Online ISSN 2424-6166. EKONOMIKA 2016 Vol. 95(2)
DOI: http://dx.doi.org/10.15388/Ekon.2016.2.10122

A NEW MODEL WITH REGIME SWITCHING ERRORS: FORECASTING GDP IN TIMES OF GREAT RECESSION

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Abstract. *This paper investigates the possibility to obtain better GDP forecasts in the early stages of Great Recession. Here, predictive performance refers to exclusively out-of-sample forecasts. Based on exploratory data analysis and general-to-specific modelling, this paper proposes a univariate predictive threshold model for the small open economy that outperforms its linear counterparts and correctly determines the course of events. This model does not explain any causal links; however, based on a set of economic arguments, it sets forward an idea regarding how a forecaster can act when principal determinant factors, responsible for a sudden, yet lasting change, are unknown, unmeasurable or cannot be influenced by national policy makers. A major dissimilarity between usual threshold models and the model presented in this paper is that while variables act differently under different conditions in the former, in this model, due to economic reasons, errors act differently. Alternatively, this paper can be viewed as a comparative GDP prediction study.*

Keywords: *univariate time series models; forecasting and prediction methods; autocorrelated errors; threshold models; GDP forecasting.*

1. Introduction

In 2010, eight prominent scientists (Colander et al., 2009) published an article that criticised the way research is conducted in economics. In response to what was at that time the failure of economics, these authors put forward well-thought propositions about how to learn from the ongoing financial crisis, how to improve research, and how our efforts should be allocated. At first sight, their article may seem like an overreaction that will pass with time. However, from a contemporary perspective, there are more arguments for than against a change, not to mention the assessment of one of the best-known economists in academia, David Romer, who also emphasised that "our models and analysis will surely change" (Romer, 2012). Consequently, this paper started with a search for an alternative empirical approach of how to incorporate the effects of the recent financial crisis into econometric models.

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The current efforts of academic economists, addressing the issue of Great Recession, are focused on the study of linkages between the financial sector and the rest of economy (Olsson, 2012; Romer, 2011). Origins of this analysis can be traced up to the influential paper, titled "The Allocation of Credit and Financial Collapse", by N. Gregory Mankiw, where he examined the case of imperfect information when a lender knows less than a borrower.

This paper offers a perspective different from that offered by macroeconomic theory; there is good reason to do so. While the direct linkage between the Great Recession and the financial sector may be appropriate for major world economies, it is not the same case for all countries. In some small open economies, e.g., Lithuania or Estonia, all financial institutions were intact, but employment and income began to decline. The main cause was that the negative impacts from the crashed economies were directly transmitted to the rest through various channels, e.g. credit tightening. Forecasters encountered a complicated problem of predicting variables of interest in the wake of the downfall. At that time, the exact transmission mechanism from crash economy was unclear; therefore, measuring the effects of the underlying factors was impossible. Exactly under these circumstances the univariate models can be of great use.

In a certain way, this article joins the ongoing discussion in academic literature on the capability of nonlinear models to surpass linear, and even suggests the possible origins of nonlinearity. In their recent article on the ability of nonlinear model to outperform linear models, Ferrara, Marcellino, and Mogliani found that "on average, non-linear models cannot outperform the linear benchmark model, even during the Great Recession period" (Ferrara et al., 2015). Their findings are not too drastic, nor do they completely deny the usefulness of nonlinear models, as the authors admit that "non-linear models do lead to an improvement in predictive accuracy" (Ferrara et al., 2015). In contradiction to the statements in this article, the authors repeat a famous insight by Stock and Watson that the current recession is a sequence of unusually large shocks and should be treated alike, rather than as a change in the stochastic macro process. The differences in the perception of what constitutes better performance lie in the subjective awareness of what is "better performance". In this paper, I present evidence to maintain the viewpoint that drastic statements from either side should be avoided since revision of the stochastic macro processes may lead to fundamental improvements. No contradictions between the opposite statements arise from the fact that the nonlinear pattern here is slightly different from the one proposed by the usual models, which were used in the study by Ferrara et al.

The structure of the paper is very concise. The first section presents the motivation for the regime switching model, the second section is devoted to the Monte Carlo study of the bias, while the third section presents several empirical applications.

The empirical analysis is based on log transformed GDP series. Irish seasonally adjusted GDP at constant factor costs was taken from the Central Statistics Office and spans the period from 1997Q1 till 2014Q4. Estonian, Latvian and Lithuanian data was taken from Eurostat. Estonian GDP at market prices from 1993Q1 till 2014Q2 was divided by the price index, logged and de-seasonalised using a seasonal-trend decomposition procedure, based on Loess and developed by Cleveland et alia (Cleveland et al., 1990). After some experimentation, the Loess window for seasonal extraction was set to be equal to 11, as this value guarantees maximum randomness of the irregular component. Latvian and Lithuanian GDPs at market prices from 1995Q1 till 2014Q1 were already seasonally adjusted and adjusted by working days. Both series were divided by the identically adjusted price indexes and logged. The main reason behind the selection of these countries is that income in these economies was severely affected by the financial crisis of 2008.

2. Model

In the early phases of the recent financial crisis, "economists have had no choice but to abandon their standard models and to produce hand-waving common-sense remedies" (Colander et al., 2009). As it will be shown, the main reason for the failure is that the stochastic structure of most popular models is appropriate for relativity at quiet times, but cannot cope with forces that operate during periods of excess instability.

2.1. Random Walk Approximation

The stochastic structure of time series may be expressed as the sum of short and long run innovations (e_t and ε_t respectively). Therefore, in a very general univariate setting, we have the following random walk representation:

$$y_t = t\delta + \sum_{i=1}^{t} \varepsilon_i + \eta(L)e_t \tag{1}$$

The division of innovations into long and short runs is very useful as we know that certain shocks, such as monetary or fiscal shocks, have temporary effects that diminish with time, while other factors, like technology, have permanent effects on the level.

The reason for retaining the popular random walk assumption is simply that we have no particular reason not to do so, especially in the context of the univariate model constraints. Random walks have been, and will be, used extensively in macroeconomics, and there are several fundamental studies where it is explicitly assumed that a certain variable (usually money aggregates or output) follows a random walk. Ball, Mankiw, and Romer assume that aggregate demand is driven by random walk movements in the money stock (Ball et al., 1988); Romer assumes that the logarithm of nominal GDP follows a random walk with a drift (Romer, 2011); Ball and Cecchetti assume "for realism" that money stock follows a random walk (Ball and Cecchetti, 1988); according to Ljungqvist and Sargent, Barro assumes that tax revenues follow a random walk

(Ljungqvist and Sargent, 2004; Barro, 1979), and so on. Also, one should not forget that many economic series, at a superficial glance, can be approximated as these processes, and in many cases random walks are the starting point for models of non-stationary variables. The models that assume a similar structure for a certain variable were useful and are likely to be useful in the future. However, for certain periods in time such as sharp downturns, these assumptions are insufficient.

