

Privatization Policy with Corporate Social Responsibility in a Mixed Duopoly

He Liang

E-mail: dreamangel_369@163.com

ORCID: <https://orcid.org/0009-0005-7639-4303>

Affiliation: College of Public Administration and Humanities, DaLian Maritime University, China

ROR: <https://ror.org/002b7nr53>

Chaoqun Sun

E-mail: suncq0505@163.com

ORCID:

Affiliation: School of Humanities and Law, Northeastern University, China

ROR: <https://ror.org/03awzbc87>

Jiaqi Gao (*corresponding author*)

E-mail: holiday1005@126.com

ORCID:

Affiliation: College of International Education, Shenyang University, China

ROR: <https://ror.org/04ddfw68>

Annotation. This paper constructs a mixed duopoly model considering privatization, the level of corporate social responsibility (CSR), and the cost of state-owned shares. It examines the equilibrium results under the Cournot model and the Stackelberg model and makes comparisons. The results show that CSR and state-owned shares have essential effects on equilibrium results. Faced with different types of competition, stakeholders make various decisions. Under the Cournot model, there is an optimal state-owned share proportion. Full privatization is optimal under the Stackelberg model and the expanded three-stage game. Therefore, state-owned shares, CSR, and competition mode should be considered in the privatization policy.

Keywords: duopoly, privatization, corporate social responsibility, state-owned shares.

JEL classification: D23, L13, L33.

Introduction

Since the 1980s, a wave of privatization reform has swept the world, affecting state-owned firms. In this process, scholars have conducted in-depth research on whether privatization should be carried out and to what extent (Wang, Chiou, 2018; Lo *et al.*, 2022). A key reason for the existence of state-owned firms is that they aim to maximize social welfare and address issues such as the inadequate supply of public goods. The primary purpose of privatization, on the other hand, is to address issues such as inefficiency and corruption in state-owned firms. State-owned firms often exhibit low efficiency and insufficient profitability, resulting in deteriorating performance, information asymmetry, and principal-agent problems. To address these issues, improving operational efficiency and optimizing governance structures are necessary (Boubakri *et al.*, 2017; Fragoudaki, Giokas, 2020; Sun *et al.*, 2025). However, private firms pursue profit maximization, and profit and social welfare are often in contradiction, and if implemented improperly, they may easily lead to excessive prices, unfair distribution and other

shortcomings. With the development of the times, the empirical goals of private firms are also evolving, and an increasing number of firms are beginning to undertake corporate social responsibility (CSR), which has become a key consensus and goal for modern firms. Increasingly firms are beginning to pay attention to CSR (Liu *et al.*, 2019; Wang *et al.*, 2020). In the context of the continuous strengthening of policies, regulations, and government supervision, as well as consumers' increasing demand for sustainable products, stakeholders of firms also have higher expectations for their firms to operate sustainably. They hope that relevant firms will reduce the negative impact on natural resources, biodiversity, and ecosystem service functions in their production processes. CSR plays an increasingly important role in supporting the strategic development of firms, enhancing the ecological environment, and ensuring social well-being. Corporate social responsibility can enable firms to bear social goals beyond profits, which are similar to the goals of state-owned firms. In this way, corporate social responsibility can serve as a remedy for the drawbacks of private firms that pursue profits excessively and damage social welfare. So how does CSR affect privatization policies? How should the government implement scientific privatization policies under the conditions of corporate social responsibility? These are the main issues to be addressed in this study.

The main contributions are as follows: first, this paper incorporates equity costs into the relationship between privatization and CSR analysis (Sun *et al.*, 2005; Matsumura, Okamura, 2015). Both state-owned shares and CSR have the characteristics of taking stakeholders into account. If private firm engages in CSR, what impacts does equity cost have on the market equilibrium results and privatization policies? These are advancements for existing research. Second, CSR decision is endogenous and the game between the market and the government is examined. The results show that public firm is squeezed out of the market. Therefore, appropriate government intervention is an effective means to maintain the regular operation of the market. Third, although some studies have analyzed the relationship between CSR and privatization, multiple competition modes are not fully considered (Dong, Barcena-Ruiz, 2021; Gioffre *et al.*, 2021). This paper explores the Cournot model and Stackelberg model simultaneously and makes comparisons to reveal the influence of firm's position and game order (Chen *et al.*, 2022).

The rest of this paper is organized as follows: A mixed duopoly market with a mixed-ownership firm and a private firm is presented in Section 2. Parameters are denoted and assumptions are proposed. In Section 3, the equilibrium results are obtained and the comparative analysis of Cournot and Stackelberg model is made. Moreover, CSR decisions are made. The last section draws the conclusions. The impacts of CSR, state-owned share costs, and competition modes are comprehensively explored.

1. Literature Review

1.1 Privatization

Studies show that a reasonable proportion of privatization exists when considering factors such as product differentiation, externalities, and production costs (Chen *et al.*, 2020; Heywood *et al.*, 2021). Beladi and Chao (2006) argue that privatization benefits society in the long term, especially with the employment effect. Bachiller (2017) analyzes the impact of privatization and believes the profit-seeking nature of private share resulted in higher short-term returns, gradually improving firms' competitiveness and efficiency. However, in reality, some private firms are solely focused on profit maximization (Tan *et al.*, 2020), resulting in economic inefficiency (Warner, Aldag, 2021). Faraz and Samad (2022) empirically investigate the privatization impact on performance and efficiency and find the efficiency of the firms is also not improved significantly. State-owned firms have several advantages over private firms, including reduced costs, controlled risk, and access to external financing (Li, Yu, 2018). They take care of other

stakeholders and easily obtain external financing, thereby maintaining stability and promoting development (D' Souza, Nash, 2017; Liu *et al.*, 2021).

Therefore, partial privatization would be a more rational choice (Du, Wang, 2020; Wang *et al.*, 2021). Xu *et al.* (2016) compare a Cournot with a Bertrand duopoly in a differentiated mixed market with emission tax and privatization policies and find that partial privatization is always optimal. Kawasaki *et al.* (2022) examine the endogenous determinants of partial privatization degree and the timing of quantity competition. When the private firm is domestic, partial privatization and private leadership emerges in equilibrium. Chen *et al.* (2023) analyze the optimal carbon emission trading prices and privatization decisions. They find that the proportion of state-owned shares and the equity efficiency gap affect the equilibrium results for different carbon emission policies. Full and partial privatization may be optimal decisions.

To summarize, privatization is influenced by various internal and external factors. This study will continue to analyze these effects and provide a basis for formulating and implementing privatization policies.

1.2 Corporate Social Responsibility

After long-term development, CSR has become a conduct code for many firms (Aloui, Jebbi, 2022; Barcena-Ruiz, Sagasta, 2021; Chen *et al.*, 2024). Canon has developed the strategy of “image public welfare” in combination with its own advantages in image technology and has carried out unique public welfare projects in environmental protection, educational enlightenment, cultural heritage, community care and humanitarian assistance, actively adopting CSR, which has been recognized by all sectors of society. Beijing-Hong Kong Metro has established a low-carbon management system to strengthen the use of clean energy. McWilliams and Siegel (2001) hypothesize the level of CSR depends on firms' size, advertising, government sales, consumer income, and so on. El Ghouli *et al.* (2011) find that the firms with better CSR scores exhibit cheaper equity financing, especially those investments in improving responsible employee relations, environmental policies, and product strategies. Kim *et al.* (2014) find CSR has a mitigating effect on crash risk. Abeysekera and Fernando (2020) argue that CSR can alleviate environmental concerns, benefit society, and reduce the firm's risk exposure. Lu *et al.* (2021) find that firms with high financial or environmental risk are able to benefit from CSR as a risk mitigation strategy. In a word, CSR plays a crucial role in the survival and development of firms.

1.3 Privatization and CSR

Privatization reform and CSR coexist in the markets where state-owned firms and private firms are mixed. Under the influence of CSR, private firm decisions take into account certain non-profit goals, which directly affect the equilibrium output and price of the market, and thus affect the decisions of competitors. For example, if private firms bear a high level of social responsibility, the responsibility they should bear towards state-owned firms is a form of sharing, which puts a lower requirement on their level of nationalization. Baron (2001) considers that privatization and CSR have a direct effect on the costs and a strategic effect on their competitive positions. Kim *et al.* (2019) argue that CSR activities are crucial in privatization policy decisions in a mixed oligopoly. Xu and Lee (2019) examine the impact of CSR on privatization policy with an international bilateral trade model and they demonstrate that both countries would choose a nationalization policy, even though the privatization policy is globally optimal under certain conditions. Khan *et al.* (2021) investigate the link between privatization and CSR performance and they find that privatization may be negatively associated with social performance. Therefore, it is necessary to incorporate CSR into the privatization policy research system. Although some results have been obtained, the in-depth impact mechanism requires further exploration.

