GOVERNMENT DEBT SUSTAINABILITY: THE LITHUANIAN CASE

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Abstract. *The paper deals with some aspects of sustainability of the government debt of Lithuania for a long, medium and short time horizon. The purpose of the study is to investigate the sustainability of the debt with respect to the nominal GDP growth, the interest rate on the debt, and the primary budget deficit. The differential equations calculus technique is used for the analysis of the debt. A scenario approach is used to simulate the sensitivity of the debt to economic shocks. An inequality for a non-increasing debt over a medium term is applied for the possibility to restore the level of the debt to a pre-crisis level. The sensitivity of the debt to a short movement of interest rate, the GDP growth rate and the primary budget deficit is investigated. A rule of debt change in a short run by one percentage point is formulated.*

Key words: *government debt, budget deficit, primary budget balance, sustainability of debt, nominal GDP growth, interest rate on debt*

1. Introduction

Currently, the sustainability of the government debt receives much attention from both economists and politicians. Financial markets very closely observe and very dramatically react to the issues related to the government debts. The reaction of financial markets is very robust and implies an extremely high volatility of the interest rates. For the countries that have no independent monetary policy, the debt sustainability is even more relevant, because such countries have no monetary tools to amortise the debt-changing inflation targets. Just before introducing the Euro, the bond yields of the Eurozone countries' converged to the yields of Germany. Currently, the spreads of the yields of different countries are extremely large, and the volatility of spreads is as high as never before. The main reason for the turbulences of the financial markets is the increased credit risk, particularly a sovereign risk. A sovereign financial risk, which is defined as the country's ability to fulfil its financial obligations, depends mainly on the development of the government debt in the future. Therefore, the sustainability of the government debt is crucial for all the countries. The European Union's (EU) financial market is a single market; nonetheless, the credit risk for each country is an individual one. The

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development of the government debt is important for the governments who issue the debt and for the investors who purchase the debt. Currently, when the volatility of the interest rate of sovereign bonds has reached an extremely high level, it is a point of issue.

In order to limit the sovereign debt, the European Union (EU) has created limits for the government debt and the budget deficit. They are called the Maastricht criteria which were implemented by the Maastricht treaty. The criteria were set forth in the Stability and Growth Pact later on. According to the Maastricht criteria, a budget deficit for a particular country should be below 3% of its GDP, and the government debt should be substantially below 60% of the GDP. The criteria are very simple to check and have a great economic point. There are many papers that justify the figures of these criteria. By using statistical data, Baum et al., (2012) have proven that the budget deficit supports the growth of the GDP when it does not exceed 67%, and a country has some pressure when the budget deficit exceeds 70%. Reinhart and Rogoff (2010) have found that the debt exceeding 60% of the GDP significantly reduces the GDP growth. By using empirical data, Kumar and Woo (2010) have proven that the amount of the debt has a nonlinear effect on the economic growth. Despite the above fact, there are some papers to suggest some adjustments to the Maastricht criteria in order to take into account other economic data such as the GDP growth (see Governatori and Eijffinger 2004).

The concept of debt sustainability was discussed by a number of authors and institutions. International institutions such as the IMF, the European Commission and the European Central Bank ECB (2011) use the common definition of sustainability. The common approach is to define the sustainability of the government debt as the government's solvency and liquidity which, in general, means an ability to refinance the debt. This paper will not deal with liquidity issues, yet it will define sustainability as solvency. According to international institutions, the debt is considered sustainable (solvent) when the present value of the future government's net income is greater than or equal to the current debt. In other words, the debt is guaranteed by the future income. This definition can be expressed in the following way:

$$b_0 \leq \sum_{t=1}^{\infty} (pb)_t \rho_t \text{ where } \rho_t = \frac{(1+g_t)}{(1+i_t)}\rho_{t-1}, \tag{1}$$

where b_0 is the initial debt to GDP, g_t is the nominal GDP growth, i_t is the interest rate on the government debt, and $(pb)_t$ is the primary budget balance.

The inequality (1) is based on the future parameters such as the primary balance, the GDP growth and the interest rate; therefore, it is extremely difficult to estimate them. Wyplosh (2007) calls such a definition non-operational. The Value at Risk approach was used by Celasum et al. (2007) to estimate the level of sustainability under a given probability. This approach is based on a long and reliable data set. Credible results can be achieved when the volatility of the parameters is low.

The sustainability of the debt depends mainly on the policy of the government. Mendoza and Ostry (2008) have developed a model which includes the government's responses to the amount of the debt in managing the deficit. Some modifications of this model were applied for the Eastern and Central European countries by Cuestas and Steahr (2010). Their analysis includes the Lithuanian case as well.