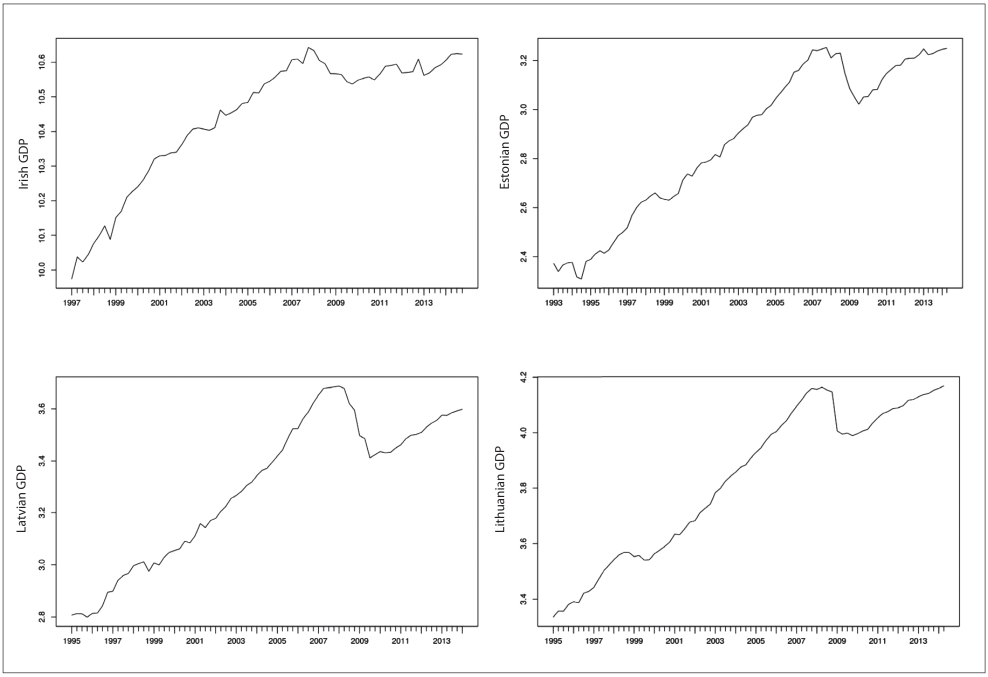

FIG. 1. **Log transformed GDP for Ireland (upper-left), Estonia (upper-right), Latvia (lower-left), and Lithuania (lower-right)**

Source: Eurostat, Central Statistics Office Ireland

As shown in the panels of figure 1, it is evident that after the crisis, growth rates for Irish, Estonian, Latvian, and Lithuanian GDPs have diminished. There is a more visible long-lasting slump in Ireland, although Ireland has not suffered a decline as sharp as that in Latvia or Lithuania in the early stages of the Great Recession. Prior to the financial crisis of 2008, all series could have been represented by the aforementioned sums of permanent and transitory components. However, as the graphs suggest, such a representation is inadequate during the initial and peak phases of the crisis.

2.2. Roles of Different Components in a Random Walk Model

One of the reasons for the failure of econometric models or the inability to keep with what was expected of them during the crisis was the misspecification of stochastic structure. Potentially, each of the elements in equation (1) can be the source of instability.

The deterministic element δ represents the long run growth rate of the variable. If the long run rate would be affected, the proper way to model this change would be to add a dummy variable that encompasses this change. All countries were severely affected by the crisis, but Lithuanian and Estonian GDPs regained (or nearly regained) their growth in a few years, while Ireland's GDP still seems to be off its long run path. Although the post-crisis growth of Ireland's GDP seems to differ from its pre-crisis growth, it does not take long to note that in the pre-crisis period, we can pick out and isolate clearly segregated growth clusters. Most probably though, what we observe in the post-crisis period is nothing else, but, again, one of these clusters. While Ireland's GDP growth is off its path for a longer time than the Lithuanian GDP growth, we cannot conclude that the long run growth rate of Ireland's GDP has changed. A relatively constant long run growth rate, onto which a variable tends to revert, is a theoretical concept and the empirical facts, in those of which we have been observing diminished growth rates for a few years, should not call into reconsideration the entire concept. In turn, the change in the deterministic component δ cannot be the source of predictive failure.

Sometimes, a wrong perception of how short run innovations act may render forecasts to be low in quality. The effects of short run innovations arise from $\eta(L)e_t$, and in the simplest univariate models they are captured via autoregressive $\alpha_i \Delta y_{t-i}$ or moving average $\beta_i \varepsilon_{t-i}$ terms. The possibility that short run innovations have changed the behaviour of GDP is not very compelling, as it suggests that, for a certain period, we have to account for a change in α or β depending on the univariate model that fits the data. From a technical point of view, this means that for certain periods, the series would merely change the form of sluggishness.

The initial guess was that the only reasonable source for the adverse effects were the ε's. By reconstructing the events, one can easily recall that the crisis hit unexpectedly, evolved with many turns that were unforeseen and unknown on their own. Growth rates changed visibly and for lasting periods, and policy results were not as expected.[1] All this perfectly coincides with the role of ε's in econometric models and their cumulative nature. From a purely technical perspective, the crisis is nothing but the negative shock in period t, that caused the subsequent array of substantial negative shocks in the forthcoming $t + i$ periods.

[1] At that time, it was very difficult to handle the consequences of the crisis and policy results, such as the fiscal stimulus of 2008 or "Cash for Clunkers" in the US (Taylor, 2010), that were not as expected. To avoid exaggeration based on the US experience, it should be mentioned that some authors conclude that certain actions reached their goals – the German Car Scrappage program helped to stabilize economy (Böckers et al., 2012).

2.3. Random Walk Approximation Revisited

A common approach in modelling time series is to assume that the errors are white noise or follow a certain stationary pattern, but if the shocks in period t trigger similar shocks in period $t + 1$, they should not be treated as uncorrelated. The following mini model will help to formalise the concept. Suppose that for the first three periods $\varepsilon \sim WN$, for the next three periods $\varepsilon_t \sim AR(1)$, and for the last period $\varepsilon_t \sim WN$ again. Therefore, there are three periods for the "usual" evolution, three periods for the recession interrelations, and then the system reverts to the usual evolution for the final period:

$$
\begin{aligned}
y_1 &= \varepsilon_1 + e_1 \\
y_2 &= \sum\nolimits_{i=1}^{2} \varepsilon_i + e_2 \\
y_3 &= \sum\nolimits_{i=1}^{3} \varepsilon_i + e_3 \\
y_4 &= \sum\nolimits_{i=1}^{3} \varepsilon_i + \rho\varepsilon_3 + v_4 + e_4 \\
&= \sum\nolimits_{i=1}^{2} \varepsilon_i + (1+\rho)\varepsilon_3 + v_4 + e_4 \\
y_5 &= \sum\nolimits_{i=1}^{4} \varepsilon_i + \rho\varepsilon_4 + v_5 + e_5 \\
&= \sum\nolimits_{i=1}^{2} \varepsilon_i + (1+\rho+\rho^2)\varepsilon_3 + (1+\rho)v_4 + v_5 + e_5 \\
y_6 &= \sum\nolimits_{i=1}^{5} \varepsilon_i + \rho\varepsilon_5 + v_6 + e_6 \\
&= \sum\nolimits_{i=1}^{2} \varepsilon_i + (1+\rho+\rho^2+\rho^3)\varepsilon_3 + (1+\rho+\rho^2)v_4 + (1+\rho)v_5 + v_6 + e_6 \\
y_7 &= \sum\nolimits_{i=1}^{2} \varepsilon_i + (1+\rho+\rho^2+\rho^3)\varepsilon_3 + (1+\rho+\rho^2)v_4 + (1+\rho)v_5 + v_6 + \varepsilon_7 + e_7
\end{aligned}
$$

The assumption, that $\varepsilon_t \sim AR(1)$ is a simplification, which implies a constant relationship among the errors with a self-evident restriction that $|\rho| < 1$. This assumption may be good enough if the system is more or less resistant to certain types of negative shocks, as it was with the impact of Russian financial crisis on the Lithuanian economy in the final years of the 20th century. At that time, economies suffered from unfavourable shocks that slowed down growth; however, as compared to the effects of the Great Recession, these shocks had minor effects on the overall state of things in Lithuania. The recent financial crisis evolved differently and the unfavourable effects were much more significant and apparent; therefore, the assumption that $|\rho| < 1$ may be too restrictive. However, letting ρ exceed unity for a longer time span would suggest the total collapse of the economy, though this possibility, with ρ being constant and larger than one, should be omitted from the analysis.

