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Stefan Banach

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Abstract. Short presentation of the life of Stefan Banach (1893–1945), his main achievements and social environment including Scottish Cafe and Scottish Book. Fatal war years.

Keywords: Stefan Banach, functional analysis, Studia Mathematica, Scottish Book.

On March 30, 1892, in a St. Lazarus hospital in Cracow, Katarzyna Banach (1868–?), a young peasant woman from the nearby village Lipnica Murowana, gave birth to a boy. A couple of days later the boy has been baptized in a hospital chapel as Stefan Banach. He got the forename Stefan after his father Stefan Greczek (1867–1967), then an orderly in the Austrian army, while surname Banach was that of his mother (in the baptism certificate one can read "father unknown"). The boy was an illegitimate son of the two, they loved each other but they could not marry (army authorities refused the consent). Soon afterwards the mother disappeared altogether from his life, leaving no traces behind (there were rumours that she married a railway worker and left Cracow with him).[1]

The boy has been taken by his grandmother Antonina Greczek to her home in the village Ostrowsko near Nowy Targ but some months later she felt ill and the boy has been given to Franciszka Płowa (1845–1926), a laundry owner in Cracow. Franciszka lived together with her niece Maria Puchalska (1880–1968) and young Stefan soon got attached to the two, loving them as mother and sister. He lived with them until maturity and even later was a frequent guest in their home. The ladies had a friend, a Frenchman Jules Mien, to whom we owe first photos of Stefan[2] and who taught him fluent French.

Stefan Banach was a brilliant student of the Gymnasium IV in Cracow, particularly interested in mathematics, and so when he got a high school diploma in 1910, it was decided that he should continue studies at the university level. Being himself convinced that mathematics is a finished area, with no new ideas thinkable, he decided to pursue engineering in the polytechnic school in Lvov. So he did, studying there mechanics up to the outbreak of World War I in 1914. Then he returned to Cracow, living there from giving private lessons in mathematics and enjoying private studies of the beloved subject. He never completed his regular studies.

In 1916 there took palce an incident, lucky for Stefan Banach. Hugo Steinhaus (1887–1972), then already an established mathematician, walked along Planty in Cra-

[1] R. Duda, *Facts and myths about Stefan Banach*, Notices Euro. Math. Soc. 71, March 2009, pp. 29–34.

[2] Some of those photos are reproduced in the book: E. Jakimowicz, A. Miranowicz (eds.), *Stefan Banach. Remarkable Life, Brilliant Mathematics*, Gdańsk Univ. Press, 2010.

cow. Suddenly, he heard the words "Lebesgue measure". Taken by a surprise, he stopped, turned around and approached the bank where two young boys were talking mathematics. They were Stefan Banach and Otto Nikodym (1887–1874). They told Steinhaus that they have some more friends and the group begun to meet on a more regular basis. Steinhaus offered an open problem and a week later Banach came with a solution. Steinhaus completed it and it was their first joint paper and the very first paper by Banach. Later Steinhaus liked to say that Banach was his "greatest scientific achievement".

In 1920 Steinhaus was nominated professor in the Jan Kazimierz University in Lvov and asked Banach to join him, asserting him at the same time a position of an assistant at the Polytechnic and somewhat later helping him to receive PhD at the University. It was not an easy task because Banach had not complete studies but the thesis he presented was brilliant and so Banach got the degree in 1920. Two years later he got the habilitation and since 1923 became professor (initially extraordinary, and since 1927 ordinary) on its own at the Jan Kazimierz University. At the age of 31 his position has become eventually secure and he could addict himself wholly to mathematics. And so he did.

Banach's thesis[3] presented an axiomatic definition of a certain "space type (B)". In modern language space of type (B) is a linear vector space with a norm, the norm allows to define a metric and the space has to be complete with respect to that metric. In short, it is a normed complete vector space. After proposal of M. Fréchet it is now commonly known as *Banach space*.