2. Model Assumptions

There is a mixed duopoly market where two firms produce homogeneous goods. Firm 1 is a mixed-ownership firm and Firm 2 is a private firm. Assuming q_1 and q_2 are their outputs. The inverse demand function is $p = a - q_1 - q_2$, where a is a positive constant, representing the size of the market. Assuming Firm 1's marginal cost is $c_1 = m\beta + (1 - \beta)n$ and Firm 2's marginal cost is $c_2 = n$, where m refers to the marginal cost of state-owned shares, n refers to the marginal cost of private shares, and m and n are constants (Kong *et al.*, 2022). To simplify the analysis, setting $a > m > n = 0$, which denotes that the efficiency of private share is higher than that of state-owned share. Based on the consideration of state-owned firms' social goals, the efficiency of state-owned firms is low. This design aims to reflect the benefits of privatization.

Suppose Firm 1's utility function is $u_1 = \beta sw + (1 - \beta)\pi_1$, meaning that Firm 1 pursues a mixed goal, it not only cares about its profit but also considers social welfare, where β denotes the proportion of state-owned shares, $0 \leq \beta \leq 1$. The larger the β , the higher the proportion of state-owned shares and the lower the degree of privatization. And Firm 2's utility function is $u_2 = \pi_2 + \lambda cs$, Firm 2 also pursues a mixed goal, it not only cares about its profits but also considers consumer surplus, where the level of CSR is measured by λ , $0 \leq \lambda \leq 1$. The larger the λ , the higher the level of CSR, and the more concerned about consumer surplus, and $0 \leq \lambda, \beta \leq 1$. The profits of the two firms are $\pi_i = pq_i - c_i q_i$, the first part is revenue and the second part is cost. cs is consumer surplus and $cs = \frac{(q_1 + q_2)^2}{2}$, the higher the total output, the more consumer surplus. sw is social welfare and $sw = \pi_1 + \pi_2 + cs$, which is the sum of producer surplus and consumer surplus. The variables are summarized in Table 1.

Table 1. List of notations and explanations

Notations	Explanations
p	Price of production
q_i	Output of Firm i
c_i	Cost of Firm i
m	Cost of state-owned shares
n	Cost of private shares
β	Proportion of state-owned shares
λ	Level of CSR
π_i	Profit of Firm i
u_i	Utility of Firm i
CS	Consumer surplus
SW	Social welfare

Source: compiled by the authors.

Under the above assumptions, in the first stage, the government formulates the proportion of state-owned shares to maximize social welfare. In the second stage, two firms compete to determine their outputs to maximize their utilities under the Cournot or Stackelberg model. The equilibrium results are represented by superscripts C and S, respectively.

3. Model Analysis

3.1 Cournot Model

Under Cournot model, the utility functions of the two firms are $u_1^C = \beta sw + (1 - \beta)\pi_1$ and $u_2^C = \pi_2 + \lambda cs$. The second derivative of utility to output is as follows: $\frac{\partial^2 u_1^C}{\partial q_1^2} = \beta - 2 < 0$ and $\frac{\partial^2 u_2^C}{\partial q_2^2} = \lambda - 2 < 0$. Therefore,

there are optimal outputs. According to the principle of utility maximization, $\frac{\partial u_i^C}{\partial q_i} = 0$ are satisfied, the equilibrium outputs are obtained as

$$q_1^{C*} = \frac{a-2\beta m+(\beta m-a)\lambda}{3-2\beta+(\beta-1)\lambda}, q_2^{C*} = \frac{a(\lambda+1)-\beta(a-m+\lambda m)}{3-2\beta+(\beta-1)\lambda} \quad (1)$$

Lemma 1. Substituting the equilibrium outputs into profit, consumer surplus and social welfare. The equilibrium results can be obtained as follows:

$$\pi_1^{C*} = \frac{(1-\beta)(a-a\lambda-2\beta m+\beta\lambda m)^2}{(3-2\beta+\lambda+\beta\lambda)^2}, \pi_2^{C*} = \frac{(a-a\beta+a\lambda+\beta m-\beta\lambda m)(a\beta\lambda-a\beta-a\lambda+\beta m+a)}{(3-2\beta+\lambda+\beta\lambda)^2}, CS^{C*} = \frac{(a\beta-2a+\beta m)^2}{2(3-2\beta+\lambda+\beta\lambda)^2}, SW^{C*} = \frac{2(1-\beta)(a-a\lambda-2\beta m+\beta\lambda m)^2+2(a-a\beta+a\lambda+\beta m-\beta\lambda m)(a\beta\lambda-a\beta-a\lambda+\beta m+a)+(a\beta-2a+\beta m)^2}{2(3-2\beta+\lambda+\beta\lambda)^2}.$$

Analyzing the impacts of λ and m on the equilibrium result, we can get Proposition 1.

Proposition 1.

(1) the effects of λ :

$$\frac{\partial \pi_1^{C*}}{\partial \lambda} \leq 0; \text{ if } 0 \leq \lambda \leq \frac{a-a\beta+\beta m}{3a-4a\beta-\beta m+a\beta^2+\beta^2 m}, \text{ then } \frac{\partial \pi_2^{C*}}{\partial \lambda} \geq 0; \text{ if } \frac{a-a\beta+\beta m}{3a-4a\beta-\beta m+a\beta^2+\beta^2 m} < \lambda \leq 1, \text{ then } \frac{\partial \pi_2^{C*}}{\partial \lambda} < 0; \\ \frac{\partial CS^{C*}}{\partial \lambda} \geq 0; \frac{\partial SW^{C*}}{\partial \lambda} \geq 0.$$

(2) the effects of m :

$$\frac{\partial \pi_1^{C*}}{\partial m} \leq 0, \frac{\partial \pi_2^{C*}}{\partial m} > 0, \frac{\partial CS^{C*}}{\partial m} \leq 0, \text{ if } 0 \leq \beta \leq \frac{-3\lambda^2+14\lambda-14+\sqrt{\lambda^4-12\lambda^3+56\lambda^2-104\lambda+48}}{-4(\lambda^2-4\lambda+4)}, \text{ then } \frac{\partial SW^{C*}}{\partial m} < 0; \text{ when } \\ \frac{-3\lambda^2+14\lambda-14+\sqrt{\lambda^4-12\lambda^3+56\lambda^2-104\lambda+48}}{-4(\lambda^2-4\lambda+4)} < \beta \leq 1, \text{ if } 0 < m \leq \frac{a(\lambda-1)[(\beta-1)\lambda-3\beta+4]}{(2\beta\lambda^2-8\beta\lambda+8\beta-2\lambda^2+10\lambda-11)\beta}, \text{ then } \frac{\partial SW^{C*}}{\partial m} \leq 0; \text{ if } \\ \frac{a(\lambda-1)[(\beta-1)\lambda-3\beta+4]}{(2\beta\lambda^2-8\beta\lambda+8\beta-2\lambda^2+10\lambda-11)\beta} < m < a, \text{ then } \frac{\partial SW^{C*}}{\partial m} > 0.$$

The higher CSR level (λ) is, the lower Firm 1's profit and the more consumer surplus and social welfare are. λ has complicated impacts on Firm 2's profit. When λ is low, continuing to increase λ increases Firm 2's profit, otherwise, its profit declines. If Firm 2 maximizes its profit, there is an optimal λ . It can be seen that Firm 2 does not blindly improve λ , but makes strategic decisions based on the current level of λ to determine whether to increase, decrease or not change the level. Moreover, Firm 1 does not support CSR activities due to loss of profit. As a competitor, although Firm 1 owns state-owned shares, from the perspective of its profit, it is not willing to support Firm 2 to improve its CSR level. Consumers and the government may encourage private firms to implement a higher λ to promote consumer surplus and social welfare. Xing et al. (2022) consider environmental R&D (ER&D) of clean technology in a polluting mixed duopoly and analyze the impacts of partial privatization on equilibrium results. They argue that an appropriate partial-privatization policy can increase social welfare. When the government formulates CSR-related policies, it also needs to consider the current level of λ . If the current level of λ is below the optimal level and the firm has the motivation to improve, it should refrain from interfering or intervening, and fully respect the market mechanism. However, if the current level of λ has exceeded the optimal level, the firm lacks the motivation to improve. If the government continues to support, it needs to implement corresponding measures, such as subsidies and concessions.

