ISSN 1392-1258. EKONOMIKA. 1997. 42

STATISTICAL–MATHEMATICAL MODELLING OF PRICES OF CONSUMER GOODS AND SERVICES IN LITHUANIA*

ŽILVINAS KALINAUSKAS, BRONISLAVA KAMINSKIENĖ, RIMANTAS RUDZKIS

Monthly data on price indices of consumer goods and services as well as groups of some goods and the principal monetary indices in Lithuania are considered in this paper using methods of mathematical statistics. The main goal of this work is to construct mathematical models of the consumer price (CPI) index, fit for short-term prediction. Statistical dependency between prices and monetary indicators is investigated in the paper. Trends and seasonal components are estimated. Random fluctuations are described using autoregression models. Regressive models of prices and monetary indicators as regressors are constructed. Errors of indicator prediction using the proposed models are estimated. An expert analysis of the state of the national economy is made, taking into account changes in price, production, and unemployment indicators. Due to data inaccuracy and frequent recalculation of indicators, only a qualitative analysis was made without applying mathematical means.

Key words: *consumer price indices, regression models, forecast*

In recent years specific attention has been paid to changes in prices of consumer goods and services in Lithuania. The price index of consumer goods and services is extensively commented upon and its prediction is also

* This paper has been written as a report for the MEET III Conference which took place on June 7–8, 1996, at Leicester University, UK

of interest. Regression and autoregression models of CPI, aimed at short-term prediction, are presented in this paper. Monetary indicators M1 and M2, currency in circulation (CC), and average monthly wages and salaries in the country are used as regressors. It is easier for government and bank experts to predict the development of these indicators than prices in Lithuania. Besides, as shown by the preliminary analysis of statistical relations between CPI and main production, employment, monetary and financial indicators in the country, the mentioned monetary indicators statistically correlated best with CPI in recent years. Statistical relations between production and price indices were distorted by production privatisation and restructuring processes. The available statistical data on employment is not exact and its usage in the regression analysis of CPI is also inexpedient at present. The data of 1993–1995 is mainly used for model construction. Due to deep political and economic changes undergone in the country of late, the data of most earlier period indicators is not useful for CPI prediction (recall that national currency was introduced in June, 1993). In addition, some of the indicators under investigation were not calculated up till 1993 or were estimated using methods other than those used at present. Only statistical CPI data is analysed throughout the period beginning in January of 1991.

Statistical Data

The following indicators are considered in the paper:

1) Consumer goods and services price index (CPI);

2) Food, alcoholic beverages and tobacco price index (FPI);

3) Clothing and footwear price index (CFPI);

4) Rent, fuel, and power price index (RPI);

5) Transport and communication price index (TPI);

6) Money M1;

7) Money M2;

8) Currency outside banks (COI);

9) Average monthly gross wages and salaries (AWSI).

All the indicators are presented on the basis of December 1993, i.e., values of the investigated indicator are divided by its value in the base month (December 1993). The statistical data is taken from the monthly publication "Economic and Social Development in Lithuania," issued by the Department of Statistics to the Government of Lithuania, and the "Bank of Lithuania" bulletin. Data diagrams are given in Figures 1–3. Note that indices 2)–5) are basic components of CPI. Let P(t) denote a CPI value at time moment t, and $P_F(t), P_{CF}(t), P_R(t)$, and $P_T(t)$ be the respective values of FPI, CFPI, RPI, TPI. Then the equality

$$P(t) = q_F P_F(t) + q_{CF} P_{CF}(t) + q_R P_R(t) + q_T P_T(t) + q_{other} P_{other}(t)$$

will hold. Here $P_{other}(t)$ denotes the price index of consumer goods and services not contained by groups 2–5, and $q_{(\cdot)}$ denotes the respective weight coefficients whose sum equals 1. These weights are determined by household surveys (and are specified no more than once a year). In recent years the sum of 2–5 group weights makes up about 90%, i.e., $q_F + q_{CF} + q_R + q_T \cong 0.9$. Food prices are particularly distinguishable in household consumption, $q_F \cong 0.65$. This indicates a low level of real income of the majority of Lithuanian inhabitants.

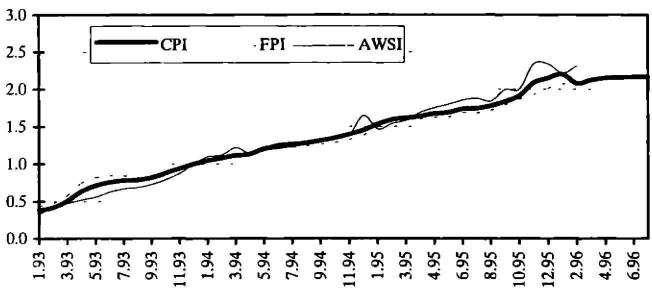


Figure 1. Changes in consumer goods and services price; food, alcoholic beverages, and tobacco price; average monthly gross wages and salaries indices (based on December 1993).

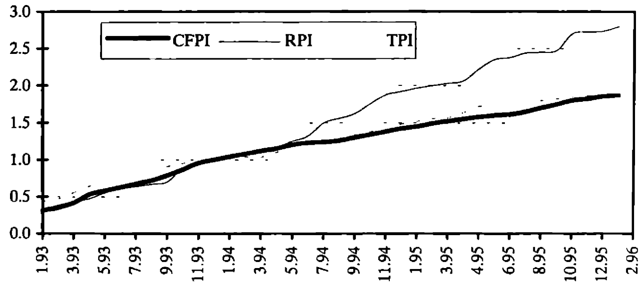


Figure 2. Changes in clothing and footwear price; rent, fuel, and power; transport and communication price indices (based on December 1993).

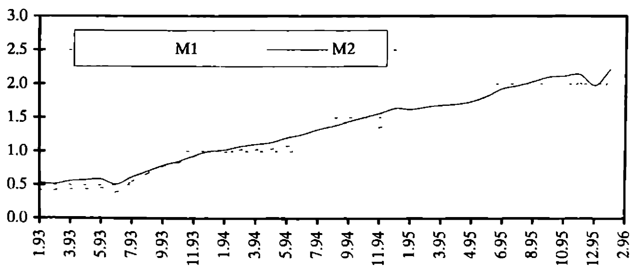


Figure 3. Changes in money indices (based on December 1993).

Trends and Seasonal Components

In making a primary analysis of statistical data it is necessary to find out the principal tendencies (trends) of indicator change. Another important characteristic is a seasonal component. Let $X(t)$ be one of the above mentioned indicators (on the basis of December 1993). Let X(t) denote a relative change of this index in a month t, i.e. $x(t) = \frac{X(t)}{X(t-1)} - 1$.