After taking first differences and letting short run Δe_t effects be captured by autoregressive terms $\alpha \Delta y_{t-1}$, we get a simple autoregression where errors for three periods suddenly become autocorrelated:

$$
\begin{aligned}
\Delta y_2 &= \alpha \Delta y_1 + \varepsilon_2 \\
\Delta y_3 &= \alpha \Delta y_2 + \varepsilon_3 \\
\Delta y_4 &= \alpha \Delta y_3 + \varepsilon_4 \\
&= \alpha \Delta y_3 + \rho \varepsilon_3 + v_4 \\
\Delta y_5 &= \alpha \Delta y_4 + \varepsilon_5 \\
&= \alpha \Delta y_4 + \rho \varepsilon_4 + v_5 \\
&= \alpha \Delta y_4 + \rho^2 \varepsilon_3 + \rho v_4 + v_5 \\
\Delta y_6 &= \alpha \Delta y_5 + \varepsilon_6 = \alpha \Delta y_5 + \rho \varepsilon_5 + v_6 \\
&= \alpha \Delta y_5 + \rho^3 \varepsilon_3 + \rho^2 v_4 + \rho v_5 + v_6 \\
\Delta y_7 &= \alpha \Delta y_6 + \varepsilon_7
\end{aligned}
\tag{2}
$$

The generalisation of the processes described above is straightforward. If we want to augment the random walk model (1) with a single switch in regime, we can add additional cumulative components that represent the behaviour of a variable after the change. Suppose we observe white noise errors till the period τ; thereafter, errors change their form and follow AR(1) process:

$$
\begin{aligned}
\Delta y_2 &= \alpha \Delta y_1 + \varepsilon_2 \\
\Delta y_3 &= \alpha \Delta y_2 + \varepsilon_3 \\
\Delta y_4 &= \alpha \Delta y_3 + \varepsilon_4 \\
&= \alpha \Delta y_3 + \rho \varepsilon_3 + v_4 \\
\Delta y_5 &= \alpha \Delta y_4 + \varepsilon_5 \\
&= \alpha \Delta y_4 + \rho \varepsilon_4 + v_5 \\
&= \alpha \Delta y_4 + \rho^2 \varepsilon_3 + \rho v_4 + v_5 \\
\Delta y_6 &= \alpha \Delta y_5 + \varepsilon_6 = \alpha \Delta y_5 + \rho \varepsilon_5 + v_6 \\
&= \alpha \Delta y_5 + \rho^3 \varepsilon_3 + \rho^2 v_4 + \rho v_5 + v_6 \\
\Delta y_7 &= \alpha \Delta y_6 + \varepsilon_7
\end{aligned}
\tag{3}
$$

If we want to revert the process to the usual autoregressive behaviour, the cumulative sum of the white noise errors should be added to obtain an equation that mimics the two switches. We begin with errors that follow a pure white noise pattern. However, things suddenly change in period τ, when the errors start to act as a AR(1) process or become interrelated in a slightly different fashion. This switch lasts for d periods; after that, the errors follow a white noise path again:

$$
y_t = t\delta + \sum_{i=1}^{\tau-1} \varepsilon_i + \varepsilon_\tau \sum_{i=0}^{d} \rho^i + \sum_{i=0}^{d-1} \sum_{j=d+i+1}^{\tau+d} \rho^i v_{j-i} + \sum_{i=\tau+d+1}^{t} \varepsilon_i + e_t \tag{4}
$$

Equation (3) is appropriate for periods when $\tau < t \leq \tau + d$, while equation (4) characterises the periods when $t > \tau + d$. Several simulated series obtained from the equation (4) are visualised in figure 2. Although the panels depict simulated series, they are strikingly similar to GDP series for Lithuania, Latvia, Estonia, and Ireland. In all plots, δ was set to be equal to 0.5, and innovations for the first 50 and for the last 40 periods were white noise. In the top panels, for the period 51:60, the errors were autocorrelated with $\rho = 0.8$, and in the bottom panels for the same period innovations had a unit root with $\rho = 1$.

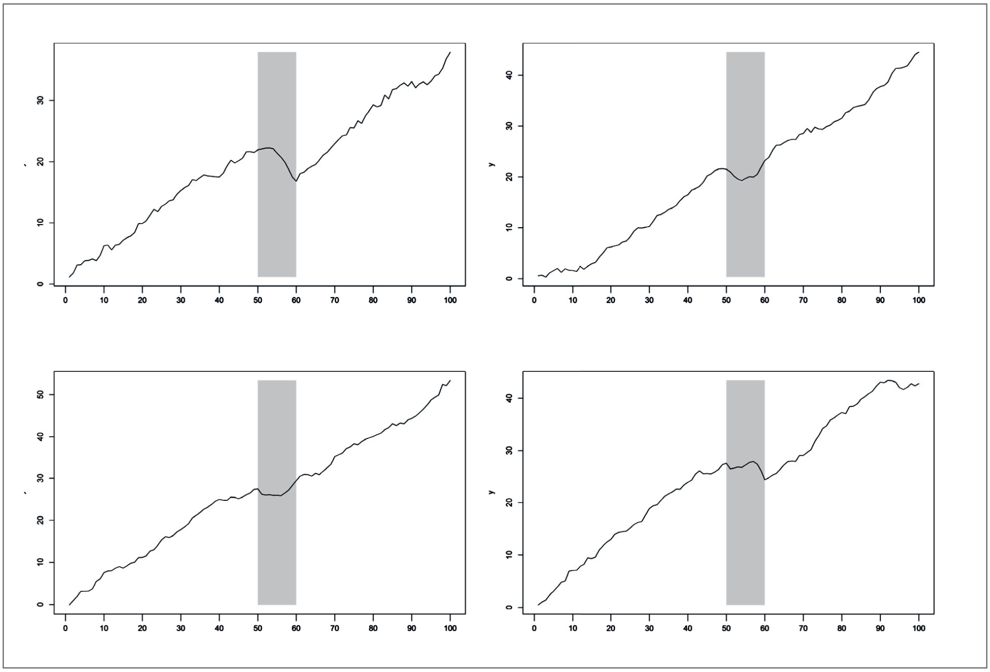

FIG. 2. **Simulated nonstationary series with regime switch**

Source: author's calculations

These graphs resemble many economic variables that were adversely affected by a certain factor. Here "crisis" effects are captured via sudden changes in the data generating process for the errors. Instead of constantly being white noise, errors suddenly became interrelated for 10 periods. This is not the only way of obtaining artificial series that are very similar to those in graphs, however slumps here stand for regularity and not for randomness.

2.4. The Final Version

The possibility of the estimation depends on the possibility of simulating similar processes. If it is possible to simulate them, it should also be possible to estimate them. The autoregressive representation in differences for the process in equation (4) is:

$$\Delta y_t = \delta + \alpha \Delta y_{t-1} + I_t(\rho \varepsilon_{t-1} + v_t) + (1 - I_t)\varepsilon_t \tag{5}$$

Here I_t is a dummy variable that is equal to 1 if $\tau < t \leq \tau + d$; otherwise, it is equal to 0. For convenience, we may consider the autocorrelation of errors as the "crisis" regime and white noise behaviour as "pre-crisis" and "post-crisis" regimes. These two error regimes yield two different autoregressive processes, respectively:

$$\begin{aligned} \Delta y_t &= \delta + \alpha \Delta y_{t-1} + \varepsilon_t \\ \Delta y_t &= \delta + \alpha \Delta y_{t-1} + \rho \varepsilon_{t-1} + v_t \end{aligned} \tag{6}$$

