3,\,-\xi_\sigma U_4\bigg\},$$

$$\Psi_{44}=\mathrm{diag}\{-\eta_1S_1,\,\xi_1S_5,\,-\eta_{12}S_2,\,\xi_{12}S_6\},\quad \Psi_{14}^{(1)}=[\eta_1K_1\ \ \xi_1L_1\ \ \eta_{12}K_2\ \ \xi_{12}L_2],$$

$$\Psi_{14}^{(2)} = [\eta_1 K_1 \;\; \xi_1 L_1 \;\; \eta_{12} K_2 \;\; \xi_{12} L_3], \quad \Psi_{14}^{(3)} = [\eta_1 K_1 \;\; \xi_1 L_1 \;\; \eta_{12} K_3 \;\; \xi_{12} L_2],$$
$$\Psi_{14}^{(4)} = [\eta_1 K_1 \;\; \xi_1 L_1 \;\; \eta_{12} K_3 \;\; \xi_{12} L_3],$$
$$\begin{aligned}\Omega_{1,1} = {}& -R_{11}G_1 - G_1^{\mathrm T}R_{11} + R_{13} + R_{13}^{\mathrm T} + P_2 + K_{11} + K_{11}^{\mathrm T} + \eta_1 S_3 \\ & + \eta_{12}S_4 + \eta_1 M_{11} + \eta_1 M_{11}^{\mathrm T} + \eta_{12}M_{12} + \eta_{12}M_{12}^{\mathrm T} - \eta_1 S_9^{\mathrm T}G_1 \\ & - \eta_1 G_1^{\mathrm T}S_9 - \eta_{12}S_{10}^{\mathrm T}G_1 - \eta_{12}G_1^{\mathrm T}S_{10},\end{aligned}$$
$$\Omega_{1,2} = -G_1^{\mathrm T}R_{12} + R_{23}^{\mathrm T} - R_{12}G_2 + R_{15},$$
$$\Omega_{1,3} = R_{12}H_2 + K_{21}^{\mathrm T} - K_{12} + K_{13} + \eta_1 M_{21}^{\mathrm T} + \eta_{12}M_{22}^{\mathrm T},$$
$$\Omega_{1,6} = R_{11}H_1 + \eta_1 S_9^{\mathrm T}H_1 + \eta_{12}S_{10}^{\mathrm T}H_1,$$
$$\Omega_{1,7} = -K_{11} + K_{12} - R_{13} + R_{14}, \quad \Omega_{1,8} = -K_{13} - R_{14},$$
$$\Omega_{1,9} = -R_{15} + R_{16}, \quad \Omega_{1,10} = -R_{16}, \quad \Omega_{1,15} = -G_1^{\mathrm T}R_{13} + R_{33} - \frac{1}{\eta_1}S_9 - M_{11},$$
$$\Omega_{1,19} = E_2R_1, \quad \Omega_{1,16} = -G_1^{\mathrm T}R_{14} + R_{34} - M_{12},$$
$$\Omega_{1,17} = -G_1^{\mathrm T}R_{15} + R_{35}, \quad \Omega_{1,18} = -G_1^{\mathrm T}R_{16} + R_{36}, \quad \Omega_{1,21} = E_1R_1,$$
$$\begin{aligned}\Omega_{2,2} = {}& -R_{22}G_2 - G_2^{\mathrm T}R_{22} + R_{25} + R_{25}^{\mathrm T} + Q_1 + Q_3 + L_{11} + L_{11}^{\mathrm T} + \xi_1 S_7 \\ & + \xi_{12}S_8 + \xi_1 M_{13} + \xi_1 M_{13}^{\mathrm T} + \eta_{12}M_{14} + \xi_{12}M_{14}^{\mathrm T} - \xi_1 S_{11}^{\mathrm T}G_2 \\ & - \xi_1 G_2^{\mathrm T}S_{11} - \eta_{12}S_{12}^{\mathrm T}G_2 - \eta_{12}G_2^{\mathrm T}S_{12},\end{aligned}$$
$$\Omega_{2,3} = R_{22}H_2 + \xi_1 S_{11}^{\mathrm T}H_2 + \xi_{12}S_{12}^{\mathrm T}H_2,$$
$$\Omega_{2,4} = L_{21}^{\mathrm T} - L_{12} + L_{13} + \xi_1 M_{23}^{\mathrm T} + \xi_{12}M_{24}^{\mathrm T},$$
$$\Omega_{2,5} = -G_2^{\mathrm T}\Omega + QZ_1 + Q_7, \quad \Omega_{2,6} = R_{12}^{\mathrm T}H_1, \quad \Omega_{2,7} = -R_{23} + R_{24},$$
$$\Omega_{2,8} = -R_{24}, \quad \Omega_{2,9} = -L_{11} + L_{12} - R_{25} + R_{26}, \quad \Omega_{2,10} = -L_{13} - R_{26},$$
$$\Omega_{2,15} = -G_2^{\mathrm T}R_{23} + R_{35}^{\mathrm T}, \quad \Omega_{2,16} = -G_2^{\mathrm T}R_{24} + R_{45}^{\mathrm T},$$
$$\Omega_{2,17} = -G_2^{\mathrm T}R_{25} - \frac{1}{\xi_1}S_{11} + R_{55} - M_{13}, \quad \Omega_{2,18} = -G_2^{\mathrm T}R_{26} + R_{56} - M_{14},$$
$$\Omega_{2,19} = E_2R_2, \quad \Omega_{2,21} = E_1R_2, \quad \Omega_{3,3} = -(1-\lambda)P_1 - K_{22} - K_{22}^{\mathrm T} + K_{23} + K_{23}^{\mathrm T},$$
$$\Omega_{3,5} = H_2^{\mathrm T}Q, \quad \Omega_{3,7} = -K_{21} + K_{22}, \quad \Omega_{3,8} = -K_{23}, \quad \Omega_{3,15} = H_2^{\mathrm T}R_{23} - M_{21},$$
$$\Omega_{3,16} = H_2^{\mathrm T}R_{24} - M_{22}, \quad \Omega_{3,17} = H_2^{\mathrm T}R_{25}, \quad \Omega_{3,18} = H_2^{\mathrm T}R_{26},$$
$$\Omega_{4,4} = -(1-\delta)Q_1 - L_{22} - L_{22}^{\mathrm T} + L_{23} + L_{23}^{\mathrm T}, \quad \Omega_{4,6} = QZ_2 - (1-\delta)Q_7,$$
$$\Omega_{4,9} = -L_{21} + L_{22}, \quad \Omega_{4,10} = -L_{23}, \quad \Omega_{4,17} = -M_{23}, \quad \Omega_{4,18} = -M_{24},$$
$$\Omega_{5,5} = Q_2 - 2Z_1, \quad \Omega_{5,19} = \Omega E_2, \quad \Omega_{6,6} = -(1-\delta)Q_2 - 2Z_2,$$
$$\Omega_{6,16} = H_1^{\mathrm T}R_{14}, \quad \Omega_{6,15} = H_1^{\mathrm T}R_{13}, \quad \Omega_{6,17} = H_1^{\mathrm T}R_{15}, \quad \Omega_{6,18} = H_1^{\mathrm T}R_{16},$$
$$\Omega_{7,7} = -(P_2 - P_1 - P_3), \quad \Omega_{7,15} = -R_{33} + R_{34} + \frac{1}{\eta_1}S_9,$$
$$\Omega_{7,16} = -R_{34} + R_{44} - \frac{1}{\eta_{12}}S_{10}, \quad \Omega_{7,18} = -R_{36} + R_{46}, \quad \Omega_{8,8} = -P_3,$$

$$\Omega_{8,15} = -R_{34}, \quad \Omega_{8,16} = -R_{44} + \frac{1}{\eta_{12}} S_{10}, \quad \Omega_{8,17} = -R_{45},$$
$$\Omega_{8,18} = -R_{46}, \quad \Omega_{9,9} = -(Q_3 - Q_4), \quad \Omega_{9,15} = -R_{35}^{\mathrm{T}} + R_{36}^{\mathrm{T}},$$
$$\Omega_{9,16} = -R_{45}^{\mathrm{T}} + R_{46}^{\mathrm{T}}, \quad \Omega_{9,17} = -R_{55}^{\mathrm{T}} + R_{56}^{\mathrm{T}} + \frac{1}{\xi_1} S_{11},$$
$$\Omega_{9,18} = -R_{56}^{\mathrm{T}} + R_{66}^{\mathrm{T}} + \frac{1}{\xi_{12}} S_{12}, \quad \Omega_{10,10} = -Q_4, \quad \Omega_{10,15} = -R_{36}^{\mathrm{T}},$$
$$\Omega_{10,16} = -R_{46}^{\mathrm{T}}, \quad \Omega_{10,17} = -R_{56}^{\mathrm{T}}, \quad \Omega_{10,18} = -R_{66} + \frac{1}{\xi_{12}} S_{12},$$
$$\Omega_{11,11} = -(P_4 - P_5), \quad \Omega_{12,12} = -P_5, \quad \Omega_{13,13} = -(Q_5 - Q_6),$$
$$\Omega_{14,14} = -Q_6, \quad \Omega_{15,15} = -\frac{1}{\eta_1} S_3, \quad \Omega_{15,19} = E_2 R_3, \quad \Omega_{15,21} = E_1 R_3,$$
$$\Omega_{16,16} = -\frac{1}{\eta_{12}} S_4, \quad \Omega_{17,17} = -\frac{1}{\xi_1} S_7, \quad \Omega_{16,19} = E_2 R_4, \quad \Omega_{16,21} = E_1 R_4,$$
$$\Omega_{17,19} = E_2 R_5, \quad \Omega_{17,21} = E_1 R_5, \quad \Omega_{18,18} = -\frac{1}{\xi_{12}} S_8, \quad \Omega_{18,19} = E_2 R_6,$$
$$\Omega_{18,21} = E_1 R_6, \quad \Omega_{19,19} = \frac{1}{l(\mathcal{T})} W_1, \quad \Omega_{20,20} = \bar{r} W_2, \quad \Omega_{21,21} = -\frac{1}{r(\mathcal{T})} W_2,$$
$$N_1 = P_4 + \eta_1 S_1 + \eta_{12} S_2 + \frac{\eta_1^2}{2} U_1 + \eta_\sigma U_2,$$
$$N_2 = Q_5 + \xi_1 S_5 + \xi_{12} S_6 + \frac{\xi_1^2}{2} U_3 + \xi_\sigma U_4,$$
$$\Lambda_1 = [-G_1 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ H_1 \ \underbrace{0 \ \ldots \ 0}_{14} \ E_1]^{\mathrm{T}}, \quad \Lambda_2 = [0 \ \ -G_2 \ \ H_2 \ \underbrace{0 \ \ldots \ 0}_{15} \ E_2 \ \ 0 \ \ 0]^{\mathrm{T}},$$
$$M_i = \begin{cases} [M_{1i}^{\mathrm{T}} \ \ 0 \ \ M_{2i}^{\mathrm{T}} \ \underbrace{0 \ \ldots \ 0}_{18}]^{\mathrm{T}} & (i = 1, 2), \\ [0 \ \ M_{1i}^{\mathrm{T}} \ \ 0 \ \ M_{2i}^{\mathrm{T}} \ \underbrace{0 \ \ldots \ 0}_{18}]^{\mathrm{T}} & (i = 3, 4), \end{cases}$$
$$K_i = \bigl[K_{1i}^{\mathrm{T}} \ \ 0 \ \ K_{2i}^{\mathrm{T}} \ \underbrace{0 \ \ldots \ 0}_{18}\bigr]^{\mathrm{T}}, \quad L_i = \bigl[0 \ \ L_{1i}^{\mathrm{T}} \ \ 0 \ \ L_{2i}^{\mathrm{T}} \ \underbrace{0 \ \ldots \ 0}_{17}\bigr]^{\mathrm{T}} \quad (i = 1, 2, 3),$$
$$\eta_{12} = \eta_2 - \eta_1, \quad \eta_\sigma = \frac{\eta_2^2 - \eta_1^2}{2}, \quad \xi_{12} = \xi_2 - \xi_1, \quad \xi_\sigma = \frac{\xi_2^2 - \xi_1^2}{2}.$$