The definition has crowned efforts of a several decades long development which produced many so called "function spaces" like the space C of real continuous functions, the space l_2 of infinite sequences such that the series consisting of squares of its elements are convergent, the space L^2 of functions the squares of which are integrable etc. All those spaces turned to be Banach spaces and in that way the thesis of Banach became the real beginning of *functional analysis*.[4] The *theory of Banach spaces* offered terminology to the field and became its fundamental part.

There appeared claims for priority on the part of H. Hahn and N. Wiener but it was Banach who won the field.[5]

A student of Banach has later commented the meaning of Banach's thesis as follows: "Functional analysis has replaced the fundamental for mathematical analysis concept of a number by a more general one, nowadays in thousands of mathematical papers called" a point of Banach's space. A generalisation of mathematical analysis, obtained in that way and called called functional analysis, has allowed a simple and uniform treatment of good many apparently different problems of mathematical analysis and has offered solutions to many of them, previously tackled unsuccessfully. It has conspicuously enlarged assistance given by mathematics to natural sciences, and particularly to physics.[6]

[3] S. Banach, *Sur les opérations dans les ensembles abstraits et leur applications aux équations integrales*, Fund. Math. 3 (1922), pp. 133–181; reprinted in: S. Banach, *Oeuvres*, volume II: *Travaux sur l'Analyse Fonctionnelle*, Warszawa 1979.

[4] J. Dieudonné, *History of Functional Analysis*, Amsredam 1981.

[5] R. Duda, *The discovery of Banach spaces*, in: W. Więsław (ed.), *European Mathematics in the Last Centuries*, Wrocław 2005, pp. 37–46.

[6] S. Mazur, *Oration on the occasion of receiving honorary degree at the Warsaw University*, Wiadom. Mat. 4.3 (1961), pp. 249–250 (in Polish).

Shortly after presentation of his thesis, Banach solved in the positive the long-standing problem of measure in dimensions 1 and 2.[7] The solution came as a surprise because the problem in higher dimensions has been answered in the negative. In the proof Banach has used the Axiom of Choice and the free use of it and of other ineffective methods has become his characteristic feature. Developing some earlier ideas, Banach and Tarski have constructed a paradoxical decomposition of the unit ball: using the Axiom of Choice one may decompose the ball into finitely many pieces which allow to compose two balls of the same radius 1. The paradox soon became famous.[8]

In 1927 there was the first Congress of Polish Mathematician in Lvov which was a success. After the Congress Steinhaus came to the idea to found a new mathematical journal devoted specifically to the functional analysis, the area of Lvov research in mathematics. He shared the idea with Banach who liked it and the two men became editors of the journal "Studia Mathematica", 9 volumes of which appeared in the years 1929–1940. In those 9 volumes there appeared 161 papers, of which 111 came from Lvov.

The journal has become an essential tool for the development of a group of young mathematicians which has already begun to accumulate around leaders Steinhaus and Banach. The group embraced Mark Kac (1914–1984), Stanisław Mazur (1905–1981), Władysław Orlicz (1903–1990), Juliusz Schauder (1899–1943), Stanisław Ulam (1909–1984) – just a few names of a dozen or so. It was a very productive group, soon to be called the Lvov School of Mathematics, well known not only for its results but also for the way of life and work. All its members were young, enthusiastic about mathematics, and very friendly to each other. They liked to work together, most often in Scottish Café, gathered around Banach's table, drinking and smoking (Steinhaus preferred a more quiet café nearby). The noise of the café did not disturb them, rather on the contrary. Since they were writing on scraps of paper and marble table tops, which were washed each night, much of their mathematics has been lost. To prevent the loss, Banach's wife Łucja has bought in 1935 a copybook in which mathematicians were supposed to write down the problems under discussion, subsequent comments and possible solutions. This they did and the copybook became known as the Scottish Book.[9]

Before the Scottish Book came to life, in 1932 there appeared Banach's monograph[10] which presented a synthesis of Lvov achievements in the area of functional analysis in the twenties and has grounded the reputation of the Lvov group. The group became known as the Lvov Mathematical School with Banach and Steinhaus as leaders.[11] Its wide significance can be seen in Banach's invitation to have a plenary talk in the International Congress of Mathematicians held in Oslo in 1936.