Thus $X(t) = X(t_0) \prod_{k=t_0+1}^{t} (1 + x(k))$, where t_0 corresponds to the month from which we consider the indicator values. In this section models of two types

$$x(t) = m_x(t) + s_x(t) + \xi_x(t) \qquad \text{(model I)} \qquad (1)$$

and

$$\log X(t) = \mu_X(t) + S_X(t) + \eta_X(t) \qquad \text{(model II)} \qquad (2)$$

were used when estimating trends and seasonal components of indicators. Here s and S are seasonal indices, i.e., $s(t) = s(t+12)$ and $\sum_{j=1}^{12} s(j) = 0$, and ξ and η are random fluctuations. Functions $\mu(t)$ of the trend are approximated by first degree polynomials or broken lines with a break point at the time moment ν, corresponding to June, 1993 (the introduction of the Litas). Thus, the complete formula of the trend is as follows:

$$\mu(t) = A + B(t - \tau) + C(t - \nu)_+ \qquad (3)$$

where τ denotes the base month (Dec. 1993).

Since $\tau - \nu = 6$, we obtain from (3) the equality

$$\mu(t) = \begin{cases} A + B(t - \tau), & t < \nu, \\ \tilde{A} + \tilde{B}(t - \tau), & t \geq \nu, \end{cases} \qquad (3a)$$

where $\tilde{A} = A + 6C$, $\tilde{B} = B + C$. The parameters A, B, and C are estimated by the *least squares* method, using standard software. Approximating trends by the second degree polynomials and investigating the significance of the parameter at the square term using the *Students* test, conclusions on its insignificance were obtained. Obviously, if the trend of x(t) is nearly constant, then log X(t) trend is close to the linear function in this case and vice versa. Therefore, if model I is employed in the estimation of the trend and seasonal index, and estimate $\hat{m}$ is obtained, assume

$$\hat{\mu}(t) = \sum_{k=\tau+1}^{t} \hat{m}(k). \tag{4}$$

If model II is used for estimating the trend and estimate $\hat{\mu}(t)$ is obtained, then the estimate

$$\hat{m}(t) = \hat{\mu}(t) - \hat{\mu}(t-1) \tag{4a}$$

of the trend will correspond to it in model I. While considering model I, the trend was approximated by a linear function

$$m(t) = a + b(t-\tau), \tag{5}$$

a jump function

$$m(t) = \begin{cases} a, & t < \nu, \\ \tilde{a}, & t \geq \nu, \end{cases} \tag{6}$$

or a function of a more general type

$$m(t) = \begin{cases} a + b(t-\tau), & t < \nu, \\ \tilde{a} + \tilde{b}(t-\tau), & t \geq \nu, \end{cases} \tag{7}$$

whose separate cases are (5) and (6). Function (7) differs from (3) in that it can be discontinued. For the comparison, along with parametric estimates, the nonparametric estimates of the trend of CPI and AWSI indicators were calculated using the moving average formulas

$$\hat{m}_x(t) = \frac{1}{2}x(t-6) + x(t-5) + \ldots + x(t+5) + \frac{1}{2}x(t+6) \tag{8}$$

and

$$\hat{\mu}_X(t) = \frac{1}{2}\ln X(t-6) + \ln X(t-5) + \ldots + \ln X(t+5) + \frac{1}{2}\ln X(t+6). \tag{9}$$

Formulas (8) and (9) eliminate seasonal influence. These estimates are not good for direct prediction; however, they can be used for the comparison and estimation of seasonal components.

The results of trend estimation are presented in Table 1. Apart from the estimates of parameters, average relative errors Δ_X and δ_x of the models are also given throughout the period after June, 1993. Here

$$\Delta_X = \frac{\sum_t \left| \frac{\hat{X}(t)}{X(t)} - 1 \right|}{\sum_t 1}, \tag{10}$$

where $\hat{X}(t) = \exp\{\hat{\mu}(t)\}$; $\sum_t$ represents the summation over all the values of $t, t \geq \nu$ for which data $X(t)$ is available and $\hat{X}(t)$ is approximated. Note that estimates (8) and (9) are calculated only for a part of the time interval observed. The error δ_x is counted analogously with (10), replacing $X(t)$ by $x(t)$ and $\hat{X}(t)$ by $\hat{x}(t) = \hat{m}(t)$. Seasonal indices are estimated using the *arithmetic mean* principle. Using model I, we have

$$\hat{s}_x(k) = \frac{\sum_t (x(t) - \hat{m}_x(t))}{\sum_t 1}, \tag{11}$$

where $\hat{m}_x(t)$ – is a selected estimate of the trend, and $\sum_t$ represents the summation over all time moments $t \geq \nu$, for which quantities $x(t)$ and $\hat{m}_x(t)$ are given and which denote the same month as k, i.e., t – k divided by 12. Following formula (4), assume in this case

$$\hat{S}_X(l) = \sum_{k=1}^{l} \hat{s}_x(k), \quad l = 1,\ldots,12. \tag{12}$$

If we use model II, then analogous to (11) we have

$$\hat{S}_X(k) = \frac{\sum_t (\log X(t) - \hat{\mu}_X(t))}{\sum_t 1}, \tag{13}$$

and, in that case, assume

$$\hat{s}_x(l) = \hat{S}_X(l) - \hat{S}_X(l-1), \quad l = 1,\ldots,12. \tag{14}$$

In Figures 4 and 5, seasonal indices for CPI and its monthly changes – calculated by various methods – are presented.

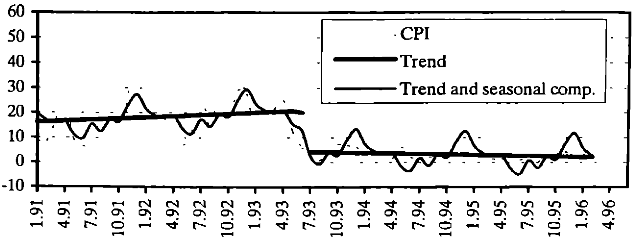
The seasonal component of relative CPI changes is as follows:

1) estimate (11) calculated using estimate (7) of the trend m(t);
2) estimate (11) calculated using estimate (8) of the trend m(t);
3) estimate (14) calculated using estimate (3) of the trend $\mu(t)$.

The seasonal component of CPI index ia as follows:

1) estimate (13) calculated using estimate (3) of the trend $\mu(t)$;
2) estimate (13) calculated using estimate (9) of the trend $\mu(t)$;
3) estimate (12) calculated using estimate (7) of the trend m(t).