Proof. Consider the following Lyapunov functional:

$$V(\mathcal{T}) = \sum_{i=1}^{6} V_i(\mathcal{T}), \tag{6}$$

where

$$V_1(\mathcal{T}) = \zeta^{\mathrm{T}}(\mathcal{T}) R \zeta(\mathcal{T}) + 2 \sum_{i=1}^{n} \mu_i \int_0^{y_i(\mathcal{T})} g_i(s) \, \mathrm{d}s,$$

$$V_2(\mathcal{T}) = \int_{\mathcal{T}-\eta(\mathcal{T})}^{\mathcal{T}-\eta_1} x^{\mathrm{T}}(s)P_1x(s)\,\mathrm{d}s + \int_{\mathcal{T}-\eta_1}^{\mathcal{T}} x^{\mathrm{T}}(s)P_2x(s)\,\mathrm{d}s + \int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} x^{\mathrm{T}}(s)P_3x(s)\,\mathrm{d}s + \int_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)P_4\dot{x}(s)\,\mathrm{d}s + \int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} \dot{x}^{\mathrm{T}}(s)P_5\dot{x}(s)\,\mathrm{d}s,$$

$$V_3(\mathcal{T}) = \int_{\mathcal{T}-\xi(\mathcal{T})}^{\mathcal{T}} \begin{bmatrix} y(s) \\ g(y(s)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_1 & Q_7 \\ * & Q_2 \end{bmatrix} \begin{bmatrix} y(s) \\ g(y(s)) \end{bmatrix} \mathrm{d}s + \int_{\mathcal{T}-\xi_1}^{\mathcal{T}} y^{\mathrm{T}}(s)Q_3y(s)\,\mathrm{d}s + \int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} y^{\mathrm{T}}(s)Q_4y(s)\,\mathrm{d}s + \int_{\mathcal{T}-\xi_1}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)Q_5\dot{y}(s)\,\mathrm{d}s + \int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} \dot{y}^{\mathrm{T}}(s)Q_6\dot{y}(s)\,\mathrm{d}s,$$

$$\begin{aligned} V_4(\mathcal{T}) = {} & \int_{-\eta_1}^{0} \int_{\mathcal{T}+\theta}^{\mathcal{T}} \begin{bmatrix} \dot{x}(s) \\ x(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1 & S_9 \\ * & S_3 \end{bmatrix} \begin{bmatrix} \dot{x}(s) \\ x(s) \end{bmatrix} \mathrm{d}s\,\mathrm{d}\theta \\ & + \int_{-\eta_2}^{-\eta_1} \int_{\mathcal{T}+\theta}^{\mathcal{T}} \begin{bmatrix} \dot{x}(s) \\ x(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_2 & S_{10} \\ * & S_4 \end{bmatrix} \begin{bmatrix} \dot{x}(s) \\ x(s) \end{bmatrix} \mathrm{d}s\,\mathrm{d}\theta \\ & + \int_{-\xi_1}^{0} \int_{\mathcal{T}+\theta}^{\mathcal{T}} \begin{bmatrix} \dot{y}(s) \\ y(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_5 & S_{11} \\ * & S_7 \end{bmatrix} \begin{bmatrix} \dot{y}(s) \\ y(s) \end{bmatrix} \mathrm{d}s\,\mathrm{d}\theta \\ & + \int_{-\xi_2}^{-\xi_1} \int_{\mathcal{T}+\theta}^{\mathcal{T}} \begin{bmatrix} \dot{y}(s) \\ y(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_6 & S_{12} \\ * & S_8 \end{bmatrix} \begin{bmatrix} \dot{y}(s) \\ y(s) \end{bmatrix} \mathrm{d}s\,\mathrm{d}\theta, \end{aligned}$$

$$\begin{aligned} V_5(\mathcal{T}) = {} & \int_{-\eta_1}^{0} \int_{\theta}^{0} \int_{\mathcal{T}+\mu}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_1\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\mu\,\mathrm{d}\theta + \int_{-\eta_2}^{-\eta_1} \int_{\theta}^{0} \int_{\mathcal{T}+\mu}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_2\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\mu\,\mathrm{d}\theta \\ & + \int_{-\xi_1}^{0} \int_{\theta}^{0} \int_{\mathcal{T}+\mu}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_3\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\mu\,\mathrm{d}\theta + \int_{-\xi_2}^{-\xi_1} \int_{\theta}^{0} \int_{\mathcal{T}+\mu}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_4\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\mu\,\mathrm{d}\theta, \end{aligned}$$

$$V_6(\mathcal{T}) = \int_{-l(\mathcal{T})}^{0} \int_{\mathcal{T}+\theta}^{\mathcal{T}} x^{\mathrm{T}}(s)W_1x(s)\,\mathrm{d}s\,\mathrm{d}\theta + \int_{-r(\mathcal{T})}^{0} \int_{\mathcal{T}+\theta}^{\mathcal{T}} J^{\mathrm{T}}(y(s))W_2J(y(s))\,\mathrm{d}s\,\mathrm{d}\theta$$

$$\zeta(\mathcal{T}) = \left\{ x(\mathcal{T}),\ y(\mathcal{T}), \int_{\mathcal{T}-\eta_1}^{\mathcal{T}} x(s)\,\mathrm{d}s, \int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} x(s)\,\mathrm{d}s, \int_{\mathcal{T}-\xi_1}^{\mathcal{T}} y(s)\,\mathrm{d}s, \int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} y(s)\,\mathrm{d}s \right\}^{\mathrm{T}}.$$

Here $R = [R_{ij}]_{6\times 6} > 0$, $P_i > 0$ $(i = 1, 2, \dots, 5)$, $Q_i > 0$ $(i = 1, 2, \dots, 6)$, $S_i > 0$ $(i = 1, 2, \dots, 8)$ and $U_i > 0$ $(i = 1, 2, ..4)$, Q_7 and S_i $(i = 9, 10, 11, 12)$ are the matrices to be determined.

Calculating $\dot{V}(x(\mathcal{T}), y(\mathcal{T}), t)$ along the solutions of (5), we have

$$\begin{aligned}
\dot{V}_1(\mathcal{T}) &= 2\zeta^{\mathrm{T}}(\mathcal{T})R\dot{\zeta}(\mathcal{T}) + 2\sum_{i=1}^{n}\mu_i g_i\big(y_i(\mathcal{T})\big)\dot{y}_i(\mathcal{T}) \\
&= 2\zeta^{\mathrm{T}}(\mathcal{T})R\begin{bmatrix} \dot{x}(\mathcal{T}) \\ \dot{y}(\mathcal{T}) \\ x(\mathcal{T}) - x(\mathcal{T}-\eta_1) \\ x(\mathcal{T}-\eta_1) - x(\mathcal{T}-\eta_2) \\ y(\mathcal{T}) - y(\mathcal{T}-\xi_1) \\ y(\mathcal{T}-\xi_1) - y(\mathcal{T}-\xi_2) \end{bmatrix} + 2g^{\mathrm{T}}\big(y(\mathcal{T})\big)\Omega\dot{y}(\mathcal{T}),
\end{aligned}$$

$$\begin{aligned}
\dot{V}_2(\mathcal{T}) &= x^{\mathrm{T}}(\mathcal{T}-\eta_1)P_1x(\mathcal{T}-\eta_1) - \big(1-\dot{\eta}(\mathcal{T})\big)x^{\mathrm{T}}\big(\mathcal{T}-\eta(\mathcal{T})\big)P_1x\big(\mathcal{T}-\eta(\mathcal{T})\big) \\
&\quad + x^{\mathrm{T}}(\mathcal{T})P_2x(\mathcal{T}) - x^{\mathrm{T}}(\mathcal{T}-\eta_1)P_2x(\mathcal{T}-\eta_1) + x^{\mathrm{T}}(\mathcal{T}-\eta_1)P_3x(\mathcal{T}-\eta_1) \\
&\quad - x^{\mathrm{T}}(\mathcal{T}-\eta_2)P_3x(\mathcal{T}-\eta_2) + \dot{x}^{\mathrm{T}}(\mathcal{T})P_4\dot{x}(\mathcal{T}) - \dot{x}^{\mathrm{T}}(\mathcal{T}-\eta_1)P_5\dot{x}(\mathcal{T}-\eta_1) \\
&\quad - \dot{x}^{\mathrm{T}}(\mathcal{T}-\eta_2)P_5\dot{x}(\mathcal{T}-\eta_2),
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3(\mathcal{T}) &= -(1-\dot{\xi}(\mathcal{T}))\begin{bmatrix} y(\mathcal{T}-\xi(\mathcal{T})) \\ g(y(\mathcal{T}-\xi(\mathcal{T}))) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} Q_1 & Q_7 \\ * & Q_2 \end{bmatrix}\begin{bmatrix} y(\mathcal{T}-\xi(\mathcal{T})) \\ g(y(\mathcal{T}-\xi(\mathcal{T}))) \end{bmatrix} \\
&\quad + \begin{bmatrix} y(\mathcal{T}) \\ g(y(\mathcal{T})) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} Q_1 & Q_7 \\ * & Q_2 \end{bmatrix}\begin{bmatrix} y(\mathcal{T}) \\ g(y(\mathcal{T})) \end{bmatrix} \\
&\quad - y^{\mathrm{T}}(\mathcal{T}-\xi_1)Q_3y(\mathcal{T}-\xi_1) + y^{\mathrm{T}}(\mathcal{T})Q_3y(\mathcal{T}) + y^{\mathrm{T}}(\mathcal{T}-\xi_1)Q_4y(\mathcal{T}-\xi_1) \\
&\quad - y^{\mathrm{T}}(\mathcal{T}-\xi_2)Q_4y(\mathcal{T}-\xi_2) + \dot{y}^{\mathrm{T}}(\mathcal{T})Q_5\dot{y}(\mathcal{T}) - \dot{y}^{\mathrm{T}}(\mathcal{T}-\xi_1)Q_5\dot{y}(\mathcal{T}-\xi_1) \\
&\quad + \dot{y}^{\mathrm{T}}(\mathcal{T}-\xi_1)Q_6\dot{y}(\mathcal{T}-\xi_1) - \dot{y}^{\mathrm{T}}(\mathcal{T}-\xi_2)Q_6\dot{y}(\mathcal{T}-\xi_2),
\end{aligned}$$

$$\begin{aligned}
\dot{V}_4(\mathcal{T}) &= \dot{x}^{\mathrm{T}}(\mathcal{T})(\eta_1S_1 + \eta_{12}S_2)\dot{x}(\mathcal{T}) - \int_{\mathcal{T}-\eta_1}^{\mathcal{T}}\dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)\,\mathrm{d}s \\
&\quad - \int_{\mathcal{T}-\eta(\mathcal{T})}^{\mathcal{T}-\eta_1}\dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)\,\mathrm{d}s - \int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta(\mathcal{T})}\dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)\,\mathrm{d}s \\
&\quad + x^{\mathrm{T}}(\mathcal{T})(\eta_1S_3 + \eta_{12}S_4)x(\mathcal{T}) - \int_{\mathcal{T}-\eta_1}^{\mathcal{T}}x^{\mathrm{T}}(s)S_3x(s)\,\mathrm{d}s \\
&\quad - \int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1}x^{\mathrm{T}}(s)S_4x(s)\,\mathrm{d}s + 2\dot{x}^{\mathrm{T}}(\mathcal{T})(\eta_1S_9 + \eta_{12}S_{10})x(\mathcal{T})
\end{aligned}$$