The sustainability of the Lithuanian government debt requires a particularly great attention, because the growth of the debt has been very high over the last years. According to the European Commission (2012), Lithuania is among the five EU countries in which the debt has more than doubled in 2008–2011. The other countries are Ireland, Latvia, Slovenia, and Romania. The issues related to the Lithuanian government debt were described by Budrytė and Tursa (2002). Jasienė and Paškevičius (2009) dealt with the impact of the government bond market on the Lithuanian financial market.

2. Debt sustainability according to the Maastricht criteria

In the paper, we use the concept of debt sustainability as it is presented by international institutions, but technically we arrive to a more general model. We use a differential equation instead of a finite-difference scheme to describe the relationship between the debt and the budget deficit. Moreover, we do not require the parameters i_i and g_i to be stable during a year. As we see from the recent history, these parameters can change dramatically in the course of a year. Instead of the compound interest we use continuously compound interest which is convenient to apply in differential calculations. It should be noted that the continuously compound interest and differential equations approach was used by Blanchard et al. (1990). The use of differential equations enables to obtain some estimations and to evaluate the risk of the debt increase by using the differential calculus technique. We define the sustainability in the short and medium perspective as a criterion for the debt to be non-increasing over time. The above-mentioned and other criteria were described by Wyplosh (2007).

First of all, we will describe a mathematical model for a relationship between the government debt and the budget deficit. The Maastricht criteria are formulated in terms of the total budget deficit, irrespective of whether the deficit occurs as the primary deficit or as a result of interest payments on the debt. Therefore, the first equation will be formulated in terms of the total budget deficit:

$$B(t) - B(t - \varDelta) = \mu(t)G(t)\varDelta,$$

where $B(t)$ is the nominal government debt at the moment t, $G(t)$ is the annual nominal GDP from the moment $(t - 1)$ to t, and $\mu(t)$ is a ratio of the government debt to GDP. occurs:

Let us divide the above equation by $G(t)$. Then we have:

$$\frac{B(t)}{G(t)} - \frac{B(t-\Delta)}{G(t)} = \mu(t)\Delta.$$

For a small Δ, the equation above can be rewritten as follows:

$$\frac{B(t)}{G(t)} - \frac{B(t-\Delta)}{G(t-\Delta)}(1 - \Delta\frac{G'(t-\Delta)}{G(t-\Delta)}) = \mu(t)\Delta,$$

where $G'(t-\Delta)$ is the derivative of G with respect of t at the moment $t-\Delta$.

Let us divide the equation above by Δ, take the limit as Δ tends to zero, and denote the ratio of the debt to GDP by $b(t)$. Then we have the following differential problem with respect to $b(t)$:

$$b'(t) = \mu(t) - b(t)\frac{G'(t)}{G(t)}, \, t > 0, \, b(0) = b_0. \tag{2}$$

Let us denote the growth of nominal GDP by $g(t)$. Then $G(t)$ satisfies the following differential equation:

$$\frac{G'(t)}{G(t)} = g(t), \; t > 0, \, G(0) = G_0.$$

The explicit solution of the above equation is equal to

$$G(t) = G_0 e^{\int_0^t g(s)ds}.$$

Taking into account the expression above, the differential equation (2) takes the following form:

$$b'(t) = \mu(t) - b(t)g(t), \; t > 0, \, b(0) = b_0. \tag{3}$$

The first-order linear differential problem (3) can be solved in the explicit way. The solution of it is equal to:

$$b(t) = e^{-\int_0^t g(s)ds}[b_0 + \int_0^t \mu(x)e^{\int_0^x g(s)ds}dx]. \tag{4}$$

The equation above is our main equation to consider later on. To test equation (5), we use the data from the Department of Statistics under the Government of the Republic of Lithuania (2012). The annual data are presented in Table 1.

We provide calculations of the debt according to formula (4). We used data from Table 1 for the value of the budget deficit (function $\mu(t)$) and data from Table 2 for the GDP growth (function $g(t)$). Our initial point was the year 2000, and we took the value of the initial debt from Table1. By using formula (4), we obtained approximate values of the debt for successive years.

TABLE 1. **Lithuanian government statistics**

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Nominal GDP in mln LTL	46003	48879	52351	57232	62997	72402	83227	99229	112084	91914	95074	106019
Nominal government debt in mln LTL	10842	11221	11620	11620	12155	13276	14939	16698	17375	26983	36132	40837
Interest paid for the debt in mln LTL	800	737	685	711	586	583	607	691	738	1160	1732	-
Budget balance as percentage of GDP	-3.2	-3.5	-1.9	-1.3	-1.5	-0.5	-0.4	-1	-3.3	-9.4	-7.2	-5.5
Debt as percentage of GDP	23.6	23	22,2	21	19.3	18.3	17.9	16.8	15.5	29.4	38	38.5

Source: Department of Statistics under the Government of the Republic of Lithuania (2012).