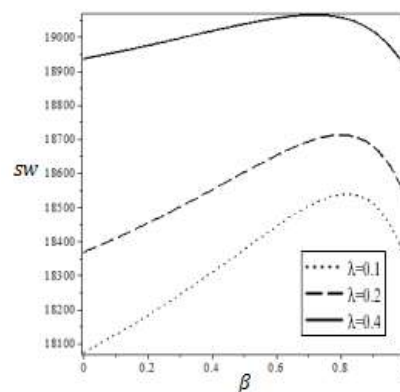
If the cost of state-owned shares (m) decreases, which means that the efficiency of state-owned shares is improved, Firm 1's profit, and consumer surplus increase, but Firm 2's profit decreases. The

relationship between m and social welfare is affected by β . When β is low, social welfare is negatively correlated with m , otherwise, social welfare first increases and then decreases as m decreases. Moreover, the reduction of m ensures that the mixed-ownership firm obtains more profit and rises in consumer surplus but declines the private firm's profit. Therefore, the improvement of the efficiency of state-owned shares can increase the total output of the industry, which is beneficial to consumers, while the impact of it on firms is related to the nature of their firms. The government should determine an appropriate β to promote consumer surplus and social welfare by taking m into consideration.

Corollary 1.

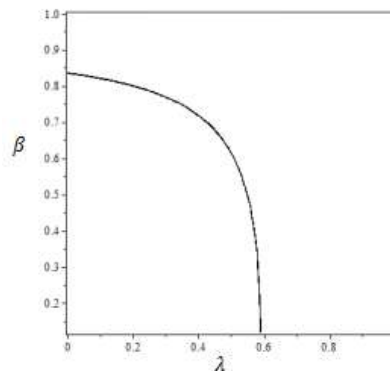
$0 \leq \beta^{C^*} \leq 1$, if $0 \leq \beta \leq \beta^{C^*}$, then $\frac{\partial sw^{C^*}}{\partial \beta} \geq 0$; if $\beta^{C^*} < \beta \leq 1$, then $\frac{\partial sw^{C^*}}{\partial \beta} < 0$.

To further illustrate Corollary 1, assuming $a = 200$ and $m = 10$, Figure 1 shows the relationship between β and λ , Figure 2 shows the relationship between sw and β when λ denotes 0.1, 0.2, 0.4.



Source: compiled by the authors.

Figure 1. The Relation between sw and β



Source: compiled by the authors.

Figure 2. The Relation between β and λ

Corollary 1 and Figure 1 imply that there is an optimal β , and Figure 2 indicates negatively related to λ . Therefore, the government should fully focus on λ when formulating privatization policy. If the CSR level of private firms increases, the government can reduce the proportion of state-owned shares in mixed-ownership firms, thereby adopting proper privatization measures to maintain social welfare at an optimal

level. Conversely, the government needs to increase the proportion of state-owned shares in mixed-ownership firms, that is, they should adopt proper nationalization. By incorporating the proportion of state-owned shares into decision-making, the government can make reasonable adjustments based on the current market situation, especially the level of CSR.

3.2 Stackelberg Model

Under Stackelberg competition, Firm 1 is the leader, using backward induction, first solve $\frac{\partial u_2^S}{\partial q_2} = 0$, and get the output of Firm 2 as

$$q_2 = \frac{a - q_1 + \lambda q_1}{2 - \lambda} \quad (2)$$

Then, the output of Firm 2 is substituted into the utility function of Firm 1. To maximize utility $\frac{\partial u_1^S}{\partial q_1} = 0$, the equilibrium output of Firm 1 is obtained, and Firm 2's output is further obtained as

$$q_1^{S*} = \frac{-2a + a\beta + 3a\lambda + 4\beta m - a\lambda^2 - 2a\beta\lambda - 4\beta\lambda m + a\beta\lambda^2 + \beta\lambda^2 m}{3\beta + 2\lambda - 2\beta\lambda - 4} \quad (3)$$

$$q_2^{S*} = \frac{-a + a\beta - 2a\lambda - 2\beta m + a\lambda^2 + a\beta\lambda + 3\beta\lambda m - a\beta\lambda^2 - \beta\lambda^2 m}{3\beta + 2\lambda - 2\beta\lambda - 4} \quad (4)$$

Lemma 2. Substituting the equilibrium outputs into profit, consumer surplus and social welfare, and the equilibrium results can be expressed as follows:

$$\pi_1^{S*} = \frac{\{\beta^2(-2\lambda+3)m + [(a+m)\lambda - a - 2m]\beta - a(\lambda-1)\}[(a+m)\lambda^2 - 2(a+2m)\lambda + a + 4m]\beta - a(\lambda-1)(\lambda-2)}{-(3\beta+2\lambda-2\beta\lambda-4)^2},$$

$$\pi_2^{S*} = \frac{\{[(a-m)\lambda - a + 2m]\beta - a(\lambda-1)\}[(a+m)\lambda^2 - (a+3m)\lambda - a + 2m]\beta - a(\lambda^2 - 2\lambda - 1)}{(3\beta+2\lambda-2\beta\lambda-4)^2},$$

$$CS^{S*} = \frac{(3a - 2a\beta - a\lambda - 2\beta m + a\beta\lambda + \beta\lambda m)^2}{(3\beta+2\lambda-2\beta\lambda-4)^2},$$

$$SW^{S*} = \frac{\{2[(a+m)\lambda^2 - 2(a+2m)\lambda + a + 4m](2\lambda-3)m\beta^3 + [-4m(2a+m)\lambda^2 + (3a^2 + 20am + 15m^2)\lambda - 4a^2 - 12am - 14m^2](\lambda-2)\beta^2 - 2[-2m\lambda^3 + (3a+11m)\lambda^2 + (-12a-19m)\lambda + 11a+10m]a\beta + (\lambda-3)(3\lambda-5)a^2\}}{2(3\beta+2\lambda-2\beta\lambda-4)^2}.$$

Proposition 2.

(1) the effects of λ :

If $0 \leq \lambda \leq \frac{E^2 + (9\beta-12)FE + 3GF}{6(\beta-1)FE}$, then $\frac{\partial \pi_1^{S*}}{\partial \lambda} \leq 0$; if $\frac{E^2 + (9\beta-12)FE + 3GF}{6(\beta-1)FE} < \lambda \leq 1$, then $\frac{\partial \pi_1^{S*}}{\partial \lambda} > 0$; if $0 \leq \lambda \leq \frac{I + (9\beta-12-31IK)J}{6(\beta-1)J}$, then $\frac{\partial \pi_2^{S*}}{\partial \lambda} \geq 0$; if $\frac{I + (9\beta-12-31IK)J}{6(\beta-1)J} < \lambda \leq 1$, $\frac{\partial \pi_2^{S*}}{\partial \lambda} < 0$; if $0 \leq \beta \leq \frac{3a}{2(a+m)}$, then $\frac{\partial CS^{S*}}{\partial \lambda} > 0$; when $\frac{3a}{2(a+m)} < \beta \leq 1$, if $0 \leq \lambda \leq \frac{2(a+m)\beta - 3a}{(a+m)\beta - a}$, then $\frac{\partial CS^{S*}}{\partial \lambda} \leq 0$; if $\frac{2(a+m)\beta - 3a}{(a+m)\beta - a} < \lambda \leq 1$, then $\frac{\partial CS^{S*}}{\partial \lambda} > 0$; $\frac{\partial SW^{S*}}{\partial \lambda} > 0$.