Combining both equations in , we get:

$$\begin{aligned} \Delta y_t &= \delta + \alpha \Delta y_{t-1} + \rho(\Delta y_{t-1} - \delta - \alpha \Delta y_{t-2}) + v_t \\ \Delta y_t &= (1-\rho)\delta + (\alpha + \rho)\Delta y_{t-1} - \rho\alpha \Delta y_{t-2} + v_t \\ \Delta y_t &= \delta^* + \alpha_1^* \Delta y_{t-1} + \alpha_2^* \Delta y_{t-2} + v_t \end{aligned} \tag{7}$$

From first sight, one can get a false impression that ordinary least squares may be used for the estimation purposes. Two issues prevent this possibility. In equation (7) $\delta^* = (1-\rho)\delta$, $\alpha_1^* = \alpha + \rho$, and $\alpha_2^* = -\rho\alpha$. Restriction $\alpha_1^* = \alpha + \rho$ can be rewritten as $\rho = \alpha_1^* - \alpha$. Combining with $\alpha_2^* = -\rho\alpha$, we get $\alpha_2^* = -(\alpha_1^* - \alpha)\alpha$ or $\alpha^2 - \alpha_1^*\alpha - \alpha_2^* = 0$. Alternatively, restriction $\alpha_2^* = -\rho\alpha$ can be rewritten as $\alpha = -\alpha_2^* / \rho$. Combining with $\alpha_1^* = \alpha + \rho$, we get $\alpha_1^* = -\alpha_2^* / \rho + \rho$ or $\rho^2 - \alpha_1^*\rho - \alpha_2^* = 0$. It follows that solutions for ρ's and α's will be identical. It does not necessarily have to be like that in real life applications.

The formula for solving this quadratic equation is $(\alpha_1^* \pm (\alpha_1^{*2} + 4\alpha_2^*)^{0.5}) / 2$. One of the roots gives us the estimate of α, while the other root is the estimate of ρ, but if Δy_t is stationary and the condition for stability $\alpha_1^* + \alpha_2^* < 1$ is fulfilled, the solutions to the quadratic equation can be complex conjugates of each other. Conversely, if $\alpha_1^* + \alpha_2^* > 1$, the two roots are real and, under usual circumstances, one root is less than 1, while the other is greater than 1. This makes sense only if errors switch from white noise into explosive behaviour, i.e., when $\rho > 1$, and it is not very likely that things would be going up that way.

The simple solution is to combine the first AR(1) model in for pre- and post-crisis periods with equation :

$$\Delta y_t = I_t(\delta + \alpha \Delta y_{t-1} + \varepsilon_t) + (1 - I_t)(\delta^* + \alpha_1^* \Delta y_{t-1} + \alpha_2^* \Delta y_{t-2} + v_t) \tag{8}$$

In this simplified version, we have analysed a stationary process that becomes autocorrelated during certain periods of time. Due to two factors, the first being the identification problems that may occur if the solutions of the above discussed quadratic equation result in complex roots, and the second being the necessary assigned explosive features of the errors, in the case of real roots, least squares are not the proper tool for the estimation of this model. Despite the similarity to the usual TAR model, model (8) is not a simple TAR. Firstly, the error sequences ε_t and v_t have distinct features: ε_t can be autocorrelated, while v_t cannot, though the model (8) cannot be written with a single "compromised" or "averaged" error term. Secondly, some coefficients in equation (7) are restricted in such a way that $\alpha_1^* = \alpha + \rho$ and $\alpha_2^* = -\rho\alpha$, which is inadequate from an economic point of view. For these reasons, it is more convenient to apply maximum likelihood routines on equation (5) than any version of least squares on equation (8) directly. Joint log likelihood function for the estimation of the model in (5) is:

$$l = \sum_{i=3}^{\tau} \ln f(\varepsilon_i) + \sum_{i=\tau+1}^{\tau+d} \ln f(v_i) + \sum_{i=\tau+d+1}^{T} \ln f(\varepsilon_i) \tag{9}$$

As mentioned before, this model is not a conventional TAR model and, despite its superficial similarity with conventional TAR models, optimisation techniques will be required for the proper estimation of the coefficients.

3. Monte Carlo

In order to determine whether the estimate of ρ is biased under maximum likelihood or not, a large-scale Monte Carlo simulation was performed. The bias function was obtained by computer simulation, using 10000 replications for different samples and different proportions of observations in an autocorrelated regime. More precisely, the samples of size 40, 80, 120, 160, 200, 240, and 280 were used, while selected values for ρ ranged from 0.1 to 1, with a 0.1 step. Selected proportions of observations in the downfall regime are 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, and 0.50. The simulated process had this form:

$$y_t = t\delta + \sum_{i=1}^{\tau-1} \varepsilon_i + \varepsilon_\tau \sum_{i=0}^{d} \rho^i + \sum_{i=0}^{d-1} \sum_{j=d+i+1}^{\tau+d} \rho^i v_{j-i} + \sum_{i=\tau+d+1}^{t} \varepsilon_i \tag{10}$$

Short run innovations were deliberately omitted for simplification, in order to bypass the estimation of the autoregressive coefficients, as in equation (10), and to save computation time.

Figure 3 summarizes the simulation results. The larger the sample, and the closer the proportion to 0.5 is, the smaller is the bias of ρ. From here, it follows that in small samples and in samples with a small proportion of observations in the "crisis regime", the estimates of ρ are substantially downward biased. Therefore, in the samples that macroeconomists work with, the bias will definitely exist. In empirical applications, the number of observations in the “crisis regime” is unlikely to be large, and the correction of bias is a must, in order to increase the reliability of forecasts.

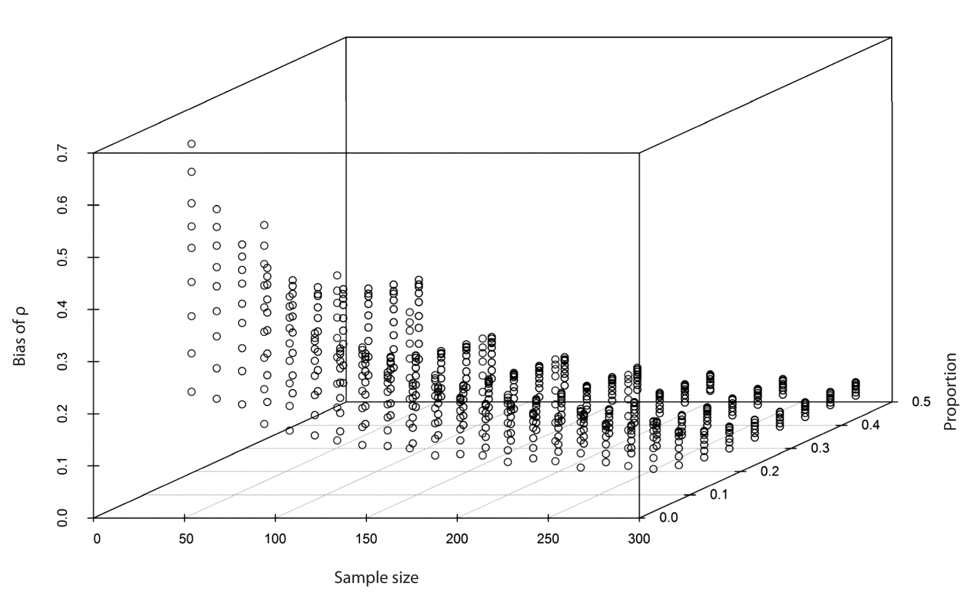

FIG. 3. **Response surface for** *ρ*

Source: author’s calculations

Initially, the response surface was calculated according to this formula:

$$B_j = \alpha + \sum_{i=1}^{4} \beta_i T_j^i + \sum_{i=1}^{4} \gamma_i P_j^i + \delta T_j P_j + \upsilon_j \tag{11}$$

Here B stands for the bias of ρ, T stands for the sample size, and P stands for the proportion of observations in the “crisis regime”. As the initial estimation of equation (11) by OLS yielded heteroscedastic errors, this equation was re-estimated by GLS with the following option for the variance:

$$\ln \sigma_j^2 = \ln \sigma^2 + \sum_{i=1}^{2} a_i T_j^i + \sum_{i=1}^{2} b_i P_j^i + c T_j P_j + u_j \tag{12}$$

Specification of powers and cross-products in equations (11) and (12) was obtained after a standard experimentation. Estimates of the coefficients, their standard errors, and t values are in table 1.

