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Dissipativity analysis of stochastic fuzzy neural networks with randomly occurring uncertainties using delay dividing approach*

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Abstract. This paper focuses on the problem of delay-dependent robust dissipativity analysis for a class of stochastic fuzzy neural networks with time-varying delay. The randomly occurring uncertainties under consideration are assumed to follow certain mutually uncorrelated Bernoulli-distributed white noise sequences. Based on the Itô's differential formula, Lyapunov stability theory, and linear matrix inequalities techniques, several novel sufficient conditions are derived using delay partitioning approach to ensure the dissipativity of neural networks with or without time-varying parametric uncertainties. It is shown, by comparing with existing approaches, that the delay-partitioning projection approach can largely reduce the conservatism of the stability results. Numerical examples are constructed to show the effectiveness of the theoretical results.

Keywords: dissipativity, stochastic fuzzy neural network, time-varying delay.

1 Introduction

Over the past few decades, dynamical behavior of neural networks (NNs)has been studied much in science and technology area, such as signal processing, parallel computing,

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optimization problems, and so on [5, 8]. This led to significant attraction of many researchers, like mathematicians, physicists, computer scientists, and biologists. So, it is valuable to investigate the stability of NNs. Time delays are likely to be present due to the finite switching speed of amplifiers that are the core elements for implementing artificial neurons in models of NNs. The existence of time delays may lead to instability, oscillation, and chaos phenomena [1,2,13]. On the basis of this point, it is greatly worthful that the stability issue of delayed NNs is researched [4,23,37,38], which has aroused much attention for their potential applications in optimization [6]. Therefore, the study of neural dynamics with consideration of time delays becomes extremely important to manufacture high quality neural networks.

In recent years, the control technique based on the so-called T–S fuzzy model has attracted lots of attention since it is regarded as a powerful solution to bridge the gap between the fruitful linear control and the fuzzy logic control targeting complex nonlinear systems. It has been efficiently developed to many applications, and it is shown to be an effective approach to neural network and its stability. Originally, Tanaka and his colleagues have provided a sufficient condition for the quadratic stability of the T–S fuzzy systems in [26] by considering a Lyapunov function of the sub-fuzzy systems of the T–S fuzzy systems, and then it is successfully used in neural network [3, 10]. The T–S fuzzy model approach is essentially a multimodel approach in which some linear models are blended into an overall single model through nonlinear membership functions to represent the nonlinear dynamics. These systems are nonlinear systems described by a set of IF-THEN rules. Recently, problems of stability analysis for fuzzy systems with time-varying delays have been discussed in [19, 24, 36].

In nonlinear systems theory [12], connections between achievable dynamic performance and some process properties linked to energy-like considerations introduce the concept of passive systems, that is, systems that cannot store more energy than that supplied by the environment and/or by other systems connected to them. A more general system-theoretic viewpoint is the concept of dissipative systems, which is a generalization of passive systems with more general process internal and supplied energies. Since the study of dissipative systems was initiated by Willems [29] and further addressed by Hill and Moylan [9], there has been a steady increase in the interest of dissipative systems in the past several decades. Many significant advances on this issue have been reported in the literature. In [31], the dissipativity of singular systems with time delay has been investigated. In addition, the dissipativity problem has been addressed for continuous-time neural networks [30, 35] and discrete-time neural networks [15, 22], but there have appeared a few works on the dissipativity analysis of T–S fuzzy neural networks with time delays [20] as it essentially generalizes the idea of a Lyapunov functional.

In real nervous systems, due to modeling and measurement errors, neural network is often disturbed by the parameter uncertainties, which may cause undesirable dynamic behaviors or poor performance. Recently, a new type of uncertainty named as randomly occurring uncertainty has been proposed to model the random changes in environmental circumstances [11], and some results related to this problem have recently published in [30, 35]. The results in [18] showed that one neural network could be stabilized or destabilized by certain stochastic inputs. It is shown that the stability analysis of stochastic neural networks has primary importance in the design and applications of neural networks. Recently, stability analysis of stochastic neural networks with time-delays has received much attention; see, for example, [14,41,42]. Although the importance of dissipativity has been widely recognized, few results have been proposed for the dissipativity of stochastic fuzzy neural networks with time-varying delay [21]. This motivates the work of this paper.

In this paper, we are concerned with the problem of dissipativity for stochastic fuzzy neural networks with time delay. By using of the delay partitioning technique and the stochastic Itô's formula, some criteria are derived to ensure the dissipativity of the considered neural networks. The obtained delay-dependent results also depend upon the partitioning size. Finally, several numerical examples are given to demonstrate the reduced conservatism of the proposed methods.

Notations. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The notation $X \geqslant 0$ (resp. $X > 0$), where X is a symmetric matrices, means that X is positive semi-definite (resp. positive definite). The superscript $^{\mathrm{T}}$ denotes the transpose of the matrix. $\Pr\{\alpha\}$ means the occurrence probability of the event α.

2 Problem formulation

Consider the following T–S fuzzy neural networks:

$$\begin{aligned}
&\text{IF} && \xi_1 \text{ is } M_{i1}, \ \ldots, \text{ and } \xi_p \text{ is } M_{ip},\\
&\text{THEN} && \mathrm{d}x(t) = \bigl[-A_i x(t) + B_i f\bigl(x(t)\bigr) + C_i f\bigl(x(t-\tau(t))\bigr) + u(t)\bigr]\,\mathrm{d}t\\
& && \qquad + \bigl[E_{1i}x(t) + E_{2i}x(t-\tau(t))\bigr]\,\mathrm{d}\omega(t),\\
& && y(t) = f\bigl(x(t)\bigr), \quad i = 1,2,\ldots,r,
\end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector associated with the neurons, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^{\mathrm{T}}$ is the activation function, $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^{\mathrm{T}}$ is the input, $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]$ is the output. r is the number of IF-THEN rules. M_{ij} are fuzzy sets and $\xi_1, \xi_2, \ldots, \xi_p$ are premise variables. The matrix $A_i = \mathrm{diag}(a_{1i}, a_{2i}, \ldots, a_{ni})$ is a diagonal matrix with positive entries. B_i, C_i are the interconnection matrices representing the weight coefficients of the neurons. $E_{1i} \in \mathbb{R}^{n\times n}$ and $E_{2i} \in \mathbb{R}^{n\times n}$ are known constant matrices. $\tau(t)$ is the time-varying delay satisfying $0 \leqslant \tau(t) \leqslant \bar{\tau}$. In order to derive some less restrictive stability criteria, we partition $\tau(t)$ into several components, i.e., $\tau(t) = \sum_{i=1}^{m} \tau_i(t)$, where m is a positive integer. In this paper, the time-varying delay components $\tau_i(t)$ satisfying the following case:

Case 1. $\tau_i(t)$ is a differentiable function satisfying

$$0 < \tau_i(t) \leqslant \bar{\tau}_i, \quad \dot{\tau}_i(t) \leqslant \tau_i \quad \forall t > 0, \tag{2}$$

where $\bar{\tau}_i$ and τ_i are constants. For convenience, we define $\bar{\tau} = \min\{\bar{\tau}_1, \bar{\tau}_2, \ldots, \bar{\tau}_m\}$, $\alpha_k(t) = \sum_{i=1}^{k} \tau_i(t)$, and $\bar{\alpha}_k = \sum_{i=1}^{k} \bar{\tau}_i$ with $\alpha_0(t) = 0$, $\bar{\alpha}_0 = 0$ in the boundary expression of the summation. That is, $\tau_i(t)$ and $\bar{\tau}_i$ indicate a partition of the lumped time-varying delay $\tau(t)$ and τ, respectively.

Remark 1. It should be pointed out that such delay-partitioning projection approach is very rational. The reasons are twofold: (i) The properties of $\tau_1(t)$ and $\tau_2(t)$ (let the partitioning number $m = 2$ for presentation simplicity) may be sharply different in many practical situations. Thus, it is not reasonable to combine them together. (ii) When $\tau(t)$ reaches its upper bound, we do not necessarily have both $\tau_1(t)$ and $\tau_2(t)$ reach their maxima at the same time. In other words, if we use an upper bound to bound the delay $\tau(t)$, we have to use the sum of the maxima of $\tau_1(t)$ and $\tau_2(t)$, however, $\tau(t)$ does not achieve this maximum value usually. Therefore, by adopting the delay-partitioning projection approach, less conservative conditions can be proposed.