Model I



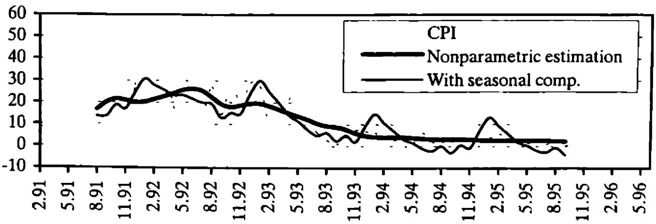


Figure 4. Changes in consumer goods and services price indiccs compared to the previous month (in per cent).

Model II

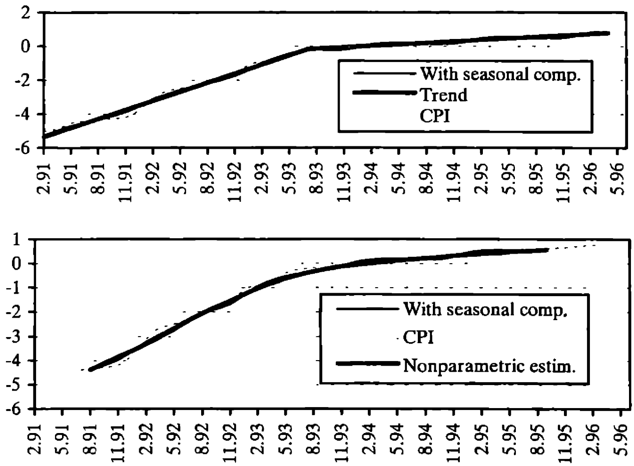


Figure 5. Changes in consumer goods and services price indices compared to December 1993.

With the view of selecting estimates of the trend and seasonal component which are most suitable for prediction of the indicator investigated, the *cross validation* method was applied. To this end, for various methods of estimation the average error estimates $\varepsilon_x(l)$ and $\varepsilon_X(l)$ of prediction were calculated using the formulas

$$\varepsilon_x(l)=\frac{1}{L}\sum_{k=T-L+1}^{T}\left|\frac{\hat{x}(k)}{x(k)}-1\right|,\quad \varepsilon_X(l)=\frac{1}{L}\sum_{k=T-L+1}^{T}\left|\frac{\hat{X}(k)}{X(k)}-1\right|, \tag{15}$$

where T is the last month of the time interval observed, L is the chosen number of prediction points (in this paper L=3), and $\hat{x}(k)$ and $\hat{X}(k)$ are predictions defined by the equations

$$\hat{x}(k)=\hat{m}(k)+\hat{s}(k),\quad \hat{X}(k)=\exp\left\{\hat{\mu}(k)+\hat{S}(k)\right\} \tag{16}$$

where estimates $\hat{m}$, $\hat{\mu}$, and $\hat{S}$ are calculated using the values of the investigated indicator at time moments $t \le k - l$. Thus, $\varepsilon(l)$ estimates the error of prediction before l months. So $\varepsilon(12)$ estimates the expectation of prediction error of the considered indicator for the same month of the next year and the quantity $\frac{1}{12}\sum_{l=1}^{12}\varepsilon(l)$ will correspond to the error expectation estimate of the average annual prediction.

Model II

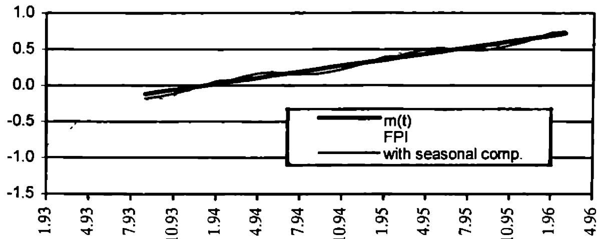


Figure 6. Changes in food, alcoholic beverages, and tobacco price indices compared to December 1993.

Model III

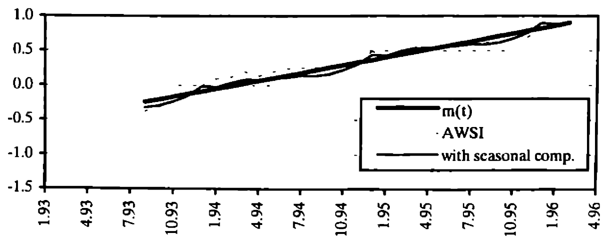


Figure 7. Changes in average monthly gross wages and salaries indices compared to December 1993.

In Table 4 prediction errors of various indicators are presented, using only the trend (in this case assume $s(t) \equiv 0$) and seasonal index.

Autoregression Models

One may expect a higher accuracy in short-term prediction not by rejecting the component of random fluctuations, but by describing it using the autoregression model:

$$\xi(t) = \sum_{k=1}^{p} \alpha_k \xi(t-k) + u(t) \qquad \text{(model I)} \qquad (17)$$

and

$$\eta(t) = \sum_{k=1}^{p} \beta_k \eta(t-k) + \hat{u}(t) \qquad \text{(model II)} \qquad (18)$$

where random variables $u(t)$ and $\hat{u}(t)$ are regarded as non-correlated with $\xi(k)$ and $\eta(\kappa)$, $k<t$. The coefficients α and β are estimated by the *least squares* method, using the values

$$\hat{\xi}_x(t) = x(t) - \hat{m}_x(t) - \hat{s}_x(t)$$

and

$$\hat{\eta}_X(t) = \log X(t) - \hat{\mu}_X(t) - \hat{S}_X(t)$$

as a sample. Since the series of available values of the investigated indicators are short, only one or two coefficients were statistically estimated in this work, the others regarded equal to 0. The following variants were considered:

a) $\hat{\alpha}_1 \neq 0$; $\hat{\beta}_1 \neq 0$;

b) $\hat{\alpha}_1 \neq 0$, $\hat{\alpha}_2 \neq 0$; $\hat{\beta}_1 \neq 0$, $\hat{\beta}_2 \neq 0$.