$$
\begin{aligned}
&- 2\int\limits_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)S_9x(s)\,\mathrm{d}s - 2\int\limits_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} \dot{x}^{\mathrm{T}}(s)S_{10}x(s)\,\mathrm{d}s \\
&+ \dot{y}^{\mathrm{T}}(\mathcal{T})(\xi_1 S_5 + \xi_{12} S_6)\dot{y}(\mathcal{T}) - \int\limits_{\mathcal{T}-\xi_1}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)S_5\dot{y}(s)\,\mathrm{d}s \\
&- \int\limits_{\mathcal{T}-\xi(\mathcal{T})}^{\mathcal{T}-\xi_1} \dot{y}^{\mathrm{T}}(s)S_6\dot{y}(s)\,\mathrm{d}s - \int\limits_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi(\mathcal{T})} \dot{y}^{\mathrm{T}}(s)S_6\dot{y}(s)\,\mathrm{d}s \\
&+ y^{\mathrm{T}}(\mathcal{T})(\xi_1 S_7 + \xi_{12} S_8)y(\mathcal{T}) - \int\limits_{\mathcal{T}-\xi_1}^{\mathcal{T}} y^{\mathrm{T}}(s)S_7 y(s)\,\mathrm{d}s \\
&- \int\limits_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} y^{\mathrm{T}}(s)S_8 y(s)\,\mathrm{d}s + 2\dot{y}^{\mathrm{T}}(\mathcal{T})(\xi_1 S_{11} + \xi_{12} S_{12})y(\mathcal{T}) \\
&- 2\int\limits_{\mathcal{T}-\xi_1}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)S_{11}y(s)\,\mathrm{d}s - 2\int\limits_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} \dot{y}^{\mathrm{T}}(s)S_{12}y(s)\,\mathrm{d}s,
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_5(\mathcal{T}) ={}& \frac{\eta_1^2}{2}\dot{x}^{\mathrm{T}}(\mathcal{T})U_1\dot{x}(\mathcal{T}) - \int\limits_{-\eta_1}^{0}\int\limits_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_1\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta + \eta_\sigma \dot{x}^{\mathrm{T}}(\mathcal{T})U_2\dot{x}(\mathcal{T}) \\
&- \int\limits_{-\eta_2}^{-\eta_1}\int\limits_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_2\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta + \frac{\xi_1^2}{2}\dot{y}^{\mathrm{T}}(\mathcal{T})U_3\dot{y}(\mathcal{T}) \\
&- \int\limits_{-\xi_1}^{0}\int\limits_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_3\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta + \xi_\sigma \dot{y}^{\mathrm{T}}(\mathcal{T})U_4\dot{y}(\mathcal{T}) \\
&- \int\limits_{-\xi_2}^{-\xi_1}\int\limits_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_4\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta,
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_6(\mathcal{T}) ={}& \bar{l}x^{\mathrm{T}}(\mathcal{T})W_1x(\mathcal{T}) - \int\limits_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}} x^{\mathrm{T}}(s)W_1x(s)\,\mathrm{d}s + \bar{r}J^{\mathrm{T}}\big(y(\mathcal{T})\big)W_2J\big(y(\mathcal{T})\big) \\
&- \int\limits_{\mathcal{T}-r(\mathcal{T})}^{\mathcal{T}} J^{\mathrm{T}}\big(y(s)\big)W_2J\big(y(s)\big)\,\mathrm{d}s.
\end{aligned}
$$

Using Assumption 1 and Lemma 1, we get

$$\dot V(\mathcal{T}) \leqslant 2\zeta^{\mathrm{T}}(\mathcal{T})R\begin{bmatrix} -G_1x(\mathcal{T}) + H_1g(y(\mathcal{T}-\xi(\mathcal{T}))) + E_1\int\limits_{\mathcal{T}-r(\mathcal{T})}^{\mathcal{T}} J(y(s))\,\mathrm{d}s \\ -G_2y(\mathcal{T}) + H_2x(\mathcal{T}-\eta(\mathcal{T})) + E_2\int\limits_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}} x(s)\,\mathrm{d}s \\ x(\mathcal{T}) - x(\mathcal{T}-\eta_1) \\ x(\mathcal{T}-\eta_1) - x(\mathcal{T}-\eta_2) \\ y(\mathcal{T}) - y(\mathcal{T}-\xi_1) \\ y(\mathcal{T}-\xi_1) - y(\mathcal{T}-\xi_2) \end{bmatrix}$$

$$\begin{aligned}
&\times 2g^{\mathrm{T}}\big(y(\mathcal{T})\big)\Omega\left(-G_2y(\mathcal{T}) + H_2x\big(\mathcal{T}-\eta(\mathcal{T})\big) + E_2\int\limits_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}} x(s)\,\mathrm{d}s\right) \\
&+ x^{\mathrm{T}}(\mathcal{T})\big[P_2 + \eta_1S_3 + \eta_{12}S_4 - \eta_1S_9G_1 - \eta_1G_1^{\mathrm{T}}S_9^{\mathrm{T}} \\
&- \eta_{12}S_{10}G_1 - \eta_{12}G_1^{\mathrm{T}}S_{10}^{\mathrm{T}} + \bar lW_1\big]x(\mathcal{T}) \\
&+ y^{\mathrm{T}}(\mathcal{T})\big[Q_3 + \xi_1S_7 + \xi_{12}S_8 - \xi_1S_{11}G_2 - \xi_1G_2^{\mathrm{T}}S_{11}^{\mathrm{T}} \\
&- \xi_{12}S_{12}G_2 - \xi_{12}G_2^{\mathrm{T}}S_{12}^{\mathrm{T}}\big]y(\mathcal{T}) \\
&+ \dot x^{\mathrm{T}}(\mathcal{T})\big[P_4 + \eta_1S_1 + \eta_{12}S_2 + \frac{\eta_1^2}{2}U_1 + \eta_\sigma U_2\big]\dot x(\mathcal{T}) \\
&+ \dot y^{\mathrm{T}}(\mathcal{T})\big[Q_5 + \xi_1S_5 + \xi_{12}S_6 + \frac{\xi_1^2}{2}U_3 + \xi_\sigma U_4\big]\dot y(\mathcal{T}) \\
&- (1-\lambda)x^{\mathrm{T}}\big(\mathcal{T}-\eta(\mathcal{T})\big)P_1x\big(\mathcal{T}-\eta(\mathcal{T})\big) \\
&- x^{\mathrm{T}}(\mathcal{T}-\eta_1)(P_2-P_1-P_3)x(\mathcal{T}-\eta_1) - x^{\mathrm{T}}(\mathcal{T}-\eta_2)P_3x(\mathcal{T}-\eta_2) \\
&- \dot x^{\mathrm{T}}(\mathcal{T}-\eta_2)P_5\dot x(\mathcal{T}-\eta_2) - \dot x^{\mathrm{T}}(\mathcal{T}-\eta_1)(P_4-P_5)\dot x(\mathcal{T}-\eta_1) \\
&- (1-\delta)\begin{bmatrix} y(\mathcal{T}-\xi(\mathcal{T})) \\ g(y(\mathcal{T}-\xi(\mathcal{T}))) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_1 & Q_7 \\ * & Q_2 \end{bmatrix} \begin{bmatrix} y(\mathcal{T}-\xi(\mathcal{T})) \\ g(y(\mathcal{T}-\xi(\mathcal{T}))) \end{bmatrix} \\
&+ \begin{bmatrix} y(\mathcal{T}) \\ g(y(\mathcal{T})) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q_1 & Q_7 \\ * & Q_2 \end{bmatrix} \begin{bmatrix} y(\mathcal{T}) \\ g(y(\mathcal{T})) \end{bmatrix} \\
&- y^{\mathrm{T}}(\mathcal{T}-\xi_1)(Q_3-Q_4)y(\mathcal{T}-\xi_1) - y^{\mathrm{T}}(\mathcal{T}-\xi_2)Q_4y(\mathcal{T}-\xi_2) \\
&- \dot y^{\mathrm{T}}(\mathcal{T}-\xi_1)(Q_5-Q_5)\dot y(\mathcal{T}-\xi_1) - \dot y^{\mathrm{T}}(\mathcal{T}-\xi_2)Q_6\dot y(\mathcal{T}-\xi_2) \\
&- \int\limits_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot x^{\mathrm{T}}(s)S_1\dot x(s)\,\mathrm{d}s - \int\limits_{\mathcal{T}-\eta(\mathcal{T})}^{\mathcal{T}-\eta_1} \dot x^{\mathrm{T}}(s)S_2\dot x(s)\,\mathrm{d}s - \int\limits_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta(\mathcal{T})} \dot x^{\mathrm{T}}(s)S_2\dot x(s)\,\mathrm{d}s \\
&- \int\limits_{\mathcal{T}-\eta_1}^{\mathcal{T}} x^{\mathrm{T}}(s)S_3x(s)\,\mathrm{d}s - \int\limits_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} x^{\mathrm{T}}(s)S_4x(s)\,\mathrm{d}s - 2\int\limits_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot x^{\mathrm{T}}(s)S_9x(s)\,\mathrm{d}s
\end{aligned}$$

$$
\begin{aligned}
&-2\int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} \dot{x}^{\mathrm{T}}(s)S_{10}x(s)\,\mathrm{d}s - \int_{\mathcal{T}-\xi_1}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)S_5\dot{y}(s)\,\mathrm{d}s - \int_{\mathcal{T}-\xi(\mathcal{T})}^{\mathcal{T}-\xi_1} \dot{y}^{\mathrm{T}}(s)S_6\dot{y}(s)\,\mathrm{d}s \\
&- \int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi(\mathcal{T})} \dot{y}^{\mathrm{T}}(s)S_6\dot{y}(s)\,\mathrm{d}s - \int_{\mathcal{T}-\xi_1}^{\mathcal{T}} y^{\mathrm{T}}(s)S_7y(s)\,\mathrm{d}s - \int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} y^{\mathrm{T}}(s)S_8y(s)\,\mathrm{d}s \\
&- 2\int_{\mathcal{T}-\xi_1}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)S_{11}y(s)\,\mathrm{d}s - 2\int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} \dot{y}^{\mathrm{T}}(s)S_{12}y(s)\,\mathrm{d}s - \int_{-\eta_1}^{0}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_1\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&- \int_{-\eta_2}^{-\eta_1}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_2\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta - \int_{-\xi_1}^{0}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_3\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&- \int_{-\xi_2}^{-\xi_1}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_4\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta - \int_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}} x^{\mathrm{T}}(s)W_1x(s)\,\mathrm{d}s + \bar{r}J^{\mathrm{T}}\big(y(\mathcal{T})\big)W_2J\big(y(\mathcal{T})\big) \\
&- \int_{\mathcal{T}-r(\mathcal{T})}^{\mathcal{T}} J^{\mathrm{T}}\big(y(s)\big)W_2J\big(y(s)\big)\,\mathrm{d}s. \qquad (7)
\end{aligned}
$$

