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Catastrophe model in nonlinear dynamical systems and working groups

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Abstract. This paper employs catastrophe theory which analyzes changes in dynamical systems depending on two or more parameters at the same time. The main purpose of this work is to investigate the dynamics of the swallowtail catastrophe depending on the values of parameters. This catastrophe could be used as a model of leadership emergence in working groups. The best model of collected data has been found by using the particle optimization algorithm.

Keywords: dynamical system, catastrophe, bifurcation, leadership emergence.

Introduction

There are seven elementary catastrophes. In this work a swallowtail catastrophe is being investigated. It is described by a function as follows [2]:

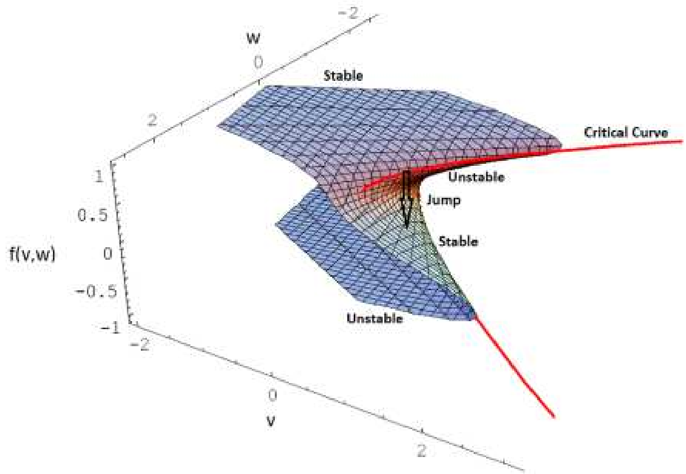
$$f(x,u,v,w) = x^5 + ux^3 + vx^2 + wx. \tag{1}$$

The parameter u is also called a bifurcation parameter because its changes may cause appearance or disappearance of the catastrophe. There is no swallowtail catastrophe for $u > 0$. If $u < 0$ holds, the behavior of the surface is depicted in Fig. 1(a). Any of the other parameters may reach their critical values and it may cause sudden jumps from one stability to another. A critical curve is found from the functions shown below [5]:

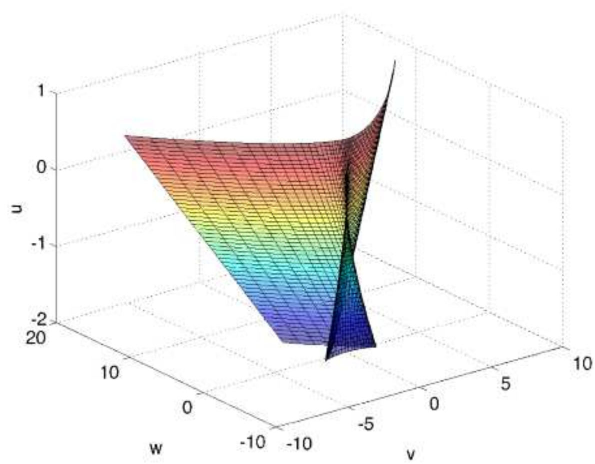
$$\begin{aligned} v(x,u) &= -10x^3 - 3ux, \\ w(x,u) &= 15x^4 + 3ux^2. \end{aligned} \tag{2}$$

The surface of the parameters shows how all three parameters depends on each other at the same time. The swallowtail surface of the parameters is shown in Fig. 1(b). Due to its appearance this catastrophe was called a swallowtail catastrophe.

In S.J. Guastello article “Nonlinear dynamics in psychology”‘ it is claimed that a leadership in working groups may be described as a dynamic of the swallowtail catastrophe. People who are not leaders are defined as one unstable area. Meantime, in two stable areas there are first and second type leaders [2]. Depending on the specificity of the task, leader may vary. This leadership model of the group is considered as a good model because all abilities of the group members may be exploited and all



(a)



(b)

Fig. 1. (a) The surface of the swallowtail catastrophe behaviour when $u < 0$; (b) The parameter surface of the swallowtail catastrophe.

the members are able to express their opinion. Otherwise, if there is no leader of the group, the chaos may appear. And that is why the task may not be finished in time. The group of a single leader is also considered as poor. If there is a single leading person, he usually will not let all of the other members to reveal themselves. This may cause worse results. The main purpose of this work is to find a model which describes leadership emergence in schoolchildren working groups.