Since seasonal indices were roughly estimated due to short sequences of data, the estimation of seasonal influence was also considered using autoregression models instead of the seasonal index (estimating the coefficients α_{12} and β_{12}). In this case the variant

c) $\hat{\alpha}_1 \neq 0, \quad \hat{\alpha}_{12} \neq 0, \quad \hat{s}(t) \equiv 0; \quad \hat{\beta}_1 \neq 0, \quad \hat{\beta}_{12} \neq 0, \quad \hat{S}(t) \equiv 0$

was considered. If x(t) is observed at time moments t_0, $t_0 + 1, \ldots, T$, then we define

$$\hat{\xi}_x(t) = x(t) - \hat{m}_x(t) - \hat{s}_x(t),$$

for $t_0 \leq t \leq T$ and

$$\hat{\xi}_x(t) = \sum_j \hat{\alpha}_j \hat{\xi}_x(t-j), \quad t>T. \tag{19}$$

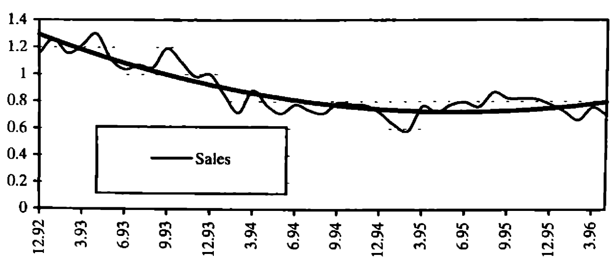
Analogously, we obtain $\hat{\eta}_X(t)$. Then

$$x(t) = \hat{m}_x(t) + \hat{s}_x(t) + \hat{\xi}_x(t),$$

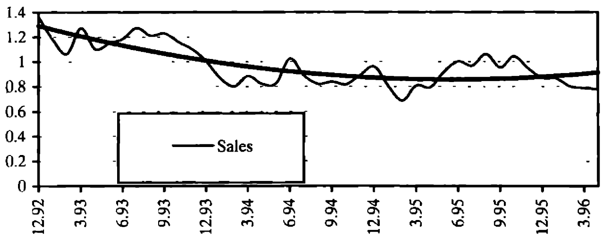
$$\hat{X}(t) = \exp\left\{\hat{\mu}_X(t) + \hat{S}_X(t) + \hat{\eta}_X(t)\right\}, \quad t>T. \tag{20}$$

In Tables 2–5 autoregression coefficients and prediction errors are presented for 1 month, 3 months, 6 months, and a year. Here ε_x and ε_X, are calculated using formula (15) and replacing (16) with (20). Diagrams of the models (20), whose usage considerably improves the predictions, are given in Figures 8–10.

a) Mining, quarrying, and manufacturing



b) Manufacturing of food products and beverages



c) Manufacturing of textiles

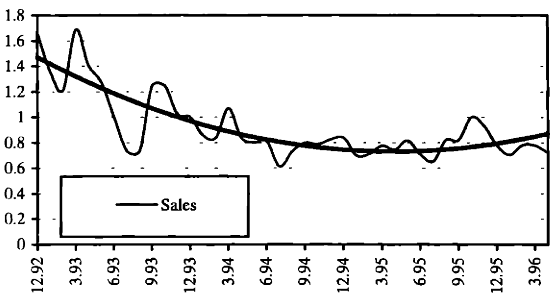
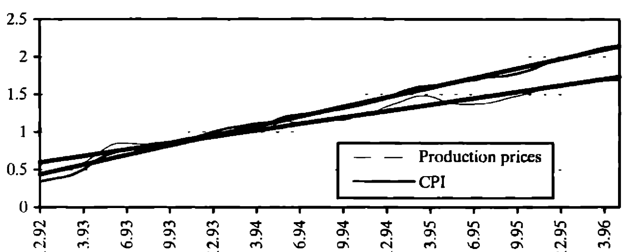
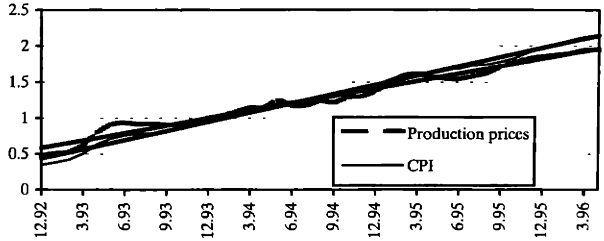


Figure 8. Changes in sales of industrial production (December 1993 equal 1.0).

a) Mining, quarrying, and manufacturing



b) Manufacturing of food products and beverages



c) Manufacturing of textiles

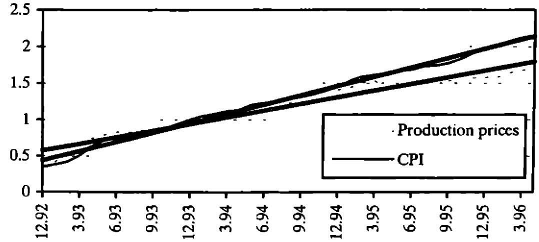


Figure 9. Changes in production prices and consumer prices (December 1993 equals 1.0).

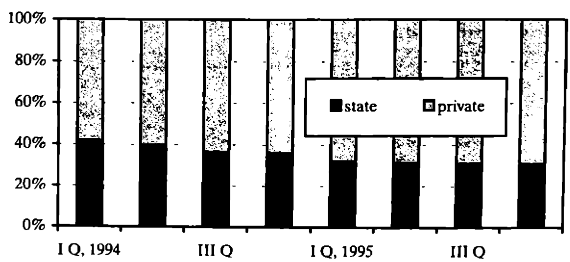


Figure 10. Share of employed by sector (in per cent).

Regression Models of Price Indices

In this section we will consider regression dependencies of price indices (CPI, FPI, CFPI, RPI, and TPI) on monetary and revenue indicators (CC, M1, M2, and AWSI). To describe the current interrelations, the statistical data mainly from 1993 can be used. Earlier data is not calculated for some indicators and the economic situation of Lithuania before 1993 was different from the present situation (it also applies to the dependencies under consideration). Since only short time series of statistical data are available, it is reasonable to consider only simple models with few parameters. More complex models, perhaps being more adequate, will yield greater prediction errors because of an inaccurate statistical parameter estimation of the model. We confined ourselves to the investigation of price dependency on one or two regressors in this work.

Let P(t) denote the considered price index, $X(t)=(X_1(t),\dots,X_k(t))$, where $X_i(t)$ denotes the values of indicators, chosen as regressors, at time moments $t-l_i$ and l_i are time lags. We considered models of the shape

$$P(t) = f(X(t)) \cdot (Q(t) + \zeta(t)) \tag{21}$$

where f is a regression function, ζ is the process of random fluctuations, and Q is the seasonal index, i.e., $Q(t+12) = Q(t)$, $\prod_{k=1}^{12} Q(k) = 1$.

Already a visual comparison of the statistical data curves under consideration shows that statistical interdependency of the investigated price and monetary indicators is close to a linear one; however, on the other hand, considering relative levels of prices and revenues with respect to the neighbouring countries, we can state that if the population income grew several times, domestic prices would increase less due to the competition of imported goods. We have, therefore, chosen a more general than linear type of regression function. The model

$$f(x_1, x_2) = \frac{x_1}{a + b \cdot x_2} \tag{22}$$

was considered in the case of two regressors.