(2) the effects of m :

When $\frac{(3\lambda^2 - 9\lambda + 7 - \sqrt{-3\lambda^4 + 16\lambda^3 - 31\lambda^2 + 24\lambda - 5})(\lambda-2)}{6\lambda^3 - 29\lambda^2 + 48\lambda - 27} < \beta < \frac{(3\lambda^2 - 9\lambda + 7 + \sqrt{-3\lambda^4 + 16\lambda^3 - 31\lambda^2 + 24\lambda - 5})(\lambda-2)}{6\lambda^3 - 29\lambda^2 + 48\lambda - 27}$, if $0 < m \leq \frac{a[(\beta-1)^2\lambda^2 - (4+2.5\beta^2+7\beta)\lambda + 1.5\beta^2 - 6\beta + 4](\lambda-1)}{-\beta(\lambda-2)^2(2\beta\lambda - 3\beta - \lambda + 2)}$, then $\frac{\partial \pi_1^{S*}}{\partial m} \leq 0$; if $\frac{a[(\beta-1)^2\lambda^2 - (4+2.5\beta^2+7\beta)\lambda + 1.5\beta^2 - 6\beta + 4](\lambda-1)}{-\beta(\lambda-2)^2(2\beta\lambda - 3\beta - \lambda + 2)} < m < a$, then $\frac{\partial \pi_1^{S*}}{\partial m} > 0$, otherwise $\frac{\partial \pi_1^{S*}}{\partial m} < 0$; $\frac{\partial \pi_2^{S*}}{\partial m} \geq 0$; if $0 \leq \beta \leq \frac{3-\lambda}{2(2-\lambda)}$, then $\frac{\partial CS^{S*}}{\partial m} < 0$; when $\frac{3-\lambda}{2(2-\lambda)} < \beta \leq 1$, if $0 <$

$$m \leq \frac{3a-2a\beta-a\lambda+a\beta\lambda}{2\beta-\beta\lambda}, \text{ then } \frac{\partial cs^{S^*}}{\partial m} \leq 0; \text{ if } \frac{3a-2a\beta-a\lambda+a\beta\lambda}{2\beta-\beta\lambda} < m < a, \text{ then } \frac{\partial cs^{S^*}}{\partial m} > 0; \text{ if } 0 \leq \beta \leq \frac{8\lambda^3-41\lambda^2+70\lambda-40+\sqrt{(\lambda-2)(16\lambda^5-128\lambda^4+429\lambda^3-750\lambda^2+684\lambda-260)}}{2(6\lambda^3-29\lambda^2+48\lambda-27)}, \text{ then } \frac{\partial sw^{S^*}}{\partial m} < 0; \text{ when } \frac{8\lambda^3-41\lambda^2+70\lambda-40+\sqrt{(\lambda-2)(16\lambda^5-128\lambda^4+429\lambda^3-750\lambda^2+684\lambda-260)}}{2(6\lambda^3-29\lambda^2+48\lambda-27)} < \beta \leq 1, \text{ if } 0 < m \leq \frac{a(\lambda-1)[2(\beta-1)^2\lambda^2-(5\beta^2+14\beta-9)\lambda+3\beta^2-12\beta+10]}{-(7-4\lambda-6\beta+4\beta\lambda)\beta(\lambda-2)^2}, \text{ then } \frac{\partial sw^{S^*}}{\partial m} \leq 0; \text{ if } \frac{a(\lambda-1)[2(\beta-1)^2\lambda^2-(5\beta^2+14\beta-9)\lambda+3\beta^2-12\beta+10]}{-(7-4\lambda-6\beta+4\beta\lambda)\beta(\lambda-2)^2} < m < a, \text{ then } \frac{\partial sw^{S^*}}{\partial m} > 0.$$

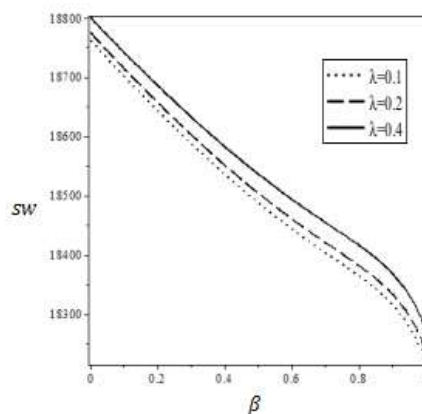
The level of CSR is first negatively correlated and then positively correlated with Firm 1's profit while the case of Firm 2's profit is the opposite. Thus, when λ is at a lower level, private enterprise may implement more CSR to increase its profit. However, if λ reaches a certain level, private firm will not increase CSR anymore. The relationship between λ and consumer surplus is affected by β . Only when β is low but λ is high, the higher λ decreases consumer surplus. In other cases, λ promotes consumer surplus. As λ increases, social welfare increases. λ has different impacts on different stakeholders. Firm 2 still has an optimal λ from the perspective of profit. To improve social welfare, the government supports private firm to rise λ .

When the proportion of state-owned shares is within a certain range, Firm 1's profit is first negatively related and then positively related to m . In other cases, reducing m increases Firm 1's profit but reduces Firm 2's profit. Consumer surplus and social welfare have similar relationships with m . They are both negatively correlated with m when β is low. If β is high enough, they are first negatively correlated and then positively correlated with m , but the critical value is different. Lower m is conducive to increasing Firm 1's profit and benefits consumers and the government to a certain extent. The influences of m and β are intertwined and must still be considered together, whether in corporate strategy or government policy.

Corollary 2.

$\frac{\partial sw^{S^*}}{\partial \beta} < 0, \beta^{S^*} = 0$, that is full privatization.

To further illustrate Corollary 2, we use a figure to explain the changes in social welfare. Assuming $a = 200, m = 10$, Figure 3 shows the relationship between sw and β when λ is respectively taken as 0.1, 0.2, 0.4.



Source: compiled by the authors.

Figure 3. The Relationship between sw and β

It can be seen from Corollary 2 and Figure 3 that social welfare decreases with β increasing, and the optimal state-owned share ratio level is 0. At this time, the optimal state-owned share is not affected by λ . Under the Stackelberg model, the government's optimal choice is full privatization, which is different from that under the Cournot model. The competition type has a significant impact on the government's decision.

3.3 Comparison

The following comparison is based on the derivation of the restriction conditions of $0 \leq \lambda \leq 1$, $0 \leq \beta \leq 1$, $q_i \geq 0$ and $\pi_i \geq 0$.

Proposition 3.

If $0 < \min\left\{\frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)}, \frac{a(\lambda-1)[(\beta-1)^2\lambda^2+(-4\beta^2+7\beta-3)\lambda+4\beta^2-7\beta+2]}{(\lambda-2)^2(2\beta^2\lambda-3\beta^2-3\beta\lambda+4\beta+\lambda-1)\beta}\right\} \leq m \leq \max\left\{\frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)}, \frac{a(\lambda-1)[(\beta-1)^2\lambda^2+(-4\beta^2+7\beta-3)\lambda+4\beta^2-7\beta+2]}{(\lambda-2)^2(2\beta^2\lambda-3\beta^2-3\beta\lambda+4\beta+\lambda-1)\beta}\right\} < a$, then $\pi_1^{C*} \leq \pi_1^{S*}$, otherwise $\pi_1^{C*} > \pi_1^{S*}$; if $0 < m \leq \frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)} < a$, then $\pi_2^{C*} \geq \pi_2^{S*}$, otherwise $\pi_2^{C*} < \pi_2^{S*}$; if $0 < \frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)} \leq m \leq \frac{a[(\beta-1)^2\lambda^2+(-6\beta^2+16\beta-10)\lambda+7\beta^2-22\beta+17]}{[(\beta-1)\lambda^2+(-6\beta+7)\lambda+7\beta-10]\beta} < a$, then $cs^{C*} \geq cs^{S*}$, otherwise $cs^{C*} < cs^{S*}$; if $0 < \min\left\{\frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)}, \frac{a(\lambda-1)[(\beta-1)^2\lambda^2+(-4\beta^2+7\beta-3)\lambda+4\beta^2-7\beta+2]}{(\lambda-2)^2(2\beta^2\lambda-3\beta^2-3\beta\lambda+4\beta+\lambda-1)\beta}\right\} \leq m \leq \max\left\{\frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)}, \frac{a(\lambda-1)[(\beta-1)^2\lambda^2+(-4\beta^2+7\beta-3)\lambda+4\beta^2-7\beta+2]}{(\lambda-2)^2(2\beta^2\lambda-3\beta^2-3\beta\lambda+4\beta+\lambda-1)\beta}\right\} < a$, then $sw^{C*} \geq sw^{S*}$, otherwise $sw^{C*} < sw^{S*}$.