TABLE 1. **The results of GLS estimation of equation**

Coefficient	Estimate	Std. Error	t value
α	0.7303	0.0345	21.1933
β_1	-5.7768×10^{-3}	7.8891×10^{-4}	-7.3226
β_2	3.7310×10^{-5}	8.0575×10^{-6}	4.6304
β_3	-1.1948×10^{-7}	3.3443×10^{-8}	-3.5728
β_4	1.4411×10^{-10}	4.8481×10^{-11}	2.9725
γ_1	-2.9232	0.3657	-7.9940
γ_2	10.7082	2.1283	5.0314
γ_3	-19.4455	5.0303	-3.8656
γ_4	13.2302	4.1479	3.1896
δ	8.5586×10^{-7}	1.2528×10^{-4}	6.8315

Source: author's calculations

The estimates of the coefficients in table 1 allow us to calculate the bias of ρ for given sample T and the proportion P in order to adjust the estimate of ρ.

To gain an impression of the consequences of ignoring the bias, consider a sample of 80 observations with 4 observations in the autocorrelated regime. Here the bias of ρ is about 0.3333. If one has 8 observations in the autocorrelated regime, the bias is smaller and is about 0.2551. With 20 observations in the autocorrelated regime, the bias is about 0.1551. With 40 observations in the autocorrelated regime and 40 observations in white noise regime, the bias is even smaller – 0.0975. In larger sample with 280 observations, 140 of which are in the autocorrelated regime and the white noise regime each, the bias is even smaller – at 0.0322. In the same sample with 280 observations, 14 of which are in the autocorrelated regime, bias is substantially larger – 0.1910. Summarizing, the larger the sample, and the closer the ratio of observations in different regimes to 0.5 is, the lower is the bias.

There are two ways to decrease the bias: increasing the sample size and (or) balancing the regimes, and it is difficult or nearly impossible to implement any of them in real life. Macroeconomic samples are usually not large and it is hard to imagine that autocorrelated errors will account for a half of the sample. The only possible way to solve the bias problem is to calculate it using the coefficients from table 1 and to alter the estimate of ρ manually. Overall, disregarding the bias is not advisable, as this may lead to severely biased forecasts.

A combination of estimation methods will be employed to "unbias" the estimates. In the first step, usual maximum likelihood routines will be applied, using the BFGS method for the initial search. After the initial estimates are obtained (denoted as $\tilde{\delta}$, $\tilde{\alpha}$, and $\tilde{\rho}$), the estimate of ρ is manually corrected. The bias for $\tilde{\rho}$, denoted as $B(\tilde{\rho})$, is calculated using the coefficients from table 1, and ρ is altered accordingly by adding the bias $B(\tilde{\rho})$ to the initial estimate $\tilde{\rho}$. This procedure yields the corrected estimate of ρ:

$\tilde{\rho}^* = \tilde{\rho} + B(\tilde{\rho})$. In the final step, maximum likelihood routines are repeated again, using $\tilde{\delta}$, $\tilde{\alpha}$, and $\tilde{\rho}^*$ as starting values for the search of δ, α and ρ. In the last step, the conjugate gradient method is preferred over BFGS as the "conjugate gradient has a tendency to converge if the starting point is very close to the desired minimum" (Shewchuk, 1994); after the bias correction, the initial estimates in the final step are close or much closer to the searched ones.

4. Estimation and Discussion

This section is devoted to the empirical illustrations of the techniques described in the previous sections. Examples will be based on GDP series for Ireland, Estonia, Latvia, and Lithuania. For GDP analysis, the univariate models are definitively misspecified in the sense that GDP cannot be adequately modelled without taking into account factors such as inflation, interest rates, money supply and etc. Here, however, misspecification is of secondary importance. Firstly, this model mimics how a forecaster could have acted in the wake of the financial crisis with information available then. Secondly, practitioners frequently observe that univariate models outperform multivariate models in short run forecasting. Recent studies by Bernardinia and Cubadda (Bernardinia et al., 2015), and Carriero, Kapetanios, and Marcellino (Carriero et al., 2011) confirm this common observation.

Forecasting with regime switching models is straightforward. First, consider the forecasts from the model with a regime switch. Updating the second equation in by one period and taking conditional expectation, conditioned on the information available in period t, we get the forecast for period $t + 1$:

$$E_t(\Delta y_{t+1}) = \delta + \alpha \Delta y_t + \rho \varepsilon_t$$

In the same way, under the assumption that $\varepsilon_{t+1} = \rho \varepsilon_t + v_{t+1}$, the forecasts for periods $t + 2$ and $t + 3$ are obtained in this manner:

$$\begin{aligned} E_t(\Delta y_{t+2}) &= \delta + \alpha E_t(\Delta y_{t+1}) + \rho E_t(\varepsilon_{t+1}) \\ &= \delta + \alpha E_t(\Delta y_{t+1}) + \rho^2 \varepsilon_t \\ E_t(\Delta y_{t+3}) &= \delta + \alpha E_t(\Delta y_{t+2}) + \rho E_t(\varepsilon_{t+2}) \\ &= \delta + \alpha E_t(\Delta y_{t+2}) + \rho^3 \varepsilon_t \end{aligned}$$

The above equations will be used for forecasting the periods in which errors are believed to be autocorrelated. For the periods with white noise errors, the usual formulae for autoregressive processes will be used.

To consider a more realistic setting, suppose the model is of the form:

$$\Delta y_t = \delta + \alpha_1 \Delta y_{t-1} + \alpha_5 \Delta y_{t-5} + I_t(\rho \varepsilon_{t-1} + v_t) + (1 - I_t)\varepsilon_t \tag{13}$$

Additionally, suppose that the switch occurred in period $t-4$ and we have to obtain the predictions for the period $t+1$. Appropriately rearranged, equation (13) takes this form:

$$E_t(\Delta y_{t+1}) = \delta + \alpha_1 \Delta y_t + \alpha_5 \Delta y_{t-4} + \rho \varepsilon_t$$
$$= \delta + \alpha_1 \Delta y_t + \alpha_5 \Delta y_{t-4} + \rho^5 \varepsilon_{t-4} + \sum\nolimits_{i=0}^{3} \rho^{i+1} \nu_{t-i}$$

For instance, if in a sample with 37 periods, the switch occurs in the 33rd period and we have to predict the GDP for the 38th period, the prediction equation will look like this:

$$E_t(\Delta y_{38}) = \delta + \alpha_1 \Delta y_{38-1} + \alpha_5 \Delta y_{38-5} + \rho^5 \varepsilon_{38-5} + \sum\nolimits_{i=1}^{4} \rho^i \nu_{38-i}$$

Next subsections will present the results for selected countries.

4.1. Irish predictions

Predictive performance of the regime switching models will be assessed with out of sample forecasts for four quarters. The comparison of predictions from the best fitting linear versus the best fitting regime switching model is essential in order to assess whether allowing errors to switch processes proved itself. Using Irish data from the 1997Q1-2008Q4 period, predictions of GDP growth rates (first differences of log transformed GDP are denoted as y_t^{ie}) are obtained for the four quarters of 2009. In the next step, the sample was augmented with the actual data from 2009Q1 and the prediction was repeated for the remaining three quarters of 2009 and the first quarter of 2010. Forecasts for subsequent periods were obtained using the same logic.