TABLE 2. **Ratios of economic indicators of Lithuania**

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Average
Nominal GDP change in percent (continuously compounded)	4.7	6.1	6.9	8.9	9.6	13.9	13.9	17.6	12.2	-19.8	3.4	10.9	7.4
Interest rate on debt	7.4	6.6	5.9	6.1	4.8	4.4	4.1	4.1	4.3	4.3	4.8	-	5.2
Interest paid for the debt as percentage of GDP	1.7	1.5	1.3	1.2	0.9	0.8	0.7	0.7	0.7	1.3	1.8	-	1.1
Primary budget balance as percentage of GDP	-1.5	-2	-0.6	-0.1	-0.6	0.3	0.3	-0.3	-2.6	-8.2	-5.4	-	-1.9

Source: author's calculations.

Diagrams of the function $b(t)$ according to formula (4) the and statistical values of $b(t)$ from Table 1 are presented in Fig. 1.

One can see in Fig. 1 that the curves are not identical. There are at least two main reasons for the differences. First of all, the government borrows funds at a lump sum by usually issuing Eurobonds, and therefore the debt is not continuous as a function of time. For instance, it can be that the bond is issued at the end of a year for the reason to finance budget needs for the next year. The model does not use the information about the timing
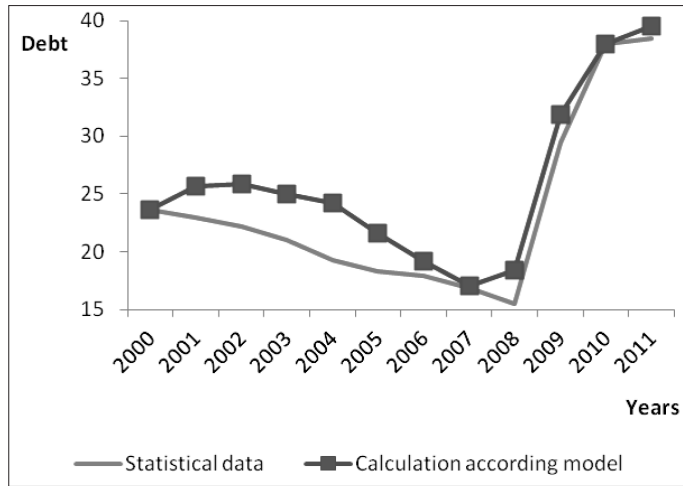
FIG. 1. **The ratio of the government debt to GDP, per cent**

Source: author's calculations.

of a concrete bond issue and the repurchase of the bond. The influence of the timing is dismissed as the time increases. The second reason for the discrepancies of the graphs is the use of privatization funds to finance the budget. This is why the model shows a higher debt in 2001–2006 than it appears from the statistical data. The model does not include the impact of privatization funds. The reason for it is a shortage of reliable statistical data.

Now, let us apply formula (4) to check the limit of the debt according to the Maastricht criteria. We will consider a simplified case. Let the relative budget deficit not to depend on t, i.e. $\mu(t)=\mu$, and the nominal GDP growth is stable, i. e. $t\,g(t) = g$ is a constant.

With the above assumptions, the integral in equation (4) may be calculated in an explicit way. After some simple calculations, we have that

$$b(t) = (b_0 - \mu/g)e^{-gt} + \mu/g$$

and, for a positive g, we have that

$$\lim_{t \to \infty} = b(t) = \mu / g.$$

This means that the relative debt tends to μ/g. If $b_0 > \mu/g$, then $b(t)$ approaches μ/g from the above, and the debt is decreasing. If $b_0 < \mu/g$, then $b(t)$ approaches μ/g from below, and the debt is increasing. In order to make numerical calculations, we can take the value of μ as an average of $\mu(t)$. In the case of Lithuania, $\mu(t)$, which is equal to the negative value of the budget balance as the percentage of GDP, can be found from Table1. The average of g can be found in Table 2 in which the have been obtained by using data from Table1. Then we have:

$\mu = 3.2\%$, $g = 7.4\%$, and $\mu/g = 43\%$.

From the findings above we see that the debt is asymptotically very close to the level of the debt at the end of 2011 when it was equal to 38.5%.

The average of the budget deficit in Lithuania is above the required Maastricht criteria (3.2 > 3); nevertheless, the amount of the debt satisfies the Maastricht criteria and converges to 43% as t increases. This is because the nominal GDP growth, which is equal to 7.4%, is significantly higher than the EU average. Upon considering the above, we can make a conclusion that the debt of Lithuania is within the limits of the Maastricht criteria, because it does not increase significantly and is asymptotically equal to 43%. The conclusion is valid, if the future economic development is on the average equal to that during the last 12 years. It is very optimistic, however, to predict that the nominal GDP growth will be as high in the future as it was before, because the catch-up process has its own limit.