Remark 2. In view of Case 1, the information of every subinterval delay is taken into account, and the derivative of the time-varying delay may have different upper bounds in various delay intervals. However, in many previous papers, such as [25, 30, 33–35], the derivative of the time-varying delay satisfies $\dot{\tau}(t) \leqslant \mu$, where μ is a constant. In fact, this treatment in [25, 30, 33–35] implies that $\dot{\tau}(t)$ in (2) is enlarged to $\dot{\tau}(t) \leqslant \mu = \max\{\tau_1, \tau_2, \dots, \tau_m\}$, which may cause conservativeness inevitably. However, by using the Lyapunov–Krasovskii functional in this paper, the case above can be taken fully into account.

The state equation is defined as follows:

$$\begin{aligned} \mathrm{d}x(t) &= \sum_{i=1}^{r} \lambda_i(\xi(t)) \big[-A_i x(t) + B_i f\big(x(t)\big) + C_i f\big(x\big(t-\tau(t)\big)\big) + u(t)\big]\,\mathrm{d}t \\ &\quad + \big[E_{1i} x(t) + E_{2i} x\big(t-\tau(t)\big)\big]\,\mathrm{d}\omega(t), \\ y(t) &= \sum_{i=1}^{r} \lambda_i\big(\xi(t)\big) f\big(x(t)\big), \end{aligned} \tag{3}$$

where

$$\lambda_i(\xi) = \frac{\beta_i(\xi)}{\sum_{i=1}^{r} \beta_i(\xi)}, \qquad \beta_i(\xi) = \prod_{j=1}^{p} M_{ij}(\xi_j),$$

and $M_{ij}(\cdot)$ is the grade of the membership function of M_{ij}. We assume

$$\beta_i\big(\xi(t)\big) \geqslant 0, \quad i = 1, 2, \dots, r, \qquad \sum_{i=1}^{r} \beta_i\big(\xi(t)\big) > 0 \quad \forall \xi(t).$$

Hence $\lambda_i(\xi(t))$ satisfy $\lambda_i(\xi(t)) \geqslant 0$, $i = 1, \dots, r$, $\sum_{i=1}^{r} \lambda_i(\xi(t)) = 1$ for any $\xi(t)$. In the sequel, for simplicity, we use λ_i to represent $\lambda_i(\xi(t))$.

Throughout this paper, it is assumed that the activation functions satisfy the following assumption.

(H1) For any $j \in 1, 2, \dots, n$, $f_j(0) = 0$, and there exist constants F_j^- and F_j^+ such that
$$F_j^- \leqslant \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leqslant F_j^+$$
for all $\alpha_1, \alpha_2 \in \mathbb{R}$ and $\alpha_1 \neq \alpha_2$.

Remark 3. It is easy to know that assumption (H1) is less conservative than that of in [25, 33, 34] since the constants are allowed to be positive, negative, or zero, that is to say, the activation function in assumption (H1) is assumed to be neither monotonic, nor differentiable, nor bounded.

There are some different definitions of dissipativity. A less restrictive definition of dissipativity is given in this paper. The quadratic energy supply function E associated with system (1) is defined by
$$E(u, y, T) = \langle y, \mathcal{Q}y\rangle_T + 2\langle y, \mathcal{S}u\rangle_T + \langle u, \mathcal{R}u\rangle_T,$$
where
$$\langle y, u\rangle_T = \int_0^T y^{\mathrm{T}} u \,\mathrm{d}t, \quad T \geqslant 0.$$
Let $L_2[0, \infty]$ be the space of square integrable functions on $[0, \infty]$. $\mathcal{Q}$, $\mathcal{S}$, and $\mathcal{R}$ are real matrices of appropriate dimensions with $\mathcal{Q}$ and $\mathcal{R}$ symmetric. Sometimes, the arguments of a function will be omitted so that no confusion can arise.

To prove our results, the following definitions and lemmas will be used in the proof of our main results.

Definition 1. Given a scalar $\gamma > 0$, real matrices $\mathcal{Q} = \mathcal{Q}^{\mathrm{T}}$, $\mathcal{R} = \mathcal{R}^{\mathrm{T}}$, and matrix $\mathcal{S}$. Neural network (1) is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\gamma$-dissipative for any $t \geqslant 0$. Under zero initial state, the following condition is satisfied:
$$\mathcal{E}\bigl\{E(u, y, t)\bigr\} \geqslant \gamma \mathcal{E}\langle u, u\rangle_t. \tag{4}$$

Definition 2. The neural network (1) is said to be passive from input $u(t)$ to output $y(t)$ if there exists a scalar $\gamma \geqslant 0$ such that the inequality
$$2\left[\int_0^{t_f} y^{\mathrm{T}}(s) u(s) \,\mathrm{d}s\right] \geqslant -\gamma \left[\int_0^{t_f} u^{\mathrm{T}}(s) u(s) \,\mathrm{d}s\right]$$
holds for all $t_f \geqslant 0$ and under the zero initial condition.

Lemma 1. (*See* [7].) *For any constant symmetric matrix $M \in \mathbb{R}^{n\times n}$, $M = M^{\mathrm{T}} > 0$, scalar $r > 0$, and vector function $g : [0, r] \to \mathbb{R}^n$ such that the integrations in the following are well defined,*
$$r \int_0^r g^{\mathrm{T}}(s) M g(s) \,\mathrm{d}s \geqslant \left[\int_0^r g(s) \,\mathrm{d}s\right]^{\mathrm{T}} M \left[\int_0^r g(s) \,\mathrm{d}s\right].$$

Lemma 2 [Schur complement]. (*See* [32].) *For a symmetric matrix*

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix},$$

the following conditions are equivalent:

(i) $S < 0$,
(ii) $S_{11} < 0$, *and* $S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0$,
(iii) $S_{22} < 0$, *and* $S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} < 0$.

Lemma 3. (*See* [32].) *For any matrices X, Y, the following matrix inequality holds*:

$$X^{\mathrm{T}} Y + Y^{\mathrm{T}} X \leqslant X^{\mathrm{T}} P^{-1} X + Y^{\mathrm{T}} P Y,$$

where P is a given positive matrix.

Lemma 4. (*See* [7].) *Given matrices $Q = Q^{\mathrm{T}}$, H, E with appropriate dimensions. Then*

$$Q + HF(t)E + E^{\mathrm{T}} F^{\mathrm{T}}(t) H^{\mathrm{T}} < 0$$

for all $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \leqslant I$ if and only if there exists a scalar $\lambda > 0$ such that

$$Q + \lambda H H^{\mathrm{T}} + \lambda^{-1} E^{\mathrm{T}} E < 0.$$

3 Main results

In this section, we will present dissipativity criteria for stochastic neural networks using delay partition approach. Based on Lyapunov functional approach, a novel delay-dependent dissipativity condition is presented in the following theorem.