Using Jensen's inequality, we get

$$
\begin{aligned}
&-\int_{\mathcal{T}-\eta_1}^{\mathcal{T}} x^{\mathrm{T}}(s)S_3x(s)\,\mathrm{d}s \leqslant -\frac{1}{\eta_1}\int_{\mathcal{T}-\eta_1}^{\mathcal{T}} x^{\mathrm{T}}(s)\,\mathrm{d}s\cdot S_3\int_{\mathcal{T}-\eta_1}^{\mathcal{T}} x(s)\,\mathrm{d}s, \\
&-\int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} x^{\mathrm{T}}(s)S_4x(s)\,\mathrm{d}s \leqslant -\frac{1}{\eta_{12}}\int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} x^{\mathrm{T}}(s)\,\mathrm{d}s\cdot S_4\int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} x(s)\,\mathrm{d}s, \\
&-\int_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)\cdot S_9x(s)\,\mathrm{d}s \leqslant -\frac{1}{\eta_1}\big[x^{\mathrm{T}}(\mathcal{T}) - x^{\mathrm{T}}(\mathcal{T}-\eta_1)\big]\,S_9\int_{\mathcal{T}-\eta_1}^{\mathcal{T}} x(s)\,\mathrm{d}s, \\
&-\int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} \dot{x}^{\mathrm{T}}(s)S_{10}x(s)\,\mathrm{d}s \leqslant -\frac{1}{\eta_{12}}\big[x^{\mathrm{T}}(\mathcal{T}-\eta_1) - x^{\mathrm{T}}(\mathcal{T}-\eta_2)\big]\,S_{10}\int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} x(s)\,\mathrm{d}s, \\
&-\int_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}} x^{\mathrm{T}}(s)W_1x(s)\,\mathrm{d}s \leqslant -\frac{1}{l(\mathcal{T})}\int_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}} x^{\mathrm{T}}(s)\,\mathrm{d}s\cdot W_1\int_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}} x(s)\,\mathrm{d}s,
\end{aligned}
$$

$$
\begin{aligned}
-\int_{\mathcal{T}-\xi_1}^{\mathcal{T}} y^{\mathrm{T}}(s) S_7 y(s)\,\mathrm{d}s &\leqslant -\frac{1}{\xi_1}\int_{\mathcal{T}-\xi_1}^{\mathcal{T}} y^{\mathrm{T}}(s)\,\mathrm{d}s \cdot S_7 \int_{\mathcal{T}-\xi_1}^{\mathcal{T}} y(s)\,\mathrm{d}s,\\
-\int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} y^{\mathrm{T}}(s) S_8 y(s)\,\mathrm{d}s &\leqslant -\frac{1}{\xi_1}\int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} y^{\mathrm{T}}(s)\,\mathrm{d}s \cdot S_8 \int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} y(s)\,\mathrm{d}s,\\
-\int_{\mathcal{T}-\xi_1}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s) S_{11} y(s)\,\mathrm{d}s &\leqslant -\frac{1}{\xi_1}\big[y^{\mathrm{T}}(\mathcal{T}) - y^{\mathrm{T}}(\mathcal{T}-\xi_2)\big]\ S_{11}\int_{\mathcal{T}-\xi_1}^{\mathcal{T}} y(s)\,\mathrm{d}s,\\
-\int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} \dot{y}^{\mathrm{T}}(s) S_{12} y(s)\,\mathrm{d}s &\leqslant -\frac{1}{\xi_{12}}\big[y^{\mathrm{T}}(\mathcal{T}-\xi_1) - y^{\mathrm{T}}(\mathcal{T}-\xi_2)\big]\ S_{12}\int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_2} y(s)\,\mathrm{d}s,\\
-\int_{\mathcal{T}-r(\mathcal{T})}^{\mathcal{T}} J^{\mathrm{T}}\big(y(s)\big) W_2 J\big(y(s)\big)\,\mathrm{d}s &\leqslant -\frac{1}{r(\mathcal{T})}\int_{\mathcal{T}-r(\mathcal{T})}^{\mathcal{T}} J^{\mathrm{T}}\big(y(s)\big)\,\mathrm{d}s \cdot W_2 \int_{\mathcal{T}-r(\mathcal{T})}^{\mathcal{T}} J\big(y(s)\big)\,\mathrm{d}s.
\end{aligned}
$$

Using Assumption 1, for $i = 1, 2, \dots, n$, we get

$$
\begin{gathered}
g_i(y_i(\mathcal{T}))\big[g_i\big(y_i(\mathcal{T})\big) - q_i y_i(\mathcal{T})\big] \leqslant 0,\\
g_i\big(y_i\big(\mathcal{T}-\xi(\mathcal{T})\big)\big)\big[g_i\big(y_i\big(\mathcal{T}-\xi(\mathcal{T})\big)\big) - q_i y_i\big(\mathcal{T}-\xi(\mathcal{T})\big)\big] \leqslant 0.
\end{gathered}
$$

Then, for any positive definite diagonal matrices, $Z_i = \operatorname{diag}\{z_{1i}, z_{2i}, \dots z_{ni}\}$ $(i = 1, 2)$, we have

$$
\begin{aligned}
0 &\leqslant -2\sum_{i=1}^{n} z_{i1} g_i\big(y_i(\mathcal{T})\big)\big[g_i\big(y_i(\mathcal{T})\big) - q_i y_i(\mathcal{T})\big]\\
&\quad - 2\sum_{i=1}^{n} z_{i2} g_i\big(y_i\big(\mathcal{T}-\xi(\mathcal{T})\big)\big)\big[g_i\big(y_i\big(\mathcal{T}-\xi(\mathcal{T})\big)\big) - q_i y_i\big(\mathcal{T}-\xi(\mathcal{T})\big)\big]\\
&= 2y^{\mathrm{T}}(\mathcal{T}) Q Z_1 g\big(t(\mathcal{T})\big) - 2g^{\mathrm{T}}\big(y(\mathcal{T})\big) Z_1 g\big(y(\mathcal{T})\big) + 2g^{\mathrm{T}}\big(\mathcal{T}-\xi(\mathcal{T})\big) Q\\
&\quad \times Z_2 g\big(y\big(\mathcal{T}-\xi(\mathcal{T})\big)\big) - 2g^{\mathrm{T}}\big(y\big(\mathcal{T}-\xi(\mathcal{T})\big)\big) Z_2 g\big(y\big(\mathcal{T}-\xi(\mathcal{T})\big)\big),
\end{aligned}
\tag{8}
$$

where $Q = \operatorname{diag}\{q_1, q_2, \dots, q_n\}$.

According to the Newton–Leibniz formula, for any matrices K_1, K_2, K_3, L_1, L_2, L_3 and M_i $(i = 1, \dots, 4)$ with appropriate dimensions, the following equations hold:

$$
0 = 2\upsilon^{\mathrm{T}}(\mathcal{T}) K_1\Bigg[x(\mathcal{T}) - x(\mathcal{T}-\eta_1) - \int_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot{x}(s)\,\mathrm{d}s\Bigg], \tag{9}
$$

$$0 = 2\upsilon^{\mathrm{T}}(\mathcal{T})K_2\left[x(\mathcal{T}-\eta_1) - x\big(\mathcal{T}-\eta(\mathcal{T})\big) - \int\limits_{\mathcal{T}-\eta(\mathcal{T})}^{\mathcal{T}-\eta_1} \dot{x}(s)\,\mathrm{d}s\right], \tag{10}$$

$$0 = 2\upsilon^{\mathrm{T}}(\mathcal{T})K_3\left[x\big(\mathcal{T}-\eta(\mathcal{T})\big) - x(\mathcal{T}-\eta_2) - \int\limits_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta(\mathcal{T})} \dot{x}(s)\,\mathrm{d}s\right], \tag{11}$$

$$0 = 2\upsilon^{\mathrm{T}}(\mathcal{T})L_1\left[y(\mathcal{T}) - y(\mathcal{T}-\xi_1) - \int\limits_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot{y}(s)\,\mathrm{d}s\right], \tag{12}$$

$$0 = 2\upsilon^{\mathrm{T}}(\mathcal{T})L_2\left[y(\mathcal{T}-\xi_1) - y\big(\mathcal{T}-\xi(\mathcal{T})\big) - \int\limits_{\mathcal{T}-\xi(\mathcal{T})}^{\mathcal{T}-\xi_1} \dot{y}(s)\,\mathrm{d}s\right], \tag{13}$$

$$0 = 2\upsilon^{\mathrm{T}}(\mathcal{T})L_3\left[y\big(\mathcal{T}-\xi(\mathcal{T})\big) - y(\mathcal{T}-\xi_2) - \int\limits_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi(\mathcal{T})} \dot{y}(s)\,\mathrm{d}s\right], \tag{14}$$

$$0 = 2\upsilon^{\mathrm{T}}(\mathcal{T})M_1\left[\eta_1 x(\mathcal{T}) - \int\limits_{\mathcal{T}-\eta_1}^{\mathcal{T}} x(s)\,\mathrm{d}s - \int\limits_{-\eta_1}^{0}\int\limits_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right], \tag{15}$$

$$\begin{aligned}0 = 2\upsilon^{\mathrm{T}}(\mathcal{T})M_2\Bigg[&\eta_{12} x(\mathcal{T}) - \int\limits_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta(\mathcal{T})} x(s)\,\mathrm{d}s - \int\limits_{\mathcal{T}-\eta(\mathcal{T})}^{\mathcal{T}-\eta_1} x(s)\,\mathrm{d}s \\ &- \int\limits_{-\eta_2}^{-\eta_1}\int\limits_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\Bigg],\end{aligned} \tag{16}$$

$$0 = 2\upsilon^{\mathrm{T}}(\mathcal{T})M_3\left[\xi_1 y(\mathcal{T}) - \int\limits_{\mathcal{T}-\xi_1}^{\mathcal{T}} y(s)\,\mathrm{d}s - \int\limits_{-\xi_1}^{0}\int\limits_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right], \tag{17}$$

$$\begin{aligned}0 = 2\upsilon^{\mathrm{T}}(\mathcal{T})M_4\Bigg[&\xi_{12} y(\mathcal{T}) - \int\limits_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi(\mathcal{T})} y(s)\,\mathrm{d}s - \int\limits_{\mathcal{T}-\xi(\mathcal{T})}^{\mathcal{T}-\xi_1} y(s)\,\mathrm{d}s \\ &- \int\limits_{-\xi_2}^{-\xi_1}\int\limits_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta\Bigg],\end{aligned} \tag{18}$$

where

$$\begin{aligned}\upsilon(\mathcal{T}) = \Big\{&x(\mathcal{T}),\ y(\mathcal{T}),\ x\big(\mathcal{T}-\eta(\mathcal{T})\big),\ y\big(\mathcal{T}-\xi(\mathcal{T})\big),\ g\big(y(\mathcal{T})\big),\ g\big(y\big(\mathcal{T}-\xi(\mathcal{T})\big)\big), \\ &x(\mathcal{T}-\eta_1),\ x(\mathcal{T}-\eta_2),\ y(\mathcal{T}-\xi_1),\ y(\mathcal{T}-\xi_2),\ \dot{x}(\mathcal{T}-\eta_1),\ \dot{x}(\mathcal{T}-\eta_2),\end{aligned}$$

$$\dot{y}(\mathcal{T}-\xi_1),\ \dot{y}(\mathcal{T}-\xi_2),\ \int_{\mathcal{T}-\eta_1}^{\mathcal{T}} x(s)\,\mathrm{d}s,\ \int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} x(s)\,\mathrm{d}s,\ \int_{\mathcal{T}-\xi_1}^{\mathcal{T}} y(s)\,\mathrm{d}s,$$
$$\left.\int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} y(s)\,\mathrm{d}s,\ \int_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}} x(s)\,\mathrm{d}s,\ J\big(y(\mathcal{T})\big),\ \int_{\mathcal{T}-r(\mathcal{T})}^{\mathcal{T}} J\big(y(s)\big)\,\mathrm{d}s\right\}^{\mathrm{T}}.$$