The asymptotical approach presented above has some weak points. Firstly, we used statistical data for 12 years, and this data set is fairly short. Secondly, the yearly volatility of the deficit, and especially of the growth rate, is very high; therefore, the use average of the data is not very reliable. In case of 5% of the average of the nominal GDP growth (approximately the average of the EU), we would arrive to the debt level of 64%, which exceeds the Maastricht criteria. To simulate the amount of the debt for a short and medium term, we use the stress scenario approach.

Let us assume that the nominal GDP growth rate and the relative budget deficit are the same in the nearest future as they were back in the years 2009, 2010, and 2011. This means that the crisis scenarios of the previous years might be repeated some time in the future. Supposedly, the GDP nominal growth compound is continuously -19.8%, 3.4%
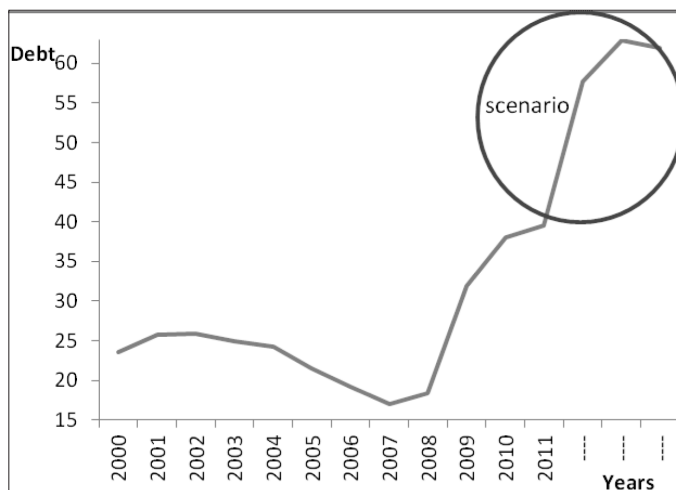

FIG. 2. **The actual data for 2000–2011. Scenario for the next years**

Source: author's calculations.

and 10.9% in the next 3 years, and the budget deficit is respectively 9.4%, 7.2%, and 5.5%, as it was in 2009, 2010 and 2011; then, according to our calculations, the debt would be 58%, 63%, and 62% of GDP in the following three years. An illustration is presented in Fig. 2.

Our calculation shows that in case of the second wing of the crisis, the government debt can reach the limit set by the Maastricht treaty, despite the government's attempts for strict saving measures as in the previous years. Therefore, new strict measures are needed.

3. Debt sustainability in a medium horizon

In section (2), we have considered the total budget deficit which in fact consists of the interest payments on the debt and the primary budget deficit. Now, we are going to distinguish between these two variables as they have different economic meanings. When the debt increases, payments of interest on the debt usually become higher; then, in order to keep the deficit within some limits, the government needs to reduce the primary deficit, which is a painful act. In this section, we consider sustainability as a condition for the debt to be non-increasing.

Let us split $\mu(t)$ in equation (2) into two parts as follows:

$$\mu(t) = i(t)b(t) + d(t).$$

In the equation, $i(t)$ is the interest rate on the debt, and $d(t)$ is the primary deficit. It should be noted that d is a deficit; therefore, it is in an opposite sign of pb in (1), where pb is a balance. Thus, the problem (2) can be rewritten in the following way:

$$b'(t) = d(t) - b(t)\,[g(t) - i(t)], \quad t > 0,\ b(0) = b_0. \tag{5}$$

The problem above can be solved in an explicit way, and the solution is equal to:

$$b(t) = e^{\int_0^t i(s) - g(s)ds}\Big[b_0 + \int_0^t d(x)e^{-\int_x^t i(s) - g(s)ds}\,dx\Big].$$

Denote $\xi(s) = i(s) - g(s)$. Then the debt $b(t)$ is equal to:

$$b(t) = e^{\int_0^t \xi(s)ds}b_0 + \int_0^t d(x)e^{\int_x^t \xi(s)ds}\,dx. \tag{6}$$

From the formula above we can arrive to the analogues of condition (1). A continuous analogue of formula (1) would be

$$b_0 \le -e^{-\int_0^t \xi(s)ds}\int_0^t d(x)e^{\int_x^t \xi(s)ds}\,dx.$$

Formula (6) is our main formula for the debt. It should be noted that if $\xi(s) = i(s) - g(s)$ does not depend on s and is equal to ξ, the formula for $b(t)$ gets simplified and takes the following form:

$$b(t) = e^{\xi t}b_0 + \int_0^t d(x)e^{\xi(t-x)}dx. \tag{7}$$

It should be noted that the formula above was presented by Blanchard et al. (1990). Formula (6) is a more general case of formula (7), because it can be used not only in cases when $\xi(s)$ is a constant. It is very important in the current environment, when the volatility of the interest rate on the debt is very high and the growth rate varies in a wide range, to have a possibility to investigate the impact of non-constant $\xi(s)$. In the case of Lithuania, the range of the growth rate lies between minus19.8% and plus17.6% .