Theorem 1. *Assume that assumption* (H1) *is hold. Then for given scalar γ, the stochastic fuzzy neural network described by* (3) *is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\gamma$-dissipative in the sense of Definition* 1 *for any time-varying delay $\tau(t)$ satisfying* (2) *if there exist positive definite matrices P, Q_k $(k = 1, 2, \dots, m)$, R_i $(i = 1, 2, 3)$, T, X, Z, positive diagonal matrices U_1, U_2, and any appropriate dimensional matrices L, $\bar{M}_1 = [M_{11}^{\mathrm{T}} \dots M_{1m}^{\mathrm{T}}]^{\mathrm{T}}$, $\bar{M}_2 = [M_{21}^{\mathrm{T}} \dots M_{2m}^{\mathrm{T}}]^{\mathrm{T}}$, $\dots$, $\bar{M}_k = [M_{k1}^{\mathrm{T}} \dots M_{km}^{\mathrm{T}}]^{\mathrm{T}}$ $(k = 1, 2, \dots, m)$ such that the following linear matrix inequalities* (*LMIs*) *hold*:

$$\begin{bmatrix} \Gamma & \Theta_1 & \Theta_2 & \Delta_1 & \cdots & \Delta_k \\ * & -Z & 0 & 0 & \cdots & 0 \\ * & * & -P & 0 & \cdots & 0 \\ * & * & * & 0 & \cdots & 0 \\ * & * & * & -Z & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & \cdots & -Z \end{bmatrix} < 0, \tag{5}$$

where

$$
\mathit{\Gamma} = \begin{bmatrix}
\mathit{\Gamma}_1 & \mathit{\Gamma}_2 & \cdots & 0 & \mathit{\Gamma}_3 & \mathit{\Gamma}_4 & \cdots & 0 & 0 & \mathit{\Gamma}_5 & \mathit{\Gamma}_6 & 0 & \mathit{\Gamma}_7 & P \\
* & \mathit{\Gamma}_8 & \cdots & 0 & 0 & \mathit{\Gamma}_9 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & \cdots & \mathit{\Gamma}_{10} & \mathit{\Gamma}_{11} & 0 & \cdots & 0 & \mathit{\Gamma}_{12} & 0 & 0 & 0 & 0 & 0 \\
* & * & \cdots & * & \mathit{\Gamma}_{13} & 0 & \cdots & 0 & 0 & 0 & \mathit{\Gamma}_{14} & 0 & 0 & 0 \\
* & * & \cdots & * & * & -X & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & \cdots & * & * & * & \cdots & -X & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \cdots & * & * & * & \cdots & * & -X & 0 & 0 & 0 & 0 & 0 \\
* & * & \cdots & * & * & * & \cdots & * & * & \mathit{\Gamma}_{15} & 0 & 0 & \mathit{\Gamma}_{16} & -\mathcal{S} \\
* & * & \cdots & * & * & * & \cdots & * & * & * & \mathit{\Gamma}_{17} & 0 & \mathit{\Gamma}_{18} & 0 \\
* & * & \cdots & * & * & * & \cdots & * & * & * & * & -T & 0 & 0 \\
* & * & \cdots & * & * & * & \cdots & * & * & * & * & * & \mathit{\Gamma}_{19} & L \\
* & * & \cdots & * & * & * & \cdots & * & * & * & * & * & * & -R+\gamma I
\end{bmatrix},
$$

$$
\begin{aligned}
&\mathit{\Gamma}_1 = -PA_i - A_i^{\mathrm{T}}P + Q_1 + R_1 - F_1U_1 + M_{11} + M_{11}^{\mathrm{T}}, \qquad \mathit{\Gamma}_2 = -M_{11} + M_{21},\\
&\mathit{\Gamma}_3 = -M_{m1}, \qquad \mathit{\Gamma}_4 = -M_{11}, \qquad \mathit{\Gamma}_5 = PB_i + R_2 + F_2U_1, \qquad \mathit{\Gamma}_6 = PC_i,\\
&\mathit{\Gamma}_7 = -A_i^{\mathrm{T}}L^{\mathrm{T}}, \qquad \mathit{\Gamma}_8 = (\tau_1 - 1)(Q_1 - Q_2) + M_{22} + M_{22}^{\mathrm{T}}, \qquad \mathit{\Gamma}_9 = -M_{m2},\\
&\mathit{\Gamma}_{10} = \left(\sum_{i=1}^{m-1}\tau_i - 1\right)(Q_{m-1} - Q_m), \qquad \mathit{\Gamma}_{11} = \mathit{\Gamma}_{12} = -M_{mm},\\
&\mathit{\Gamma}_{13} = \left(\sum_{i=1}^{m}\tau_i - 1\right)(Q_m + R_1) - F_1U_2 + M_{mm} + M_{mm}^{\mathrm{T}},\\
&\mathit{\Gamma}_{14} = \left(\sum_{i=1}^{m}\tau_i - 1\right)R_2 + F_2U_2, \qquad \mathit{\Gamma}_{15} = R_3 + \bar{\tau}^2T - U_1 - \mathcal{Q},\\
&\mathit{\Gamma}_{16} = B_i^{\mathrm{T}}L^{\mathrm{T}}, \qquad \mathit{\Gamma}_{17} = \left(\sum_{i=1}^{m}\tau_i - 1\right)R_3 - U_2, \qquad \mathit{\Gamma}_{18} = C_i^{\mathrm{T}}L^{\mathrm{T}},\\
&\mathit{\Gamma}_{19} = \bar{\alpha}_mX - L - L^{\mathrm{T}},\\
&\mathit{\Theta}_1 = \big[\sqrt{\bar{\alpha}_m}ZE_{1i} \ \ \underbrace{0 \ \ldots \ 0}_{m-1} \ \ \sqrt{\bar{\alpha}_m}ZE_{2i} \ \ \underbrace{0 \ \ldots \ 0}_{m} \ \ 0 \ 0 \ 0 \ 0 \ 0\big]^{\mathrm{T}},\\
&\mathit{\Theta}_2 = \big[PE_{1i} \ \ \underbrace{0 \ \ldots \ 0}_{m-1} \ \ PE_{2i} \ \ \underbrace{0 \ \ldots \ 0}_{m} \ \ 0 \ 0 \ 0 \ 0 \ 0\big]^{\mathrm{T}},\\
&\mathit{\Delta}_1 = \operatorname{col}\big[0 \ \ M_{11} \ \ M_{12} \ \ \ldots \ \ M_{1m} \ \ \underbrace{0 \ \ldots \ 0}_{m+5}\big],\\
&\mathit{\Delta}_2 = \operatorname{col}\big[0 \ \ M_{21} \ \ M_{22} \ \ \ldots \ \ M_{2m} \ \ \underbrace{0 \ \ldots \ 0}_{m+5}\big],
\end{aligned}
$$

$$\dots$$
$$\Delta_k = \mathrm{col}\big[0 \;\; M_{k1} \;\; M_{k2} \;\; \dots \;\; M_{km} \;\; \underbrace{0 \;\; \dots \;\; 0}_{m+5}\big], \quad k = 1, \dots, m.$$

Proof. For simplicity, we denote

$$g(t) = -A_i x(t) + B_i f\big(x(t)\big) + C_i f\big(x\big(t - \tau(t)\big)\big) + u(t),$$
$$\alpha(t) = E_{1i} x(t) + E_{2i} x\big(t - \tau(t)\big).$$

Then system (3) can be rewritten as

$$\mathrm{d}x(t) = \sum_{i=1}^{r} \lambda_i(\xi(t)) g(t)\,\mathrm{d}t + \alpha(t)\,\mathrm{d}\omega(t),$$
$$y(t) = \sum_{i=1}^{r} \lambda_i\big(\xi(t)\big) f\big(x(t)\big).$$

Choose a Lyapunov functional candidate for system (3) as follows:

$$V(x_t) = x^{\mathrm{T}}(t) P x(t) + \sum_{k=1}^{m} \int_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} x^{\mathrm{T}}(s) Q_k x(s)\,\mathrm{d}s$$
$$+ \int_{t-\tau(t)}^{t} \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix} \mathrm{d}s$$
$$+ \bar{\tau} \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} f^{\mathrm{T}}\big(x(s)\big) T f\big(x(s)\big)\,\mathrm{d}s\,\mathrm{d}\theta + \int_{-\bar{\alpha}_m}^{0} \int_{t+\theta}^{t} g^{\mathrm{T}}(s) X g(s)\,\mathrm{d}s\,\mathrm{d}\theta$$
$$+ \int_{-\bar{\alpha}_m}^{0} \int_{t+\theta}^{t} \mathrm{Trace}\big[\alpha^{\mathrm{T}}(s) Z \alpha(s)\big]\,\mathrm{d}s\,\mathrm{d}\theta.$$

