Online ISSN 2424-6166. EKONOMIKA 2017 Vol. 96(2)
DOI: https://doi.org/10.15388/Ekon.2017.2.10998

INVESTMENT PORTFOLIO OPTIMIZATION BY APPLYING A GENETIC ALGORITHM-BASED APPROACH

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Abstract. *The investment portfolio optimization issues have been widely discussed by scholars for more than 60 years. One of the key issues that emerge for researchers is to clarify which optimization approach helps to build the most efficient portfolio (in this case, the efficiency refers to the minimization of the investment risk and the maximization of the return). The objective of the study is to assess the fitness of a genetic algorithm approach in optimizing the investment portfolio. The paper analyzes the theoretical aspects of applying a genetic algorithm-based approach, then it adapts them to practical research. To build an investment portfolio, four Lithuanian enterprises listed on the OMX Baltics Stock Exchange Official List were selected in accordance with the chosen criteria. Then, by applying a genetic algorithm-based approach and using MatLab software, the optimum investment portfolio was constructed from the selected enterprises. The research results showed that the genetic algorithm-based portfolio in 2013 reached a better risk-return ratio than the portfolio optimized by the deterministic and stochastic programing methods. Also, better outcomes were achieved in comparison with the OMX Baltic Market Index. As a result, the hypothesis of the superiority of a portfolio, optimized on the basis of a genetic algorithm, is not rejected. However, it should be noted that in seeking for more reliable conclusions, further research should include more trial periods as the current study examined a period of one year. In this context, the operation of the approach in the context of a market downturn could be of particular interest.*

Keywords: *artificial intelligence, genetic algorithm, stochastic programming, investment portfolio optimization.*

Introduction

The investment portfolio optimization is a process by which an investor seeks to maximize the investment returns or minimize the risks. Due to a number of factors, such as a limited amount of resources or random changes in the prices of financial instruments, the appliance of the existence or optimization approaches to the investment process appears to be a rather complex procedure (Berntsson 2009; Markowitz 1952; Siddiqui and Maribu 2009 and others).

A large part of the problems, related to the investment portfolio optimization, are most commonly solved using deterministic models, i.e., when the search for the optimal solution is conducted under strictly defined conditions (Baumert 2005; Markowitz 1952).

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However, deterministic models are usually hard to implement in practice, since the global economic financial system is a complex, non-linear (*non-linearity* is progressive taxation, limited resources etc.) system consisting of a number of interrelated subsystems – enterprises, banks, stock exchanges – and exposed to external noise, such as political events or external disasters (Plikynas, Daniušis 2010).

For this reason, some economists are trying to describe this continuously changing system by means of differential, usually stochastic, equation systems (differential equations containing noise members), perform computer simulations and compare the dynamics of simulation systems with the dynamics observed in the real market. This application of artificial intelligence not only facilitates more accurate solutions but also increases the solution-making efficiency in investment processes.

Therefore, this paper describes the main research problem, i.e., the efficiency of computer intelligence (which is a part of artificial intelligence), applied to optimizing the investment portfolio. As computer intelligence encompasses many fields, such as evolutionary computing, neural networks, fuzzy logic, hybrid and multiagent systems etc., the study focuses on one branch of evolutionary computation, i.e., the genetic algorithms that can be applied to construct the optimal portfolio.

The claim that genetic algorithms are more suitable in building the optimal investment portfolio, rather than the classical deterministic and stochastic programming algorithms, is the core hypothesis of the research described in this article. In this case, the concept of *suitability* is defined as the ability of the portfolio, built on the basis of genetic algorithms, to earn higher returns with a lower risk, as compared with the portfolio that is formed by classical optimization methods. The second hypothesis states that the optimal investment portfolio, formed on the basis of the genetic algorithm, is more likely to earn a higher return when compared with the market.

1. Genetic Algorithms and Their Application in the Field of Finance

Genetic algorithms belong to a branch of evolutionary computing. Evolutionary computation is understood as an area of computer intelligence, addressing combinatorial optimization problems. Their goal is to apply the principles of natural evolution, in accordance to which only the best-adjusted to the given environment survive (Plikynas, Daniušis 2010). The intensive exploration of this area was launched in the fifties of the twentieth century, along with the appearance of the first computers. For example, R.M. Friedberg was exploring automatic programming in terms of its support to create a program/software capable to perform a certain function by the given input and output values (Friedberg 1958; Friedberg et al. 1959). Fraser (1957) carried out genetic processes simulations by using a computer. Later, the development of evolutionary computation foundation was reinforced by H.J. Bremermann, who applied the evolutionary model to solve equation systems (Bremermann 1962) and was the first to introduce the theory of evolutionary algorithms (Bremermann et al. 1965).