Substituting Eqs. (8)–(18) into Eq. (7), we have

$$\begin{aligned}
\dot{V}(\mathcal{T}) \leqslant{}& \upsilon^{\mathrm{T}}(\mathcal{T})\Psi_{11}\upsilon(\mathcal{T}) + \dot{x}^{\mathrm{T}}(\mathcal{T})N_1\dot{x}(\mathcal{T}) + \dot{y}^{\mathrm{T}}(\mathcal{T})N_2\dot{y}(\mathcal{T}) \\
&- \int_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)\,\mathrm{d}s - \int_{\mathcal{T}-\eta(\mathcal{T})}^{\mathcal{T}-\eta_1} \dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)\,\mathrm{d}s - \int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta(\mathcal{T})} \dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)\,\mathrm{d}s \\
&- \int_{\mathcal{T}-\xi_1}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)S_5\dot{y}(s)\,\mathrm{d}s - \int_{\mathcal{T}-\xi(\mathcal{T})}^{\mathcal{T}-\xi_1} \dot{y}^{\mathrm{T}}(s)S_6\dot{y}(s)\,\mathrm{d}s - \int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi(\mathcal{T})} \dot{y}^{\mathrm{T}}(s)S_6\dot{y}(s)\,\mathrm{d}s \\
&- \int_{-\eta_1}^{0}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_1\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta - \int_{-\eta_2}^{-\eta_1}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_2\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&- \int_{-\xi_1}^{0}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_3\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta - \int_{-\xi_2}^{-\xi_1}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_4\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&- 2\upsilon^{\mathrm{T}}(\mathcal{T})K_1\int_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot{x}(s)\,\mathrm{d}s - 2\upsilon^{\mathrm{T}}(\mathcal{T})L_3\int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi(\mathcal{T})} \dot{y}(s)\,\mathrm{d}s \\
&- 2\upsilon^{\mathrm{T}}(\mathcal{T})K_2\int_{\mathcal{T}-\eta(\mathcal{T})}^{\mathcal{T}-\eta_1} \dot{x}(s)\,\mathrm{d}s - 2\upsilon^{\mathrm{T}}(\mathcal{T})K_3\int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta(\mathcal{T})} \dot{x}(s)\,\mathrm{d}s \\
&- 2\upsilon^{\mathrm{T}}(\mathcal{T})L_1\int_{\mathcal{T}-\eta_1}^{\mathcal{T}} \dot{y}(s)\,\mathrm{d}s - 2\upsilon^{\mathrm{T}}(\mathcal{T})L_2\int_{\mathcal{T}-\xi(\mathcal{T})}^{\mathcal{T}-\xi_1} \dot{y}(s)\,\mathrm{d}s \\
&- 2\upsilon^{\mathrm{T}}(\mathcal{T})M_1\int_{-\eta_1}^{0}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta - 2\upsilon^{\mathrm{T}}(\mathcal{T})M_2\int_{-\eta_2}^{-\eta_1}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&- 2\upsilon^{\mathrm{T}}(\mathcal{T})M_3\int_{-\xi_1}^{0}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta - 2\upsilon^{\mathrm{T}}(\mathcal{T})M_4\int_{-\xi_2}^{-\xi_1}\int_{\mathcal{T}+\theta}^{\mathcal{T}} \dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\theta.
\end{aligned}$$

Subsequently, by using the preliminary inequality $-2e^{\mathrm{T}}f \leqslant e^{\mathrm{T}}S^{-1}e + f^{\mathrm{T}}Sf$ with $S = S^{\mathrm{T}} > 0$, we arrive at

$$\begin{aligned}
\dot{V}(\mathcal{T}) &\leqslant \upsilon^{\mathrm{T}}(\mathcal{T})\Psi_{11}\upsilon(\mathcal{T}) + \upsilon^{\mathrm{T}}(\mathcal{T})\Lambda_1 N_1 \Lambda_1^{\mathrm{T}}\upsilon^{\mathrm{T}}(\mathcal{T}) + \upsilon^{\mathrm{T}}(\mathcal{T})\Lambda_2 N_2 \Lambda_2^{\mathrm{T}}\upsilon(\mathcal{T}) \\
&\quad + \eta_1\upsilon^{\mathrm{T}}(\mathcal{T})K_1 S_1^{-1}\upsilon(\mathcal{T}) + (\eta(\mathcal{T}) - \eta_1)\upsilon^{\mathrm{T}}(\mathcal{T})K_2 S_2^{-1}K_2^{\mathrm{T}}\upsilon(\mathcal{T}) \\
&\quad + \big(\eta_2 - \eta(\mathcal{T})\big)\upsilon^{\mathrm{T}}(\mathcal{T})K_3 S_2^{-1}K_3^{\mathrm{T}}\upsilon(\mathcal{T}) + \xi_1\upsilon^{\mathrm{T}}(\mathcal{T})L_1 S_5^{-1}L_1^{\mathrm{T}}\upsilon(\mathcal{T}) \\
&\quad + \big(\xi(\mathcal{T}) - \xi_1\big)\upsilon^{\mathrm{T}}(\mathcal{T})L_2 S_6^{-1}L_2^{\mathrm{T}}\upsilon(\mathcal{T}) + \big(\xi_2 - \xi(\mathcal{T})\big)\upsilon^{\mathrm{T}}(\mathcal{T})L_3 S_6^{-1}L_3^{\mathrm{T}}\upsilon(\mathcal{T}) \\
&\quad + \frac{\eta_1^2}{2}\upsilon^{\mathrm{T}}(\mathcal{T})M_1 U_1^{-1}M_1^{\mathrm{T}}\upsilon(\mathcal{T}) + \eta_\sigma\upsilon^{\mathrm{T}}(\mathcal{T})M_2 U_2^{-1}M_2^{\mathrm{T}}\upsilon(\mathcal{T}) \\
&\quad + \frac{\xi_1^2}{2}\upsilon^{\mathrm{T}}(\mathcal{T})M_3 U_3^{-1}M_3^{\mathrm{T}}\upsilon(\mathcal{T}) + \xi_\sigma\upsilon^{\mathrm{T}}(\mathcal{T})M_4 U_4^{-1}M_4^{\mathrm{T}}\upsilon(\mathcal{T}) \\
&= \upsilon^{\mathrm{T}}(\mathcal{T})\Psi\upsilon(\mathcal{T}),
\end{aligned}$$

where

$$\begin{aligned}
\Psi &= \Psi_{11} + \Lambda_1 N_1 \Lambda_1^{\mathrm{T}} + \Lambda_2 N_2 \Lambda_2^{\mathrm{T}} + \frac{1}{2}\eta_1^2 M_1 U_1^{-1}M_1^{\mathrm{T}} + \eta_\sigma M_2 U_2^{-1}M_2^{\mathrm{T}} \\
&\quad + \frac{1}{2}\xi_1^2 M_3 U_3^{-1}M_3^{\mathrm{T}} + \xi_\sigma M_4 U_4^{-1}M_4^{\mathrm{T}} + \eta_1 K_1 S_1^{-1}K_1^{\mathrm{T}} + \xi_1 L_1 S_5^{-1}L_1^{\mathrm{T}} \\
&\quad + \big(\eta(\mathcal{T}) - \eta_1\big)K_2 S_2^{-1}K_2^{\mathrm{T}} + (\eta_2 - \eta(\mathcal{T}))K_3 S_2^{-1}K_3^{\mathrm{T}} \\
&\quad + \big(\xi(\mathcal{T}) - \xi_1\big)L_2 S_6^{-1}L_2^{\mathrm{T}} + (\xi_2 - \xi(\mathcal{T}))L_3 S_6^{-1}L_3^{\mathrm{T}}.
\end{aligned}$$

Noting $\eta_1 \leqslant \eta(\mathcal{T}) \leqslant \eta_2$, the term $((\eta(\mathcal{T}) - \eta_1)/\eta_{12})K_2 S_2^{-1}K_2^{\mathrm{T}} + ((\eta_2 - \eta(\mathcal{T}))/\eta_{12}) \times K_3 S_2^{-1}K_3^{\mathrm{T}}$ is a convex combination of $K_2 S_2^{-1}K_2^{\mathrm{T}}$ and $K_3 S_2^{-1}K_3^{\mathrm{T}}$ with respect to $\eta(\mathcal{T})$. Similarly, it follows from $\xi_1 \leqslant \xi(\mathcal{T}) \leqslant \xi_2$, the term $((\xi(\mathcal{T}) - \xi_1)/\xi_{12})L_2 S_6^{-1}L_2^{\mathrm{T}} + ((\xi_2 - \xi(\mathcal{T}))/\xi_{12})L_3 S_6^{-1}L_3^{\mathrm{T}}$ is a convex combination of $L_2 S_6^{-1}L_2^{\mathrm{T}}$ and $L_3 S_6^{-1}L_3^{\mathrm{T}}$ with respect to $\xi(\mathcal{T})$.

By the convex combination method, $\Psi < 0$ holds if the following inequalities hold:

$$\begin{aligned}
&\Psi_{11} + \Lambda_1 N_1 \Lambda_1^{\mathrm{T}} + \Lambda_2 N_2 \Lambda_2^{\mathrm{T}} + \frac{1}{2}\eta_1^2 M_1 U_1^{-1}M_1^{\mathrm{T}} + \eta_\sigma M_2 U_2^{-1}M_2^{\mathrm{T}} \\
&\quad + \frac{1}{2}\xi_1^2 M_3 U_3^{-1}M_3^{\mathrm{T}} + \xi_\sigma M_4 U_4^{-1}M_4^{\mathrm{T}} + \eta_1 K_1 S_1^{-1}K_1^{\mathrm{T}} \\
&\quad + \xi_1 L_1 S_5^{-1}L_1^{\mathrm{T}} + \eta_{12}K_2 S_2^{-1}K_2^{\mathrm{T}} + \xi_{12}L_2 S_6^{-1}L_2^{\mathrm{T}} < 0, \qquad (19)
\end{aligned}$$

$$\begin{aligned}
&\Psi_{11} + \Lambda_1 N_1 \Lambda_1^{\mathrm{T}} + \Lambda_2 N_2 \Lambda_2^{\mathrm{T}} + \frac{1}{2}\eta_1^2 M_1 U_1^{-1}M_1^{\mathrm{T}} + \eta_\sigma M_2 U_2^{-1}M_2^{\mathrm{T}} \\
&\quad + \frac{1}{2}\xi_1^2 M_3 U_3^{-1}M_3^{\mathrm{T}} + \xi_\sigma M_4 U_4^{-1}M_4^{\mathrm{T}} + \eta_1 K_1 S_1^{-1}K_1^{\mathrm{T}} \\
&\quad + \xi_1 L_1 S_5^{-1}L_1^{\mathrm{T}} + \eta_{12}K_2 S_2^{-1}K_2^{\mathrm{T}} + \xi_{12}L_3 S_6^{-1}L_3^{\mathrm{T}} < 0, \qquad (20)
\end{aligned}$$

$$\begin{aligned}
&\Psi_{11} + \Lambda_1 N_1 \Lambda_1^{\mathrm{T}} + \Lambda_2 N_2 \Lambda_2^{\mathrm{T}} + \frac{1}{2}\eta_1^2 M_1 U_1^{-1}M_1^{\mathrm{T}} + \eta_\sigma M_2 U_2^{-1}M_2^{\mathrm{T}} \\
&\quad + \frac{1}{2}\xi_1^2 M_3 U_3^{-1}M_3^{\mathrm{T}} + \xi_\sigma M_4 U_4^{-1}M_4^{\mathrm{T}} + \eta_1 K_1 S_1^{-1}K_1^{\mathrm{T}} \\
&\quad + \xi_1 L_1 S_5^{-1}L_1^{\mathrm{T}} + \eta_{12}K_3 S_2^{-1}K_3^{\mathrm{T}} + \xi_{12}L_2 S_6^{-1}L_2^{\mathrm{T}} < 0, \qquad (21)
\end{aligned}$$

$$\begin{aligned}&\Psi_{11}+\Lambda_1N_1\Lambda_1^{\mathrm T}+\Lambda_2N_2\Lambda_2^{\mathrm T}+\frac{1}{2}\eta_1^2M_1U_1^{-1}M_1^{\mathrm T}+\eta_\sigma M_2U_2^{-1}M_2^{\mathrm T}\\&\quad+\frac{1}{2}\xi_1^2M_3U_3^{-1}M_3^{\mathrm T}+\xi_\sigma M_4U_4^{-1}M_4^{\mathrm T}+\eta_1K_1S_1^{-1}K_1^{\mathrm T}\\&\quad+\xi_1L_1S_5^{-1}L_1^{\mathrm T}+\eta_{12}K_3S_2^{-1}K_3^{\mathrm T}+\xi_{12}L_3S_6^{-1}L_3^{\mathrm T}<0.\end{aligned}\tag{22}$$

Using Schur complement lemma, it is easy to view that inequalities (19)–(22) are equivalent to $\Psi_1<0$, $\Psi_2<0$, $\Psi_3<0$ and $\Psi_4<0$.