In a separate case, when b=0, we obtained a linear dependency. The model of one regressor is obtained from (22), assuming $x_1=x_2$. We shall discuss the statistical parameter estimation, estimating coefficients of function (22) by using standard software. It is natural to apply the *least squares* method. If we have a sufficiently exact estimate of seasonal index Q we can say the following:

$$(\hat{a},\hat{b}) = \arg\min_{a,b} \sum_t \left[\frac{P(t)}{X_1(t)}\cdot a + \frac{P(t)X_2(t)}{X_1(t)}\cdot b - \hat{Q}(t)\right]^2 \tag{23}$$

where $\sum_t$ stands for the summation over all available observations

If the time series of the regression indicator $X_2(t)$ have no explicitly expressed seasonal prevalence or the value of the coefficient b is small, we may use the statistic

$$\hat{Q}(t) = \exp\left\{\hat{S}_P(t) - \hat{S}_{X_1}(t)\right\} \tag{24}$$

as an estimate of Q(t) in formula (23), where $\hat{S}_P$ and $\hat{S}_{X_1}$ are seasonal index estimates of the respective indicators that were obtained in the earlier section. Another way of getting $\hat{a}$, $\hat{b}$, and $\hat{Q}$ is recurrent. First suppose $\hat{Q}(t)\equiv 1$. Afterwards, employing (22) and (24) we obtain the estimate $\hat{f}$ of the regression function. Denoting $Y(t)=\dfrac{P(t)}{f(X(t))}$ we get its

estimates $\hat{Y}(t)=\dfrac{P(t)}{\hat{f}(X(t))}=\dfrac{P(t)\cdot\left[\hat{a}+\hat{b}X_2(t)\right]}{X_1(t).}$. Next, since $Y(t)=Q(t)+\zeta(t)$ analogously, as in the previous section, we may calculate the estimate of seasonal index Q(t) using the formula

$$\hat{Q}(j) = \frac{\sum_t \hat{Y}(t)}{\sum_t 1},\ j=1,\ldots,12, \tag{25}$$

where $\sum_t$ denotes the summation over all time moments for which $\hat{Y}(t)$ are calculated and denote the same month as j. Now that we have the estimate $\hat{Q}$, we can revise and specify estimates $\hat{a}$ and $\hat{b}$ using formula (23). After some iterations we obtain the estimates $\tilde{f}$ and $\hat{Q}$.

In order to assess the accuracy of prediction we use the same *cross validation* method:

$$\varepsilon_t = \frac{1}{L} \sum_{k=T-L+1}^{T} \left| \frac{\hat{P}(k)}{P(k)} - 1 \right|,$$

where $\hat{P}(k) = \hat{f}(X(k)) \cdot \hat{Q}(k)$, and estimates $\tilde{f}$ and $\hat{Q}$ are obtained using the observations up till time moment t (see the results of regression analysis in Tables 6–8 and Figures 11–13).

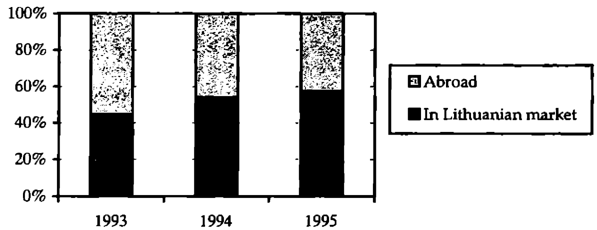


Figure 11. Share of export in sales of industrial production.

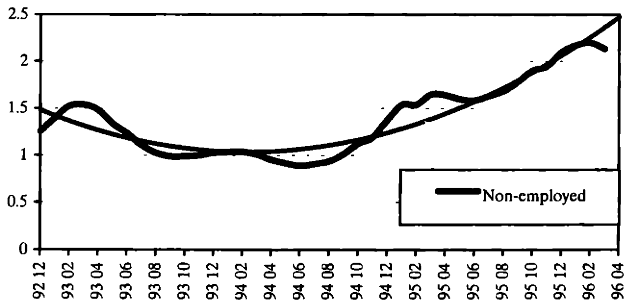


Figure 12. Non-employed, seeking work.

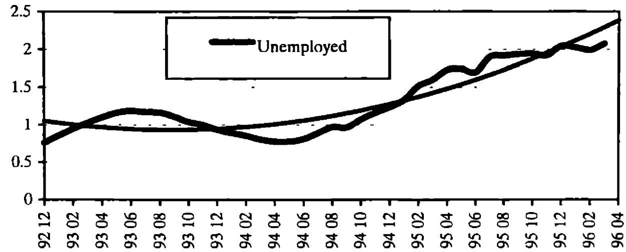


Figure 13. Changes of the unemployed having received unemployment benefits.

Prices, Production, and Employment

During the processes of privatisation and restructuring, statistical relations between production and price indicators were destroyed. Apart from that, the accuracy and reliability of production indicators is insufficient due to a great fluctuation in the number of enterprises submitting data and a variation of their part occupied in the market; therefore, it is not reasonable at present to apply methods of mathematical statistics in the analysis of production and prices.

Similar problems are also met when considering employment indicators. Many enterprises were not functioning in aggregate productive capacity during the privatisation process and the work time was shorter; thus, a latent unemployment dominated in industry which distorted the general view.

Considering the above mentioned conditions, a qualitative-expert analysis of the state national economy has been made, estimating the changes and tendencies in price, production, and employment indicators.

Changes in sales of industrial production are presented in Figure 8 (in this figure and in all others the polynomial trend is presented by a solid line). The volume of sale production constantly decreased during the period 1993–1994. The main cause stipulating the decrease may be regarded as a decrease in aggregate demand within the country. With a decrease in real income of the population, less production was sold in the Lithuanian market and more of it was exported (see Figure 11). Meanwhile, the volume of import extended, especially from the West. Production of public (state) enterprises could not stand the import competition, because state production used out-of-date technologies while prices were higher than those of imported products. A great part of our production lay long in warehouses even without getting to the market; therefore, state enterprises were forced to diminish the volume of output, thereby diminishing the sale as well.

Privatisation and restructurization processes also had a great influence on the decline of industry. According to the law, 50% of enterprise shares could be acquired by employees; thus, an enterprise had no actual host and shareholders oriented themselves towards keeping their working places rather than increasing production capacities and the volume of output or towards a competitive fight against cheap imported products. This situation caused the bankruptcy of many enterprises in 1994, which caused a further decline in industry. The textile industry was in a difficult state (see Figure 8c). The main reason was the loss of market in the East and the incapability of competing with imported products. By attracting foreign investments, however, and introducing new technologies, the volume of sale production of the textile industry considerably increased from the middle of 1995, mainly due to an increase of products meant for export.