Even though scholars have developed a comparatively wide range of evolutionary algorithms that use computers to describe task-solving systems (the key elements applied by these systems are mathematical models derived from the natural evolution methods), all these algorithms are based on a few basic principles:

- The collective training of each evolutionary algorithm is being carried out through a population of individuals. Each individual corresponds to one solution of the problem under consideration.
- Each individual is assigned the value that evaluates the quality of the solution. In terms of this value, in accordance with the probability, the participation of higher-quality individuals in the selection process is stronger than that of the lower quality individuals.
- Individuals are randomly generated by modelling crossover and mutation. A crossover is the exchange of information between two or more individuals, whereas mutation corresponds to a false self-copying (Narvydas 2006).

Genetic algorithms are usually employed by technologies targeted at the optimal solution search. During the search, the principles of selection, inheritance, mutation and crossover are applied. Computer simulation tools assist in creating a population of individuals. Every individual in this population is characterized by a set of chromosomes. The optimization process searches for the best individual, i.e., a set of signs that characterizes the best individual and shows the solution of the problem (Plikynas, Daniušis 2010).

The application of genetic algorithms for solving financial problems has become popular relatively recently. Roberto Pereira (2000) argues that these algorithms are suitable for solving practical financial problems, especially in cases where it is necessary to use effective and sophisticated optimization techniques. The application of genetic algorithms in the field of finance can cover such areas as return forecasting, portfolio optimization, trading strategy setting and other instances (Pereira 2000). The success of these algorithms, applied to optimizing an investment portfolio, has been confirmed by a great number of scholars (Aranba and Iba 2009; Divya and Kumar 2012; Garkaz 2011; Sefiane and Benbouziane 2012). Nevertheless, it should be remembered that the research results may differ significantly depending on the nature of the market, the type of the genetic algorithm, the selected function of the objective etc.

2. Methodology of the Investment Portfolio Optimization by Applying a Genetic Algorithm

2.1. Creating a Genetic Algorithm

The application of genetic algorithms to optimize the investment portfolio has the following advantages: a) These algorithms work with a set of parameters, rather than with each parameter separately; b) A parallel search can be carried out for the optimal solution through a number of points, not through a single point; c) A direct presentation of the problem domain could be used without any additional parameters; d) Probabilistic rules may be used instead of deterministic search algorithms (Augusto et al. 2006; Plikynas, Daniušis 2010). Table No. 1 shows the steps of a classical genetic algorithm (Plikynas, Daniušis 2010).

At the end of the algorithm, it is important to obtain at least some of the chromosomes that are close to the optimal solution.

TABLE No. 1. **The steps of the classical genetic algorithm**

	The steps of a classical genetic algorithm
1	The problem area of the optimization task is defined by identifying the key variables, on the basis of which the area of variables is formed. This area is of a fixed size. In a genetic algorithm, it is called a chromosome. Next, a set of such initial chromosomes, the number N (population size, topology of the population variance), the probability of crossover p_k and the probability of mutation p_m are determined.
2	The fitness function is formed by means of which the fitness of an individual (the chromosome) of the population is determined according to the selected criteria of the optimization task. In this way, it is possible to select those individuals in the population that will be used for the purpose of mating reproduction (creation of an adaptive topology map, in order to find optimal solutions).
3	The initial N chromosomes (investment subjects) population: $x_1, x_2, x_3 \ldots x_N$ is randomly generated.
4	The fitness of each chromosome is evaluated: $f(x_1), f(x_2), f(x_3) \ldots f(x_N)$.
5	The pairs of chromosomes, the fitness values of which show the maximum of the probability, are selected out of the population by means of the fitness function (see step no. 2).
6	A new pair of offspring chromosomes is created by means of genetic crossover and mutation operators.
7	The created offspring are entered into the population, while their parents are eliminated (the most suitable parents remain so that their chromosomes were retained).
8	Steps 5-7 are repeated until the size of a new population corresponds to the old one. The old generation is replaced by a new one.
9	The procedure is repeated from step no. 4 until the signs of the ending algorithm become visible.

It is necessary to have in mind that genetic algorithms belong to a class of stochastic search algorithms, so only a probabilistic evaluation, concerning which generation a solution will be found in, is possible in this case. As a rule, a general practice is applied, where the algorithm is interrupted at the predetermined number of generations. If the correct solution is not found, the algorithm is relaunched (Plikynas, Daniušis 2010).