On the other hand, from (6) and Theorem 1 conditions, we note that

$$\begin{aligned}V_1\big(\mathcal{T}_k,\zeta(\mathcal{T}_k),j\big)-V_1(\mathcal{T}_k^-,\zeta(\mathcal{T}_k^-),i)&=\zeta^{\mathrm T}(\mathcal{T}_k)B_j\zeta(\mathcal{T}_k)-\zeta^{\mathrm T}(\mathcal{T}_k^-)A_j\zeta(\mathcal{T}_k^-)\\&=\zeta^{\mathrm T}(\mathcal{T}_k^-)F_{ik}^{\mathrm T}B_jF_{ik}\zeta(\mathcal{T}_k^-)-\zeta^{\mathrm T}(\mathcal{T}_k^-)A_j\zeta(\mathcal{T}_k^-)\\&=\zeta^{\mathrm T}(\mathcal{T}_k^-)(F_{ik}^{\mathrm T}B_jF_{ik}-B_j)\zeta(\mathcal{T}_k^-)\leqslant 0,\\V_1\big(\sqcup_k,\zeta(\sqcup_k),j\big)&\leqslant V_1\big(\sqcup_k^-,\zeta(\sqcup_k^-),i\big),\quad k\in\mathbb{Z}_+,\end{aligned}$$

which implies that

$$V\big(\sqcup_k,x(\sqcup_k),j\big)\leqslant V\big(\sqcup_k^-,x(\sqcup_k^-),i\big),\quad k\in\mathbb{Z}_+.$$

System (5) with impulsive effect is globally asymptotically stable. Hence, the proof is completed. □

Consider the following impulsive GRNs with leakage delays, distributed delays and parameter uncertainties:

$$\begin{aligned}\dot{x}(\mathcal{T})&=-(G_1+\Delta G_1)x(\mathcal{T})+(H_1+\Delta H_1)g\big(y\big(\mathcal{T}-\xi(\mathcal{T})\big)\big)\\&\quad+(E_1+\Delta E_1)\int_{\mathcal{T}-r(\mathcal{T})}^{\mathcal{T}}J\big(y(s)\big)\,\mathrm{d}s,\\\dot{y}(\mathcal{T})&=-(G_2+\Delta G_2)y(\mathcal{T})+(H_2+\Delta H_2)x\big(\mathcal{T}-\eta(\mathcal{T})\big)\\&\quad+(E_2+\Delta E_2)\int_{\mathcal{T}-l(\mathcal{T})}^{\mathcal{T}}x(s)\,\mathrm{d}s,\\x(\mathcal{T}_k)&=\mathcal{D}_1x(\mathcal{T}_k)^-,\quad y(\mathcal{T}_k)=\mathcal{D}_2y(\mathcal{T}_k)^-,\quad k\in\mathbb{Z}^+,\\x_0&=x(\theta)=\psi(\theta),\ \ y_0=y(\theta)=\pi(\theta)\quad\forall\theta\in[-\varpi,0],\end{aligned}\tag{23}$$

where ΔG_1, ΔH_1, ΔE_1, ΔG_2, ΔH_2, ΔE_2 denotes the time-varying parameter uncertainties, which is defined as

$$[\Delta G_1\ \Delta H_1\ \Delta E_1\ \Delta G_2\ \Delta H_2\ \Delta E_2]=AC(\sqcup)[F_1\ F_2\ F_3\ F_4\ F_5\ F_6],$$

where F_i $(i=1,\dots,6)$ and G are notable constant matrices, and $C(\sqcup)$ denotes the unspecified time-changing matrix-valued function satisfying $C^{\mathrm T}(\sqcup)C(\sqcup)\leqslant I$. Then, the following theorem will give the stability criterion for GRNs with parameter uncertainties.

Theorem 2. *With the help of Assumptions* 1 *and* 2*, for given positive scalars* $\eta_2 > \eta_1$*,* $\xi_2 > \xi_1$*,* λ *and* δ*, system* (23) *becomes globally asymptotically stable if there exists positive-definite matrices* $R = [R_{ij}]_{6\times6}$*,* P_i $(i = 1,2,\dots,5)$*,* Q_i $(i = 1,2\dots,6)$*,* S_i $(i = 1,2,\dots,8)$ *and* U_i $(i = 1,\dots,4)$*, matrices* Q_7*,* S_i $(i = 9,10,11,12)$*,* K_1*,* K_2*,* K_3*,* L_1*,* L_2*,* L_3*,* M_i $(i = 1,\dots,4)$ *and positive definite diagonal matrices* $\Omega = \mathrm{diag}\{z_{1i}, z_{2i},\dots,z_{ni}\}$ $(i = 1,2)$ *such that the following LMIs hold*:

$$F_{ik}^{\mathrm{T}} B_i F_{ik} - B_i < 0,$$

$$\begin{bmatrix} Q_1 & Q_7 \\ * & Q_2 \end{bmatrix} \geqslant 0, \qquad \begin{bmatrix} S_1 & S_9 \\ * & S_3 \end{bmatrix} \geqslant 0, \qquad \begin{bmatrix} S_2 & S_{10} \\ * & S_4 \end{bmatrix} \geqslant 0,$$

$$\begin{bmatrix} S_5 & S_{11} \\ * & S_7 \end{bmatrix} \geqslant 0, \qquad \begin{bmatrix} S_6 & S_{12} \\ * & S_8 \end{bmatrix} \geqslant 0,$$

$$\Psi_i = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14}^{(i)} \\ * & \Psi_{22} & 0 & 0 \\ * & * & \Psi_{33} & 0 \\ * & * & * & \Psi_{44} \end{bmatrix} < 0, \quad (i = 1,2,3,4),$$

with

$$\begin{aligned}
\Psi_{11} &= [\Omega_{ij}]_{21\times21}, \quad \Psi_{12} = [\Lambda_1 N_1 \ \Lambda_2 N_2], \quad \Psi_{13} = \left[\frac{\eta_1^2}{2} M_1 \ \eta_\sigma M_2 \ \frac{\xi_1^2}{2} M_3 \ \ \xi_\sigma M_4\right],\\
\Psi_{22} &= \mathrm{diag}\{-N_1, -N_2\}, \quad \Psi_{33} = \mathrm{diag}\left\{-\frac{\eta_1^2}{2} U_1, -\eta_\sigma U_2, -\frac{\xi_1^2}{2} U_3, -\xi_\sigma U_4\right\},\\
\Psi_{44} &= \mathrm{diag}\{-\eta_1 S_1, \xi_1 S_5, -\eta_{12} S_2, \xi_{12} S_6\}, \quad \Psi_{14}^{(1)} = [\eta_1 K_1 \ \xi_1 L_1 \ \eta_{12} K_2 \ \xi_{12} L_2],\\
\Psi_{14}^{(2)} &= [\eta_1 K_1 \ \xi_1 L_1 \ \eta_{12} K_2 \ \xi_{12} L_3], \quad \Psi_{14}^{(3)} = [\eta_1 K_1 \ \xi_1 L_1 \ \eta_{12} K_3 \ \xi_{12} L_2],\\
\Psi_{14}^{(4)} &= [\eta_1 K_1 \ \xi_1 L_1 \ \eta_{12} K_3 \ \xi_{12} L_3],\\
\Omega_{1,1} &= -R_{11} G_1 - R_{11} \varepsilon^{-1} A A^{\mathrm{T}} - G_1^{\mathrm{T}} R_{11} - \varepsilon F_1 F_1^{\mathrm{T}} R_{11} + R_{13} + R_{13}^{\mathrm{T}} + P_2\\
&\quad + K_{11} + K_{11}^{\mathrm{T}} + \eta_1 S_3 + \eta_{12} S_4 + \eta_1 M_{11} + \eta_1 M_{11}^{\mathrm{T}} + \eta_{12} M_{12} + \eta_{12} M_{12}^{\mathrm{T}}\\
&\quad - \eta_1 S_9^{\mathrm{T}} G_1 - \eta_1 S_9^{\mathrm{T}} \varepsilon^{-1} A A^{\mathrm{T}} - \eta_1 G_1^{\mathrm{T}} S_9 - \eta_1 S_9 \varepsilon F_1 F_1^{\mathrm{T}} - \eta_{12} S_{10}^{\mathrm{T}} G_1\\
&\quad - \eta_{12} S_{10}^{\mathrm{T}} \varepsilon^{-1} A A^{\mathrm{T}} - \eta_{12} G_1^{\mathrm{T}} S_{10} - \eta_{12} S_{10} \varepsilon F_1 F_1^{\mathrm{T}},\\
\Omega_{1,2} &= -G_1^{\mathrm{T}} R_{12} - R_{12} \varepsilon F_1 F_1^{\mathrm{T}} - R_{12} + R_{23}^{\mathrm{T}} - R_{12} G_2 - R_{12} \varepsilon^{-1} A A^{\mathrm{T}} + R_{15},\\
\Omega_{1,3} &= R_{12} H_2 + R_{12} \varepsilon^{-1} A A^{\mathrm{T}} + K_{21}^{\mathrm{T}} - K_{12} + K_{13} + \eta_1 M_{21}^{\mathrm{T}} + \eta_{12} M_{22}^{\mathrm{T}},\\
\Omega_{1,6} &= R_{11} H_1 + R_{11} \varepsilon^{-1} A A^{\mathrm{T}} + \eta_1 S_9^{\mathrm{T}} H_1 + \eta_{12} S_{10}^{\mathrm{T}} H_1,\\
\Omega_{1,15} &= -G_1^{\mathrm{T}} R_{13} - R_{13} \varepsilon F_1 F_1^{\mathrm{T}} + R_{33} - \frac{1}{\eta_1} S_9 - M_{11},\\
\Omega_{1,19} &= E_2 R_1 + R_1 \varepsilon^{-1} A A^{\mathrm{T}}, \quad \Omega_{1,16} = -G_1^{\mathrm{T}} R_{14} - R_{14} \varepsilon F_1 F_1^{\mathrm{T}} + R_{34} - M_{12},\\
\Omega_{1,17} &= -G_1^{\mathrm{T}} R_{15} - R_{15} \varepsilon F_1 F_1^{\mathrm{T}} + R_{35},
\end{aligned}$$