Now, we will analyse the sustainability of the debt in a certain finite period. We define the sustainability as a condition for the debt to be non-increasing.

Admittedly, the function does not increase if the derivative of the function is less than or equal to zero. From equation (5), we see that $b'(t) \leq 0$ if and only if

$$d(t) + b(t)\,\xi(t) \leq 0.$$

This is a condition for the debt not to increase at the given moment t. From the inequality above, we derive that the primary deficit should satisfy the inequality

$$d(t) \leq -b(t)\,\xi(t)$$

to ensure the debt to be non-increasing at the moment t. Let us integrate the differential equation (5) from t to $t + \Delta$. Then, we have the following relationship:

$$\int_t^{t+\Delta} b'(x)dx = b(t + \Delta) - b(t) = \int_t^{t+\Delta}[d(x) + b(x)\xi(x)]dx.$$

From the equation above, we derive that the debt is non-increasing over the period from t to $t + \Delta$ t if and only if

$$\int_t^{t+\Delta}[d(x) + b(x)\xi(x)]dx \leq 0.$$

From the inequality above, we see that the following proposition is possible:

Proposition 1. To keep the debt non-increasing in the period from t to $t + \Delta$ it is necessary and sufficient that the primary budget deficit *d(x)* is at the level at which the inequality

$$\int_t^{t+\Delta} d(x)dx \leq -\int_t^{t+\Delta} b(x)\xi(x)dx, \tag{8}$$

holds. There, *d* is the primary deficit, *b* is the debt, and *ξ(x) = i(x) − g(x)*, where *i(x)* is the interest rate on the debt and *g(x)* is the nominal GDP growth.

According to the Mean Value Theorem for Integrals, if $\xi(x)$ does not change its sign in $[t, t + \Delta]$, then there exists x^* from the interval $[t, t + \Delta]$, which allows

$$\int_t^{t+\Delta} b(x)\xi(x)dx = b(x^*)\int_t^{t+\Delta} \xi(x)dx.$$

Then the inequality (8), which represents the condition for the debt to be non-increasing, can be rewritten in the following way:

$$\int_t^{t+\Delta} d(x)dx \le -b(x^*)\int_t^{t+\Delta} \xi(x)dx.$$

The above is a very important inequality, and we are going to comment on it in more detail.

If the function $\xi(x)$ is such that the integral at the right-hand side is positive, then the integral at the left side should be negative. This means that if the debt financing rate is higher than the nominal GDP growth rate at the interval $[t, t+\Delta]$, then the budget deficit should be on the average negative at the interval $[t, t+\Delta]$. We derive that the primary budget should run the surplus on the average in order to ensure a non-increasing debt.

The situation changes if the integral at the right-hand side is negative. This may be in the case when the GDP growth rate is higher than the debt financing rate. Blanchard et al. (1990) claim that the interest rate on the debt lower than the GDP growth rate is uncommon in a long-term period. We agree with this argument concerning a stable economy in a long run. Lithuania is still in a catch-up process, and statistical data show that the Lithuanian GDP growth rate is higher on the average than the debt financing cost.

A high GDP growth allows the government to run some small primary budget deficit even without increasing the ratio of the debt to the GDP. In 2007 and 2008, Lithuania had a deficit of the prime budget; nonetheless, the debt was decreasing (see Tables 1 and 2). Izák (2009) illustrates that Poland was the only country in the Eastern and Central Europe where the interest rate on the debt was higher than the nominal GDP growth in 1999–2006. Unfortunately, this pleasant phenomenon cannot last for ever.

The Proposition 1 could be applied directly to the Lithuanian debt. According to the forecast by the Ministry of Finance of the Republic of Lithuania (see Lietuvos Respublikos Finansų Ministerija 2012), the actual GDP growth will be 4.3% in 2012, i.e. 3.9% compounded continuously. Let us presume that the interest rate on the debt is equal to 4.8% as it was in 2010. Then we see that $\xi = 0.9$, which is positive. Therefore, in order to keep the debt non-increasing in 2012, the primary deficit should be negative, i.e. the primary budget should be in surplus.