[7] S. Banach, *Sur le problème de mesure*, Fund. Math. 4 (1923), pp. 7–33; reprinted in: S. Banach, *Oeuvres I*, Warszawa 1967.

[8] S. Banach, A. Tarski, *Sur la décomposition des ensembles de points en parties respectivement congruentes*, Fund. Math. 6 (1924), pp. 244–277; reprinted in: S. Banach, *Oeuvres I*, Warszawa 1967; A. Tarski, *Collected Papers I*, Birkhäuser 1986. See also monograph: S. Wagon, *The Banach–Tarski Paradox*, Cambridge 1985.

[9] R.D. Mauldin (ed.), *The Scottish Book. Mathematics from the Scottish Café*, Birkhäuser 1981.

[10] S. Banach, *Théorie des opérations linéaires*, Monografie Matematyczne 1, Warszawa 1932.

[11] G. Köthe, *Banach und die Lemberger Schule*, Math. Semesterber. 36 (1989), pp. 145–158. See also: R. Duda, *Lwowska Szkoła Matematyczna*, Wrocław 2007 (english translation: *Pearls from the Lost City. The Lvov Mathematical School*, will be printed by the American Mathematical Society).

In the thirties Lvov mathematics enjoyed already an international reputation. Widely known was its enjoyable social atmosphere: original lifestyle with the Scottish Café and its Scottish Book as the centre, vivid sessions of the local section of the Polish Mathematical Society, renown journal "Studia Mathematica", excursions beyond the city (Banach was particularly fond of them), popular lectures for wide audience etc. Members of the group fully enjoyed their life. They knew the value of their mathematics and were taking a pride in what they were doing. One evidence of that was that the group attracted many visitors from within the country, particularly from Warsaw (A. Tarski, W. Sierpiński and others) and from Vilnius (A. Zygmund, J. Marcinkiewicz), as well as from abroad (H. Lebesgue, J. von Neuman, E. Zermelo and many others).

Uncommon feature of the Lvov group was its involvement in writing manuals for schools. Particularly known was the trio consisting of S. Banach and W. Stozek from Lvov and W. Sierpiński from Warsaw. They have written altogether 7 such books, later many times reedited.

Happy two decades came to an abrupt end with the outbreak of World War II. On September 1, 1939, Germans attacked Poland and a week later they already were at the gates of the city, starting the siege. The city has been defended but when the Soviets, then allied with Germans, took over the siege, the situation became hopeless and the city capitulated on September 24. There begun the Soviet occupation which lasted until the end of June 1941. The occupation has been marked by arrests, mass deportations, plunder and common misery, but Banach was among those who enjoyed some privileges: he became the dean of the faculty of mathematics and physics at the newly established Ukrainian university and collaborator of the Ukrainian Academy of Sciences. His position, however, became much worse with the advance of Germans in 1941 who closed the university and all secondary schools. Banach earned his living by feeding louses with his blood in the institute producing anti-typhus vaccines for Wehrmacht. When three years later, in the summer 1944, Soviets came back, Banach has been immediately restored to all of his previous positions. He took them up, even went with a visit to Moscow, but all of it was too late for him: he was already weak and exhausted, quickly loosing his vital force. He died on August 31, 1945, and was buried at the Łyczakowski cemetery in Lvov. His wife Łucja and their only son Stefan jr. left Lvov for Poland within her new borders.

The memory of Banach is still vivid. He lives in mathematics, where many notions and theorems still bear his name. He is remembered by Polish and Ukrainian mathematicians, who organize conferences on his work. There are his monuments in Cracow and some other Polish cities, many streets are named after him, there are postal stamps and medals with him. And his grave in Lvov is always covered with fresh flowers.

REZIUMĖ

Stefanas Banachas

R. Duda

Straipsnyje pateikiama trumpa Stefano Banacho (1893–1945) gyvenimo apžvalga, jo pagrindiniai atsiekimai bei socialinės aplinkos etiudai, apimantys Škotišką kavinę ir Škotų knygą.

Raktiniai žodžiai: Stefanas Banachas, funkcinė analizė, Studia Mathematica, Škotų knyga.