When m is in a certain range, Firm 1 has higher profits under Stackelberg model. In other cases, Cournot model can make more profits. When m is lower, Firm 2 makes more profit under Cournot model. Otherwise, more profit is obtained under Stackelberg model. This indicates that when the difference of equity cost between the two firms is small, making decisions at the same time can obtain higher profit for private firm. Consumer surplus and social welfare are both higher under Cournot model within a certain range, and in other cases, they are higher under Stackelberg model. Different stakeholders make different decisions by considering m . As share cost of state-owned, m make various influences not only on privatization policy formulation but also on market competition. Owing to the order of competition and the cost of equity, stakeholders prefer to compete with Cournot more intensively, while those who prefer to compete with Stackelberg are more scattered and complex. These provide theoretical support for government policy formulation and market order regulation.

3.4 Expansion

The above models are extended to three stages, including the CSR decision of private firm. The first stage remains unchanged, and the original second stage becomes the third stage. In the added second stage, Firm 2 determines λ through maximizing its utility. By incorporating CSR decision-making into the game stage, this section makes CSR endogenous, and the impacts of privatization and CSR are deeply examined.

3.4.1 Cournot Model

Under Cournot competition, the equilibrium outputs in the third stage are $q_1^{C*} = \frac{a-2\beta m+(\beta m-a)\lambda}{3-2\beta+(\beta-1)\lambda}$ and $q_2^{C*} = \frac{a(\lambda+1)-\beta(a-m+\lambda m)}{3-2\beta+(\beta-1)\lambda}$, substituting them into the utility function of the private firm, we can get

$$u_2^{C*} = \frac{(2-\lambda)[a^2+2m(\lambda-1)a+m^2]\beta^2+2a(\lambda-2)[a(\lambda+1)+m(\lambda-1)]\beta-2a^2(\lambda^2-2\lambda-1)}{2(3-2\beta+\lambda+\beta\lambda)^2} \quad (5)$$

Because $\frac{\partial u_2^{C*}}{\partial \lambda} > 0$, $\lambda^{C*} = \frac{a-2\beta m}{a-\beta m}$ can be derived, substitute it into social welfare

$$sw^{C*} = \frac{a^2-\beta^2 m^2}{2} \quad (6)$$

$$\frac{\partial sw^{C*}}{\partial \beta} = -\beta m^2 < 0, \text{ so } \beta^{C*} = 0.$$

3.4.2 Stackelberg Model

Under Stackelberg model, substituting the equilibrium outputs in the third stage into Firm 2's utility function, we can get

$$u_2^{S*} = \frac{(2-\lambda)[(3a^2+2am-m^2)\lambda^3-2(4a^2+5am-3m^2)\lambda^2+4(a^2+4am-3m^2)\lambda+2m(\lambda-1)a+m^2+2(a-2m)^2]\beta^2-2a(\lambda-2)[(3a+m)\lambda^2-(4a+3m)\lambda-a+2m]\beta-(\lambda-2)a^2(3\lambda^2-6\lambda-1)}{2(4-3\beta-2\lambda+2\beta\lambda)^2} \quad (7)$$

$\frac{\partial u_2^{S*}}{\partial \lambda} > 0$, $\lambda^{S*} = \frac{2\beta a+4\beta m-3a+\sqrt{a(a-4\beta^2 m)}}{2(a\beta+\beta m-a)}$ can be derived, bring it into the social welfare

$$sw^{S*} = \frac{\{-4(a-m)\beta^2+3(3a-m)\beta-5a\}\sqrt{a(a-4\beta^2 m)}+2(a^2-6am+m^2)\beta^3-6a(a-3m)\beta^2+a(7a-5m)\beta-3a^2\}a[(a+m)\beta-a]}{4[(1-\beta)\sqrt{a(a-4\beta^2 m)}+(a-m)\beta^2-2\beta a+a]^2} \quad (8)$$

$$\frac{\partial sw^{S*}}{\partial \beta} < 0, \text{ so } \beta^{S*} = 0.$$

Under two competition models, if the government first determines the proportion of state-owned shares in a three-stage game, then the optimal β is zero, which means that Firm 1 is fully privatized. The output and profit of Firm 1 are both zero, indicating that private firm squeezes Firm 1 out of the market by implementing CSR. However, λ and outputs decisions of private firms are different. The reason is that the utility of Firm 2 increases with its CSR level, when it makes CSR decisions independently, it will select the highest level of CSR within the feasible range. With the dual competitive advantages of production cost and social identity, private firms can successfully capture the entire market.

Conclusions

By constructing a mixed oligopoly model that includes mixed ownership and private firms, this paper examines the impacts of the level of CSR and the cost of state-owned shares on the equilibrium results and the optimal proportion of state-owned shares, and expands the analysis by endogenizing CSR. The conclusions are as follows:

First, the level of CSR (λ) has impacts on the equilibrium results. Under Cournot competition, with λ increases, the profit of Firm 1 (mixed ownership firm) decreases, and the profit of Firm 2 (private firm) first decreases and then increases. Under Stackelberg competition, there is a critical value that affects Firm 1's profit. When λ is less than the critical value, Firm 1's profit decreases with the increase of λ , and when it is greater than the critical value, Firm 1's profit increase with λ increasing. Additionally, in the Cournot model, consumer surplus and social welfare increase as λ increases. In the Stackelberg model, the relationship between consumer surplus and λ is affected by the proportion of state-owned shares. When the proportion of state-owned shares is high but the level of social responsibility is low, increasing λ

reduces consumer surplus. Therefore, when Firm 2 undertakes social responsibility, it needs to consider the various impacts fully.

Second, the cost of state-owned shares (m) has impacts on the equilibrium results. In the Cournot model, with the increase of m , the profit of Firm 1 decreases, and the profit of Firm 2 increases. In the Stackelberg model, there is a critical value for the impact on the profit of Firm 1 and is related to the proportion of state-owned shares. In addition, in the Cournot model, consumer surplus decreases with m increasing. In the Stackelberg model, the relationship between m and consumer surplus as well as social welfare is affected by the proportion of state-owned shares. When the proportion of state-owned shares is high but the level of social responsibility is low, reducing m increases consumer surplus and social welfare. Reducing the cost of state-owned shares is beneficial to mixed-ownership firms, consumers, and the government in most cases, but has a lowering effect on the profit of private firm. Therefore, when Firm 2 undertakes social responsibility, it needs to fully consider the various impacts. Hence, from the perspective of consumers and society, private firms should be encouraged to implement CSR, mixed-ownership firms should be supported to reduce m , and government should guide firms in the market to compete with appropriate type accordingly.

Third, under different competition modes, the equilibrium results for different stakeholders vary. Thus, the government should implement its privatization policy by different situations. When m is in a specific range, Firm 1 tends to adopt Stackelberg competition from its profit, and in other cases, chooses Cournot competition. Firm 2 gets more profit under Cournot competition when m is low. Consumers and the government benefit more from the Cournot model within a specific range, while in other cases, the Stackelberg model is more advantageous.

Fourth, there is an optimal proportion of state-owned shares. In the Cournot model, the optimal proportion of state-owned shares is negatively correlated with the level of social responsibility; in the Stackelberg model, the optimal proportion of state-owned shares is zero. After expanding to three stages, the optimal proportion of state-owned shares in the two models is 0, indicating full privatization. Therefore, for the government, when conducting Cournot competition in the market, policy formulation should consider the social responsibility of private firms in the market. In other cases, full privatization can be implemented.

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PRIVATIZAVIMO POLITIKA IR BENDROVĖS SOCIALINĖ ATSAKOMYBĖ MIŠRIOJE DUOPOLIJOJE

He Liang, Chaoqun Sun, Jiaqi Gao

Santrauka. Tyrimo metu buvo sukonstruotas mišrus duopolijos modelis, kuriame atsižvelgiama į privatizavimą, socialinės įmonių atsakomybės (SĮA) lygį ir valstybei priklausančių akcijų kainą. Pagal Cournot ir Stackelberg modelį išnagrinėti ir palyginti pusiausvyros rezultatai. Rezultatai atskleidė, kad SĮA ir valstybei priklausančios akcijos iš esmės lemia pusiausvyros rezultatus. Suinteresuotos šalys, susidūrusios su įvairaus pobūdžio konkurencija, priima įvairius sprendimus. Pagal Cournot modelį egzistuoja optimali valstybės turimų akcijų dalis. O visiškai privatizavimas laikomas optimaliu remiantis Stackelberg modeliu ir išplėstu trijų etapų žaidimo modeliu. Todėl formuojant privatizavimo politiką būtina atsižvelgti į valstybės akcijų proporciją, SĮA veiksnius ir taikomą konkurencijos režimą.