Then the stochastic differential of $V(x_t)$ along with (3) can be obtained as follows:

$$\mathrm{d}V(x_t) = \mathcal{L}V(x_t)\,\mathrm{d}t + \sum_{i=1}^{r} \lambda_i 2x^{\mathrm{T}}(t) P g(t)\,\mathrm{d}\omega(t),$$

where

$$\mathcal{L}V(x_t) = \sum_{i=1}^{r} \lambda_i \Big\{ 2x^{\mathrm{T}}(t) P\big[-A_i x(t) + B_i f\big(x(t)\big) + C_i f\big(x\big(t - \tau(t)\big)\big) + u(t)\big]$$
$$+ \mathrm{Trace}\big[\alpha^{\mathrm{T}}(t) P \alpha(t)\big] + x^{\mathrm{T}}(t) Q_1 x(t)$$

$$
\begin{aligned}
&- \sum_{k=1}^{m-1} \left[\left(1 - \sum_{i=1}^{k} \dot{\tau}_i(t) \right) x^{\mathrm{T}}\big(t-\alpha_k(t)\big)(Q_k - Q_{k+1})x\big(t-\alpha_k(t)\big) \right] \\
&- \left(1 - \sum_{i=1}^{m} \dot{\tau}_i(t) \right) x^{\mathrm{T}}\big(t-\alpha_m(t)\big)Q_m x\big(t-\alpha_m(t)\big) \\
&+ \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \\
&- \big(1-\dot{\tau}(t)\big) \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix} \\
&+ \bar{\tau}^2 f^{\mathrm{T}}\big(x(t)\big)Tf\big(x(t)\big) - \bar{\tau} \int_{t-\bar{\tau}}^{t} f^{\mathrm{T}}\big(x(s)\big)Tf\big(x(s)\big)\,\mathrm{d}s \\
&+ \bar{\alpha}_m g^{\mathrm{T}}(t)Xg(t) - \sum_{k=1}^{m} \int_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} g^{\mathrm{T}}(s)Xg(s)\,\mathrm{d}s - \int_{t-\bar{\alpha}_m}^{t-\alpha_m(t)} g^{\mathrm{T}}(s)Xg(s)\,\mathrm{d}s \\
&+ \bar{\alpha}_m \operatorname{Trace}\big[\alpha^{\mathrm{T}}(t)Z\alpha(t)\big] - \int_{t-\bar{\alpha}_m}^{t} \operatorname{Trace}\big[\alpha^{\mathrm{T}}(s)Z\alpha(s)\big]\,\mathrm{d}s \Bigg\}.
\end{aligned}
$$

It is clear that $\operatorname{Trace}[\alpha^{\mathrm{T}}(t)P\alpha(t)] = [\alpha^{\mathrm{T}}(t)P\alpha(t)]$ and $\operatorname{Trace}[\alpha^{\mathrm{T}}(t)Z\alpha(t)] = [\alpha^{\mathrm{T}}(t)\times Z\alpha(t)]$. In view of (2) and by Lemma 1, we have

$$
\begin{aligned}
&\mathcal{L}V(x_t) + \gamma u^{\mathrm{T}}(t)u(t) - \big[y^{\mathrm{T}}(t)\mathcal{Q}y(t) + 2y^{\mathrm{T}}(t)\mathcal{S}u(t) + u^{\mathrm{T}}(t)\mathcal{R}u(t)\big] \\
&\quad \leqslant \sum_{i=1}^{r} \lambda_i \Bigg\{ -2x^{\mathrm{T}}(t)PA_i x(t) + 2x^{\mathrm{T}}(t)PB_i f\big(x(t)\big) \\
&\quad + 2x^{\mathrm{T}}(t)PC_i f\big(x(t-\tau(t))\big) + 2x^{\mathrm{T}}(t)Pu(t) \\
&\quad + \big[E_{1i}x(t) + E_{2i}x\big(t-\tau(t)\big)\big]^{\mathrm{T}} P \big[E_{1i}x(t) + E_{2i}x\big(t-\tau(t)\big)\big] + x^{\mathrm{T}}(t)Q_1 x(t) \\
&\quad - \sum_{k=1}^{m-1} \left[\left(1 - \sum_{i=1}^{k} \tau_i \right) x^{\mathrm{T}}\big(t-\alpha_k(t)\big)(Q_k - Q_{k+1})x\big(t-\alpha_k(t)\big) \right] \\
&\quad - \left(1 - \sum_{i=1}^{m} \tau_i \right) x^{\mathrm{T}}\big(t-\alpha_m(t)\big)Q_m x\big(t-\alpha_m(t)\big) \\
&\quad + \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \\
&\quad - \left(1 - \sum_{i=1}^{m} \tau_i \right) \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 & R_2 \\ R_2^{\mathrm{T}} & R_3 \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix} \\
&\quad + \bar{\tau}^2 f^{\mathrm{T}}\big(x(t)\big)Tf\big(x(t)\big)
\end{aligned}
$$

$$
\begin{aligned}
&- \left(\int\limits_{t-\bar{\tau}}^{t} f\big(x(s)\big)\,\mathrm{d}s \right)^{\mathrm{T}} T \left(\int\limits_{t-\bar{\tau}}^{t} f\big(x(s)\big)\,\mathrm{d}s \right) + \bar{\alpha}_m g^{\mathrm{T}}(t) X g(t) \\
&- \sum_{k=1}^{m} \left(\int\limits_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} g(s)\,\mathrm{d}s \right)^{\mathrm{T}} X \left(\int\limits_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} g(s)\,\mathrm{d}s \right) \\
&- \left(\int\limits_{t-\bar{\alpha}_m}^{t-\alpha_m(t)} g(s)\,\mathrm{d}s \right)^{\mathrm{T}} X \left(\int\limits_{t-\bar{\alpha}_m}^{t-\alpha_m(t)} g(s)\,\mathrm{d}s \right) \\
&+ \bar{\alpha}_m \big[E_{1i}x(t) + E_{2i}x\big(t-\tau(t)\big)\big]^{\mathrm{T}} Z \big[E_{1i}x(t) + E_{2i}x\big(t-\tau(t)\big)\big] \\
&- \sum_{k=1}^{m} \int\limits_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} \mathrm{Trace}\big[\alpha^{\mathrm{T}}(s) Z \alpha(s)\big]\,\mathrm{d}s - \int\limits_{t-\bar{\alpha}_m}^{t-\alpha_m(t)} \mathrm{Trace}\big[\alpha^{\mathrm{T}}(s) Z \alpha(s)\big]\,\mathrm{d}s \Bigg\} \\
&+ u^{\mathrm{T}}(t)[\gamma I - \mathcal{R}]u(t) - f^{\mathrm{T}}\big(x(t)\big)\mathcal{Q}f\big(x(t)\big) - 2f^{\mathrm{T}}\big(x(t)\big)\mathcal{S}u(t). \qquad (6)
\end{aligned}
$$

From assumption (H1) we have

$$
\big(f_i\big(x_i(t)\big) - F_i^- x_i(t)\big)\big(f_i\big(x_i(t)\big) - F_i^+ x_i(t)\big) \leqslant 0, \quad i = 1, 2, \dots, n,
$$

which are equivalent to

$$
\begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} F_i^- F_i^+ e_i e_i^{\mathrm{T}} & -\frac{F_i^- + F_i^+}{2} e_i e_i^{\mathrm{T}} \\ -\frac{F_i^- + F_i^+}{2} e_i e_i^{\mathrm{T}} & e_i e_i^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix} \leqslant 0, \quad i = 1, 2, \dots, n,
$$

where e_r denotes the unit column vector having 1 on its rth row and zeros elsewhere.

Let $U_1 = \mathrm{diag}\{u_{11}, u_{12}, \dots, u_{1n}\}$ and $U_2 = \mathrm{diag}\{u_{21}, u_{22}, \dots, u_{2n}\}$.

