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Modelling of the nodes immunity change process in network system

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Abstract. In this paper we analyze the additive hazard distribution modelling in the network system with non-constant nodes' immunities. Nodes' immunities depend on number of cycle and unknown parameter that is defined as random variable. Bayesian approach is applied for the updating of the estimate of this random variable. In this study we are interested in how many cycles can system work under influence of the hazard in the network or how many cycles are required to reduce amount of the nodes' hazard to the safe level. Obtained results can be used in the prediction of system' lifetime and accident analysis. Numerical experiment was performed to illustrate application of developed algorithm.

Keywords: Bayesian approach, non-constant immunity modelling.

1. Introduction

More and more attention is focused on the problems related to quantitative assessment of the behaviour of systems/components/infrastructures in terms of dependability and security and, more generally, quality of service indicators. People, technical equipments, computer (hardware, software) and etc. or various their combinations structure network system (i.e., nodes and network lines/channels) Fig. 1.

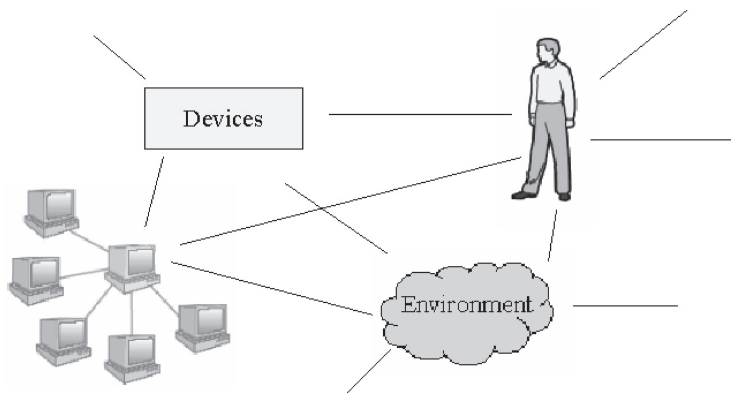


Fig. 1. An example of a general network system.

An hazard/error coming, its propagation and negative effect, natural physical ageing of the system, influence of external factors to the system vitality and so on affects network system' dependability and quality of service. In other hand, in network system some (or all) nodes can have immunity form hazard (for example, in computers we have firewalls, antivirus programs against hacker attacks, computer viruses), according to hazard propagation immunity of the system' nodes changes (could be increasing).

Analyzing network systems under the influence of hazard propagation process one point of interests is modeling of nodes resistances (immunity changes). Numerical experiment of proposed algorithm application for updating node' immunity function is presented in this paper.

This study could be used for:

- forecast how many cycles are necessary to eliminate hazard or to reduce to safe level; how long (how many cycles) system can work normally under the influence of hazard.
- modeling of hazard distribution in the network with non-constant nodes' immunities.

2. Definitions of hazard, immunity, flows and others concepts

Let's define several concepts that will be used in the paper:

A number of network nodes. The number of network nodes is marked as N.

Additive hazard. It is a sort of hazard, when hazards in the nodes of the network can be added to or a part of hazard moved to the other nodes. The examples of the additive hazard are: collection of hazardous materials, amount of fake money in the supermarkets, transport intensity at the crossroads, etc, marked as H.

Flow intensity. g_{ij}: $g_{ij} \geqslant 0$ – coefficient of flow intensity in the network lines or channel; it marks the part of the hazard in the node i that will be transmitted to the node j. The intensity of the flow to the node j and the intensity of the flow from the node j are defined respectively

$$\breve{g}_j = \sum_{\substack{i=1\\ i\neq j}}^{N} g_{ij}, \quad \frown{g}_j = \sum_{\substack{k=1\\ k\neq j}}^{N} g_{jk}. \tag{1}$$

Hazard transfer cycle. Hazard transfer in the network from one node to the other is regarded as one hazard transfer cycle.

Network node immunity. $I_j(\cdot)$ is immunity coefficient of network node j in the ith cycle. It marks which part of the hazard is stopped, before getting in the node j ($0 \leqslant I_{ji} \leqslant 1$, i.e., percentage). Node immunity can be created by the security systems, antvirus computer software, etc. "Observed" value of node j immunity could be obtained

$$I_{ji} = \frac{\breve{g}_{ji} - P_{ji}}{\breve{g}_{ji}}, \tag{2}$$

$i = 1, 2, \ldots$, here i – number of cycles, $\breve{q}_{ji}$ – the flow of hazard to the node j in the ith cycle, P_{ji} – amount of hazard that gets into the node j during the ith cycle.