Before starting the creation of the genetic algorithm process, it is necessary to determine the function of the objective. In this case, the investor wants to maximize the return on the portfolio and, at the same time, minimize the risk. The return on the investment portfolio is calculated as follows:

$$R_p = \sum_{i=1}^{n} w_i r_i \quad (1)$$

Meanwhile, the portfolio risk is assessed by:

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \, \sigma_i^2 + \sum \sum 2 w_i w_j cov(i, j) \quad (2)$$

where: w_i – weigh of stock i in the portfolio, r – the average return on stock i, σ_i – standard deviation of stock i return, $cov(i, j)$ – the covariance between stock i and j returns.

Due to the fact that the goal is to achieve the maximum return and take the minimal risk at the same time, the function of the objective is as follows:

$$f(chromosome) = \frac{R_p}{\sigma_p} \rightarrow max \quad (3)$$

This function can be called the fitness function, the result of which is the fitness value.

In order to create a reproduction pool out of the initial population of chromosomes, it is necessary to select the chromosomes with their maximum fitness probability values. These values are calculated by means of the evaluation function. This is a relative indicator, which can be expressed as follows:

$$f(i \; evaluation) = \frac{f(chromosome \; i)}{\sum f(chromosome \; i)} \quad (4)$$

So, by knowing the fitness and probabilistic evaluation functions, the creation of the initial population, consisting of chromosomes, can be launched. The chromosome consists of genes. In this case, the gene is seen as weight w_i, attributed to the stock included in the investment portfolio. Therefore, each chromosome can be seen as one combination of the investment portfolio. While creating the initial population, the genes are formed by assigning them random values in accordance with the normal probability distribution. Random values (V_i) are used to calculate weights (w_i) of the selected stock, i.e., $V_i / \sum V_i$. Considering the fact that a weight is perceived as a percentage of the total amount of the money invested, which is recommended to be invested in the selected stock, it follows that the population of 10 random chromosomes represents 10 random solutions (values of the objective function).

Having formed the initial population, the next step is the formation of the reproductive pool. The reproductive pool is a set of chromosomes, which will later become mates. There are a number of possible approaches employed to form the reproductive pool, such as the tournament selection, roulette wheel selection, stochastic-based selection etc. This study applies a roulette wheel selection approach. This means that each chromosome is evaluated by the fitness function (3). In a normal distribution, random numbers (r_i) are generated and compared with a composite distribution of the chromosome. Chromosome i, with its composite distribution $p_{i-1} < r_i < p_i$, will be selected into the reproductive pool.

The strongest chromosomes (those with a higher fitness function value) have a higher probability of entering the reproductive pool.

The chromosomes of the reproduction pool are mated, i.e., they cross over. Crossover can occur through various methods, such as a single point crossover, the two-point crossover, uniform crossover, heuristic crossover etc. This study applies the arithmetic crossover, as in terms of the portfolio optimization, it appears to be more accurate (Seflane, Benbouziane 2012; Sinha, Chandwani et al. 2013). This crossover is carried out in accordance with the following principles:

$$Offspring\ 1 = a \times CH1 + (1 + a) \times CH2 \tag{5}$$

$$Offspring\ 2 = (1 - a) \times CH1 + a \times CH2 \tag{6}$$

where CH1 and CH2 are parents; we also assume that the chromosomes can mutate.

However, in this case, a question arises of when the chromosome has to be crossed, and when can it mutate? The crossover is more concerned with explorative procedures. It creates a chromosome which has the characteristics of both parents. Meanwhile, the mutation includes exploitative procedures. At this time, the chromosome features of one of the parents are just altered. As a result, the crossover transmits information from both parents, while the mutation provides new information to the offspring. In this study, it is assumed that the mutation probability is $p_m = 0.4$, while that of crossover is $p_c = 0.6$. In the case of identical entering the reproduction pool, the shares must mutate in order to gain a genetic advantage in the population of the next generation.

2.2. Investment Portfolio Building Algorithm

The investment portfolio building algorithm starts with the stock selection process. Since one of the objectives is to minimize the risk, diversification should be used for the stock selection process. As a result, the shares are broken down by sector. The fundamental data of each enterprise, which will be reviewed later, are comparable not with all enterprises listed on the stock exchange, but only with those that belong to the same sector.