Then

$$
\sum_{i=1}^{n} u_{1i} \begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} F_i^- F_i^+ e_i e_i^{\mathrm{T}} & -\frac{F_i^- + F_i^+}{2} e_i e_i^{\mathrm{T}} \\ -\frac{F_i^- + F_i^+}{2} e_i e_i^{\mathrm{T}} & e_i e_i^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_i(x_i(t)) \end{bmatrix} \leqslant 0.
$$

That is,

$$
\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} F_1 U_1 & -F_2 U_1 \\ -F_2 U_1 & U_1 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \leqslant 0. \qquad (7)
$$

Similarly, one has

$$
\begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} F_1 U_2 & -F_2 U_2 \\ -F_2 U_2 & U_2 \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix} \leqslant 0. \qquad (8)
$$

By Leibniz–Newton formula, the following equations hold for any matrices $L, \bar{M}_1, \bar{M}_2, \dots, \bar{M}_k$ ($k = 1, 2, \dots, m$) with appropriate dimensions:

$$0 = 2\xi^{\mathrm{T}}(t)\bar{M}_1\left[x(t) - x\bigl(t-\alpha_1(t)\bigr) - \int_{t-\alpha_1(t)}^{t} g(s)\,\mathrm{d}s - \int_{t-\alpha_1(t)}^{t} \alpha(s)\,\mathrm{d}\omega(s)\right], \tag{9}$$

$$\dots$$

$$\begin{aligned} 0 = 2\xi^{\mathrm{T}}(t)\bar{M}_k\Biggl[& x\bigl(t-\alpha_{k-1}(t)\bigr) - x\bigl(t-\alpha_k(t)\bigr) - \int_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} g(s)\,\mathrm{d}(s) \\ & - \int_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} \alpha(s)\,\mathrm{d}\omega(s)\Biggr], \end{aligned} \tag{10}$$

$$0 = 2g^{\mathrm{T}}(t)L\bigl[-A_i x(t) + B_i f\bigl(x(t)\bigr) + C_i f\bigl(x\bigl(t-\tau(t)\bigr)\bigr) + u(t) - g(t)\bigr], \tag{11}$$

where

$$\xi(t) = \bigl[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}\bigl(t-\alpha_1(t)\bigr)\ \ \dots\ \ x^{\mathrm{T}}\bigl(t-\alpha_{m-1}(t)\bigr)\ \ x^{\mathrm{T}}\bigl(t-\alpha_m(t)\bigr)\bigr]^{\mathrm{T}}.$$

It follows from Lemma 3 that

$$\begin{aligned} -2\xi^{\mathrm{T}}(t)\bar{M}_1 \int_{t-\alpha_1(t)}^{t} \alpha(s)\,\mathrm{d}\omega(s) \leqslant{}& \xi^{\mathrm{T}}(t)\bar{M}_1 Z^{-1}\bar{M}_1^{\mathrm{T}}\xi(t) + \left(\int_{t-\alpha_1(t)}^{t} \alpha(s)\,\mathrm{d}\omega(s)\right)^{\mathrm{T}} Z \\ & \times \left(\int_{t-\alpha_1(t)}^{t} \alpha(s)\,\mathrm{d}\omega(s)\right), \end{aligned} \tag{12}$$

$$\dots$$

$$\begin{aligned} -2\xi^{\mathrm{T}}(t)\bar{M}_k \int_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} \alpha(s)\,\mathrm{d}\omega(s) \leqslant{}& \xi^{\mathrm{T}}(t)\bar{M}_k Z^{-1}\bar{M}_k^{\mathrm{T}}\xi(t) + \left(\int_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} \alpha(s)\,\mathrm{d}\omega(s)\right)^{\mathrm{T}} Z \\ & \times \left(\int_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)} \alpha(s)\,\mathrm{d}\omega(s)\right). \end{aligned} \tag{13}$$

On the other hand, by using the Itô isometry in [18], we can obtain

$$\begin{aligned} & \mathcal{E}\left\{\left[\int_{t-\alpha_1(t)}^{t} \alpha(s)\,\mathrm{d}\omega(s)\right]^{\mathrm{T}} Z \left[\int_{t-\alpha_1(t)}^{t} \alpha(s)\,\mathrm{d}\omega(s)\right]\right\} \\ & \quad = \mathcal{E}\int_{t-\alpha_1(t)}^{t} \mathrm{Trace}\bigl[\alpha^{\mathrm{T}}(s)Z\alpha(s)\bigr]\,\mathrm{d}s, \end{aligned} \tag{14}$$

$$
\begin{aligned}
&\dots\\
&\mathcal{E}\Biggl\{\Biggl[\int\limits_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)}\alpha(s)\,\mathrm{d}\omega(s)\Biggr]^{\mathrm{T}} Z\Biggl[\int\limits_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)}\alpha(s)\,\mathrm{d}\omega(s)\Biggr]\Biggr\}\\
&\quad=\mathcal{E}\int\limits_{t-\alpha_k(t)}^{t-\alpha_{k-1}(t)}\mathrm{Trace}\bigl[\alpha^{\mathrm{T}}(s)Z\alpha(s)\bigr]\,\mathrm{d}s.
\end{aligned}
\tag{15}
$$

Combining (6)–(15), we can obtain

$$
\begin{aligned}
&\mathcal{L}V(x_t)-y^{\mathrm{T}}(t)\mathcal{Q}y(t)-2y^{\mathrm{T}}(t)\mathcal{S}u(t)+u^{\mathrm{T}}(t)[\gamma I-\mathcal{R}]u(t)\\
&\quad\leqslant\sum_{i=1}^{r}\lambda_i\zeta^{\mathrm{T}}(t)\bigl[\varGamma+\varTheta_1 Z\varTheta_1^{\mathrm{T}}+\varTheta_2 P\varTheta_2^{\mathrm{T}}+\varDelta_1 Z\varDelta_1^{\mathrm{T}}+\dots+\varDelta_k Z\varDelta_k^{\mathrm{T}}\bigr]\zeta(t),
\end{aligned}
\tag{16}
$$

where

$$
\begin{aligned}
\zeta(t)=\Biggl[&\xi^{\mathrm{T}}(t)\ \int\limits_{t-\alpha_1(t)}^{t}g^{\mathrm{T}}(s)\,\mathrm{d}s\ \dots\ \int\limits_{t-\alpha_m(t)}^{t-\alpha_{m-1}(t)}g^{\mathrm{T}}(s)\,\mathrm{d}s\ \int\limits_{t-\bar{\alpha}_m}^{t-\alpha_m(t)}g^{\mathrm{T}}(s)\,\mathrm{d}s\\
&f^{\mathrm{T}}\bigl(x(t)\bigr)\ f^{\mathrm{T}}\bigl(x\bigl(t-\tau(t)\bigr)\bigr)\ \Biggl(\int\limits_{t-\bar{\tau}}^{t}f\bigl(x(s)\bigr)\,\mathrm{d}s\Biggr)^{\mathrm{T}}\ g^{\mathrm{T}}(t)\ u^{\mathrm{T}}(t)\Biggr]^{\mathrm{T}}.
\end{aligned}
$$

Applying Schur complement, (16) is equivalent to (5). Since $\varGamma<0$, it is easy to get

$$
\begin{aligned}
&\mathcal{E}\bigl\{y^{\mathrm{T}}(t)\mathcal{Q}y(t)+2y^{\mathrm{T}}(t)\mathcal{S}u(t)+u^{\mathrm{T}}(t)\mathcal{R}u(t)\bigr\}\\
&\quad>\mathcal{E}\bigl\{\mathcal{L}V(x_t)+\gamma u^{\mathrm{T}}(t)u(t)\bigr\}.
\end{aligned}
\tag{17}
$$

Integrating (17) from 0 to t, under zero initial conditions we obtain

$$
\mathcal{E}\bigl\{E(y,u,t)\bigr\}\geqslant\mathcal{E}\bigl\{\gamma\langle u,u\rangle_t+V(x_t)-V(0)\bigr\}\geqslant\gamma\mathcal{E}\langle u,u\rangle_t
$$

for all $t\geqslant 0$. Therefore, when condition (4) is satisfied, the neural network (3) is strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-$\gamma$-dissipative according to Definition 1. □

Remark 4. Recently, the authors of [30, 35] have investigated the dissipativity of neural networks with randomly occuring uncertainties by using Lyapunov–Krasovskii functional methods and LMI approaches. Further, Pan et al. [21] have studied the problem of dissipativity condition of stochastic fuzzy neural networks with distributed time-varying delays and derived a new stability condition based on a Lyapunov–Krasovskii functional including double-integral term and free weighting matrix method. However, as a different aspect, by construction of a new Lyapunov–Krasovskii functional based on a time-delay partitioning approach, which are expected to be less conservative than the results discussed in the some existing literature, an improved delay-dependent stability criterion is proposed in this paper. Although the Lyapunov stability theory and LMI technique were widely used to consider the dynamic behaviors of delayed neural networks. To the best of our knowledge, it is the first time to investigate the dissipativity of stochastic fuzzy neural networks using delay-decomposition method. The condition obtained in Theorem 1 is in the form of LMI, which can be easily solved by using LMI toolbox in Matlab.

