MODELING OF TAXATION THAT CONSIDERS REPRODUCTION

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The article considers the modifications of taxation models that define the influence of tax rate on the production growth and the minimal net margin. The aim of the article is to analyse the development of taxation models that combine total tax allocation and value added, remuneration of labour, amortization.

The models generalize the introduction of figurers', heterogeneity and the production function complication, namely the two-factor production function while considering capital and labour is used. As a result, the estimation of the production growth depending on the tax rate was established, the minimal net margin that allows simple reproduction was defined, the total tax allocation frontier was found.

A tax rate optimization model that maximizes allocation to the budget was considered. Calculations for Ukrainian macroeconomic data were conducted. Statistic estimations of the model's main macroeconomic components allow defining the potential level of the economic growth, comparing it with the actual growth, and concluding on the efficiency of the national development model.

Keywords: *taxation models, value added, minimal net margin, reproduction, economic growth*

Aims

A study of tax rate influence on the process of reproduction, using simple macroeconomic models. Determination of the total tax allocation frontier that allows to keep an economic unit functioning.

Methods

The macroeconomic model that combines total tax allocation and value added, remuneration of labour and amortization is built. This model differs from the conceptual taxation models (Kostina, 2002) by a sufficient coordination with the national accounts practice. The model is the generalization of the model proposed by Balatsky (2000), because the heterogeneity of figures is used and the production function is completed by using two factors of production function that consider capital and labour.

The optimization approach is used when the tax rate is modelled by maximizing the budget allocation.

Results

Let us consider the main taxation scheme that uses the two-factor production function of Cobb-Douglas:

$$Y_t = a_t F_t^{\alpha} L_t^{1-\alpha}, \quad t = 0,1,2,\ldots, \tag{1}$$

where: Y_t – production volume, value added;

F_t – capital investments in fixed assets;

L_t – labour;

a_t, α – coefficients of technology and technical progress influence on production.

Production growth rates:

$$1+\gamma_t = \frac{Y_t}{Y_{t-1}} = \frac{a_t}{a_{t-1}} \cdot \frac{L_t}{L_{t-1}} \left(\frac{F_t}{L_t} \cdot \frac{L_{t-1}}{F_{t-1}} \right)^{\alpha} \tag{2}$$

Long-term observations revealed a strong relationship between labour and capital investments (Kostina, 1998): 2–3 units of past labour (raw material, equipment, transport, education, etc.) should be spent on each unit of labour in the current production. The american economist P. Douglas determined that the ratio between the income from labour and the income from capital practically do not change in the national income structure with time. This fact was proven by further studies in 1946–1990 (Manque, 1994). Thus, it could be assumed that

$$c_t = \frac{F_t}{L_t} = c = const, \tag{3}$$

whence $\dfrac{c_t}{c_{t-1}} = \dfrac{F_t}{L_t} \cdot \dfrac{L_{t-1}}{F_{t-1}} = 1,$

i.e. $$\frac{L_t}{L_{t-1}} = \frac{F_t}{F_{t-1}}. \tag{4}$$

If to assume that $$a_t = a_0 \prod_{i=1}^{t} q_i, \tag{5}$$

where q_i, $i = 1,\ldots,t$, – the coefficient of technology development in the corresponding year, then from (2) we derive:

$$1+\gamma_t = q_t \cdot \frac{L_t}{L_{t-1}} = q_t \cdot \frac{F_t}{F_{t-1}}. \tag{6}$$

The coefficient of the production capital capacity, that includes labour costs is calculated using the following formula:

$$k_t = \frac{L_t + F_t}{Y_t}. \tag{7}$$

Taking into consideration (3) and (6), the following equation can be derived:

$$\frac{k_{t-1}}{k_t} = \frac{Y_t}{Y_{t-1}} \cdot \frac{L_{t-1} + F_{t-1}}{L_t + F_t} = q_t = \frac{a_t}{a_{t-1}}, \tag{8}$$

As from (3):

$$a_0 k_0 = a_0 \cdot \frac{L_0 + F_0}{Y_0} = \frac{L_0 + F_0}{F_0^{\alpha} L_0^{1-\alpha}} = \frac{1+c}{c^{\alpha}}, \tag{9}$$

then from (5):

$$k_t = \frac{k_{t-1}}{q_t} = \frac{k_0}{\prod_{i=1}^{t} q_i} = \frac{a_0 k_0}{a_t} = \frac{1+c}{c^{\alpha} a_t}. \tag{10}$$

Let us consider the fund forming process (Balatsky, 2000):

$$F_t = (1-\varepsilon) F_{t-1} + m\left(\Pi_t + \varepsilon F_{t-1}\right), \tag{11}$$

where: F_t – fixed assets at the end of the period (year);

ε – the coefficient of yearly assets retirement (equal to depreciation rate);

Π_t – enterprise net income (after all taxes);

m – the enterprise's inclination to invest in fixed assets $(0 < m < 1)$.