This will ensure that the enterprises entering the portfolio are the leaders in their sector and the portfolio is successfully diversified, because it includes enterprises from a variety of business sectors. The stock selection is performed by five parameters offered by Sinha, Chandwani et al. (2013): P/E ratio, earnings per share, wealth creation (which is defined as the difference between ROIC and WACC), stock valuation (shares become undervalued when their market value is less than the fair value) and PEG ratio.

Having selected the parameters, the construction of priority index receives further attention. The priority index function uses all five factors listed above that assist in ranking shares based on how many points they have accumulated. The more favorable is the parameter value, the more points the share is awarded. If the maximum of the parameter is desired, the function is as follows:

$$S_{ij} = 100\left(X_{ij} - Min\right)/(Max - Min) \quad (7)$$

where: S_{ij} – the number of points attributable to share i by parameter j, X_{ij} – the value of the function of share i, Min – minimum value of share i, Max – the maximum value of share i.

If an investor requires the minimum of the parameter, then the function would be as follows:

$$S_{ij} = 100\left(X_{ij} - Max\right)/(Min - Max) \quad (8)$$

Pritority index (PI_i) is calculated by summing up all the share points earned by all parameters:

$$PI_i = \sum S_{ij} \quad (9)$$

The shares are selected according to the priority index (10). It is recommended that the algorithm constructs a portfolio of only those shares with a priority index greater than 3.8 applying a 5.0-point scale (Sinha, Chandwani et al. 2013).

The stocks, selected in accordance with the priority index, are included in the portfolio, while their weights are determined by means of optimization. As mentioned above, the share in the portfolio is considered as a gene, while a set of all the shares is considered to be a chromosome. The initial generation is comprised of 10 of these chromosomes randomly. Then, the reproduction pool is constructed. The chromosomes of the pool are crossed or mutated. These procedures enable to create a new generation. Each generation has the same number of chromosomes. For example, 100 of generations will construct the population of 1000 chromosomes (while 10 chromosomes comprise one generation). The chromosome with the highest fitness function value will be a solution for the problem of the investment portfolio optimization.

2.3. Constructing a Portfolio by Deterministic and Stochastic Programing Methods

In order to verify the hypothesis of the study with regard to the ability of an optimal portfolio based on a genetic algorithm, to generate a more favorable return-risk ratio than other optimization methods or the market, it is necessary to describe the principles of applying deterministic and stochastic approaches.

While constructing a deterministic approach, the same stock selection procedure can be applied as in the case of a genetic algorithm. The objective function (3) and variables (w_i) also will be the same. In this case, only the search methodology for optimal weights will be different. Instead of a genetic algorithm, nonlinear programming principles (as the objective function is nonlinear) will be applied, so the limitations of the situation are as follows:

The total weight amount must be equal to one, i.e., $\Sigma w_i = 1$, i.e., it is assumed that the investor invests all the funds allocated in the budget. The weights may be negative. This means that the investor can borrow securities, i.e., to open short positions.

However, as mentioned at the beginning of this study, the deterministic optimization method has certain disadvantages. The most prevailing shortcoming is that the deterministic approach is based on an assumption that the stock price does not change, i.e., the return and the risk are constant. Obviously, it contradicts the actual situation, so it is worth to check the advantage of the portfolio optimized on the basis of a genetic algorithm by comparing it with a portfolio constructed on the basis of not only deterministic but also of a stochastic programing approach. A portfolio constructed on the basis of a stochastic programing method is much closer to reality, because many of the variables existing here have not been known in advance. Consequently, the return of shares, the risk and the available resources can also be random variables. The problems where random variables dominate instead of the determined variables may be called the problems arising from stochastic programing. In order to solve them, the theory of probability needs to be addressed. For example, while optimizing the investment portfolio, the stock price is not known in advance.

Stochastic programing tasks comprise random variables that have the following characteristics: mathematical hope μ; mean standard deviation σ; coefficient of variation v; normal distribution.

The mathematical hope is defined as the objective function value, which is obtained under the assumption that stock prices are stable. In other words, the mathematical hope is the result of investment (return/risk ratio), which is obtained by solving a task that is deterministic in its nature. The probability of the occurrence of the mathematical hope is equal to 0.5, so very often it is seen as the arithmetic average of the result of the objective function (in this case, the relationship between the return and the risk). Apart from the mathematical hope, it is necessary to know the mean standard deviation of the objective function. This value is obtained by multiplying the mathematical hope by the objective function coefficient of variation (11):

$$\sigma[TF] = \mu \times v[TF] \tag{10}$$

The coefficient of variation of the objective function can be found by calculating the variations of stock returns of all enterprises included in the portfolio and deriving the arithmetic average of these variations. In this way, it is assumed that the objective function is equal to the mean of variation of stock returns of all enterprises involved in the portfolio (11). Undoubtedly, there are other possible ways to assess the variation of the objective function.