1 The experiment

On the 9-th and 16-th of March, 2015 the grade 6 and grade 10 schoolchildren of the KTU engineering lyceum were asked to participate in a team session. The schoolchildren were randomly separated into groups. All groups had to do mathematical-logical

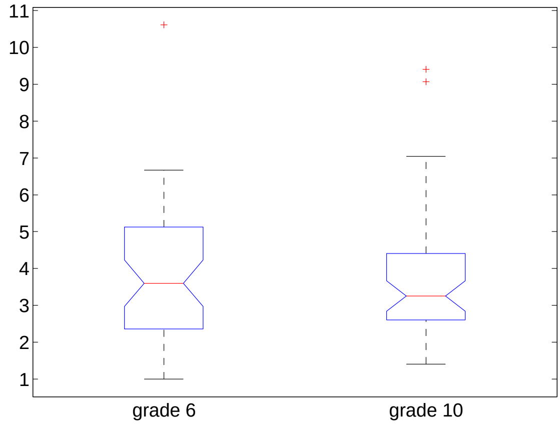


Fig. 2. The box plot for the data of the grade 6 and grade 10 schoolchildren.

exercises depending on accuracy and time of the performance. Children had to fill a form concerning the character traits of their team members. They also were asked to fill in the questionnaire which was used in additional research. The total number of participants was 102. Each question was scored by one point. This way all the children were evaluated by the sum of total points. All of the collected data was normalized by the number of schoolchildren and the total number of questions.

2 Model building and further analysis

The data collected during the process of the teamwork was analysed using statistical methods such as analysis of variance, correlation analysis and hypothesis verification for the equality of probability distributions. The box plot of the data is shown in Fig. 2.

The Shapiro Wilk [1] test was used to identify if the data of grade 6 and grade 10 schoolchildren are distributed normally. The results have shown that the null hypothesis about this distribution is rejected. Therefore the Kruskal Wallis analysis of variance [3] was used to determine if the data of grade 6 and grade 10 schoolchildren has the same distribution. Using $\alpha = 0.05$ critical value the R^2 value is 0.6969. That means there is no significant difference in distribution of the data. Therefore, the data of grade 6 and grade 10 schoolchildren were analyzed as a single data set.

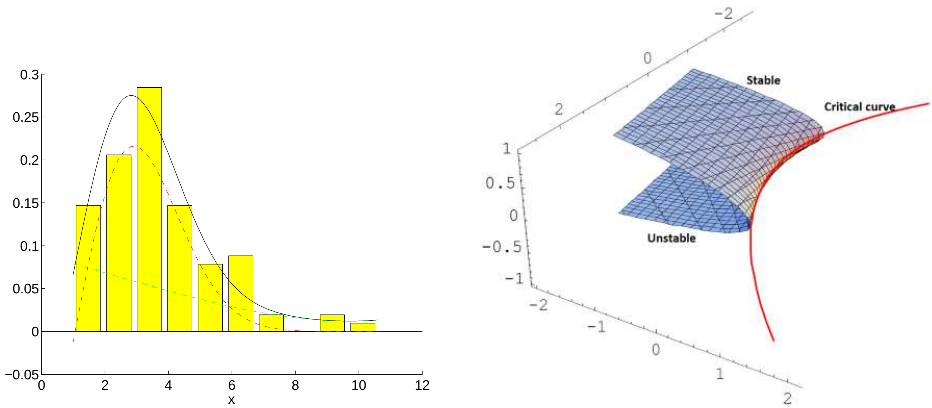
The model best describing the probability distribution of the leadership score is defined by Eq. (3).

$$g(x) = p(a_1 \cdot x + a_2) + a_3 \cdot \big(f(x \cdot a_1 + a_4 + a_2) + a_5\big), \tag{3}$$

where $p(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$ is Rayleigh‘s distribution with parameter σ, and $f(x)$ is the function of the swallowtail catastrophe with parameters u, v, w. Furthermore, parameters a_1 ,a_2, a_3, a_4, a_5 are transformation parameters of the functions. The particle swarm optimization (PSO) algorithm [4] was used to find the best parameter values. A set of all analysed parameters (see Table 1) are particles in PSO realization. In this work the minimized fitness function is the root mean square error between histogram and model density function.

Table 1. The best parameter values.

Parameters	Best values
u	6.2472
v	-0.0669
w	-5.7073
σ	0.1342
a_1	0.0727
a_2	-1.0631
a_3	0.5345
a_4	-0.0912
a_5	2.5457



(a) (b)

Fig. 3. The model found by PSO algorithm is depicted in (a); the surface of the corresponding swallowtail catastrophe behaviour ($u > 0$) is shown in (b). The red line is Rayleigh's probability density function; the green – the swallowtail catastrophe function.

The graphical view of the best model is shown in Fig. 3(a).

In the further investigation all the questions of the form were divided into three groups in respect of the character features and correlation analysis [1]:

(1) **Communication**

Depending questions: 2, 6, 4, 11, 16, 19. Communication, reduction of disagreements allows all members of the group to feel more stressed. As a result the task might be completed less effectively.

(2) **Creativity**

Depending questions: 5, 7, 10, 13, 18. A leader of the group not only has to generate new ideas but also has to know how to persuade other team members.