$$\begin{aligned}
\Omega_{1,18} &= -G_1^{\mathrm{T}} R_{16} - R_{16}\varepsilon F_1 F_1^{\mathrm{T}} + R_{36}, \quad \Omega_{1,21} = E_1 R_1 + R_1 \varepsilon^{-1} A A^{\mathrm{T}},\\
\Omega_{2,2} &= -R_{22} G_2 - R_{22}\varepsilon^{-1} A A^{\mathrm{T}} - G_2^{\mathrm{T}} R_{22} - R_{22}\varepsilon F_1 F_1^{\mathrm{T}} + R_{25} + R_{25}^{\mathrm{T}}\\
&\quad + Q_1 + Q_3 + L_{11} + L_{11}^{\mathrm{T}} + \xi_1 S_7 + \xi_{12} S_8 + \xi_1 M_{13} + \xi_1 M_{13}^{\mathrm{T}} + \eta_{12} M_{14}\\
&\quad + \xi_{12} M_{14}^{\mathrm{T}} - \xi_1 S_{11}^{\mathrm{T}} G_2 - \xi_1 S_{11}^{\mathrm{T}} \varepsilon^{-1} A A^{\mathrm{T}} \xi_1 G_2^{\mathrm{T}} S_{11} - \xi_1 S_{11} \varepsilon F_2 F_2^{\mathrm{T}}\\
&\quad - \eta_{12} S_{12}^{\mathrm{T}} G_2 - \eta_{12} S_{12}^{\mathrm{T}} \varepsilon^{-1} A A^{\mathrm{T}} - \eta_{12} G_2^{\mathrm{T}} S_{12} - \eta_{12} S_{12} \varepsilon F_2 F_2^{\mathrm{T}},\\
\Omega_{2,3} &= R_{22} H_2 + R_{22}\varepsilon^{-1} A A^{\mathrm{T}} + \xi_1 S_{11}^{\mathrm{T}} H_2 + \xi_1 S_{11}^{\mathrm{T}} \varepsilon^{-1} A A^{\mathrm{T}} + \xi_{12} S_{12}^{\mathrm{T}} H_2\\
&\quad + \xi_{12} S_{12}^{\mathrm{T}} \varepsilon^{-1} A A^{\mathrm{T}}, \Omega_{2,5} = -G_2^{\mathrm{T}} \Omega - \Omega \varepsilon F_2 F_2^{\mathrm{T}} + Q Z_1 + Q_7,\\
\Omega_{2,6} &= R_{12}^{\mathrm{T}} H_1 + R_{12}^{\mathrm{T}} \varepsilon^{-1} A A^{\mathrm{T}}, \quad \Omega_{2,19} = E_2 R_2 + R_2 \varepsilon^{-1} A A^{\mathrm{T}},\\
\Omega_{2,16} &= -G_2^{\mathrm{T}} R_{24} - R_{24}\varepsilon F_2 F_2^{\mathrm{T}} + R_{45}^{\mathrm{T}},\\
\Omega_{2,17} &= -G_2^{\mathrm{T}} R_{25} - R_{25}\varepsilon F_2 F_2^{\mathrm{T}} - \frac{1}{\xi_1} S_{11} + R_{55} - M_{13},\\
\Omega_{2,18} &= -G_2^{\mathrm{T}} R_{26} - R_{26}\varepsilon F_2 F_2^{\mathrm{T}} + R_{56} - M_{14}, \quad \Omega_{2,21} = E_1 R_2 + R_2 \varepsilon^{-1} A A^{\mathrm{T}},\\
\Omega_{3,5} &= H_2^{\mathrm{T}} Q + Q \varepsilon F_6 F_6^{\mathrm{T}}, \quad \Omega_{3,15} = H_2^{\mathrm{T}} R_{23} + R_{23}\varepsilon F_6 F_6^{\mathrm{T}} - M_{21},\\
\Omega_{3,16} &= H_2^{\mathrm{T}} R_{24} + R_{24}\varepsilon F_6 F_6^{\mathrm{T}} - M_{22}, \quad \Omega_{3,17} = H_2^{\mathrm{T}} R_{25} + R_{25}\varepsilon F_6 F_6^{\mathrm{T}},\\
\Omega_{3,18} &= H_2^{\mathrm{T}} R_{26} + R_{26}\varepsilon F_6 F_6^{\mathrm{T}}, \quad \Omega_{5,19} = \Omega E_2 + \Omega \varepsilon^{-1} A A^{\mathrm{T}},\\
\Omega_{6,15} &= H_1^{\mathrm{T}} R_{13} + R_{13}\varepsilon F_2 F_2^{\mathrm{T}}, \quad \Omega_{6,16} = H_1^{\mathrm{T}} R_{14} + R_{14}\varepsilon F_2 F_2^{\mathrm{T}},\\
\Omega_{6,17} &= H_1^{\mathrm{T}} R_{15} + R_{15}\varepsilon F_2 F_2^{\mathrm{T}}, \quad \Omega_{6,18} = H_1^{\mathrm{T}} R_{16} + R_{16}\varepsilon F_2 F_2^{\mathrm{T}},\\
\Omega_{15,19} &= E_2 R_3 + R_3 \varepsilon^{-1} A A^{\mathrm{T}}, \quad \Omega_{15,21} = E_1 R_3 + R_3 \varepsilon^{-1} A A^{\mathrm{T}},\\
\Omega_{16,19} &= E_2 R_4 + R_4 \varepsilon^{-1} A A^{\mathrm{T}}, \quad \Omega_{16,21} = E_1 R_4 + R_4 \varepsilon^{-1} A A^{\mathrm{T}},\\
\Omega_{17,19} &= E_2 R_5 + R_5 \varepsilon^{-1} A A^{\mathrm{T}}, \quad \Omega_{17,21} = E_1 R_5 + R_5 \varepsilon^{-1} A A^{\mathrm{T}},\\
\Omega_{18,19} &= E_2 R_6 + R_6 \varepsilon^{-1} A A^{\mathrm{T}}, \quad \Omega_{18,21} = E_1 R_6 + R_6 \varepsilon^{-1} A A^{\mathrm{T}}.
\end{aligned}$$

Proof. By replacing D_1, E_1, H_1, D_2, E_2, H_2 in (4) with $D_1 + \Delta D_1$, $E_1 + \Delta E_1$, $H_1 + \Delta H_1$, $D_2 + \Delta D_2$, $E_2 + \Delta E_2$, $H_2 + \Delta H_2$, respectively, and using Lemmas 1, 2 and Theorem 1, follows the proof. □

Corollary 1. *With the help of Assumptions* 1 *and* 2*, for given positive scalars* $\eta_2 > \eta_1$, $\xi_2 > \xi_1$, *system* (5) *becomes globally asymptotically stable, if there exists positive-definite matrices* $R = [R_{ij}]_{6\times 6}$, P_i $(i = 2, \dots, 5)$, Q_i $(i = 3, \dots, 6)$, S_i $(i = 1, 2, \dots, 8)$ *and* U_i $(i = 1, \dots, 4)$, *matrices* S_i $(i = 9, 10, 11, 12)$, K_1, K_2, K_3, L_1, L_2, L_3, M_i $(i = 1, \dots, 4)$ *and positive definite diagonal matrices* $\Omega = \operatorname{diag}\{z_{1i}, z_{2i}, \dots, z_{ni}\}$ $(i = 1, 2)$ *such that the following LMIs hold*:

$$F_{ik}^{\mathrm{T}} B_i F_{ik} - B_i < 0,$$

$$\begin{bmatrix} S_1 & S_9 \\ * & S_3 \end{bmatrix} \geqslant 0, \qquad \begin{bmatrix} S_2 & S_{10} \\ * & S_4 \end{bmatrix} \geqslant 0, \qquad \begin{bmatrix} S_5 & S_{11} \\ * & S_7 \end{bmatrix} \geqslant 0, \qquad \begin{bmatrix} S_6 & S_{12} \\ * & S_8 \end{bmatrix} \geqslant 0,$$

$$\Psi_i = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14}^{(i)} \\ * & \Psi_{22} & 0 & 0 \\ * & * & \Psi_{33} & 0 \\ * & * & * & \Psi_{44} \end{bmatrix} < 0 \quad (i = 1, 2, 3, 4),$$

where

$$\begin{aligned} &\Psi_{11} = [\Omega_{ij}]_{21\times 21}, \\ &\Omega_{3,3} = -K_{22} - K_{22}^{\mathrm{T}} + K_{23} + K_{23}^{\mathrm{T}}, \quad \Omega_{4,4} = -L_{22} - L_{22}^{\mathrm{T}} + L_{23} + L_{23}^{\mathrm{T}}, \\ &\Omega_{4,6} = QZ_2,\ \Omega_{5,5} = -2Z_1, \quad \Omega_{6,6} = -2Z_2, \quad \Omega_{7,7} = -(P_2 - P_3). \end{aligned}$$

Other elements of Ω and Ψ are same as in Theorem 1.

Proof. The proof follows from Theorem 1. □

Remark 1. In the Lyapunov–Krasovskii functional, the triple integral terms

$$\int_{-\eta_1}^{0}\int_{\theta}^{0}\int_{\mathcal{T}+\mu}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_1\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\mu\,\ \mathrm{d}\theta, \qquad \int_{-\eta_2}^{-\eta_1}\int_{\theta}^{0}\int_{\mathcal{T}+\mu}^{\mathcal{T}} \dot{x}^{\mathrm{T}}(s)U_2\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\mu\,\mathrm{d}\theta,$$

$$\int_{-\xi_1}^{0}\int_{\theta}^{0}\int_{\mathcal{T}+\mu}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_3\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\mu\,\mathrm{d}\theta \quad \text{and} \quad \int_{-\xi_2}^{-\xi_1}\int_{\theta}^{0}\int_{\mathcal{T}+\mu}^{\mathcal{T}} \dot{y}^{\mathrm{T}}(s)U_4\dot{y}(s)\,\mathrm{d}s\,\mathrm{d}\mu\,\ \mathrm{d}\theta$$

are introduced with hope to reduce the less conservativeness of the advanced results. In addition, the improved vector $\upsilon(\mathcal{T})$ consists of the terms

$$\int_{\mathcal{T}-\eta_1}^{\mathcal{T}} x(s)\,\mathrm{d}s, \qquad \int_{\mathcal{T}-\eta_2}^{\mathcal{T}-\eta_1} x(s)\,\mathrm{d}s, \qquad \int_{\mathcal{T}-\xi_1}^{\mathcal{T}} y(s)\,\mathrm{d}s, \qquad \int_{\mathcal{T}-\xi_2}^{\mathcal{T}-\xi_1} y(s)\,\mathrm{d}s^{\mathrm{T}}.$$

Then, the integral terms are different compared with the existing works [33, 34].

Remark 2. In this work, some convex combination technique and free-weighting matrix method is approached. Because, convex combination method helps us to reduce the decision variables in LMIs, which is the relevance lemma of Jensen's inequality and free-weighting matrix assist to decrease the conservatism of stability criterion than the existing literature.

Remark 3. As much as know, all the existing results concerning the dynamical behaviors of genetic regulatory networks [20, 33, 41] have not considered the global asymptotic stability performance in the mean square and time-varying delayed situation, which are investigated via LMI approach in this paper. Therefore, our conclusions are new when compared to the previous results.

Remark 4. In this paper, we also consider the relationship between time-varying delays and their upper bounds. In order to obtain the maximum upper bounds of distributed delays and time-varying delays, we used some inequality techniques, see Example 1. Hence, the techniques and methods used in this paper may lead to less conservative criterions. To this evident, Table 1 shows the maximum upper bound of ξ, which guarantees the global asymptotic stability of the addressed genetic networks (5). These tables demonstrate the effectiveness of our proposed method.

4 Numerical simulations

In this portion, twin examples with simulations are provided to demonstrate the usefulness of the obtained results.