Reikšminiai žodžiai: duopolija; privatizacija; socialinė įmonių atsakomybė; valstybei priklausančios akcijos.

APPENDIX A

$$\begin{aligned} \frac{\partial \pi_1^{C^*}}{\partial \lambda} &= \frac{2(\beta-1)[a(\lambda-1)-m(\lambda-2)\beta][(a+m)\beta-2a]}{(3-2\beta-\lambda+\beta\lambda)^3}, \quad \text{if } 0 \leq \lambda \leq 1, \quad \text{then } \frac{\partial \pi_1^{C^*}}{\partial \lambda} \leq 0; \quad \frac{\partial \pi_2^{C^*}}{\partial \lambda} = \\ &= \frac{(a+m)\beta^2\lambda + [(a+m)-(4a+m)\lambda]\beta + a(3\lambda-1)][(a+m)\beta-2a]}{(3-2\beta-\lambda+\beta\lambda)^3}, \quad \text{if } 0 \leq \lambda \leq \frac{a-a\beta+\beta m}{3a-4a\beta-\beta m+5\beta n+a\beta^2+\beta^2 m}, \quad \text{then } \frac{\partial \pi_2^{C^*}}{\partial \lambda} \geq 0; \quad \text{if} \\ &\frac{a-a\beta+\beta m}{3a-4a\beta-\beta m+5\beta n+a\beta^2+\beta^2 m} < \lambda \leq 1, \quad \text{then } \frac{\partial \pi_2^{C^*}}{\partial \lambda} < 0; \quad \frac{\partial cs^{C^*}}{\partial \lambda} = \frac{(1-\beta)(2a-a\beta-\beta m)^2}{(3-2\beta-\lambda+\beta\lambda)^3}, \quad \text{if } 0 \leq \lambda \leq 1, \quad \text{then } \frac{\partial cs^{C^*}}{\partial \lambda} \geq 0; \\ \frac{\partial sw^{C^*}}{\partial \lambda} &= \frac{[[a(\lambda-1)-m(\lambda-3)]\beta^2 + [2(1-\lambda)a+m(\lambda-4)]\beta + a(\lambda-1)][(a-m)\beta-2a]}{(3-2\beta-\lambda+\beta\lambda)^3}, \quad \text{if } 0 \leq \lambda \leq 1, \quad \text{then } \frac{\partial sw^{C^*}}{\partial \lambda} > 0. \\ \frac{\partial \pi_1^{C^*}}{\partial m} &= \frac{2\beta(\beta-1)(\lambda-2)[a(\lambda-1)-m(\lambda-2)\beta]}{(3-2\beta-\lambda+\beta\lambda)^2} \leq 0; \quad \frac{\partial \pi_2^{C^*}}{\partial m} = \frac{\beta(2a-2a\beta-a\lambda+2\beta m+a\lambda^2+2a\beta\lambda-2\beta\lambda m-a\beta\lambda^2)}{(3-2\beta-\lambda+\beta\lambda)^2} > 0, \quad \frac{\partial cs^{C^*}}{\partial m} = \\ &= \frac{\beta(-2a+a\beta+\beta m)}{(3-2\beta-\lambda+\beta\lambda)^2} \leq 0; \quad \frac{\partial sw^{C^*}}{\partial m} = \frac{[-2m(\lambda-2)^2\beta^2 + [(a+2m)\lambda^2-2(2a+5m)\lambda+3a+11m]\beta - a(\lambda-1)(\lambda-4)]\beta}{(3-2\beta-\lambda+\beta\lambda)^2}, \quad \text{if } 0 \leq \beta \leq \\ &\frac{-3\lambda^2+14\lambda-14+\sqrt{\lambda^4-12\lambda^3+56\lambda^2-104\lambda+48}}{-4(\lambda^2-4\lambda+4)}, \quad \text{then } \frac{\partial sw^{C^*}}{\partial m} < 0; \quad \text{if } \frac{-3\lambda^2+14\lambda-14+\sqrt{\lambda^4-12\lambda^3+56\lambda^2-104\lambda+48}}{-4(\lambda^2-4\lambda+4)} < \beta \leq 1 \quad \text{and } 0 < \\ m &\leq \frac{a(\lambda-1)[(\beta-1)\lambda-3\beta+4]}{(2\beta\lambda^2-8\beta\lambda+8\beta-2\lambda^2+10\lambda-11)\beta}, \quad \text{then } \frac{\partial sw^{C^*}}{\partial m} \leq 0; \quad \text{if } \frac{a(\lambda-1)[(\beta-1)\lambda-3\beta+4]}{(2\beta\lambda^2-8\beta\lambda+8\beta-2\lambda^2+10\lambda-11)\beta} < m < a, \quad \text{then } \frac{\partial sw^{C^*}}{\partial m} > 0. \end{aligned}$$

APPENDIX B

$$\begin{aligned} \beta C^* &= \frac{-B^2+12(\lambda-2)(\lambda-3)mB+12C(\lambda-2)-\sqrt{-3(B^4+24B^2C\lambda+144C^2\lambda^2-48B^2C-576C^2\lambda+576C^2)}}{12(\lambda-2)^2mB}, \quad \text{where } B = \{108m[(3- \\ &\lambda)m + a(\lambda-1)]^2 + 12\sqrt{3A}\}^{\frac{1}{3}}(\lambda-2)^{\frac{2}{3}}, \quad C = m(1-m)\lambda^3 + (a^2-7am+8m^2)\lambda^2 - 2(a^2-8am+ \\ &11m^2)\lambda + (a-3m)(a-7m), \quad A = \\ &4m^3(a-m)^3\lambda^9 + 12m^2(a^2-7am+8m^2)(a-m)^2\lambda^8 + 12m(a-m)(a^4-16a^3m+83a^2m^2-150am^3+86m^4)\lambda^7 + \\ &4(a^6-33a^5m+366a^4m^2-1795a^3m^3+4086a^2m^4-4248am^5+1631m^6)\lambda^6 - 3(8a^6-200a^5m+1911a^4m^2- \\ &8812a^3m^3+20146a^2m^4-21828am^5+8903m^6)\lambda^5 + 6(10a^6-3240a^5m+2225a^4m^2-10256a^3m^3+24276a^2m^4 \\ &-27880am^5+12217m^6)\lambda^4 - 2(40a^6-990a^5m+9441a^4m^2-45260a^3m^3+113646a^2m^4-140790am^5 \\ &+67241m^6)\lambda^3 + 12(a-3m)(5a^5-116a^4m+974a^3m^2-3834a^2m^3+6845am^4-4402m^5)\lambda^2 - 3(a-3m)^2(8a^4- \\ &176a^3m+1295a^2m^2-4026am^3+4015m^4)\lambda + 2(a-3m)^3(2a^3-42a^2m+267am^2-605m^3) \\ &\lambda-2. \end{aligned}$$