$$v[TF] = \frac{\sum v(R_i)}{n} \tag{11}$$

Given the mathematical hope and standard deviation of the objective function, the maximum and minimum values of the objective function with a 99% probability can be calculated:

$$TF_{min} = \mu - 3 \times \sigma[TF] \tag{12}$$

$$TF_{max} = \mu + 3 \times \sigma[TF] \tag{13}$$

Also, by using a normal probability distribution function, it is possible to determine the probability of occurrence of the value of the respective objective function.

Further, having constructed the optimal portfolios, the next step is to carry out their analysis and compare them with the results of the market index.

3. The Results of the Study

The stock selection is carried out from Lithuanian enterprises listed on the NASDAQ OMX Baltic Official list of companies. In 2012, this list quoted shares of 18 stock companies. Two of them (AB Ūkio bankas and AB Sanitas) were not included in the sample, because the financial statements of these enterprises were not made public. The investment portfolio was intended to be constructed of four enterprises. As already mentioned above, each of them has to represent different business sectors. The financial indicators for the year 2012 are presented in the Annex to this article. Having performed their assessment in accordance with functions (7) and (8), and calculated the priority index, the following results were obtained (see Table No. 2).

TABLE No. 2. **Example of a table caption (Use AR Table Caption)**

Company	Priority index
Linas Agro Group	3.86
Rokiškio sūris	3.63
Vilkyškių pieninė	3.61
Vilniaus baldai	3.54
Panevėžio statybos trestas	3.21
City service	3.16
Apranga	3.15
…	…

It is obvious that only one enterprise exceeds (Sinha, Chandwani et al. 2013) the recommended priority index threshold of 3.8. Thus, in order to diversify the investment, the portfolio has to include enterprises whose priority index by a 5-point system is less than 3.8. In this case, it should be Rokiškio sūris, Vilkyškių pieninė and Vilniaus baldai. However, Rokiškio sūris and Vilkyškių pieninė belong to the same business sector. As a result, Vilkyškių pieninė, having a lower priority index than that of Rokiškio sūris, is replaced by the construction company Panevėžio statybos trestas.

Thus, the investment portfolio consists of four enterprises: Linas Agro Group (agriculture), Rokiškio sūris (food industry), Vilniaus baldai (furniture) and Panevėžio statybos trestas (construction). Next, this portfolio is optimized by using a genetic algorithm approach. The optimization is performed using the MatLAB (Global Optimization Tools) software.

Before performing optimization, it should be noted that the MatLAB (Global Optimization Tools) software package is designed to address the problems of minimizing the objective function; thus, the numerator and denominator of the objective function (3) should be reversed. In this case, the objective function will be as follows:

$$f(chromosome) = \frac{\sigma_p}{R_p} \rightarrow min \tag{14}$$

In the area of restrictions, it will be assumed that the portfolio allows short positions, i.e., the investor can borrow shares and sell them right away. The only restriction is associated with the amount of stock portfolio weight, which must be equal to one.

The following assumption is that the population consists of 10 chromosomes, i.e., 10 investment portfolios. While generating the population, the double vector population type is used. The optimization process involves the roulette selection, while the arithmetic method is applied for crossover procedures. The optimization process is arrested at the end of the 100th generation.

Also, it is assumed that the investor forms an investment portfolio in the amount of 100 000 EUR, and carries out transactions at stock market prices of 31 December 2012. Then in 2013, he carries out the *buy and hold* strategy and observes the changes in the results that take place in the portfolio in 2013. An optimal portfolio is constructed on 31 December 2012, while on 31 December 2013 changes in the value of the portfolio over the past year are analyzed. The results are presented in Table No. 3.

The results show that the annual return of the portfolio, constructed by a genetic algorithm over the period of 2013, is 34.64%. This is significantly more than the rate of market return, which, in 2013, amounted to 9.27%.

TABLE No. 3. **The optimal portfolio using genetic algorithm approach**

Company	Position	Purchased shares, units	Value, 12.31.2012, EUR	Weight	Value, 12.31.2013, EUR
Linas Agro Group	Long	115 789	66 000	66.0	79 432
Rokiškio sūris	Long	110 000	154 000	154.0	174 900
Vilniaus baldai	Short	8 028	(114 000)	(114.0)	(112 394)
Panevėžio statybos trestas	Short	6 459	(6 000)	(6.0)	(7 298)
			100 000	100.0	134 639

Further, the portfolio is optimized by a deterministic programing method. As in the case of genetic algorithms, the same assumptions are used. MS Excel Solver software is applied for the process of optimization. The structure of the portfolio, optimized by means of a deterministic programing approach, is presented in Table No. 4.