Example 1. Consider the GRN (5) with the following parameters:

$$G_1 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.2 \end{pmatrix}, \qquad G_2 = \begin{pmatrix} -0.5 & 0.2 \\ 0.2 & 0.8 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.1 \end{pmatrix},$$
$$H_2 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.9 \end{pmatrix} \qquad E_1 = \begin{pmatrix} 0.36 & 0 \\ 0 & 0.4 \end{pmatrix}, \qquad E_2 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.8 \end{pmatrix}.$$

Let $g(y) = y^2/(1+y^2)$ is taken as the regulatory function. It can be easily checked that the derivative of $g(y)$ is less than 0.65. Assume that the feedback regulation delay $\eta(\mathcal{T}) = 2$ and the translation delay $\xi(\mathcal{T}) = 2$. Then $\eta_1 = 0.3$, $\eta_2 = 0.5$, $\xi_1 = 0.5$, $\xi_2 = 2.5$, $\lambda = 0.2$ and $\delta = 0.4$ can be obtained.

By Theorem 1 we can obtain the following feasible parameters. From Table 1 our work is more effective and less conservative than the existing works. Due to space consideration, we only provide a part of the feasible solutions here.

$$R_1 = \begin{pmatrix} 0.0331 & -0.0309 \\ -0.0309 & 0.1319 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} 0.0485 & -0.1063 \\ -0.1063 & 0.0238 \end{pmatrix},$$
$$R_3 = \begin{pmatrix} 0.0798 & 0.2106 \\ 0.2106 & 0.0689 \end{pmatrix}, \qquad R_4 = \begin{pmatrix} 0.5440 & 0.0003 \\ 0.0003 & 1.7873 \end{pmatrix},$$
$$P_1 = \begin{pmatrix} 0.0050 & 0.0000 \\ 0.0000 & 0.0081 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 0.2461 & 0.0186 \\ 0.0186 & 0.1769 \end{pmatrix},$$
$$Q_1 = \begin{pmatrix} 1.9658 & -0.0575 \\ -0.0575 & 2.0320 \end{pmatrix}, \qquad Q_2 = \begin{pmatrix} 0.0050 & 0.0000 \\ 0.0000 & 0.0081 \end{pmatrix},$$
$$S_1 = \begin{pmatrix} 1.7391 & 0.1795 \\ 0.1795 & -0.3801 \end{pmatrix}, \qquad U_1 = \begin{pmatrix} 0.1639 & 0.0001 \\ 0.0001 & 0.1249 \end{pmatrix},$$
$$Z_1 = \begin{pmatrix} 0.1001 & 0.0020 \\ 0.0020 & 0.1615 \end{pmatrix}, \qquad W_1 = \begin{pmatrix} 0.1001 & 0.0020 \\ 0.0020 & 0.1615 \end{pmatrix},$$

From Theorem 1 one can conclude that the continuous-time GRNs (5) with impulsive effects are globally asymptotically stable. The concentrations of mRNAs and

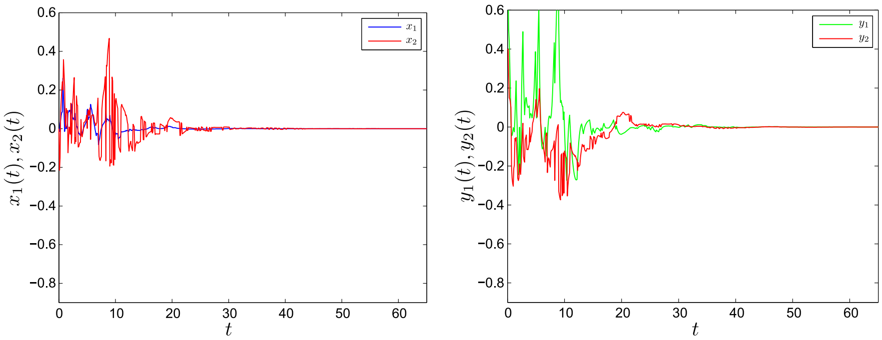

Figure 1. mRNA and Protein concentrations with impulsive effects.

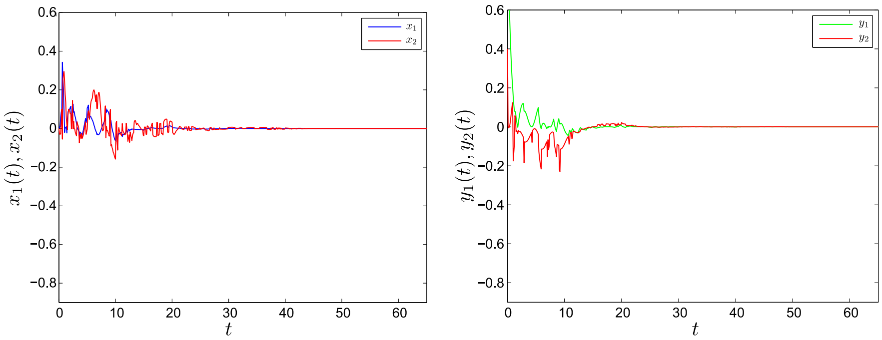

Figure 2. mRNA and Protein concentrations without impulsive effects.

Table 1. Comparisons of upper bounds of time-delay $\xi(\mathcal{T})$ for various ξ_1.

Methods	$\xi_1 = 0.5$	$\xi_1 = 1$
In [33]	3.78	2.50
In [41]	5.91	6.41
In [20]	6.15	6.62
In Theorem 1	7.18	7.98

proteins with impulsive effects are illustrated in Fig. 1 with the initial conditions $x(0) = [0.01\ 0.02]^{\mathrm{T}}$, $y(0) = [0.1\ 0.2]^{\mathrm{T}}$ and the concentrations of mRNAs and proteins without impulsive effects are illustrated in Fig. 2 with the initial conditions $x(0) = [0.01\ 0.1]^{\mathrm{T}}$ and $y(0) = [0.1\ 0.3]^{\mathrm{T}}$.

Example 2. Consider the parameter uncertainty GRN (23) with the following parameters:

$$G_1 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.2 \end{pmatrix}, \qquad G_2 = \begin{pmatrix} -0.5 & 0.2 \\ 0.2 & 0.8 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.1 \end{pmatrix},$$
$$H_2 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.9 \end{pmatrix}, \qquad E_1 = \begin{pmatrix} 0.36 & 0 \\ 0 & 0.4 \end{pmatrix}, \qquad E_2 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.8 \end{pmatrix},$$

$$F_1 = F_2 = F_3 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix}, \qquad F_4 = F_5 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix}.$$

The regulatory function is taken as $g(y) = y^2/(1 + y^2)$. It can be easily checked that the derivative of $g(y)$ is less than 0.65. Assume that the feedback regulation delay $\eta(\mathcal{T}) = 2$ and the translation delay $\xi(\mathcal{T}) = 2$. Then $\eta_1 = 0.3$, $\eta_2 = 0.5$, $\xi_1 = 0.45$, $\xi_2 = 2.5$, $\lambda = 0.2$ and $\delta = 0.4$ can be obtained.

By Theorem 2 we can obtain the following feasible parameters. Due to space consideration, we only provide a part of the feasible solutions here.

$$R_1 = \begin{pmatrix} 0.6770 & 0.0005 \\ 0.0005 & 1.2181 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} 0.0328 & 0.0031 \\ 0.0031 & 0.0730 \end{pmatrix},$$

$$R_3 = \begin{pmatrix} 0.2942 & 0.0000 \\ 0.0000 & 0.4591 \end{pmatrix}, \qquad R_4 = \begin{pmatrix} 0.1440 & 0.0003 \\ 0.0003 & 0.7873 \end{pmatrix},$$

$$P_1 = \begin{pmatrix} 0.1504 & 0.0002 \\ 0.0002 & 0.0081 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 0.1461 & 0.0186 \\ 0.0186 & 0.1269 \end{pmatrix},$$

$$Q_1 = \begin{pmatrix} 0.3440 & 0.0003 \\ 0.0003 & 0.2873 \end{pmatrix}, \qquad Q_2 = \begin{pmatrix} 0.2052 & 0.0001 \\ 0.0001 & 0.1081 \end{pmatrix},$$

$$S_1 = \begin{pmatrix} 0.5461 & 0.0186 \\ 0.0186 & 0.6769 \end{pmatrix}, \qquad U_1 = \begin{pmatrix} 0.2639 & 0.0001 \\ 0.0001 & 0.2249 \end{pmatrix},$$

$$Z_1 = \begin{pmatrix} 0.3001 & 0.0020 \\ 0.0020 & 0.5615 \end{pmatrix}, \qquad W_1 = \begin{pmatrix} 0.2001 & 0.0020 \\ 0.0020 & 0.1615 \end{pmatrix},$$

From Theorem 2 one can conclude that the continuous-time GRNs (23) with impulsive effects are globally asymptotically stable. The concentrations of mRNAs and proteins with impulsive effects are illustrated in Fig. 3 with the initial conditions $x(0) = [0.01 \ -0.01]^{\mathrm{T}}$, $y(0) = [0.3 \ -0.2]^{\mathrm{T}}$, and the concentrations of mRNAs and proteins without impulsive effects are illustrated in Fig. 4 with the initial conditions $x(0) = [0.01 \ -0.02]^{\mathrm{T}}$ and $y(0) = [0.3 \ 0.1]^{\mathrm{T}}$.

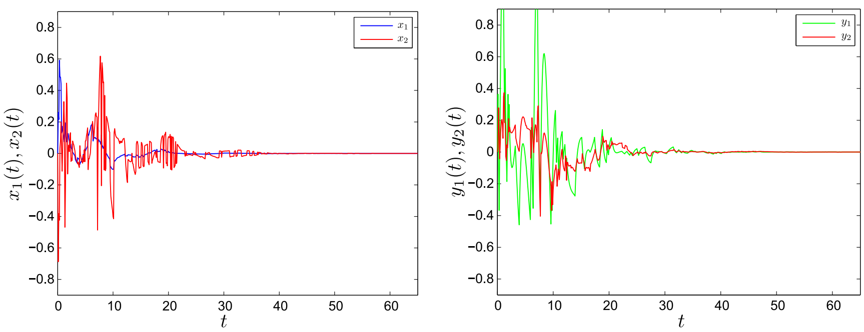

Figure 3. mRNA and Protein concentrations with impulsive effects.

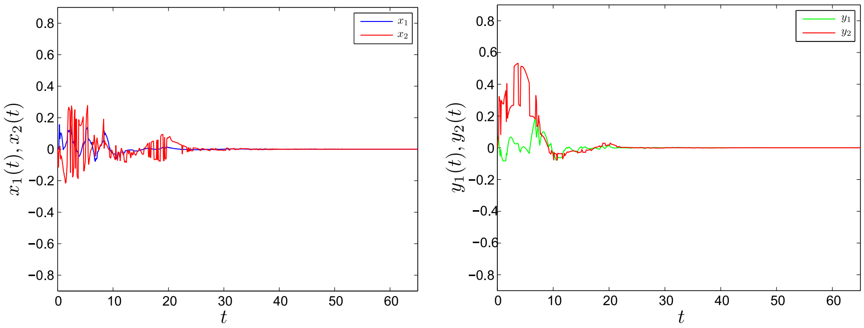

Figure 4. mRNA and Protein concentrations without impulsive effects.

5 Conclusions

In this work, we have investigated the global asymptotic stability problem for a class of uncertain genetic regulatory networks with distributed delays, time-varying delays and impulses. By constructing new Lyapunov–Krasovskii functional with triple integral terms, sufficient stability analysis has been rooted in terms of LMIs. By applying convex combination technique and free-weighting matrix method, conservatism of the stability criteria have been diminished greatly. Lastly, the feasibility and advantages of the developed results have been demonstrated by the numerical simulation examples.

In the near future, we plan to work with stabilization of stochastic genetic regulatory networks with leakage and impulsive effects in finite-time stable sense. Also, we will try to present a real life model to justify our theoretical concepts for the considered GRN.

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