The mechanism of hazard propagation in network systems in the case of single hazard, evolved in one of the network nodes and in case when hazard arises during each cycle (with constant nodes‘ immunities) was analyzed by Augutis [1]. In their study the marginal hazard distribution mathematical model the hazard caused by fuel transportation by fuel trucks was estimated in the fragment of Lithuanian roadway network. In this study non-constant immunity is analyzed.

3. Updating of node’ immunity function by application of Bayesian approach

Commonly Bayesian approach is applied to update estimated parameters of stationary process when more statistical information becomes available. But often there is need to deal with problems that are related to non-stationary processes. In such case available statistical data can not be used to update characteristics of previous period, because it represents the other state of system. Analysing non-stationary process required information is

- distribution of statistical data;
- form of the trend of system’ dynamics describing characteristics (as functions of some factors and parameters), for example, it is exponential, polynomial, linear, etc.

In the presented task immunity of chosen node depends on r different factors $F_1, \ldots, F_r$, and the trend of immunity is a priori known, so expected value of immunity satisfies this equality

$$\mathrm{E}I(\theta, F_1, ..., F_r) = f(\theta, F_1, ..., F_r), \tag{3}$$

here $\theta = (\theta_1, \ldots, \theta_s)^T$ – multidimensional parameter. Prior information can lead some uncertainty and these parameters are assumed as random independent variables with their prior probability distributions.

Assume that parameters $\theta_1, \ldots, \theta_s$ density functions of a prior distributions – $p_l(x_l),\ l = 1, \ldots, s$, distribution of statistical data $I_{ji},\ i = 1, \ldots, m$, (immunity of node j in the ith cycle) is also known, i.e., likelihood function – $L(\cdot)$ that satisfies (3). Posterior multidimensional density function is obtained by application of Bayesian formula for this information [2].

$$\begin{aligned}&\varphi(x_1, ..., x_s | I_{j1}, ..., I_{ji})\\&\quad = \frac{\prod_{l=1}^{s} p_l(x_l) \cdot L(I_{j1}, ..., I_{ji} | x_1, ..., x_s)}{\int_{R_1} \cdots \int_{R_s} \prod_{i=1}^{s} p_l(u_l) \cdot L(I_{j1}, ..., I_{ji} | u_1, ..., u_s)\, \mathrm{d}u_1 \ldots \mathrm{d}u_s},\end{aligned} \tag{4}$$

$i = 2, \ldots, m$, R_l – range (set of all possible values) of parameter $\theta_l,\ l = 1, \ldots, s$.

So Bayesian estimate (expected value of posterior distribution) of parameter θ_l is

$$\hat{\theta}_{li} = \int_{R_1} \ldots \int_{R_l} \ldots \int_{R_s} x_l \cdot \varphi(x_1, \ldots, x_l, \ldots, x_s | I_1, \ldots, I_j)\, \mathrm{d}x_1 \ldots \mathrm{d}x_l \ldots \mathrm{d}x_s, \qquad (5)$$

$i = 2, \ldots, m$.

There was performed numerical example to illustrate application of proposed algorithm. Lets' assume that we analyze network system (Fig. 2) that is compounded of

- six nodes (with non-constant immunity $I(i)$ (i number of cycle): $0 \leqslant I(i) \leqslant 1$, mean $\mathrm{E}I(a, i) = 1 - \mathrm{e}^{-a \cdot i}$, $a > 0$; the same for all nodes);
- channels (with coefficients of flow intensity $g_{jl} > 0$).

Distribution of immunity must have these properties (Fig. 3):

1) obtained values of random variable belong to the interval [0; 1];
2) variance is decreasing depending on number of cycle i, mean is increasing (to 1).

Beta distribution satisfies those requirements. In this case variation of immunity is bounded (see lemma).