In 1995 a slight increase in sales of industrial production was achieved. Along with a narrowing of economic relations with the East and their extension with the West, foreign capital investments acquired an important role by bringing new technologies into the national economy of Lithuania and enlarging the competitiveness of her production as well as exporting possibilites. With an increase in real income of the population, the enterprises could sell more of their products in the domestic market (see Figure 11). The establishment of joint ventures with foreign capital, the attraction of direct foreign investments, and foreign credits played and are still playing an important role in the recovery of industry.

When analysing changes in producer and consumer prices, it is possible to draw some conclusions about the current situation in the national economy of Lithuania in the period 1993–1995, as well as the reasons for inflation. As seen from the curves described in Figure 9, producer prices were growing more rapidly than consumer prices in 1993. This growth was stipulated by the fact that in the neighbouring countries producer prices were a little higher than in Lithuania, which affected the prices of Lithuania's exported products. Profit-making producers increased product prices instead of extending the scope of production. Inflation dependent upon demand prevailed in Lithuania. Consumer prices grew slower, since, due to the lack of real income of the inhabitants, the aggregate demand for domestic goods decreased. This resulted in a decrease of product sale and production volume.

In 1994–1995 the situation changed. Consumer prices began growing more rapidly than that of production prices. With an increase in real income of the population, more goods were purchased and the aggregate demand increased. With an increase in demand, consumer prices also increased. Meanwhile, with the appearance of more imported goods in the domestic market, Lithuanian producers were forced to compete with importers, and this slowed down the growth in production prices. Another reason for a slower growth in production prices was that after investing more funds into the process of production and using better technologies, it was rather worth it not to raise production prices but to orient oneself towards the extension of the output volume. That is why producers were aimed at increasing production costs rather than raising prices. Cost-dependent inflation became predominant and continues up till now.

It is inexpedient to include employment indicators in the modelling of inflation processes at present, since we face the problems of data inaccuracy, unreliability, and variation in estimation methods. Therefore, we are only able to make a qualitative analysis of employment indicators.

During the process of privatisation, the percentage of employed in the public and private sector changed a great deal (see Figure 10). The percentage employed in the private sector constantly increased; meanwhile, the variation in the number of non-employed persons seeking work and unemployed persons is much more interesting (see Figures 12 and 13). From the diagrams we see that, up till the middle of 1993, the number of unemployed grew and later on began diminishing. This can be explained by the prevalence of latent unemployment in Lithuania. At that time enterprises had to decrease volumes of output and couldn't work for a full week. In addition, the persons did not apply to the labour exchange after losing their work, but he/she tried to find an illegal job (for instance, he went to buy goods in adjacent countries and sold them in the Lithuanian market). In 1994, after operation records became stricter and after defining labour relations by law, the number of unemployed began to increase. In addition to this, when enterprises began extending the volumes of production and more free work places appeared, more and more people were interested in finding a legal job and getting higher wages or salaries, whereas those not wishing to work wanted to apply to the labour exchange and get unemployment benefits in accordance with the established legal order.

Conclusions

Taking into account the research carried out by using Lithuanian economic data from the years 1993–1995, we can assert that, in recent years, there has been a strong dependency between consumer prices and monetary indicators as well as income indicators in Lithuania. This dependency is close to linear but a more exact result is received using a non-linear model in the regression analysis of prices.

It is expedient to use a broken line trend (break point at July, 1993) to forecast prices and monetary indicators that were explored by time series methods. The use of statistical estimation of seasonal components improves the forecast's exactness of model I in FPI and AWSI and model II in AWSI. The seasonal prevalence manifests itself no stronger for the other indicators. To forecast these indicators it is, at present, useful to use an estimation of trend and autoregression models without seasonal components, because of the shortness of data series.

Privatisation and restructurization processes had a great influence on the decline of industry in the period 1992–1995 in Lithuania. In 1992–1993 inflation dependent upon demand prevailed in Lithuania. Cost-dependent inflation became predominant in 1994–1995 and continues up till now.

REFERENCES

1. Bergman, L., and O. Olsen. Economic Modelling in the Nordic Countries Oslo: Elsemer Science Publishers B.V., 1992.
2. Fischer, S. and R. Dornbusch. Macroeconomics. Oxford: McGraw-Hill, 1994.
3. Lithuania. Department of Statistics to the Government of the Republic of Lithuania. Economic and Social Development in Lithuania. Monthly Bulletin. Vilnius: 1993 – 1996.
4. The Bank of Lithuania. The Bank of Lithuania Monthly Bulletin. Vilnius: 1996.

T a b l e 1. **The results of trend estimation**

Index	Model I						Model II				
	parameters				error		parameters			error	
	a	b	$\tilde{a}$	$\tilde{b}$	δ_x	Δ_x	A	B	C	δ_x	Δ_x
CPI	0.1226	–0.0015	0.0393	–0.0007	0.4080	0.0918	0.7256	0.1551	–0.1246	0.6961	0.0368
FPI	0.1226	–0.0011	0.0272	0.0001	0.6889	0.1218	0.5924	0.1272	–0.0994	2.2463	0.0335
CFPI	0.1215	–0.0023	0.0562	–0.0020	0.5101	0.2097	0.8949	0.1917	–0.1610	0.6981	0.0655
RPI	0.0891	–0.0052	0.0727	–0.0022	0.6608	0.1833	0.6278	0.1620	–0.1137	0.6981	0.0784
TPI	0.15637	0.0032	0.0356	–0.0008	0.5589	0.1053	0.6819	0.1403	–0.1129	1.0763	0.0265
M1	–0.2291	–0.0272	0.1031	–0.0049	0.8750	0.2851	0.2519	0.1137	–0,0755	6.3226	0.1033
M2	–0,1955	–0,0238	0,0846	–0,0039	0,5771	0,3713	0,1256	0,0768	–0,0368	1.1801	0.0809
AWSI	0,1109	–0,0007	0,0559	–0,0011	0,6872	0,1184	0,5956	0,1521	–0,1140	3.3545	0.0629

T a b l e 2. **The results of AR estimation** (model I)

Index	AR(1)		AR(2)		
	parameters	error	parameters		error
	a_1	δ_x	a_1	a_2	δ_x
CPI	0.2810	0.3692	0.4439	–0.6309	0.3684
FPI	0.2666	0.8239	0.4142	–0.5827	0.6371
CFPI	0.2561	0.4083	0.3324	–0.3369	0.4347
RPI	–0.0817	0.6749	–0.1148	–0.1997	0.6232
TPI	–0.2100	0.5787	–0.2269	–0.1110	0.5804
M1	0.2484	0.5890	0.1431	0.4078	0.5732
M2	0.0021	0.4817	0.0144	0.2415	0.4225
AWSI	–0.3981	0.7108	–0.2157	0.2365	0.8029