Remark 5. Consider the following neural networks without fuzzy and stochastic effects. However, for the general neural network, one has achieved some results in [25,30,33–35]. We have

$$\begin{aligned} \mathrm{d}x(t) &= -Ax(t) + Bf\big(x(t)\big) + Cf\big(x(t-\tau(t))\big) + u(t),\\ y(t) &= f\big(x(t)\big). \end{aligned} \tag{18}$$

According to Theorem 1, we have the following corollary for the delay-dependent dissipativity of the neural networks (18).

Corollary 1. *Assume that assumption* (H1) *is hold. Then for given scalar γ, the neural network described by* (18) *is strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-$\gamma$-dissipative for any time-varying delay $\tau(t)$ satisfying* (2) *if there exist positive definite matrices P, Q_k $(i = 1, 2, \dots, m)$, R_i $(i = 1, 2, 3)$, T, X, positive diagonal matrices U_1, U_2, and any appropriate dimensional matrices L, $\bar{M}_1 = [M_{11}^{\mathrm{T}}\, M_{12}^{\mathrm{T}}\, \dots\, M_{1m}^{\mathrm{T}}]^{\mathrm{T}}$, $\bar{M}_2 = [M_{21}^{\mathrm{T}}\, M_{22}^{\mathrm{T}}\, \dots\, M_{2m}^{\mathrm{T}}]^{\mathrm{T}}$, $\dots$, $\bar{M}_k = [M_{k1}^{\mathrm{T}}\, M_{k2}^{\mathrm{T}}\, \dots\, M_{km}^{\mathrm{T}}]^{\mathrm{T}}$ $(k = 1, 2, \dots, m)$ such that the following LMIs hold*:

$$\bar{\Gamma} = \begin{bmatrix}
\Gamma_1 & \Gamma_2 & \cdots & 0 & \Gamma_3 & \Gamma_4 & \cdots & 0 & 0 & \Gamma_5 & \Gamma_6 & 0 & \Gamma_7 & P\\
* & \Gamma_8 & \cdots & 0 & 0 & \Gamma_9 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
* & * & \cdots & \Gamma_{10} & \Gamma_{11} & 0 & \cdots & 0 & \Gamma_{12} & 0 & 0 & 0 & 0 & 0\\
* & * & \cdots & * & \Gamma_{13} & 0 & \cdots & 0 & 0 & 0 & \Gamma_{14} & 0 & 0 & 0\\
* & * & \cdots & * & * & -X & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
* & * & \cdots & * & * & * & \cdots & -X & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & \cdots & * & * & * & \cdots & * & -X & 0 & 0 & 0 & 0 & 0\\
* & * & \cdots & * & * & * & \cdots & * & * & \Gamma_{15} & 0 & 0 & \Gamma_{16} & -\mathcal{S}\\
* & * & \cdots & * & * & * & \cdots & * & * & * & \Gamma_{17} & 0 & \Gamma_{18} & 0\\
* & * & \cdots & * & * & * & \cdots & * & * & * & * & -T & 0 & 0\\
* & * & \cdots & * & * & * & \cdots & * & * & * & * & * & \Gamma_{19} & L\\
* & * & \cdots & * & * & * & \cdots & * & * & * & * & * & * & -R+\gamma I
\end{bmatrix} < 0, \tag{19}$$

$$\begin{aligned}
&\Gamma_1 = -PA - A^{\mathrm{T}}P + Q_1 + R_1 - F_1U_1 + M_{11} + M_{11}^{\mathrm{T}}, \qquad \Gamma_2 = -M_{11} + M_{21},\\
&\Gamma_3 = -M_{m1}, \qquad \Gamma_4 = -M_{11}, \qquad \Gamma_5 = PB_i + R_2 + F_2U_1, \qquad \Gamma_6 = PC,\\
&\Gamma_7 = -A^{\mathrm{T}}L^{\mathrm{T}}, \qquad \Gamma_8 = (\tau_1 - 1)(Q_1 - Q_2) + M_{22} + M_{22}^{\mathrm{T}}, \qquad \Gamma_9 = -M_{m2},
\end{aligned}$$

$$
\begin{aligned}
&\mathit{\Gamma}_{10} = \left(\sum_{i=1}^{m-1}\tau_i - 1\right)(Q_{m-1} - Q_m), \qquad \mathit{\Gamma}_{11} = \mathit{\Gamma}_{12} = -M_{mm},\\
&\mathit{\Gamma}_{13} = \left(\sum_{i=1}^{m}\tau_i - 1\right)(Q_m + R_1) - F_1U_2 + M_{mm} + M_{mm}^{\mathrm{T}},\\
&\mathit{\Gamma}_{14} = \left(\sum_{i=1}^{m}\tau_i - 1\right)R_2 + F_2U_2, \qquad \mathit{\Gamma}_{15} = R_3 + \bar{\tau}^2T - U_1 - \mathcal{Q},\\
&\mathit{\Gamma}_{16} = B^{\mathrm{T}}L^{\mathrm{T}}, \qquad \mathit{\Gamma}_{17} = \left(\sum_{i=1}^{m}\tau_i - 1\right)R_3 - U_2,\\
&\mathit{\Gamma}_{18} = C^{\mathrm{T}}L^{\mathrm{T}}, \qquad \mathit{\Gamma}_{19} = \bar{\alpha}_mX - L - L^{\mathrm{T}}.
\end{aligned}
$$

Remark 6. When $\mathcal{Q} = 0$, $\mathcal{S} = I$, and $\gamma I - \mathcal{R} = -\gamma I$, our result in Corollary 1 corresponds to a passive problem [25, 33, 34], which can be obtained in the following corollary directly. Thus, the investigation in this paper improves the existing literature.

Corollary 2. *Assume that assumption* (H1) *is hold. Then the neural network described by* (18) *is passive in the sense of Definition* 2 *for any time-varying delay* $\tau(t)$ *satisfying* (2) *if there exist positive definite matrices* P, Q_k $(i = 1, 2, \dots, m)$, R_i $(i = 1, 2, 3)$, T, X, *positive diagonal matrices* U_1, U_2, *and any appropriate dimensional matrices* L, $\bar{M}_1 = [M_{11}^{\mathrm{T}}\, M_{12}^{\mathrm{T}} \dots M_{1m}^{\mathrm{T}}]^{\mathrm{T}}$, $\bar{M}_2 = [M_{21}^{\mathrm{T}}\, M_{22}^{\mathrm{T}} \dots M_{2m}^{\mathrm{T}}]^{\mathrm{T}}$, $\dots$, $\bar{M}_k = [M_{k1}^{\mathrm{T}}\, M_{k2}^{\mathrm{T}} \dots M_{km}^{\mathrm{T}}]^{\mathrm{T}}$ $(k = 1, 2, \dots, m)$ *such that the following LMIs hold* :