The quality of the predictions will be assessed by weighting up the accuracy of the predictions and the significance, constancy, and stability of the parameters in linear and regime switching models. The root-mean-square error (RMSE) was chosen as the main quantitative accuracy measure. The estimates and RMSEs of corresponding linear and regime switching models are in table 2, while the actual and predicted GDPs for Ireland are depicted in the six panels of figure 4.

The first lags in all of the best fitting autoregressions (top rows of table 2) for Ireland are insignificant, and the values of first order coefficients tend to decrease from 0.2184 to 0.1122. Therefore, as the size of the sample increases, the significance declines. All this signals about the loss of accuracy and decreased measurement precision. The fifth lag does not reveal anything of particular interest, the estimate of which fluctuates with no clear tendency peaking at 0.3639 or falling till 0.2863. The changes in the sample size do not affect the significance of the fifth lag in any clear direction.

There are several reasons why the first lag is included, despite being insignificant. Firstly, it is hard to believe that it takes five quarters for the growth rates to adjust, especially since it is difficult to imagine what regularly happens with a frequency of five quarters. Taken together, the first and the fifth lags in quarterly data analysis make sense, as there are four quarters between them and this choice perfectly coincides with the type of data analysed. Secondly, knowledge of the history of the model build-up is necessary to fully justify exactly this selection. It is quite unbelievable that the analysis of ACFs and PACFs has revealed no signs of self-dependency for the Irish growth rates. Despite the inability to find at least one significant autocorrelation, the fifth lag, as the highest, was preselected for the estimation of the model. Even though the first lag was insignificant, it was added on conceptual grounds, already knowing the dynamic pattern of a regime switching model. Contrary to the situation with a single fifth lag, jointly considering the first and the fifth lags is quite meaningful as they indicate the dependency between the current and the preceding quarter with a one year lag.

TABLE 2**. Estimates and RMSEs of Irish predictions**

Sample	**Predictions**	**Estimates and Std. Errors**				**RMSE**
		δ	α_1	α_5	ρ	
Autoregression $y_t^{ie} = \delta + \alpha_1 y_{t-1}^{ie} + \alpha_5 y_{t-5}^{ie} + \varepsilon_t^{ie}$						
1997Q1:2008Q4	2009Q1:2009Q4	0.0123 (0.0033)	−0.2184 (0.1473)	0.3479 (0.1597)		2045.52
1997Q1:2009Q1	2009Q2:2010Q1	0.0117 (0.0033)	−0.1547 (0.1446)	0.2863 (0.1596)		1286.78
1997Q1:2009Q2	2009Q3:2010Q2	0.0115 (0.0033)	−0.1482 (0.1427)	0.3004 (0.1558)		1081.55
1997Q1:2009Q3	2009Q4:2010Q3	0.0109 (0.0035)	−0.1333 (0.1413)	0.3479 (0.1470)		336.42
1997Q1:2009Q4	2010Q1:2010Q4	0.0106 (0.0036)	−0.1122 (0.1374)	0.3639 (0.1444)		344.53
1997Q1:2013Q4	2014Q1:2014Q4	0.0094 (0.0029)	−0.1606 (0.1192)	0.3621 (0.1269)		592.24
Regime switching model $y_t^{ie} = \delta + \alpha_1 y_{t-1}^{ie} + \alpha_5 y_{t-5}^{ie} + I_t(\rho\varepsilon_{t-1}^{ie} + v_t^{ie}) + (1 - I_t)\varepsilon_t^{ie}$						
1997Q1:2008Q4	2009Q1:2009Q4	0.0157 (0.0028)	−0.3449 (0.0966)	0.2823 (0.1023)	0.7942 (0.3362)	246.05
1997Q1:2009Q1	2009Q2:2010Q1	0.0139 (0.0028)	−0.2257 (0.0979)	0.2830 (0.0991)	0.882 (0.3193)	576.28
1997Q1:2009Q2	2009Q3:2010Q2	0.0142 (0.0028)	−0.2353 (0.0957)	0.2829 (0.0964)	0.8255 (0.2616)	846.14
1997Q1:2009Q3	2009Q4:2010Q3	0.0136 (0.0028)	−0.2066 (0.0966)	0.3194 (0.0962)	0.8427 (0.2529)	196.68
1997Q1:2009Q4	2010Q1:2010Q4	0.0135 (0.0028)	−0.2250 (0.0947)	0.3151 (0.0946)	0.8339 (0.2243)	444.04
1997Q1:2013Q4	2014Q1:2014Q4	0.0106 (0.0021)	−0.2011 (0.0818)	0.3492 (0.0841)	0.7717 (0.22345)	482.15

Source: author's calculations

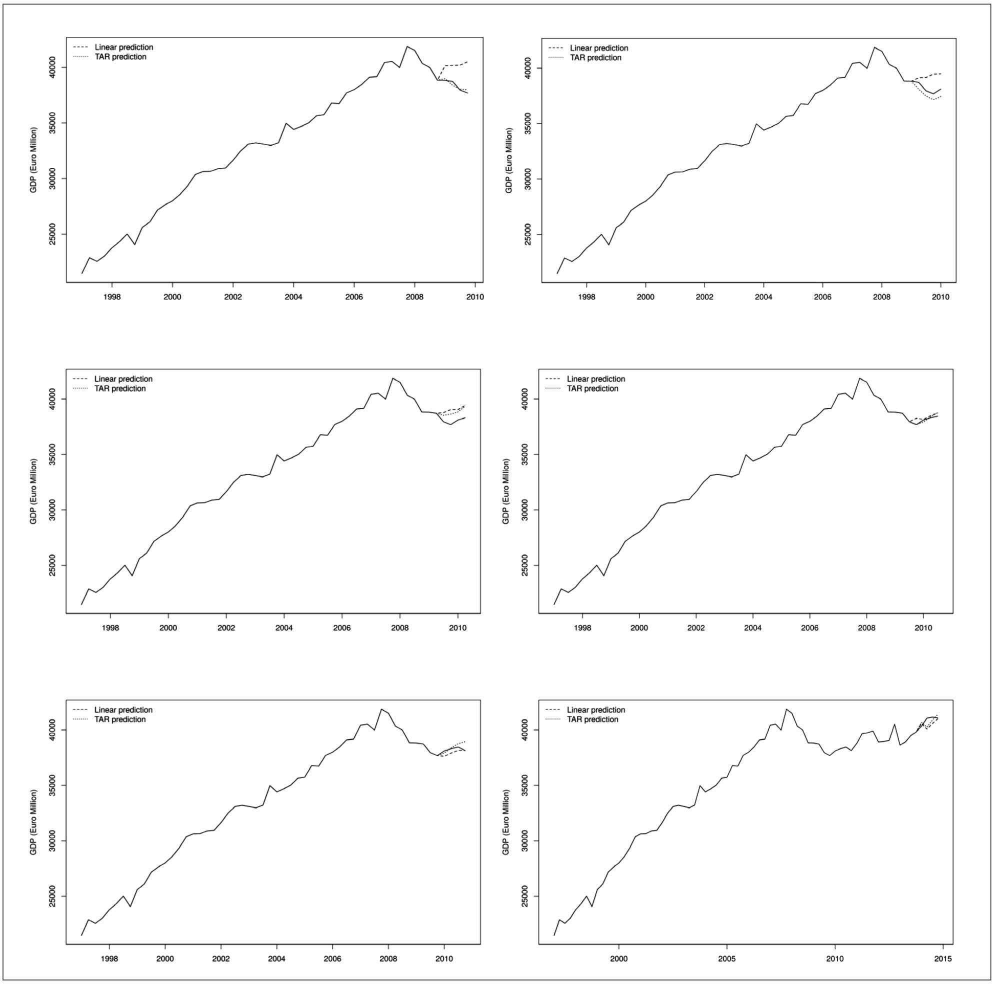

FIG. 4. **Actual and predicted GDP for Ireland for the corresponding periods**

Source: the Central Statistics Office of Ireland and author's calculations

Alternative models for the same samples, but with a regime switch for the errors that occur in the last quarter of 2007, are in bottom rows of table 2. The subjective selection of precisely this date for the regime switch can be motivated by the fact that in Ireland, GDP started to decline since the mentioned date and what was supposed to be a simple recession later turned into an unprecedented downfall. Turning dates for Estonia, Latvia, and Lithuania were selected analogously. The fact that I do not provide any statistical test for the detection of turning points does not mean that tests like these can not be constructed; it may only indicate that in this particular analysis, the subjective judgements gave more than satisfactory results.