T a b l e 3. **The results of AR estimation** (model II)

Index	AR(1)		AR(2)		
	parameters	error	parameters		error
	a_1	Δx	a_1	a_2	Δx
CPI	0.7806	0.0215	0.8041	–0.5052	0.0212
FPI	0.5781	0.0205	0.9866	–0.1310	0.0175
CFPI	0.8754	0.0323	0.9484	–0.1280	0.0312
RPI	0.8382	0.0448	0.9484	–0.1280	0.0445
TPI	0.6578	0.0218	0.6078	0.0851	0.0212
M1	0.7998	0.0547	0.9276	–0.1486	0.0515
M2	0.8013	0.0412	0.8227	–0.0293	0.0402
AWSI	0.8079	0.0328	0.6816	0.1762	0.0328

T a b l e 4. **The average relative errors** (model II)

a) forecast for one year

Index	Trend	Trend with seasonal term	Trend with seasonal term and AR(1)	Trend with seasonal term and AR(2)
1	2	3	4	5
CPI	0.0618	0.0790	0.0342	0.0367
FPI	0.0327	0.0306	0.0283	0.0244
CFPI	0.1311	0.1605	0.0623	0.0659
RPI	0.1848	0.2549	0.1006	0.1058
TPI	0.0409	0.0554	0.0295	0.0274
M1	0.1749	0.1963	0.0855	0.0896
M2	0.1785	0.2111	0.0795	0.0827
AWSI	0.1131	0.1083	0.0668	0.0680

b) forecast for a half year

CPI	0.0765	0.0886	0.0346	0.0372
FPI	0.0322	0.0292	0.0145	0.0177
CFPI	0.1269	0.1425	0.0820	0.0843
RPI	0.2078	0.3024	0.1246	0.1273
TPI	0.0693	0.0913	0.0408	0.0391
M1	0.1054	0.1292	0.0999	0.1006
M2	0.1633	0.1881	0.0819	0.0836
AWSI	0.1032	0.0549	0.0549	0.0544

c) forecast for three months

1	2	3	4	5
CPI	0.0624	0.0973	0.0430	0.0442
FPI	0.0435	0.0310	0.0114	0.0168
CFPI	0.1178	0.1837	0.1413	0.1428
RPI	0.2106	0.3158	0.1878	0.1823
TPI	0.0824	0.1062	0.0262	0.0254
M1	0.0973	0.1215	0.1335	0.1326
M2	0.1779	0.1593	0.1063	0.1062
AWSI	0.0864	0.0718	0.0873	0.0864

d) forecast for one month

CPI	0.0563	0.1310	0.0591	0.0623
FPI	0.0273	0.0234	0.0138	0.0147
CFPI	0.1232	0.3358	0.2236	0.2207
RPI	0.2259	0.4617	0.2539	0.2390
TPI	0.0880	0.1042	0.0250	0.0248
M1	0.1202	0.2970	0.3384	0.3399
M2	0.2224	0.3010	0.2872	0.2862
AWSI	0.0508	0.0457	0.1778	0.1781

a) forecast for one year

Index	Trend	Trend and AR(1)	Trend and AR(2)
CPI	0.0618	0.0710	0.0200
FPI	0.0327	0.0710	0.0323
CFPI	0.1311	0.0698	0.0187
RPI	0.1848	0.0647	0.0291
TPI	0.0409	0.0584	0.0215
M1	0.1749	0.0576	0.0546
M2	0.1785	0.0382	0.0332
AWSI	0.1131	0.0524	0.0514

b) forecast for a half year

CPI	0.0765	0.0675	0.0193
FPI	0.0322	0.0680	0.0312
CFPI	0.1269	0.0706	0.0165
RPI	0.2078	0.0499	0.0309
TPI	0.0693	0.0448	0.0166
M1	0.1054	0.0710	0.0640
M2	0.1633	0.0362	0.0292
AWSI	0.1032	0.0506	0.0494

c) forecast for three months

1	2	3	4
CPI	0.0624	0.0688	0.0147
FPI	0.0135	0.0707	0.0258
CFPI	0.1178	0.0668	0.0098
RPI	0.2106	0.0486	0.0392
TPI	0.0824	0.0441	0.0172
M1	0.0973	0.0642	0.0609
M2	0.1779	0.0266	0.0240
AWSI	0.0864	0.0540	0.0531

d) forecast for one month

CPI	0.0563	0.0625	0.0083
FPI	0.0073	0.0647	0.0196
CFPI	0.1232	0.0591	0.0061
RPI	0.2259	0.0299	0.0256
TPI	0.0880	0.0399	0.0139
M1	0.1202	0.0664	0.0716
M2	0.2224	0.0334	0.0320
AWSI	0.0508	0.0596	0.0583

T a b l e 5. **The average relative errors** (model I)

a) forecast for one year

Index	Trend	Trend with seasonal term	Trend with seasonal term and AR(1)	Trend with seasonal term and AR(2)
1	2	3	4	5
CPI	0.0618	0.0790	0.0342	0.0367
FPI	0.0327	0.0306	0.0283	0.0244
CFPI	0.1311	0.1605	0.0623	0.0659
RPI	0.1848	0.2549	0.1006	0.1058
TPI	0.0409	0.0554	0.0295	0.0274
M1	0.1799	0.1870	0.0855	0.0896
M2	0.1785	0.1963	0.0795	0.0827
AWSI	0.1131	0.1083	0.0668	0.0680

b) forecast for a half year

CPI	0.0705	0.0886	0.0346	0.0373
FPI	0.0322	0.0292	0.0145	0.0141
CFPI	0.1269	0.1425	0.0826	0.0823
RPI	0.2078	0.3024	0.1246	0.1243
TPI	0.0693	0.0913	0.0408	0.0391
M1	0.1054	0.1292	0.0499	0.1006
M2	0.1633	0.1881	0.0819	0.0816
AWSI	0.1032	0.0549	0.0549	0.0544

c) forecast for three months

CPI	0.0624	0.0973	0.0430	0.0442
FPI	0.0435	0.0310	0.0114	0.0168
CFPI	0.1178	0.1837	0.1413	0.1428
RPI	0.2106	0.3158	0.1878	0.1823
TPI	0.0824	0.1062	0.0262	0.0254
M1	0.0973	0.1215	0.1335	0.1326
M2	0.1779	0.1593	0.1063	0.1062
AWSI	0.0864	0.0718	0.0873	0.0864