Let us denote the income margin as

$$r_t = \frac{\Pi_t}{Y_t}. \tag{12}$$

According to (11), (7), (3),

$$F_t = (1-\varepsilon(1-m))F_{t-1} + mr_tY_t =$$

$$= (1-\varepsilon(1-m))F_{t-1} + mr_t\frac{L_t+F_t}{k_t}, \tag{13}$$

$$\left(1-\frac{mr_t}{k_t}\right)F_t = (1-\varepsilon(1-m))F_{t-1} + \frac{mr_t}{k_t}L_t, \tag{14}$$

$$\frac{F_t}{F_{t-1}} = \frac{1-\varepsilon(1-m)}{1-\frac{mr_t}{k_t}\left(1+\frac{1}{c}\right)}. \tag{15}$$

Then, taking into consideration (6),

$$1+\gamma_t = q_t \cdot \frac{1-\varepsilon(1-m)}{1-\frac{mr_t}{k_t}\left(1+\frac{1}{c}\right)}. \tag{16}$$

From (16), the dependence of income margin r_t and production growth rate $(1+\gamma_t)$ can be found:

$$r_t = \frac{k_t}{m\left(1+\frac{1}{c}\right)}\left(1-q_t\frac{1-\varepsilon(1-m)}{1+\gamma_t}\right). \tag{17}$$

For a simple reproduction $(\gamma_t = 0)$, the minimal income margin is

$$r_t^* = \frac{k_t}{m\left(1+\frac{1}{c}\right)}(1-q_t+q_t\varepsilon(1-m)). \tag{18}$$

Note that the model (15) represents the potential production ability of an economic structure without considering demand constraints, price changes and other factors.

Let us consider the value added equation (Balatsky, 2000):

$$Y_t = L_t + A_t + \Pi_t + T_t, \tag{19}$$

where T_t is total taxes.

Then $\theta_t = 1-\sigma_t - r_t - \frac{A_t}{Y_t}$, (20)

where $\theta_t = \frac{T_t}{Y_t}$ – the weight of taxes in value added;

$\sigma_t = \frac{L_t}{Y_t}$ is the weight of wage.

According to (1), (2), (3), (5), (10), (16), we can derive:

$$\frac{A_t}{Y_t} = \frac{\varepsilon F_{t-1}}{Y_t} = \frac{\varepsilon F_{t-1}}{Y_{t-1}}\cdot\frac{Y_{t-1}}{Y_t} =$$

$$= \frac{\varepsilon}{1+\gamma_t}\cdot\frac{F_{t-1}}{a_{t-1}F_{t-1}^{\alpha}L_{t-1}^{1-\alpha}} = \frac{\varepsilon c^{1-\alpha}}{a_{t-1}(1+\gamma_t)}, \tag{21}$$

$$\sigma_t = \frac{L_t}{Y_t} = \frac{L_t}{a_tF_t^{\alpha}L_t^{1-\alpha}} = \frac{1}{a_tc^{\alpha}}, \tag{22}$$

$$r_t = k_t \cdot \frac{1-q_t\cdot\frac{1-\varepsilon(1-m)}{1+\gamma_t}}{m\left(1+\frac{1}{c}\right)}. \tag{23}$$

Then from (20) – (23), (10) the following formula can be obtained:

$$\theta_t = 1-\frac{1}{a_tc^{\alpha}} - \frac{1+c}{a_tc^{\alpha}}\cdot\frac{1-q_t\cdot\frac{1-\varepsilon(1-m)}{1+\gamma_t}}{m\left(1+\frac{1}{c}\right)} -$$

$$-\frac{\varepsilon c^{1-\alpha}}{a_{t-1}(1+\gamma_t)}. \tag{24}$$

Taking (8) into consideration, we can find:

$$a_t c^\alpha (1-\theta_t) = 1 + \frac{c}{m}\left(1 - q_t \cdot \frac{1-\varepsilon(1-m)}{1+\gamma_t}\right) +$$

$$+\frac{\varepsilon q_t c}{1+\gamma_t},$$

from which

$$1+\gamma_t = \frac{q_t(1-\varepsilon)}{1+\frac{m}{c} - mc^{\alpha-1}a_t(1-\theta_t)}. \tag{25}$$

From (25) we can derive the ratio that allows defining the tax rate:

$$\theta_t = \frac{1}{mc^{\alpha-1}a_t}\left(\frac{q_t(1-\varepsilon)}{1+\gamma_t} - 1 - \frac{m}{c} + mc^{\alpha-1}a_t\right), \tag{26}$$

which depends on the influence of technology and technical progress on the production, on the labour and capital ratio, on the production technology development, on the investment trend, on the fixed assets deterioration and on the production growth rate.