The are several benefits to letting the errors change the regimes. First, the significance of the coefficients increased and the first lag became significant. Keeping in mind that in initial data analysis all ACFs for y_t^{ie} were insignificant, omission of the regime switching part for the errors caused that essential autocorrelations having been overlooked. Second, the constancy of the estimates of the fifth lag increased (estimates range from 0.2823 to 0.3194). Third, the estimates gained extra vitality and started to reflect the agent perspectives based on the then-current state of the affairs. The equation in table 2, which was based on the 1997Q2-2008Q4 sample, reflected the pessimistic moods (the sum of the autoregressive coefficients was negative) that prevailed at that time. In the remaining equations, the sums of autoregressive coefficients were already positive. These models echo the change in people's expectations, which we can vividly express as being from "things will be worse and worse" to "with time, we will overcome that as well". The estimate of ρ fluctuates from 0.8882 to 0.7942, being large in magnitude and mirroring the response that the negative impact of the financial crisis could diminish only with time.

It is important to note that the expansion of the sample leads to more drastic changes in the constant regime model than with the regime switching model, yet the coefficients in regime switching models are more stable as compared to those in the constant regime models.

It is evident that the regime switching portion added a visible amount of robustness and significance to the coefficients. Despite this, in terms of predictability, regime switching models overcame constant regime models only in the initial stages of the crisis. Predicting the growth rates for 2009Q1-2009Q4 and 2009Q2-2010Q1, linear models failed to guess the direction of the change, while threshold models were more or less correct. This can be clearly seen from the two upper panels in figure 4. For later periods, threshold models were not as effective and their predictions did not differ as considerably as of the linear model. RMSEs only confirm the conclusions of the graphical analysis.

It is clear that better predictions ar yielded by those models who are more stable on the long run, yet more responsive to the fundamental changes, namely the drastic change in the coefficients switching from an equation based on the 1997Q1:2008Q4 sample to an equation based on the 1997Q1:2009Q1 sample. Regime switching models are useful if the economy is still in the phase of downturn or it overcame this period in a very near past. Otherwise, the forecasts from constant regime and regime switching models are similar.

The regime switching part was the fundamental missing link in the equation, the omission of which caused incorrect short run forecasts, a loss of estimation precision and misspecified dynamics (single fifth lag, instead of two, first and fifth). By summarising the Irish predictions, we can conclude that the main advantage of the regime switching models is the accuracy of short-term forecasts. If we compare the predictions of constant and regime switching models, we can clearly see that models with changing regimes give more reliable estimates of what is likely to happen in the initial phases of the crisis. For later periods, their predictions do not differ very much.

4.2. Estonian, Latvian, and Lithuanian predictions

The strategy is similar to the one proved and tested with the Irish data. We begin with a 1993Q1-2008Q4 Estonian GDP sample and update it again by one observation every time a new data point becomes available. The estimates and RMSEs of best fitting autoregressions and regime switching models for the growth rates of Estonian GDP (y_t^{ee}) are in table 3.

TABLE 3. **Estimates and RMSEs of Estonian predictions**

Sample	Predictions	Estimates and Std. Errors			RMSE
		δ	α_1	ρ	
Autoregression $y_t^{ee} = \delta + \alpha_1 y_{t-1}^{ee} + \varepsilon_t^{ee}$					
1993Q2:2008Q4	2009Q1:2009Q4	0.0121 (0.0034)	0.0913 (0.1490)		2.6889
1993Q2:2009Q1	2009Q2:2010Q1	0.0105 (0.0042)	0.2625 (0.1318)		1.0086
1993Q2:2009Q2	2009Q3:2010Q2	0.0099 (0.0045)	0.3083 (0.1228)		0.3107
1993Q2:2009Q3	2009Q4:2010Q3	0.0092 (0.0046)	0.3358 (0.1197)		0.8453
1993Q2:2009Q4	2010Q1:2010Q4	0.0100 (0.0044)	0.3059 (0.1182)		0.3101
1993Q2:2013Q2	2013Q3:2014Q2	0.0101 (0.0037)	0.2714 (0.1098)		0.1236
Regime switching model $y_t^{ee} = \delta + \alpha_1 y_{t-1}^{ee} + I_t(\rho\varepsilon_{t-1}^{ee} + \nu_t^{ee}) + (1 - I_t)\varepsilon_t^{ee}$					
1993Q2:2008Q4	2009Q1:2009Q4	0.0134 (0.0024)	0.0853 (0.0928)	0.7001 (0.7231)	0.4081
1993Q2:2009Q1	2009Q2:2010Q1	0.0113 (0.0025)	0.2361 (0.0978)	0.9505 (0.4820)	1.6464
1993Q2:2009Q2	2009Q3:2010Q2	0.0107 (0.0025)	0.2744 (0.0985)	0.9280 (0.4206)	0.8731
1993Q2:2009Q3	2009Q4:2010Q3	0.0103 (0.0025)	0.2985 (0.0998)	0.9413 (0.3925)	1.2888
1993Q2:2009Q4	2010Q1:2010Q4	0.0107 (0.0025)	0.2726 (0.0972)	0.8322 (0.3338)	0.2897
1993Q2:2013Q2	2013Q3:2014Q2	0.0107 (0.0022)	0.2285 (0.0915)	0.8636 (0.3297)	0.2567

Source: author's calculations

Here, the first quarter of 2008 was chosen for the regime switch as that is the date when GDP in Estonia began to decline. Similarly to Ireland, what was supposed to be a simple recession turned into abnormal downfall. The actual and predicted GDPs for Estonia are depicted in the six panels of the figure 5.

Similar to the case of Ireland, the regime switching portion added a visible amount of robustness and significance for the coefficients, but in terms of prediction capacity, regime switching models overcame constant regime models only in very initial stages of the crisis. For the later periods, there are no visible differences between the predictions from competing models. Predicting the growth rates for 2009Q1-2009Q4, the linear models failed to guess the direction of change, while threshold models were correct. This can be clearly seen from the upper left panel in figure 5.

The estimates of both Estonian and Irish regime switching models are more robust and precise than their linear counterparts, even though the possibility for the errors to change regimes increased the overall reliability of the models and helped to extract the main dynamic features of the variables.