d) forecast for one month

CPI	0.0705	0.0886	0.0346	0.0373
FPI	0.0322	0.0292	0.0145	0.0141
CFPI	0.1269	0.1425	0.0826	0.0823
RPI	0.2078	0.3024	0.1246	0.1243
TPI	0.0693	0.0913	0.0408	0.0391
M1	0.1054	0.1292	0.0499	0.1006
M2	0.1633	0.1881	0.0819	0.0816
AWSI	0.1032	0.0549	0.0549	0.0544

a) forecast for one year

Index	Trend	Trend and AR(1)	Trend and AR(2)
CPI	0.0132	0.0119	0.0121
FPI	0.0131	0.0115	0.0126
CFPI	0.0175	0.0181	0.0184
RPI	0.0197	0.0159	0.0212
TPI	0.0249	0.0251	0.0251
M1	0.0148	0.0153	0.0150
M2	0.0747	0.0653	0.0516
AWSI	0.0342	0.0354	0.0308

b) forecast for a half year

CPI	0.0128	0.0120	0.0132
FPI	0.0141	0.0131	0.0150
CFPI	0.0232	0.0186	0.0241
RPI	0.0227	0.0226	0.0218
TPI	0.0120	0.0134	0.0154
M1	0.0770	0.0653	0.0478
M2	0.0363	0.0366	0.0309
AWSI	0.0390	0.0403	0.0384

c) forecast for three months

CPI	0.0624	0.0688	0.0735
FPI	0.0135	0.0707	0.0749
CFPI	0.1178	0.0668	0.0753
RPI	0.2106	0.0486	0.0552
TPI	0.0824	0.0441	0.0494
M1	0.0973	0.0642	0.0746
M2	0.1779	0.0266	0.0404
AWSI	0.0864	0.0540	0.0528

d) forecast for one month

CPI	0.0563	0.0625	0.0742
FPI	0.0073	0.0647	0.0771
CFPI	0.1232	0.0591	0.0740
RPI	0.2259	0.0299	0.0513
TPI	0.0880	0.0399	0.0456
M1	0.1202	0.0664	0.0688
M2	0.2224	0.0334	0.0351
AWSI	0.0508	0.0596	0.0548

T a b l e 6. **The estimation of the regression model**

Index	Currency in circulation				Money M1				Money M2			
	parameters		error		parameters		error		parameters		error	
	a	b	Δx	with seasonal term	a	b	Δx	with seasonal term	a	b	Δx	with seasonal term
CPI	1.301	0.191	0.0543	0.0331	1.376	0.277	0.0561	0.0314	1.351	0.223	0.0545	0.0316
FPI	1.479	0.332	0.0739	0.0315	1.632	0.503	0.0790	0.0291	1.576	0.395	0.0686	0.0305
CFPI	1.123	0.100	0.0597	0.0427	1.157	0.143	0.0564	0.0405	1.163	0.129	0.0566	0.0381
RPI	0.968	–0.141	0.1045	0.0472	0.928	–0.189	0.1106	0.0497	0.926	–0.166	0.1104	0.0517
TCI	1.434	0.288	0.0557	0.0268	1.563	0.432	0.0582	0.0244	1.516	0.340	0.0484	0.0260

T a b l e 7. **The estimation of regression model**

Index	parameters		error	
Currency in circulation				
	a	b	Δ_X	with seasonal term
CPI	2.055	0.743	0.0982	0.0311
FPI	2.375	1.004	0.1150	0.0342
CFPI	1.790	0.603	0.0668	0.0286
RPI	1.430	0.135	0.0989	0.0345
TPI	2.297	0.925	0.0858	0.0290
Money M1				
CPI	1.358	0.169	0.0675	0.0411
FPI	1.598	0.362	0.0754	0.0382
CFPI	1.148	0.052	0.0935	0.0539
RPI	0.948	–0.230	0.1572	0.0613
TPI	1.541	0.306	0.0458	0.0343
Money M2				
CPI	1.159	0.128	0.0819	0.0522
FPI	1.359	0.287	0.0814	0.0406
CFPI	0.988	0.038	0.0964	0.0696
RPI	0.809	–0.212	0.1692	0.0760
TPI	1.307	0.237	0.0572	0.0431
Average monthly wages and salaries				
CPI	1.352	0.234	0.0494	0.0331
FPI	1.581	0.416	0.0721	0.0308
CFPI	1.154	0.127	0.0580	0.0421
RPI	0.940	–0.163	0.1071	0.0472
TPI	1.524	0.362	0.0482	0.0261

T a b l e 8. **The error of forecast using the regression model**

	Currency in circulation	M1	M2	AWS	CC with AWS	M1 with AWS	M2 with AWS
CPI	0.1072	0.0632	0.0804	0.0582	0.0567	0.0587	0.0571
FPI	0.1241	0.0820	0.0910	0.0832	0.0783	0.0839	0.0724
CFPI	0.0747	0.0848	0.0882	0.0537	0.0637	0.0603	0.0602
RPI	0.1117	0.1401	0.1527	0.1003	0.1114	0.1191	0.1175
TPI	0.0928	0.0479	0.0562	0.0532	0.0589	0.0619	0.0589

VARTOJIMO PREKIŲ IR PASLAUGŲ KAINŲ LIETUVOJE STATISTINIS-MATEMATINIS MODELIAVIMAS

Žilvinas Kalinauskas, Bronislava Kaminskienė, Rimantas Rudzkis

Santrauka

Darbe matematinės statistikos metodais tiriami vartojimo prekių ir paslaugų kainų indekso (VKI), kai kurių prekių grupių kainų indeksų ir svarbiausių monetarinių rodiklių mėnesiniai duomenys Lietuvoje. Pagrindinis darbo tikslas – sudaryti VKI matematinius modelius, tinkamus trumpalaikiam prognozavimui. Straipsnyje tiriama statistinė kainų ir monetarinių rodiklių priklausomybė. Įvertinami nagrinėjamų rodiklių trendai ir sezoninės komponentės, atsitiktinės fluktuacijos aprašomos autoregresiniais modeliais. Sudaryti kainų regresiniai modeliai, kai regresoriais yra monetariniai rodikliai. Įvertintos rodiklių prognozavimo, panaudojant siūlomus modelius, paklaidos. Buvo atlikta Lietuvos ekonomikos ekspertinė analizė, atsižvelgiant į kainų, gamybos ir užimtumo pokyčius. Esant netiksliems duomenims perskaičiuojant rodiklius, dažnai buvo atlikta tik kokybinė analizė, netaikant matematinių priemonių.