Now, we will define the conditions for the debt to return back to the level of the beginning of 2009. With the help of historical data on the development of Lithuania during 2009, 2010 and 2011, we will determine the condition for the primary deficit in the next three years to ensure the debt to be at the same level during the period of six years as it was at the end of 2008. According to Proposition 1, the debt will be non-increasing in the period of six years, if

$$\int_{2009}^{2012} d(x)dx + \int_{2012}^{2015} d(x)dx \le -\int_{2009}^{2012} b(x)\xi(x)dx - \int_{2012}^{2015} b(x)\xi(x)dx.$$

The values of the variables in 2009, 2010 and 2011 are presented in Tables 1 and 2. We took the values of *i* and *d* in 2011 as an average of these values in 200–2010. Let us assume that $b(x)$ gradually reduces from the current level of 38.5% to the level of 15.5% as it was at the beginning of 2009. Let the value of ξ be equal to the average of *i–g* in 2000–2011.Then ξ = -1.7% (see Table 2). Then, from the equality above, we see, that

$$\int_{2012}^{2015} d(x)dx \le -3.9.$$

On the other hand, if the inequality above occurs, then the increase of the debt will be less than or equal to zero in 2009–2015. This means that the primary budget surplus in the next three years should be higher than or equal to 1.3 on the average. We should note that the Lithuanian primary budget deficit was in surplus only in 2005 and 2006 (see Table 2).

4. Sensitivity of the debt to the GPD growth, the interest rate on the debt and the budget deficit in a short run

There are two main parameters that influence the amount of the debt. They are the difference between the debt financing rate and the nominal GDP growth rate ξ and the primary budget deficit *d.*

The variable ξ consists of two components, and we will provide an overview of their influence individually. First of all, we will consider the interest rate variable. Since the debt of the government of Lithuania consists of the internal and external debts, the average of the interest rate on the debt depends on the interest rate in the Lithuanian bond market and the Eurobond market. According to the Report of the Ministry of Finance (Finansų Ministerija, 2012), the share of the internal debt is 26% and of the external debt 74%. The Lithuanian internal market is not very liquid, yet the interest rates are quite stable, mainly because of the monetary policy of Lithuania. Yields of Lithuanian Eurobonds depend mainly on the ratings of Lithuania, which depend on the level of the government debt as well. Here we have some nonlinearity. Sovereign yields in the Eurobonds market react immediately to the increase of the government debt, which causes a higher interest rate on the debt and at the same time a bigger amount of the debt. Therefore, the problems of servicing the debt double and offer new challenges for the government to manage the sustainability of the debt. According to the report of the Ministry of Finance (Finansų Ministerija, 2012), the duration of the debt is around 3.6 years. Considering the above, we can claim that the debt which Lithuania needs to refinance during a year is on the average less than 1/3 of the amount of the debt. Therefore, the influence of the possibility of a sudden increase in the refinancing interest rate *i* by *Δi* changes ξ by no more than *Δi/3*. We can conclude that the increase of the interest rate on the debt has a moderate influence on the debt, and the risk of an increase of the interest rate is not high.

The slowdown of the economy effect is three times higher in Lithuania. Considering the historical volatility of the nominal growth of its GDP, a substantial risk for the sustainability of the debt, even in a short run, is created.

Let us take the average interest rate on the debt in Lithuania from Table1, which is equal to 5.2%, and the average of the prime deficit as 1.9% and the debt equal to 38.5%. The graph of the debt concerned as a function of the GDP growth, which is calculated according to formula (7), is presented in Fig. 3. The one-year perspective is used in the figure. As we see in Fig. 3, the debt changes dramatically, if the GDP growth is in the range of–20% to 20%. It should be noted that the figures close to the figures above were observed in the development of the Lithuanian economy in 2007 and 2009. The change of the relative debt varied from 36% to 51.5% just because of the dramatic change in the nominal GDP growth.

The sensitivity of the relative debt to the interest rate of the debt is much more moderate. If the interest rate of the debt varies from 0 to 10%, which is much higher than that observed in the Lithuanian practice, the debt amount changes from 35.5% to 39%. The graph is presented in Fig. 4. The GDP growth there is equal to 7.4%, which is the average of 2000–2011 (see Table 2), and the debt is equal to 38.5%.
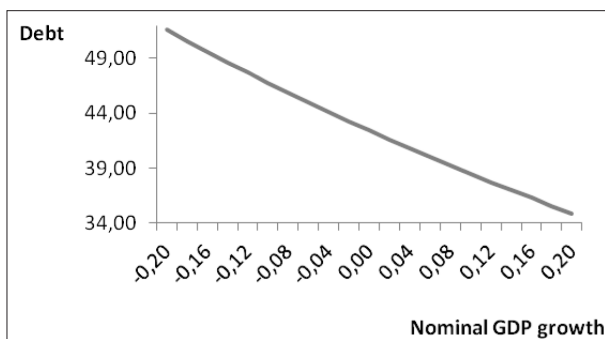
FIG. 3. Dependence of the relative size of debt on the nominal GDP growth in one-year period
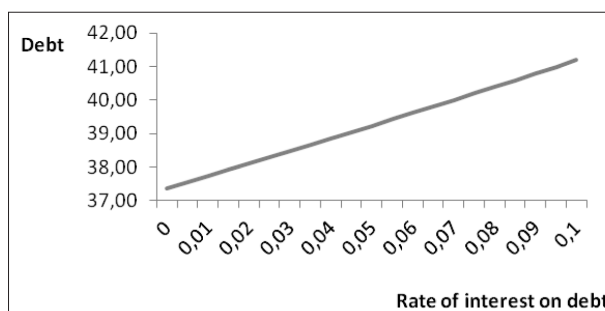
Source: author's calculations.