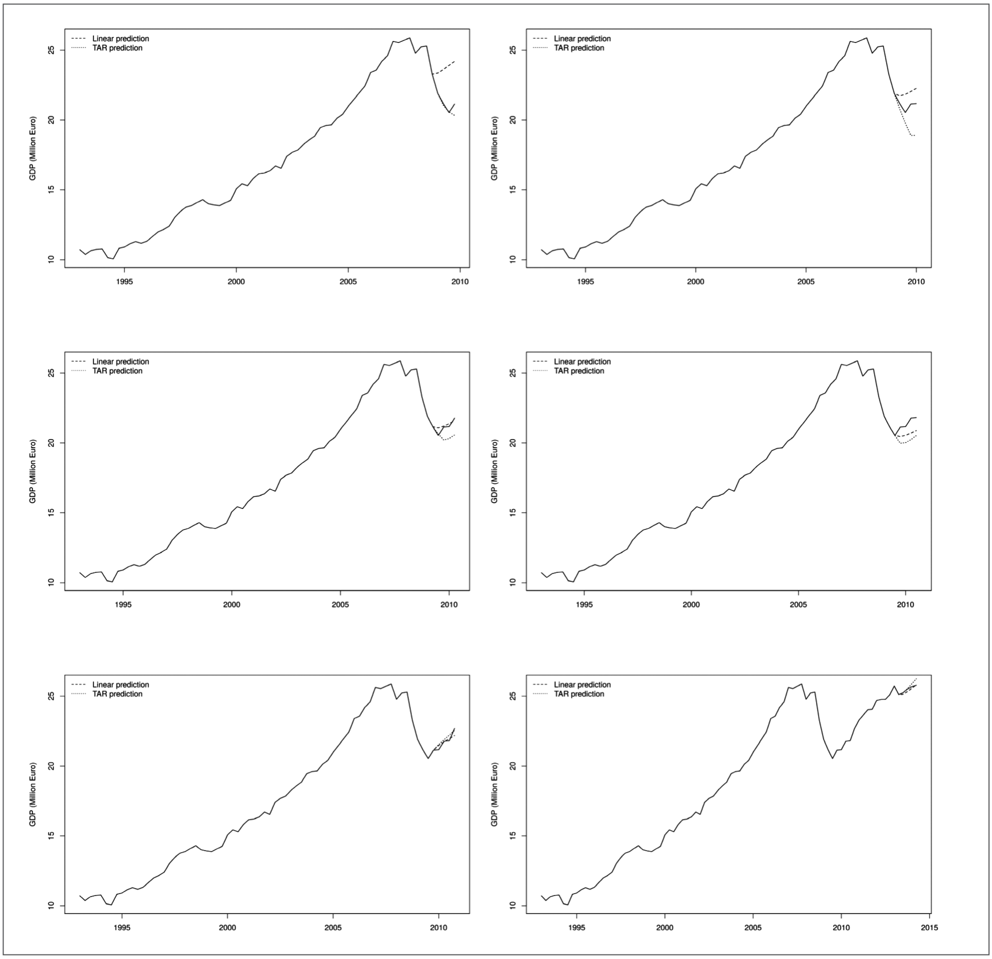

FIG. 5. **Actual and predicted GDP for Estonia**

Source: Eurostat and author's calculations

If one compares Estonian and Irish models, two distinctions become evident. First, the coefficients in models for Estonia are slightly less stable, compared with those from the models for Ireland. This disparity may be related to policy changes. Estonian politicians responded to the crisis immediately and introduced austerity measures (Dudzińska, 2013), while the Irish politicians paused and lingered (Whelan, 2013). Different challenges in the economic and political environment forced politicians in these countries to act differently. The different timing choices for political actions caused differences in stability of Estonian and Irish autoregressive coefficients. As was stressed in the first section, the possibility that short run innovations have caused the change in the behaviour of GDP is not very compelling. Changes in short run innovations can cause changes in autoregressive or moving average coefficients, depending on the univariate model that fits the data. However, there is a second distinction – innovations in Estonia were autocorrelated for two periods, while Irish innovations were autocorrelated for a longer time span. These differences can be explained by the fact that Estonia was hit by the crisis harder than Ireland, but the transition into a milder phase of recession began sooner in Estonia.

Irish and Estonian examples are very illustrative in that they both demonstrate sufficient predictability with the early warning signs, whereas the Latvian and Lithuanian cases demonstrate limited predictability. Ireland and Lithuania are two antipode economies: the downturn in Ireland begun with clear signs of recession, and the recession turned into depression, whereas in Lithuania, when the crisis started, the economy was still in peak condition.

Models for Latvia and Lithuania were not very promising as the initial estimation revealed the explosive features of the errors that are not consistent with any economic logic, or perhaps only reflect the crash expectations that prevailed in both of these countries. The negative expectations, pertaining to the belief that everything will only be worse, were common at that time. The unsatisfactory estimates for Latvia (see equation 14) and Lithuania (equation 15), using the data from 1995Q2-2008Q4, are given below. The bias correction is omitted as there is no need to correct the inadequate coefficients.

$$y_t^{lv} = \underset{(0.0031)}{0.0139} - \underset{(0.0975)}{0.0863}\, y_{t-1}^{lv} + \underset{(0.0979)}{0.2293}\, y_{t-2}^{lv} + I_t(\underset{(0.5264)}{1.1460}\, \varepsilon_{t-1}^{lv} + v_t^{lv}) + (1 - I_t)\varepsilon_t^{lv} \tag{14}$$

$$y_t^{lt} = \underset{(0.0020)}{0.0128} + \underset{(0.0991)}{0.2099}\, y_{t-1}^{lt} + I_t(\underset{(0.4649)}{1.0472}\, \varepsilon_{t-1}^{lt} + v_t^{lt}) + (1 - I_t)\varepsilon_t^{lt} \tag{15}$$

We know that the downturn in Latvia and Lithuania was harder, particularly with the first two quarters being harsh. At that time, the crisis showed no clear signs of retreat. After the lowest possible boundary was reached, a long lasting recovery process began and the severe recession turned milder. In this case, the estimates of ρ for Latvia and Lithuania should be larger in magnitude and may exceed unity, but should diminish with time. These possibilities require a different treatment and are left for the forthcoming papers.

The findings and the results of this analysis do not contradict the research done by other authors and should be treated as complements to the studies already done. In the aftermath of the crisis, many authors have presented their forecasting innovations. Marcin Kolasa and Michał Rubaszek found that financial frictions increased the quality of forecasts for the crisis period, but reduced the quality of forecasts during non-crisis times (Kolasa et al, 2015). Although the threshold model from this paper cannot be directly compared with the model employed by Kolasa and Rubaszek on conceptual grounds, their findings match the general conclusion from this paper – if the predictions fall into the regime switch or if the switch occurred in the near past, threshold models outperform their linear competitors, but if the regime switch occurred long ago, predictions from both the linear and threshold models are equally accurate. In a recent paper, Huber presented enough evidence that non-linear and combinations of linear and non-linear models yielded more accurate results (Huber, 2016). Fady Barsoum and Sandra Stankiewicz predicted the GDP growth, but the main innovation in their research was the Markov-switching model for the mixed frequency data. Even though the financial crisis was of secondary importance in the model, their analysis led to the conclusion that it is possible to find a setting that would be helpful in the periods of crises, but not particularly useful in forecasting GDP growth during periods of stable growth (Barsoum et al., 2015).

Keeping in mind the substantial and positive improvements in forecasts and comparing the analysis and the results of this article with the studies done by other authors, re-specification of the error term fulfilled its purpose.

5. Concluding remarks

The analysis presented here has revealed that if the predictions fall into the regime switch or if the regime switch occurred in the near past, threshold models outperform their linear counterparts, but if the regime switch occurred long ago, predictions from both the linear and threshold models are equally accurate. Regime switching models are useful if: a) The economy is still in the phase of a downturn; b) The downturn is in its initial phases; c) Constant regime models failed to predict it.

Contrary to the viewpoint that the recent financial crisis can not be considered as a change in macro process, the predictions made with threshold models that assume this change are far superior to those who do not. Consequently, the effects of the financial crisis or recession can be modelled by relaxing the classical assumptions and letting the errors change the regimes. Even if this conclusion is false, letting the errors switch patterns proved to be a fruitful choice in terms of forecast quality. The clear-cut shortcoming of retaining the assumption of the stable data generating pattern is the diminished predictive power.

This paper is introductory in the sense that it presents a concept. Further studies are necessary to obtain a broader picture of the possibilities that are offered by the regime switching errors. High frequency financial data may be a proper choice, as that data can contain frequent and relatively long lasting switches.

All calculations were done with statistical software R.

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