$$
\hat{\mathit{\Gamma}} = \begin{bmatrix}
\mathit{\Gamma}_1 & \mathit{\Gamma}_2 & \cdots & 0 & \mathit{\Gamma}_3 & \mathit{\Gamma}_4 & \cdots & 0 & 0 & \mathit{\Gamma}_5 & \mathit{\Gamma}_6 & 0 & \mathit{\Gamma}_7 & P\\
* & \mathit{\Gamma}_8 & \cdots & 0 & 0 & \mathit{\Gamma}_9 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
* & * & \cdots & \mathit{\Gamma}_{10} & \mathit{\Gamma}_{11} & 0 & \cdots & 0 & \mathit{\Gamma}_{12} & 0 & 0 & 0 & 0 & 0\\
* & * & \cdots & * & \mathit{\Gamma}_{13} & 0 & \cdots & 0 & 0 & 0 & \mathit{\Gamma}_{14} & 0 & 0 & 0\\
* & * & \cdots & * & * & -X & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
* & * & \cdots & * & * & * & \cdots & -X & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & \cdots & * & * & * & \cdots & * & -X & 0 & 0 & 0 & 0 & 0\\
* & * & \cdots & * & * & * & \cdots & * & * & \mathit{\Gamma}_{15} & 0 & 0 & \mathit{\Gamma}_{16} & -I\\
* & * & \cdots & * & * & * & \cdots & * & * & * & \mathit{\Gamma}_{17} & 0 & \mathit{\Gamma}_{18} & 0\\
* & * & \cdots & * & * & * & \cdots & * & * & * & * & -T & 0 & 0\\
* & * & \cdots & * & * & * & \cdots & * & * & * & * & * & \mathit{\Gamma}_{19} & L\\
* & * & \cdots & * & * & * & \cdots & * & * & * & * & * & * & -\gamma I
\end{bmatrix} < 0,
$$

where

$$
\mathit{\Gamma}_{15} = R_3 + \bar{\tau}^2T - U_1,
$$

and other terms are same as defined in Corollary 1.

Proof. The proof immediately follows from the similar way of proof of Corollary 1, hence it is omitted. □

4 Dissipativity with randomly occurring uncertainty

In this section, based on Theorem 1 and Corollary 1, we are now ready to develop delay-dependent criterion for the neural networks with time-varying parameter and randomly occurring uncertainties in the form

$$\begin{aligned} \mathrm{d}x(t) &= -\big(A + \alpha(t)\Delta A(t)\big)x(t) + \big(B + \beta(t)\Delta B(t)\big)f\big(x(t)\big) \\ &\quad + \big(C + \delta(t)\Delta C(t)\big)f\big(x(t-\tau(t))\big) \\ x(t) &= \phi(t) \quad \forall t \in [-\bar{\tau}, 0], \end{aligned} \tag{20}$$

where $\Delta A(t)$, $\Delta B(t)$ and $\Delta C(t)$ denotes the uncertain matrices and takes the following form:

$$\big[\Delta A(t)\ \ \Delta B(t)\ \ \Delta C(t)\big] = HF(t)[E_1\ \ E_2\ \ E_3], \tag{21}$$

where H, E_1, E_2 and E_3 are known real constant matrices with appropriate dimensions and $F(t)$ is an unknown real matrix function satisfying

$$F^{\mathrm{T}}(t)F(t) \leqslant I \quad \forall t \geqslant 0.$$

In this paper, the parameter uncertainties denoted as in (21) are randomly occurring, which was firstly introduced in [11]. The stochastic variables $\alpha(t), \beta(t)$ and $\delta(t)$ are mutually independent Bernoulli-distributed white sequences. A natural assumption on the these stochastic variables can be made as follows:

$$\Pr\{\alpha(t) = 1\} = \alpha, \qquad \Pr\{\alpha(t) = 0\} = 1 - \alpha, \tag{22}$$

$$\Pr\{\beta(t) = 1\} = \beta, \qquad \Pr\{\beta(t) = 0\} = 1 - \beta, \tag{23}$$

$$\Pr\{\delta(t) = 1\} = \delta, \qquad \Pr\{\delta(t) = 0\} = 1 - \delta, \tag{24}$$

where $\alpha \in [0, 1]$, $\beta \in [0, 1]$ and $\delta \in [0, 1]$ are known constants.

Based on Corollary 1, the following criteria can be readily derived.

Theorem 2. *Assume that assumption* (H1) *is hold. Then for given scalar γ, the neural network described by* (20) *is strictly* $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-$\gamma$*-dissipative for any time-varying delay $\tau(t)$ satisfying* (2) *if there exist a scalar λ, positive definite matrices P, Q_k ($i = 1, 2, \dots, m$), R_i ($i = 1, 2, 3$), T, X, positive diagonal matrices U_1, U_2 and any appropriate dimensional matrices L, $\bar{M}_1 = [M_{11}^{\mathrm{T}}\ \dots\ M_{1m}^{\mathrm{T}}]^{\mathrm{T}}$, $\bar{M}_2 = [M_{21}^{\mathrm{T}}\ \dots\ M_{2m}^{\mathrm{T}}]^{\mathrm{T}}$, $\dots$, $\bar{M}_k = [M_{k1}^{\mathrm{T}}\ \dots\ M_{km}^{\mathrm{T}}]^{\mathrm{T}}$ ($k = 1, 2, \dots, m$) such that the following LMIs hold*:

$$\begin{bmatrix} \bar{\Gamma} & \Gamma_d & \lambda\Gamma_e^{\mathrm{T}} \\ * & -\lambda I & 0 \\ * & * & -\lambda I \end{bmatrix} < 0, \tag{25}$$

where $\bar{\Gamma}$ is defined in Corollary 1.

Proof. Replace A, B, C in (19) with $A+\alpha(t)\Delta A(t)$, $B+\beta(t)\Delta B(t)$, $C+\delta(t)\Delta C(t)$, respectively. Then the uncertain system (20) is equivalent to the following condition:

$$\bar{\Gamma}+\Gamma_d F(t)\Gamma_e+\Gamma_e^{\mathrm{T}}F^{\mathrm{T}}(t)\Gamma_d^{\mathrm{T}}<0, \tag{26}$$

where

$$\Gamma_d=\big[PH\ \underbrace{0\ \ldots\ 0}_{m}\ \underbrace{0\ \ldots\ 0}_{m}\ 0\ 0\ 0\ LH\ 0\big]^{\mathrm{T}},$$
$$\Gamma_e=\big[-\alpha E_1\ \underbrace{0\ \ldots\ 0}_{m}\ \underbrace{0\ \ldots\ 0}_{m}\ \beta E_2\ \delta E_3\ 0\ 0\ 0\big].$$

By using Lemma 4, we obtain the necessary and sufficient condition to satisfy inequality (26), and there exist a scalar $\lambda>0$ such that

$$\bar{\Gamma}+\lambda^{-1}\Gamma_d\Gamma_d^{\mathrm{T}}+\lambda\Gamma_e^{\mathrm{T}}\Gamma_e<0. \tag{27}$$

Now, by applying Schur complement, (27) is equivalent to (25). This completes the proof of Theorem 2. □

Remark 7. In view of delay-partitioning idea employed in this work, with integer m increasing, the dimension of the derived LMIs will become higher, and it will take more computing time to check the stability criteria. It should be pointed out that the conservatism will be reduced as the decomposition becomes thinner. Therefore, the more effective results can be obtained if we employ Lyapunov–Krasovskii functional with $m>2$.

5 Numerical examples

In this section, we are analyzing some numerical examples to show the effectiveness of the proposed methods.