Lemma. *Let us say, that network nodes' immunity.* $I(i)$ *in the* i*th cycle is random parameter distributed by Beta distribution. Mean of immunity is* $\mathrm{E}I(i) = 1 - \mathrm{e}^{-ai}$

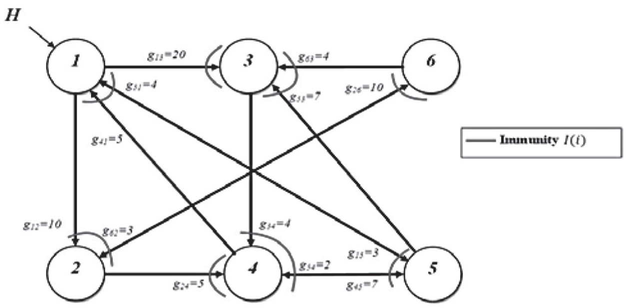


Fig. 2. Network system of numerical example. – – Immunity $I(i)$.

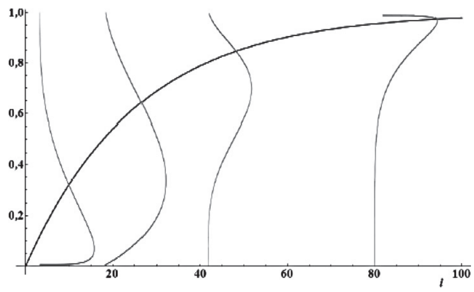


Fig. 3. Transformation of density function. – – mean of immunity.

(parameter a is a real number, $a \in (0;\infty)$). Then variation of immunity is bounded $VarI(i) < (1-\mathrm{e}^{-ai})\mathrm{e}^{-ai}$ (in the ith cycle) and maximum of variation value is $VarI(i) < 0.25$ (independent of parameter a).

Immunity of each node is approximates by Beta distribution, i.e., $I(i) \sim Be(\alpha(i), \beta(i))$, with mean (priori known dependence function)

$$\mathrm{E}I(a,i) = 1 - \mathrm{e}^{-a\cdot i}, \quad a > 0, \tag{6}$$

and variance

$$\mathrm{Var}\, I(a,i) = b(1-\mathrm{e}^{-a\cdot i})\mathrm{e}^{-ai}, \quad 0 < b < 1. \tag{7}$$

Statistical data (Fig. 4) $I(i)$: $0 \leqslant I_i \leqslant 1$, $i = 1, 2, \ldots, m$, were simulated by Beta distribution (with parameters $\alpha(a,i) = (1-\mathrm{e}^{-a^*\cdot i})(1-b^*)/b^*$ ir $\beta(a,i) = \mathrm{e}^{-a^*\cdot i}(1-b^*)/b^*$), for the simulation the values of parameter a and b were chosen $a^* = 0.1$, $b^* = 0.5$.

In the mathematical model parameter a is assumed as random variable with a prior known non-informative distribution, i.e., density function is

$$p(x) = \mathrm{const}, \quad x > 0. \tag{8}$$

In the case of vague prior knowledge and large amount of data are available for updating a prior distribution, the usage of so-called non-informative prior distribution is useful, i.e., posterior distribution is based on the information of likelihood function [3, 4].

Posterior distribution of parameter a is obtained by formula (4) using its a prior density function and simulated data. So density function of posterior distribution is

$$\begin{aligned} p(x|I_1,\ldots,I_i) &= \frac{p(x)\cdot\prod_{l=1}^{i} L(x,I_l)}{\int_0^\infty p(u)\cdot\prod_{i=1}^{j} L(u,I_i)\,\mathrm{d}u} \\ &= \frac{p(x)\cdot\prod_{l=1}^{i}\frac{1}{B(\alpha(x,l),\beta(x,l))} I_l^{\alpha(x,l)}(1-I_l)^{\beta(x,l)}}{\int_0^\infty p(u)\cdot\prod_{l=1}^{i}\frac{1}{B(\alpha(u,l),\beta(u,l))} I_l^{\alpha(u,l)}(1-I_l)^{\beta(u,l)}\,\mathrm{d}u}, \end{aligned} \tag{9}$$

$i = 2, \ldots, m$, $L(\cdot)$ – likelihood function.