FIG. 4. Dependence of the relative size of debt on the interest rate on the debt in one-year period
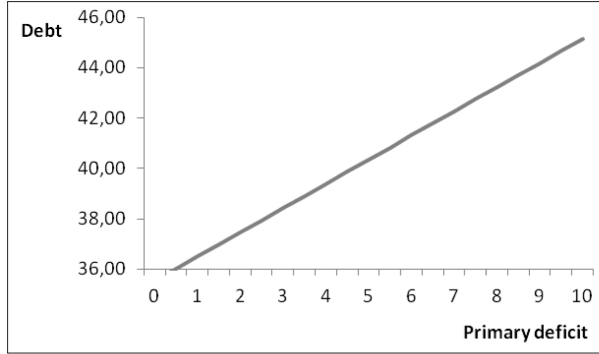
Source: author's calculations.

FIG. 5. **Dependence of the relative size of debt on the primary deficit in one-year period**

Source: author's calculations.

Sensitivity of the debt to the amount of the prime deficit in a one-year perspective is almost linear (see Fig. 5).

Moreover, we are going to estimate the impact of the debt on small movements of the parameters.

For simplicity, we will consider a short period of time from t_1 to t_2 and assume that $\xi(t)$ and $d(t)$ do not depend on t during this short period. Then we can use an analogue of formula (7) for the development of the debt over time. Now, let us assume that at the moment t_1 the debt b is equal to $b(t_1)$. If $d(t)$ is constant, the formula can be simplified in the following way:

$$b(t,\xi,d) = b(t_1)e^{\xi(t-t_1)} + \frac{d[e^{\xi(t-t_1)} - 1]}{\xi}.$$

A partial derivative of b with respect to ξ and d can be found as follows:

$$\frac{\partial b}{\partial \xi} = b(t_1)(t-t_1)e^{\xi(t-t^1)} + d[\frac{(t-t_1)e^{\xi(t-t^1)}}{\xi} - \frac{e^{\xi(t-t^1)}}{\xi^2} + \frac{1}{\xi^2}] \approx$$

$$\approx (t-t_1)[e^{\xi(t-t_1)}b(t_1) + (t-t_1)d/2],$$

$$\frac{\partial b}{\partial d} = \frac{e^{\xi(t-t_1)} - 1}{\xi} \approx (t-t_1) + (t-t_1)^2\xi/2.$$

We use approximate values of the derivatives to simplify the formulas.

If at the moment t_1, the parameters ξ and d change respectively to $\xi + \Delta\xi$ and $d + \Delta d$, then at the moment t_2 we have:

$$b(t_2, \xi + \Delta\xi, d) \approx b(t_2, \xi, d) + \Delta\xi \frac{\partial b}{\partial \xi},$$

$$b(t_2, \xi, d + \Delta d) \approx b(t_2, \xi, d) + \Delta d \frac{\partial b}{\partial d}.$$

Let us comment on the equalities above. The derivatives $\frac{\partial b}{\partial d}$ and $\frac{\partial b}{\partial \xi}$ are positive; therefore, the debt increases if $\Delta\xi$ and Δd are positive. If intuitively clear that a increase of the debt financing rate and a decrease in the nominal GDP growth implies a increase of the relative debt. Moreover, a increase of the budget deficit increases the debt. The formulas above show the scale of this phenomenon. Considering the above, we can formulate the following proposition:

Proposition 2. Let at the moment t_1 the debt financing interest to stand at the level i, the average yearly nominal GDP growth for the period from t_1 to t_2 to be g, and the budget deficit for the period to be d. The increase in $i - g$ by

$$b_{\Delta\xi} = b(t_2, \xi + \Delta\xi, d) - b(t_2, \xi, d) \approx \Delta\xi(t_2 - t_1)[e^{\xi(t_2 - t_1)}b(t_1) + (t_2 - t_1)d/2].$$