APPENDIX C

$$\begin{aligned} \frac{\partial \pi_1^{S^*}}{\partial \lambda} &= \frac{2(a+m)(2\lambda-3)(\lambda-2)m(\lambda-1)\beta^4 + [-2(a^2+6am+3m^2)\lambda^3 + 3(3a^2+20am+11m^2)\lambda^2 - 12(a^2+8am+5m^2)\lambda + 5a^2 + \\ &50am+36m^2]\beta^3 + [2(3a^2+6am+m^2)\lambda^3 - 6(5a+m)(a+2m)\lambda^2 + 4(11a^2+30am+6m^2)\lambda - 4(5a^2+18am+4m^2)]\beta^2 - \\ &[2(3a+2m)\lambda^3 - 3(11a+8m)\lambda^2 + 6(9a+8m)\lambda - (27a+32m)]a\beta + 2a^2(\lambda-1)(\lambda-2)(\lambda-3)}{(3\beta+2\lambda-2\beta\lambda-4)^3}, \quad \text{if } 0 \leq \lambda \leq \\ \frac{E^2+(9\beta-12)FE+3GF}{6(\beta-1)FE}, \quad \text{then } \frac{\partial \pi_1^{S^*}}{\partial \lambda} &\leq 0; \quad \text{if } \frac{E^2+(9\beta-12)FE+3GF}{6(\beta-1)FE} < \lambda \leq 1, \quad \text{then } \frac{\partial \pi_1^{S^*}}{\partial \lambda} > 0, \quad \text{where } E = \{27\beta[(a+m)\beta^2 - \\ &3a\beta + 2a]^2 + 6\sqrt{D}(\beta-1)\}^{\frac{1}{3}}[(a+m)\beta - a]^{\frac{2}{3}}[-2m\beta^2 + (a+m)\beta - a]^{\frac{2}{3}}, \quad F = [-2m\beta^2 + (a+m)\beta - \\ &a][(a+m)\beta - a], \quad G = [-2m\beta^3 + (3a-m)\beta^2 - 5a\beta + 2a][(a+m)\beta^2 - 3a\beta + 2a], \quad D = \\ &[-6m^3\beta^7 + 3m^2(9a-7m)\beta^6 + 117am^2\beta^5 - 36am(3a+m)\beta^4 + 90a^2m\beta^3 + \\ &9a^2(3a-m)\beta^2 - 33a^3\beta + 66a^3][(a+m)\beta^2 - 3a\beta + 2a]^3, \quad \frac{\partial \pi_2^{S^*}}{\partial \lambda} = \\ &\frac{(a-a\beta-\beta m)(-2\beta^2m+a\beta+\beta m-a)}{(a+m)(\lambda-2)[2(a-m)\lambda^2-5(a-m)\lambda+a-2m]\beta^3 + [-2(3a^2-m^2)\lambda^3 + 6(5a^2-2m^2)\lambda^2 - 4(11a^2-am-6m^2)\lambda + \\ &15a^2-10am-16m^2]\beta^2 + [6a\lambda^3-33a\lambda^2+2(27a-2m)\lambda-19a+8m]a\beta-2a^2(\lambda^3-6\lambda^2+11\lambda-4)}{-(3\beta+2\lambda-2\beta\lambda-4)^3}, \quad \text{if } 0 \leq \lambda \leq \\ \frac{I+(9\beta-12-31IK)J}{6(\beta-1)J}, \quad \text{then } \frac{\partial \pi_2^{S^*}}{\partial \lambda} &\geq 0; \quad \text{if } \frac{I+(9\beta-12-31IK)J}{6(\beta-1)J} < \lambda \leq 1, \quad \text{then } \frac{\partial \pi_2^{S^*}}{\partial \lambda} < 0, \quad \text{where } I = \{-27(\beta-2)[(a+ \\ &m)\beta^2 - 3a\beta + 2a]^2 + 6\sqrt{H}(\beta-1)\}^{\frac{1}{3}}[(a+m)\beta - a]^{\frac{2}{3}}[-2m\beta^2 + (a+m)\beta - a]^{\frac{2}{3}}, \quad J = [(a+m)\beta - \\ &a][(a-m)\beta - a], \quad K = [(a+m)\beta^2 - 3a\beta + 2a][(3a+m)\beta^2 - (5a+4m)\beta + 2a], \quad H = \\ &[9m^2(9a+7m)\beta^4 + (81a^3-27a^2m-153am^2-48m^3)\beta^3 + (-297a^3+36a^2m+234am^2)\beta^2 \\ &+12a^2(31a-3m)\beta-156a^3][(a+m)\beta^2-3a\beta+2a]^3; \quad \frac{\partial cs^{S^*}}{\partial \lambda} = \\ &\frac{(a-a\beta-\beta m)(a\beta-\beta m-a)}{(2a-3a\beta+a\beta^2+\beta^2 m)(3a-2a\beta-a\lambda-2\beta m+a\beta\lambda+\beta\lambda m)}, \quad \text{if } 0 \leq \beta \leq \frac{3a}{2(a+m)}, \quad \text{then } \frac{\partial cs^{S^*}}{\partial \lambda} > 0; \quad \text{if } \frac{3a}{2(a+m)} < \beta \leq 1 \quad \text{and } 0 \leq \lambda \leq \end{aligned}$$

$$\begin{aligned}
& \frac{2(a+m)\beta-3a}{(a+m)\beta-a}, \quad \text{then} \quad \frac{\partial cs^{S*}}{\partial \lambda} \leq 0; \quad \text{if} \quad \frac{2(a+m)\beta-3a}{(a+m)\beta-a} < \lambda \leq 1, \quad \text{then} \quad \frac{\partial cs^{S*}}{\partial \lambda} > 0; \quad \frac{\partial sw^{S*}}{\partial \lambda} = \\
& \frac{2(a+m)(2\lambda-3)(\lambda-2)m(\lambda-1)\beta^4 + [(-8\lambda^3+42\lambda^2-73\lambda+42)m^2 - 2(-60\lambda^3+30\lambda^2+48\lambda+25)am + a^2(\lambda-1)]\beta^3 +}{[4(\lambda-2)^3m^2 + (12\lambda^3-66\lambda^2+120\lambda-73)am - 4a^2(\lambda-1)]\beta^2 + [(-4\lambda^3+24\lambda^2-50\lambda+36)m + a(\lambda-1)]a\beta - 2a^2(\lambda-1)} \\
& \frac{\partial sw^{S*}}{\partial \lambda} > 0. \\
& \frac{\partial \pi_1^{S*}}{\partial m} = \frac{\beta[(a+2m)\lambda^2 - 2(a+4m)\lambda + a + 8m](2\lambda-3)\beta^2 - (\lambda-2)[(2a+m)\lambda^2 - (5a+4m)\lambda + 3a + 4m]\beta + a(\lambda-1)(\lambda-2)^2}{(3\beta+2\lambda-2\beta\lambda-4)^2}, \quad \text{when} \quad \frac{(3\lambda^2-9\lambda+7-\sqrt{-3\lambda^4+16\lambda^3-31\lambda^2+24\lambda-5})(\lambda-2)}{6\lambda^3-29\lambda^2+48\lambda-27} < \beta < \\
& \frac{(3\lambda^2-9\lambda+7+\sqrt{-3\lambda^4+16\lambda^3-31\lambda^2+24\lambda-5})(\lambda-2)}{6\lambda^3-29\lambda^2+48\lambda-27}, \quad \text{if} \quad 0 < m \leq \frac{a[(\beta-1)^2\lambda^2 - (4+2.5\beta^2+7\beta)\lambda + 1.5\beta^2-6\beta+4](\lambda-1)}{-\beta(\lambda-2)^2(2\beta\lambda-3\beta-\lambda+2)}, \quad \text{then} \quad \frac{\partial \pi_1^{S*}}{\partial m} \leq 0; \quad \text{if} \\
& \frac{a[(\beta-1)^2\lambda^2 - (4+2.5\beta^2+7\beta)\lambda + 1.5\beta^2-6\beta+4](\lambda-1)}{-\beta(\lambda-2)^2(2\beta\lambda-3\beta-\lambda+2)} < m < a, \quad \text{then} \quad \frac{\partial \pi_1^{S*}}{\partial m} > 0, \quad \text{otherwise} \quad \frac{\partial \pi_1^{S*}}{\partial m} < 0; \quad \frac{\partial \pi_2^{S*}}{\partial m} = \\
& \frac{\beta(\lambda-2)[(\lambda-2)(2\lambda m + a - 2m)\beta + 2a]}{-(3\beta+2\lambda-2\beta\lambda-4)^2}, \quad \text{when} \quad a > m > n = 0, \quad \frac{\partial \pi_2^{S*}}{\partial m} \geq 0; \quad \frac{\partial cs^{S*}}{\partial m} = \frac{\beta(\lambda-2)[(\lambda-2)(a+m)\beta - a(\lambda-3)]}{(3\beta+2\lambda-2\beta\lambda-4)^2}, \quad \text{if} \quad 0 \leq \beta \leq \\
& \frac{3-\lambda}{2(2-\lambda)}, \quad \text{then} \quad \frac{\partial cs^{S*}}{\partial m} < 0; \quad \text{when} \quad \frac{3-\lambda}{2(2-\lambda)} < \beta \leq 1, \quad \text{if} \quad 0 < m \leq \frac{3a-2a\beta-a\lambda+a\beta\lambda}{2\beta-\beta\lambda}, \quad \text{then} \quad \frac{\partial cs^{S*}}{\partial m} \leq 0; \quad \text{if} \quad \frac{3a-2a\beta-a\lambda+a\beta\lambda}{2\beta-\beta\lambda} < \\
& m < a, \quad \text{then} \quad \frac{\partial cs^{S*}}{\partial m} > 0; \quad \frac{\partial sw^{S*}}{\partial m} = \frac{[(a+2m)\lambda^2 - 2(a+4m)\lambda + a + 8m](2\lambda-3)\beta^2 - [2(a+m)\lambda^2 - 5(a+0.5m)\lambda + 3a + 7m](\lambda-2)\beta}{(3\beta+2\lambda-2\beta\lambda-4)^2}, \quad \text{if} \quad 0 \leq \\
& \beta \leq \frac{8\lambda^3-41\lambda^2+70\lambda-40+\sqrt{(\lambda-2)(16\lambda^5-128\lambda^4+429\lambda^3-750\lambda^2+684\lambda-260)}}{2(6\lambda^3-29\lambda^2+48\lambda-27)}, \quad \text{then} \quad \frac{\partial cs^{S*}}{\partial m} < 0; \quad \text{when} \\
& \frac{8\lambda^3-41\lambda^2+70\lambda-40+\sqrt{(\lambda-2)(16\lambda^5-128\lambda^4+429\lambda^3-750\lambda^2+684\lambda-260)}}{2(6\lambda^3-29\lambda^2+48\lambda-27)} < \beta \leq 1, \quad \text{if} \quad 0 < m \leq \\
& \frac{a(\lambda-1)[2(\beta-1)^2\lambda^2 - (5\beta^2+14\beta-9)\lambda + 3\beta^2-12\beta+10]}{-(7-4\lambda-6\beta+4\beta\lambda)\beta(\lambda-2)^2}, \quad \text{then} \quad \frac{\partial sw^{S*}}{\partial m} \leq 0; \quad \text{if} \quad \frac{a(\lambda-1)[2(\beta-1)^2\lambda^2 - (5\beta^2+14\beta-9)\lambda + 3\beta^2-12\beta+10]}{-(7-4\lambda-6\beta+4\beta\lambda)\beta(\lambda-2)^2} < m < \\
& a, \quad \text{then} \quad \frac{\partial sw^{S*}}{\partial m} > 0.
\end{aligned}$$