Bayesian estimate (expected value) of parameter a is

$$\hat{a}_i = \int_0^\infty x\cdot p(x|I_1,\ldots,I_i)\,\mathrm{d}x, \quad i = 2,\ldots,m. \tag{10}$$

The results (the estimates of unknown parameter using Bayesian approach) of considered numerical experiment are shown in Fig. 5. In this case total sum of error squares is

$$\Delta = \sum_{i=1}^{m}(\hat{a}_i - a^*)^2 = 0.0232. \tag{11}$$

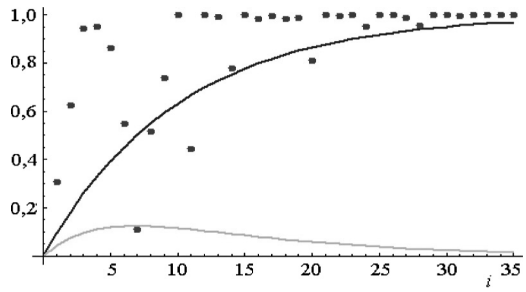


Fig. 4. • – simulated statistical data, – – graphic of true mean of immunity, – – graphic of variation of immunity, i – number of the cycle.

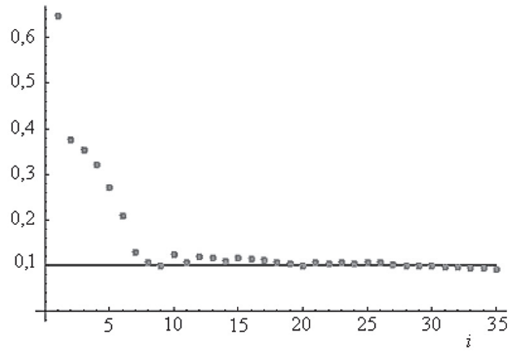


Fig. 5. • – Bayesian estimates of random parameter a, – – true value of parameter $a(a^* = 0.1)$, i – number of Bayesian approach applications.

Updated immunity' functions

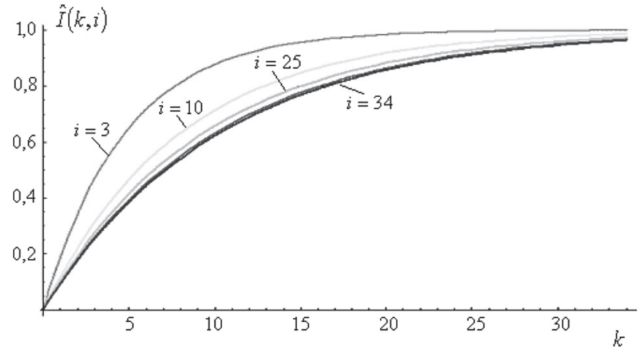
$$\hat{I}(k,i) = 1 - \mathrm{e}^{-\hat{a}_i \cdot k}, \quad k = 1, 2, ..., \; i = 1, 2, ..., \tag{12}$$

are presented in Fig. 6.

By obtained results we can prognosticate how many cycles are necessary to eliminate hazard or to reduce to safe level.

4. Results and conclusions

The main aim of the paper is to present the developed mathematical model for network nodes immunity changes updating by Bayesian approach and new available observations. Usage of the presented algorithm – nodes' immunity forecast is more and more precise (i.e., convergence to true value); it gives possibility to perform modeling of hazard distribution in the network with non-constant nodes' immunities.



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In the paper illustration of the developed model applicability is presented by numerical experiment. It was shown that Bayesian approach application gives descending uncertainty of immunity describing distribution.

References

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REZIUMĖ

B. Jokšas, J. Augutis, I. Žutautaitė-Šeputienė. Tinklinės sistemos viršūnių imuniteto kitimo modeliavimas

Šiame darbe nagrinėjama pavojaus sklaida tinklinėse sistemose ir tinklo viršūnių pasipriešinimas sklindančiam pavojui. Darbe apibrėžiama adityvaus pavojaus sklidimas, kiekvienos viršūnės gebėjimas pasipriešinti (imunitetas) priklausomai nuo ciklo skaičiaus. Bajeso metodu realizuota galimybė patikslinti sudarytą modelį, kai gaunami nauji stebėjimai. Patikslintas modelis yra naudojamas prognozavimui, po kiek laiko pavojus viršūnėje bus sumažintas iki leistino lygio. Pasiūlytas algoritmas gali būti taikomi įvairių apsaugos sistemų saugaus eksploatavimo laiko įvertinimui, avarijų analizei ir pan. Sudaryto algoritmo realizacijai buvo atliktas skaitinis eksperimentas.