Example 1. Consider the stochastic fuzzy neural networks (3) with the parameters as follows:

$$\begin{aligned}\mathrm{d}x(t)&=\big[-A_ix(t)+B_if\big(x(t)\big)+C_if\big(x\big(t-\tau(t)\big)\big)+u(t)\big]\,\mathrm{d}t\\&\quad+\big[E_{1i}x(t)+E_{2i}x\big(t-\tau(t)\big)\big]\,\mathrm{d}\omega(t),\quad i=1,2,\end{aligned}$$

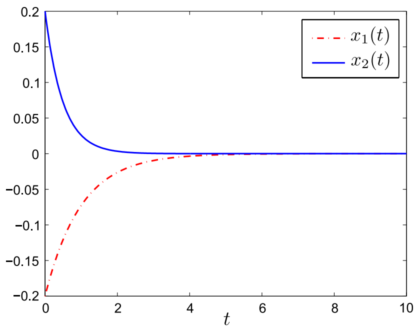
where

$$A_1=\begin{bmatrix}1&0\\0&2\end{bmatrix},\qquad A_2=\begin{bmatrix}2&0\\0&3\end{bmatrix},\qquad B_1=\begin{bmatrix}0.14&-0.04\\-0.05&0.06\end{bmatrix},$$

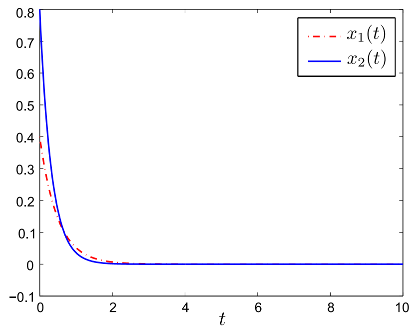
$$B_2=\begin{bmatrix}-0.09&-0.16\\0.09&-0.24\end{bmatrix},\qquad C_1=\begin{bmatrix}-0.01&0.12\\0.03&0.01\end{bmatrix},\qquad C_2=\begin{bmatrix}0.1&0.06\\0.02&-0.1\end{bmatrix},$$

Table 1. Maximum value of $\bar{\tau}$ for different values of τ (Example 1).

Methods	$\tau = 0.1$	$\tau = 0.3$	$\tau = 0.5$	$\tau = 0.7$	$\tau = 0.9$
[21]	2.0649	1.1296	1.0550	1.0550	1.0550
Theorem 1 ($m = 2$)	2.3116	1.5278	1.3648	1.1537	1.1537



(a) Model 1



(b) Model 2

Figure 1. The state response in Example 1.

$$E_{11} = \begin{bmatrix} 0.2 & -0.1 \\ 0.1 & -0.2 \end{bmatrix}, \qquad E_{12} = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & 0.1 \end{bmatrix},$$

$$E_{21} = \begin{bmatrix} -0.2 & 0.01 \\ 0.2 & -0.02 \end{bmatrix}, \qquad E_{22} = \begin{bmatrix} 0.1 & -0.02 \\ 0.1 & 0.1 \end{bmatrix},$$

$$\mathcal{Q} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix}, \qquad \mathcal{R} = \begin{bmatrix} 5 & 0 \\ 0 & 6 \end{bmatrix}, \qquad \mathcal{S} = \begin{bmatrix} 0.7 & 0.5 \\ 2.2 & -0.4 \end{bmatrix}.$$

The activation functions are taken as $f_1(x_1) = \tanh(-x_1)$, $f_2(x_2) = \tanh(0.3x_2)$. The time-varying delay satisfies (2), and for given scalar $\gamma = 1$, by solving LMI (5) in Theorem 1, the obtained upper bounds $\bar{\tau}$ for different τ are listed in Table 1. From Table 1 it can be easily seen that the method proposed in this paper is much less conservative than the corresponding method in [21]. Figure 1(a) gives the state response of the neural network (3) with the initial condition $x(t) = [-0.2\,0.2]^{\mathrm{T}}$. Figure 1(b) gives the state response of the neural network (3) with the initial condition $x(t) = [0.4\,0.8]^{\mathrm{T}}$, which shows that the neural network is stable.

Example 2. Consider the neural networks (20) with the parameters as follows:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 & 1 \\ 0.5 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} -0.5 & 0.6 \\ 0.7 & 0.8 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, \qquad E_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.15 \end{bmatrix},$$

Table 2. Maximum value of γ for different values of a (Example 2).

Methods	$a = 0$	$a = 0.25$	$a = 0.50$	$a = 1$
[30]	1.5871	1.5807	1.5739	1.5597
[35]	1.7183	1.7138	1.7090	1.6990
Theorem 2 ($m = 2$)	1.9832	1.9342	1.8812	1.8524

Table 3. Maximum value of γ for different values of $\bar{\tau}$ (Example 2).

Methods	$\bar{\tau} = 0.5$	$\bar{\tau} = 1.0$	$\bar{\tau} = 1.5$	$\bar{\tau} = 2.0$
[30]	1.5182	1.5179	1.5153	1.5104
[35]	1.7006	1.6379	1.5922	1.5583
Theorem 2 ($m = 2$)	1.9252	1.8687	1.8176	1.7825

$$E_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad \mathcal{Q} = \begin{bmatrix} -0.9 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad \mathcal{S} = \begin{bmatrix} 0.5 & 0 \\ 0.3 & 1 \end{bmatrix}, \qquad \mathcal{R} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

$$F_1^+ = F_2^+ = 0.9, \qquad F_1^- = F_2^- = -0.1.$$

In this example, we choose $\alpha = \beta = \delta = a$, $\bar{\tau} = \tau = 0.4$. By using Theorem 2, the maximum value of γ for different values of a are shown in Table 2. In addition, for $a = \tau = 0.5$ and different $\bar{\tau}$, the maximum values of γ calculated by Theorem 2 are listed in Table 3. From Tables 2 and 3 it can be easily seen that the method proposed in this paper is much less conservative than the corresponding methods in [30, 35].

Example 3. Consider the neural networks (18) with the parameters as follows:

$$A = \begin{bmatrix} 2.2 & 0 \\ 0 & 1.8 \end{bmatrix}, \qquad B = \begin{bmatrix} 1.2 & 1 \\ -0.2 & 0.3 \end{bmatrix}, \qquad C = \begin{bmatrix} 0.8 & 0.4 \\ -0.2 & 0.1 \end{bmatrix}.$$

The activation functions are taken as follows:

$$f_i(x_i) = 0.5\bigl(|x_i + 1| - |x_i - 1|\bigr), \quad i = 1, 2.$$

It can be verified that assumption (H1) is satisfied with $F_1^+ = F_2^+ = 1$, $F_1^- = F_2^- = 0$. For various values of τ, the computed upper bounds $\bar{\tau}$, which guarantee the passivity of neural network (18), are listed in Table 4. It can be seen that the passivity result we proposed is less conservative than that in [25, 30, 33–35]. Figure 2 gives the state response of the neural network (18) with the initial condition $x(t) = [-1\,1]^{\mathrm{T}}$.

Table 4. Maximum value of $\bar{\tau}$ for different values of τ (Example 3).

Methods	$\tau = 0.5$	$\tau = 0.9$	$\tau \geqslant 1$
[25]	0.5227	0.4613	0.4613
[34]	1.3752	1.3027	1.3027
[33]	1.4693	1.4243	1.4240
[30]	1.8450	1.7647	1.7313
[35]	2.2058	2.0366	2.0000
Corollary 3.3 ($m = 2$)	2.6784	2.4246	2.3137

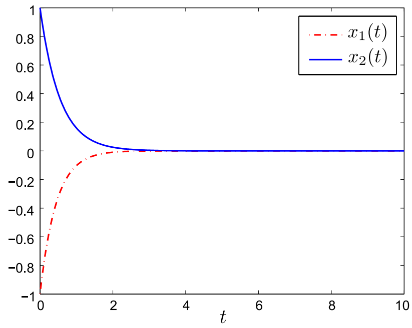


Figure 2. State trajectory of the neural network in Example 3.

6 Conclusion

In this paper, the problem of dissipativity analysis has been investigated for stochastic fuzzy neural networks with time delay using delay partitioning approach. Several delay-dependent sufficient conditions have been proposed to guarantee the dissipativity of the considered neural networks. All the results given in this paper are delay-dependent as well as partition dependent. The effectiveness as well as the reduced conservatism of the derived results has been shown by several numerical examples. By utilizing the proposed idea of this paper, future works will focus on stabilization for various dynamic systems with time-delays, such as adaptive dynamic surface control design systems [16], distributed adaptive consensus tracking control for nonlinear multi-agent systems [39], fuzzy fault-tolerant control systems [17], output-feedback control systems [28], fuzzy hierarchical sliding-mode control systems [40], and fuzzy adaptive tracking control systems [27].

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