APPENDIX D

$$\begin{aligned}
& \pi_1^{C*} - \pi_1^{S*} = \frac{-(3\beta+2\lambda-2\beta\lambda-4)^2(1-\beta)(a-a\lambda-2\beta m+\beta\lambda m)^2 - \{\beta^2(-2\lambda+3)m + [(a+m)\lambda - a - 2m]\beta}{-a(\lambda-1)\{[(a+m)\lambda^2 - 2(a+2m)\lambda + a + 4m]\beta - a(\lambda-1)(\lambda-2)\}(3-2\beta+\lambda+\beta\lambda)^2}}{-(3\beta+2\lambda-2\beta\lambda-4)^2(3-2\beta+\lambda+\beta\lambda)^2}, \quad \text{if} \quad 0 < \\
& \min\left\{\frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)}, \frac{a(\lambda-1)[(\beta-1)^2\lambda^2 + (-4\beta^2+7\beta-3)\lambda + 4\beta^2-7\beta+2]}{(\lambda-2)^2(2\beta^2\lambda-3\beta^2-3\beta\lambda+4\beta+\lambda-1)\beta}\right\} \leq m \leq \\
& \max\left\{\frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)}, \frac{a(\lambda-1)[(\beta-1)^2\lambda^2 + (-4\beta^2+7\beta-3)\lambda + 4\beta^2-7\beta+2]}{(\lambda-2)^2(2\beta^2\lambda-3\beta^2-3\beta\lambda+4\beta+\lambda-1)\beta}\right\} < a, \quad \text{then} \quad \pi_1^{C*} \leq \pi_1^{S*}, \quad \text{otherwise} \quad \pi_1^{C*} > \pi_1^{S*}; \quad \pi_2^{C*} - \\
& \pi_2^{S*} = \frac{(a-a\beta+a\lambda+\beta m-\beta\lambda m)(a\beta\lambda-a\beta-a\lambda+\beta m+a)(3\beta+2\lambda-2\beta\lambda-4)^2 - \{[(a-m)\lambda - a + 2m]\beta}{-a(\lambda-1)\{[(a+m)\lambda^2 - (a+3m)\lambda - a + 2m]\beta - a(\lambda^2-2\lambda-1)\}(3-2\beta+\lambda+\beta\lambda)^2}}{(3\beta+2\lambda-2\beta\lambda-4)^2(3-2\beta+\lambda+\beta\lambda)^2}, \quad \text{if} \quad 0 < m \leq \frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)} < a, \quad \text{then} \\
& \pi_2^{C*} \geq \pi_2^{S*}, \quad \text{otherwise} \quad \pi_2^{C*} < \pi_2^{S*}; \quad cs^{C*} - cs^{S*} = \\
& \frac{(a\beta-2a+\beta m)^2(3\beta+2\lambda-2\beta\lambda-4)^2 - 2(3-2\beta+\lambda+\beta\lambda)^2(3a-2a\beta-a\lambda-2\beta m+a\beta\lambda+\beta\lambda m)^2}{2(3-2\beta+\lambda+\beta\lambda)^2(3\beta+2\lambda-2\beta\lambda-4)^2}, \quad \text{if} \quad 0 < \frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)} \leq m \leq \\
& \frac{a[(\beta-1)^2\lambda^2 + (-6\beta^2+16\beta-10)\lambda + 7\beta^2-22\beta+17]}{[(\beta-1)\lambda^2 + (-6\beta+7)\lambda + 7\beta-10]\beta} < a, \quad \text{then} \quad cs^{C*} \geq cs^{S*}, \quad \text{otherwise} \quad cs^{C*} < cs^{S*}; \quad sw^{C*} - sw^{S*} = \\
& \frac{\{2(1-\beta)(a-a\lambda-2\beta m+\beta\lambda m)^2 + 2(a-a\beta+a\lambda+\beta m-\beta\lambda m)(a\beta\lambda-a\beta-a\lambda+\beta m+a) + (a\beta-2a+\beta m)^2\}(3\beta+2\lambda-2\beta\lambda-4)^2 - \{2[(a+m)\lambda^2 - 2(a+2m)\lambda + a + 4m](2\lambda-3)m\beta^3 + [-4m(2a+m)\lambda^2 + (3a^2+20am+15m^2)\lambda - 4a^2-12am-14m^2](\lambda-2)\beta^2 - 2[-2m\lambda^3 + (3a+11m)\lambda^2 + (-12a-19m)\lambda + 11a+10m]a\beta + (\lambda-3)(3\lambda-5)a^2\}(3-2\beta+\lambda+\beta\lambda)^2}}{2(3-2\beta+\lambda+\beta\lambda)^2(3\beta+2\lambda-2\beta\lambda-4)^2}, \quad \text{if} \quad 0 < \\
& \min\left\{\frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)}, \frac{a(\lambda-1)[(\beta-1)^2\lambda^2 + (-4\beta^2+7\beta-3)\lambda + 4\beta^2-7\beta+2]}{(\lambda-2)^2(2\beta^2\lambda-3\beta^2-3\beta\lambda+4\beta+\lambda-1)\beta}\right\} \leq m \leq \\
& \max\left\{\frac{a(\beta-1)^2(1-\lambda)}{\beta(\beta\lambda-\lambda-\beta+2)}, \frac{a(\lambda-1)[(\beta-1)^2\lambda^2 + (-4\beta^2+7\beta-3)\lambda + 4\beta^2-7\beta+2]}{(\lambda-2)^2(2\beta^2\lambda-3\beta^2-3\beta\lambda+4\beta+\lambda-1)\beta}\right\} < a, \quad \text{then} \quad sw^{C*} \geq sw^{S*}, \quad \text{otherwise} \quad sw^{C*} < sw^{S*}.
\end{aligned}$$