(3) **Responsibility**

Depending questions: 1, 3, 8, 9, 12, 14, 15, 17, 20. Focusing on the task, decision making, absorption of the questions are very important for the whole team. Without all these character traits the exercise might not be done in time.

Table 2. A questionnaire for the schoolchildren.

Number	Question	Number	Question
1	The most decision making	11	Asked the most questions
2	The most supportive	12	Had the most desire to win
3	The most trustful	13	Provided the most ideas
4	Tried to reduce disagreements	14	Followed the instructions the most
5	Expressed the most convincing ideas	15	The most sought to finish the exercises in time
6	Was the most supportive in other ideas	16	Reminded that the members deviate from the path
7	Gave the most suggestions	17	Summarize the results
8	The most responsible	18	Tried to simplify the exercises
9	The most focused on exercises	19	Kidding the most
10	Was the fastest in generating new ideas	20	Acted like a leader the most

Table 3. Analysis for groups of questions and parameters u, v, w.

Group	Results	u	v	w
First	Best value	4.2080	−2.0717	−6.2909
	Error of group 1	0.1224	0.1200	0.1215
	General error	0.0645	**0.0753**	**0.0696**
Second	Best value	−9.3530	−1.7783	−6.6522
	Error of group 2	0.0897	0.0884	0.0958
	General error	0.0593	**0.0730**	**0.0756**
Third	Best value	4.0459	−1.1065	−6.2495
	Error of group 3	0.0857	0.0829	0.0818
	General error	**0.0701**	0.0641	0.0642

Using Kruskal–Wallis analysis of variance [3] it was found that all the groups of the questions have the same distribution. For this reason the same model (3)can be used for all question groups.

Another aim of this work is to find the dependence of swallowtail catastrophe function parameters u, v, w on recently analyzed question groups. There are 9 different cases. Using the data of the question groups the better value of the fitness function was sought comparing it to general model case.

The mean square error (MRSE) are shown in Table 3. The minimum value of the fitness function in general model (3) is 0.0650. The improvement of general MRSE means that a parameter describes the analyzed group well and vice versa. According to the Table 3 we can claim that parameter u describes the data of the third group well. However, it is not possible to say which one, v or w, better describes first or second groups. The reason of that might have been the children comprehension of communication not as a good, but as a bad character trait, which bothers and distracts other team members.

This study revealed that the leadership in grade 6 and grade 10 schoolchildren working groups of the KTU engineering lyceum can be described as a simpler case of swallowtail catastrophe. The parameter u is always positive. Thus the behavior of the children working groups could be described as the surface in Fig. 3(b). All the analyzed schoolchildren could be separated into two groups: leaders and leaderless group. The second type of leader does not exist. The possible reason of this phenomenon may be the fact that schoolchildren usually consider the leader to be the one who gives the right answer and not the one who stimulates the whole group. In the swallowtail model a leader and a second leader are the people who encourage all team members work together to achieve the better results and to promote improvements. Meanwhile, for a child the leader is a person who takes the whole control of the group and others may not be heat or understood.

Surely, the model found relations on character traits. However, much depends on parenting, teaching and children age. It is likely that the swallowtail catastrophe in students working group could cause different results.

3 Conclusions

1. The analysis has shown that the sum of the Rayleigh distribution and the swallowtail catastrophe density function describes the data collected the best. The finest parameters values are: $u = 6.2472$, $v = -0.0669$, $w = -5.7073$.
2. All the questions of the form were divided into three groups in respect of the character features and correlation analysis: communication, creativity and responsibility. The study has shown that parameter u well describes the third questions group. Other parameter could not be separated. This may appeared because the child which talks constantly has the least probability to be a leader.
3. There are no second leaders among the researched schoolchildren working groups. Thus the children questioned should be taught to work together as a team to reach better results.

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REZIUMĖ

Katastrofos netiesinėse sistemose ir žmonių darbo grupėse

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Katastrofų teorija nagrinėja, kaip dinaminė sistema elgiasi priklausomai ne nuo vieno, o nuo kelių parametrų vienu metu. Pati katastrofa, taip pat kaip ir bifurkacija, yra staigus dinaminės sistemos kokybinis pasikeitimas parametrų reikšmėms pakitus per nykstamą dydį. Darbo tikslas yra pritaikyti kregždes uodegos katastrofos dinamiką grupių lyderystės modeliams formuoti. Šios katastrofos pagalba yra formuojamas modelis, kuris aprašo šeštos ir dešimtos klasių moksleivių grupių lyderystės atsiradimą. Ištiriama, kokios savybės gali lemti vieno ar kelių lyderių formavimąsi tarp vaikų.

Raktiniai žodžiai: dinaminė sistema, katastrofa, bifurkacija, lyderio atsiradimo modelis.