Raktiniai žodžiai: Bajeso metodas, kintamo imuniteto modeliavimas.

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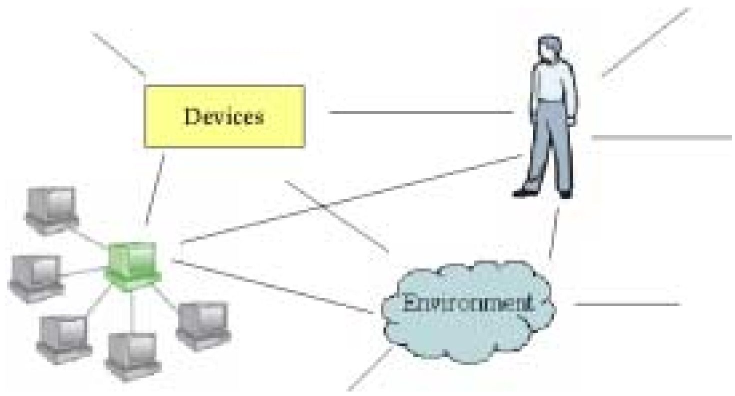

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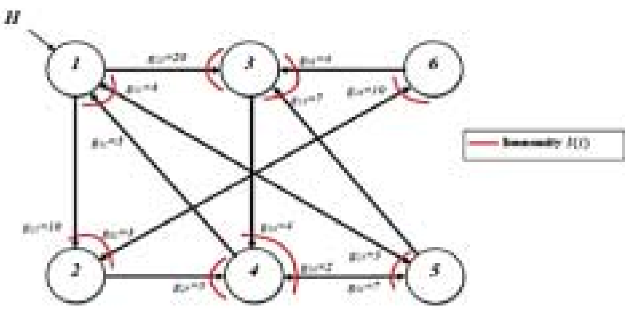

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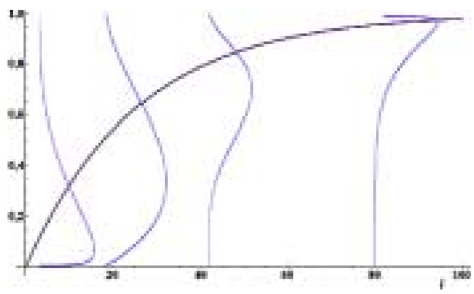

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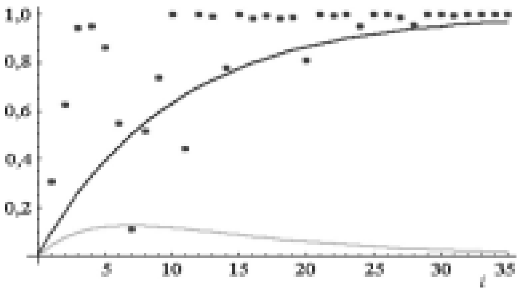

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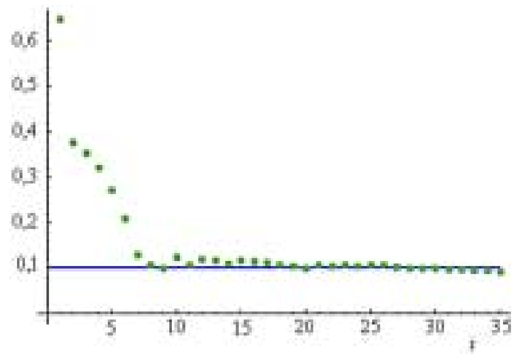

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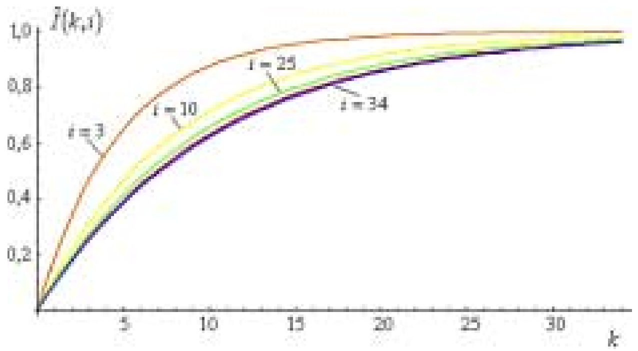

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