TABLE No. 4. **The optimal portfolio using a deterministic programing approach**

Company	Position	Purchased shares, units	Value, 12.31.2012, EUR	Weight	Value, 12.31.2013, EUR
Linas Agro Group	Long	159 474	90 900	90.9	109 399
Rokiškio sūris	Long	843	1 180	1.2	1 340
Vilniaus baldai	Long	3 372	47 880	47.9	47 206
Panevėžio statybos trestas	Short	(43 014)	(39 960)	(40.0)	(48 606)
			100 000	100.0	109 339

Apparently, in this case, the return on investment (+9.34%) is much lower in comparison with the portfolio optimized by a genetic algorithm method. The return of the portfolio optimized by a deterministic programing method is almost identical to the market return. It may be noted that, although the portfolio generated a lower return, at the same time, it had a lower systemic risk in comparison with the genetic algorithm portfolio. The beta coefficient of the portfolio built by a deterministic programing method, was only 0.18 in 2013, while the beta coefficient of a genetic algorithm portfolio was 0.73. A slightly smaller gap was observed in assessing the Treynor ratio. It was 0.50 and 0.469, respectively.

The evaluation of both the investment returns and the systemic risk at the same time showed that the portfolio optimized by a genetic algorithm proved to be superior. The Jensen's alpha ratio of the latter was 0.28, while this ratio of the determined portfolio was only 0.07.

Finally, the optimization of the portfolio is being carried out by applying a stochastic programing approach. In this case, first of all, the mathematical hope needs to be known, which is equal to the objective function value obtained by a determined programing approach. The latter value, i.e., the risk and return ratio, is minimized. It is observed that the minimum ratio of risk and return of the investment portfolio optimized by determined programing is 7.4034. The variation of the objective function (the average of the variations of all companies in a portfolio) is 0.000241717. Knowing both of these values, the standard deviation of the objective function can be calculated, i.e., 7.4034 x 0.000241717 = 0.001788.

Knowing the average and the standard deviation of the objective function, we can calculate the minimum objective function value (using function 12), which is equal to 7.3980. Having optimized the investment portfolio so that the objective function value is equal to 7.3980, its structure does not change and remains the same, as in the case of the deterministic programing. This may occur due to a relatively small standard deviation of returns of all enterprises involved in the portfolio. This is confirmed by the low beta coefficients of the enterprises included in the portfolio (in all cases, they are smaller than

1.0). This means that the systemic risk or uncertainty has a small effect on the stock price of the enterprises included in the portfolio. As a result, the stochastic programming results are almost identical to the results obtained in the case of deterministic programming.

Conclusions

An investigation showed that the investment portfolio optimized by applying a genetic algorithm approach generates higher returns than the portfolio constructed by means of deterministic or stochastic programming methods. The difference between the returns is quite significant. The return of the portfolio optimized by a genetic algorithm earned an annual return of 34.64% the over the year 2013, while the portfolio formed by means of deterministic and stochastic programming methods earned an annual return of 9.34% (the earnings were quite close to the rate of the market return, i.e., 9.27%).

In assessing a systemic risk, in all cases, it was less than 1.00, but portfolios optimized by deterministic as well as stochastic programming were less risky in comparison with the genetic algorithm portfolio. This probably explains the reason why the portfolio optimized by a genetic algorithm was more profitable. Taking into account these two aspects, i.e., both the returns and risks, the portfolio optimized by means of a genetic algorithm approach was more efficient as its Jensen's alpha ratio was 0.28, while the alpha of the portfolios constructed by applying deterministic and stochastic optimization methods was only 0.07. Thus, in conclusion, it can be stated that the hypothesis, with regard to the advantage of the portfolio formed by a genetic algorithm over the portfolio constructed by means of deterministic and stochastic programming methods, cannot be rejected. Also, the second hypothesis, which states that the performance of the portfolio that is optimized by a genetic algorithm may exceed the market performance, cannot be rejected.

However, it should be noted that the study tested the results of only one year, i.e., the year 2013. In seeking for more reliable conclusions, it is necessary to examine a larger number of periods, with a special emphasis on the application of a genetic algorithm over a period of market downturn.

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