The boundary level of total taxes subject to the minimal income margin can be determined by the following ratio:

$$\theta_t^* = \frac{1}{mc^{\alpha-1}a_t}\left(q_t(1-\varepsilon) - 1 - \frac{m}{c}(1-c^\alpha a_t)\right). \tag{27}$$

The equation (25) can be rewritten as follows:

$$c + m - mc^\alpha a_t(1-\theta_t) = \frac{cq_t(1-\varepsilon)}{1+\gamma_t}, \tag{28}$$

From it, taking (22) into account, we can obtain:

$$\frac{Y_t}{L_t}(1-\theta_t) - 1 = \frac{c}{m}\left(1 - \frac{q_t(1-\varepsilon)}{1+\gamma_t}\right). \tag{29}$$

Finally, the equation for defining the relative tax rate is:

$$\theta_t = 1 - \sigma_t\left(1 + \frac{c}{m}\left(1 - \frac{q_t(1-\varepsilon)}{1+\gamma_t}\right)\right). \tag{30}$$

Then, the boundary level of total taxes can be determined from the following expression:

$$\theta_t^* = 1 - \sigma_t\left(1 + \frac{c}{m}(1 - q_t(1-\varepsilon))\right). \tag{31}$$

Let us consider the optimization tax rate problem subject to maximization of the budget allocation:

$$\theta \prod_{k=1}^{t}(1+\gamma_k) \to \max, \tag{32}$$

where t is the planning horizon.

According to (2), (6), (25) and at a constant θ we can obtain:

$$1+\gamma_k = \frac{Y_k}{Y_{k-1}} = \frac{q_t(1-\varepsilon)}{1+\frac{m}{c} - mc^{\alpha-1}a_t(1-\theta)}.$$

Denote

$$A = 1-\varepsilon, \quad B = 1+\frac{m}{c}, \quad C = mc^{\alpha-1},$$

then

$$1+\gamma_k = \frac{Y_k}{Y_{k-1}} = \frac{Aq_t}{B + Ca_t(\theta-1)}, \tag{33}$$

wherefrom $Y_t = Y_0 \prod_{k=1}^{t} \frac{Aq_k}{B + Ca_k(\theta-1)}$. (34)

According to (33), the problem (32) reduces to maximization of the following expression:

$$H_t = \theta \prod_{k=1}^{t} \frac{q_k}{D + Ga_k(\theta-1)}, \tag{35}$$

where $D = \frac{B}{A}$, $G = \frac{C}{A}$.

Using the necessary condition of function extreme,

$$\frac{dH_t}{d\theta}=\left(1-\theta\sum_{k=1}^{t}\frac{Ga_k q_k}{D+Ga_k(\theta-1)}\right)$$

$$\prod_{k=1}^{t}\frac{q_k}{D+Ga_k(\theta-1)}=0. \tag{36}$$

From (36) we can obtain the equation for finding the optimal θ:

$$\frac{1}{\theta}=\sum_{k=1}^{t}\frac{q_k}{Ea_k^{-1}-1+\theta}, \tag{37}$$

where $E=\frac{D}{G}=\frac{B}{C}=\frac{1+\frac{m}{c}}{mc^{\alpha-1}}$.

Using the Ukrainian macroeconomic data for 2000–2007, modeling accounts were conducted in order to define the boundary level of the tax rate. Also, tax rate figures for a 1% and 2% economic growth were computed. A comparison of computation results is shown in Figure 1. Let us note that the more the tax rate level increases the stronger its influence on the economic growth rate.

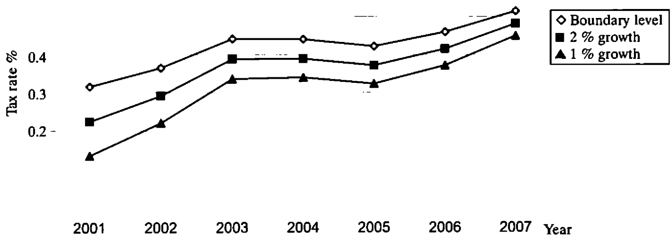

*Figure 1. **The boundary level of tax rate and the tax rates in case of 1% and 2% production growth***

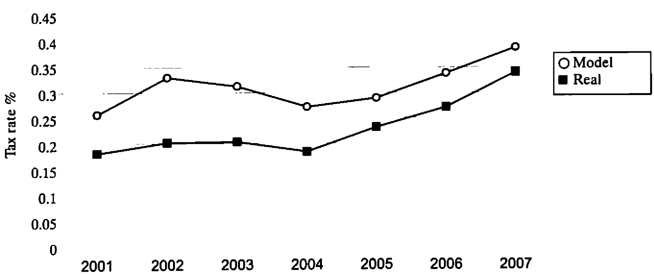

*Figure 2. **Modelled and real levels of tax rate***

According to (30), the relative total tax rate for Ukraine for 2001–2007 was computed. Its comparison with the real tax rate level for the mentioned period (Figure 2) shows that the Government has a small fiscal reserve, i. e. the economic system is not expedient by taxes. The further economic growth depends more on the production capital capacity decrease and on investment increase.

Conclusions

The proposed model enables to estimate the influence of fiscal, technological and investment macroeconomic factors on the process of the national economic growth. The modelled boundary level of the total tax rate and its comparison with the real level show that there is a fiscal reserve. Use of statistic estimations of the main macroeconomic components of the model allows forecasting the potential economic growth rate, its comparison with the real growth rate and a conclusion about the efficiency of the national development model.

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