The increase in d by Δd causes an increase of the debt at the moment by

$$\Delta b_{\Delta d} = (t_2, \xi, d + \Delta d) - b(t_2, \xi, d) \approx \Delta d(t_2 - t_1)[1 + (t_2 - t_1)\xi/2].$$

For the period of one year, $t_2 - t_1 = 1$. Considering the above, we can simplify the formulas in Proposition 2 in the following way:

$$\Delta b_{\Delta\xi} \approx \Delta\xi(e^\xi b(t_1) + d/2), \ \ b_{\Delta d} \approx \Delta d(1 + \xi/2). \tag{9}$$

In 2011, the nominal GDP growth in Lithuania was higher than the interest rate paid for the debt; therefore, the value of ξ was negative. This implies that $e^\xi < 1$. Evidently, the effect on the debt, caused by a small increase of ξ, is partially amortised by the coefficient e^ξ. In the year 2012, according the forecast of the Ministry of Finance (Lietuvos Respublikos Finansų Ministerija, 2012), ξ is going to be positive (therefore $e^\xi > 1$), and the effect of the increase of ξ would have even a greater effect on the increase of the relative debt.

Now, we apply the findings of Proposition 2 by using the data of Lithuania. We will determine conditions for the debt to change by 1 percentage point. We use data from Tables 1 and 2 at the end of 2011 (data for the primary budget deficit are taken at the end of 2010 because of the lack of statistical data). Then we have

$$b(t_1) = 38.5\% \ \xi = (5.2\% - 10.9\%) = -5.7\%, d = 5.4\%.$$

Let us take $b_{\Delta\xi} = 1\%$. Then, from formula (9) we have that $\Delta\xi \approx 2.6\%$. This means that if the parameter ξ decreases by 0.026 or 2.6 percentage points, the relative debt

decreases by 1 percentage point during one year, i.e. from 38.5% to 37.5%; and vice versa, if the parameter ξ increases by 2.6 percentage points, the relative debt increases by 1 percentage point during one year, i.e. from 38.5% to 39.5%. The example indicates the sensitivity of the debt to the interest rate on the debt and the nominal GDP growth in a one-year period of time.

Let us take $b_{\Delta d}=1\%$. Then, from formula (7) we have that $\Delta d \approx 0.97\%$. This means that if the parameter d, which is the primary deficit, decreases by 0.0097 or 0.97 percentage points, the relative debt decreases by 1 percentage point during one year, i.e. from 38.5% to 37.5%. This shows the sensitivity of the debt to the primary deficit in a one-year period of time.

We can formulate a one percentage point rule for the amount of the debt. For the debt to be lower by one percentage point during one year, we need to have the GDP growth higher by 2.6%, or the interest rate on the debt lower by 2.6%, or the primary budget deficit lower by 0.97% percentage point. Moreover, an opposite statement takes place. For the debt to be higher by one percentage point during one year, we need to have the GDP growth rate lower by 2.6%, or the interest rate on the debt higher by 2.6%, or the primary budget deficit higher by 0.97% percentage point.

The above rule is an operational one. It can be applied to estimate the sensitivity of the debt to the debt financing rate, the nominal GDP growth rate, and the primary deficit.

5. Conclusions

The debt of Lithuania would be around 44% in a long-term period, which is substantially less than the level set by the Maastricht criteria, if Lithuania continues to have the average growth rate as in 2000–2011 and the average of the budget deficit is the same as during this period.

The scenario of the years 2009–2011 is very risky for Lithuania in the future. If the GDP dropped to the same level and the budget deficit was in the same range as it was in the mentioned years, the government debt would reach the level of 60% in two years. This would imply big expenses for servicing the debt and would force a bigger restriction of the primary budget balance. The primary budget balance should be in a substantial surplus later on in order to fulfil the Maastricht criteria.

At the current level of the debt and the policy which is applied for the debt management, the amount of the debt is not very sensitive to a small movement of the interest rate on the debt in a short run. The negative impact of the slowdown of the economy is three times higher. A high volatility of the nominal GDP growth in Lithuania creates a big risk for a substantial increase in the relative amount of the debt in a short run. The change of the GDP growth or the interest rate on the debt by 2.6% has caused the change of the debt by 1 percentage point. The change of the primary deficit by 0.97% has caused the change of the debt by 1 percentage point.

For the debt to return to the level of the beginning of 2009, the total surplus the primary budget should be 3.9% in the next year.

The primary budget balance of Lithuania was negative, except for the years of 2005 and 2006. This creates no substantial problem for the relative increase of the debt, until the nominal GDP growth is on a strongly positive side. In the period of economic shocks, it is extremely difficult to keep the relative debt non-increasing. In order to ensure a non-increasing debt in the medium term, during the period of a strong economic growth the primary balance should be strongly positive in order to compensate a negative balance in the period of economic shocks.

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